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A Modified Exponential Estimator for Estimating the Population Mean in Simple Random Sampling

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Abstract: In this article, an improved exponential estimator of study variable has been suggested. The properties like Mean Square Error (MSE), the conditions for minimum (MSE) and Percent Relative Efficiency (PRE) have been derived. To approve the results empirical study is carried out.

Keywords: Exponential estimator, Auxiliary variable, Mean square error (MSE) and Percent Relative Efficiency (PRE).

1 Introduction

In literature many authors have studied the various methods of estimation like the ratio and product estimators vis-à-vis [12], [13], [7], [9],[8],[10],[11], [14] and [15]. Recently [3] and [4] have contributed in this field.

Let N be the population size of different units $U=\{U_1,U_2,....,U_N\}$. Suppose y and x be the study and the ancillary variable, with means, $\bar{Y}=\frac{1}{N}\sum_{i=1}^{N}y_i$ and $\bar{X}=\frac{1}{N}\sum_{i=1}^{N}x_i$ respectively. The standard ratio estimator of $\bar{Y}$

$$T_R=\frac{\bar{y}}{\bar{x}}\bar{X}, \tag{1}$$

and the MSE is,

$$MSE(T_R)=\gamma\bar{Y}^2\left[C_y^{\,2}+C_x^{\,2}-2\rho C_xC_y\right], \tag{2}$$

Where C_y and C_x be the coefficient of variation (CV) of y and x respectively and ρ is correlation coefficient between y and x. The usual product estimator of $\bar{Y}$ is

$$T_P=\bar{y}\frac{\bar{x}}{\bar{X}}, \tag{3}$$

And the MSE is,

$$MSE(T_P)=\gamma\bar{Y}^2\left[C_y^{\,2}+C_x^{\,2}+2\rho C_xC_y\right]. \tag{4}$$

The standard linear regression estimator and its variance is,

$$T_{lr}=\bar{y}+\hat{\beta}\left(\bar{X}-\bar{x}\right), \tag{5}$$

$$V(T_{lr})=\frac{1-f}{n}S_y^{\,2}\left(1-\rho^2\right). \tag{6}$$

[1] proposed the exponential ratio estimator given as,

$$T_E=\bar{y}\exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right), \tag{7}$$

And the MSE is

$$MSE(T_E)=\gamma\bar{Y}^2\left[C_y^{\,2}+\frac{C_x^{\,2}}{4}-2\rho C_xC_y\right]. \tag{8}$$

[10] proposed a generalized exponential ratio estimator of $\bar{Y}$,

$$t=\bar{y}\exp\left\{\frac{(a\bar{X}+b)-(a\bar{x}+b)}{(a\bar{X}+b)+(a\bar{x}+b)}\right\}, \tag{9}$$

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Where $a(\neq 0), b$ are any real numbers or the functions of the well-known parameters of the supplementary variable X. The MSE is,

$$MSE(t) = \gamma \bar{Y}^2 \left[C_y^{\ 2} + \theta^2 C_x^{\ 2} - 2\theta\rho C_x C_y \right] \tag{10}$$

2 Proposed Estimator

We have proposed a modified generalized class of exponential ratio estimator of $\bar{Y}$,

$$t^* = \bar{y}\left(\frac{\bar{x}}{\bar{X}}\right)^{\alpha} \exp\left\{\frac{(a\bar{X}+b)-(a\bar{x}+b)}{(a\bar{X}+b)+(a\bar{x}+b)}\right\}. \tag{11}$$

To acquire the MSE, we write

$$\bar{y} = \bar{Y}(1+e_0), \bar{x} = \bar{X}(1+e_1),$$

Such that

$$E(e_0) = E(e_1) = 0,$$

and

$$E(e_0^{\ 2}) = Var(\bar{y}) = \gamma.C_y^{\ 2}, \quad E(e_1^{\ 2}) = Var(\bar{x}) = \gamma.C_x^{\ 2}$$

$$, \quad E(e_0 e_1) = Cov(\bar{y},\bar{x}) = \gamma\rho C_y C_x.$$

Writing equation (11) in e's we get,

$$t^* = \bar{Y}(1+e_0)(1+e_1)^{\alpha} \exp\left\{\frac{-a\bar{X}e_1}{2(a\bar{X}+b)+\bar{X}e_1}\right\}$$

$$= \bar{Y}(1+e_0)(1+e_1)^{\alpha} \exp\left\{\frac{-\theta e_1}{1+\theta e_1}\right\}$$

$$= \bar{Y}(1+e_0)(1+e_1)^{\alpha} \exp\left\{-\theta e_1 (1+\theta e_1)^{-1}\right\}, \tag{12}$$

Where $\theta = \dfrac{a\bar{X}}{2(a\bar{X}+b)}$.

Expanding (12) and ignoring e's with higher power,

$$t^* = \bar{Y}(1+e_0)\left[\begin{array}{l} 1-\theta e_1 + \theta^2 e_1^{\ 2} + \frac{1}{2}\theta^2 e_1^{\ 2} \\ -\alpha\theta e_1^{\ 2} + \alpha e_1 + \frac{\alpha(\alpha-1)}{2} e_1^{\ 2} \end{array}\right]. \tag{13}$$

Subtracting $\bar{Y}$ and taking expectations of (13), we get,

$$E(t^* - \bar{Y}) = E\left[\bar{Y}\left\{e_0 - \theta e_1 e_0 + \alpha e_1 e_0 - \theta e_1 + \alpha e_1\right\}\right]. \tag{14}$$

Squaring equation (14) we get,

$$E(t^* - \bar{Y})^2 = E\left[\bar{Y}^2 \left\{\begin{array}{l} e_0 - \theta e_1 e_0 + \alpha e_1 e_0 \\ -\theta e_1 + \alpha e_1 \end{array}\right\}^2\right],$$

The MSE given as,

$$MSE(t^*) = \bar{Y}^2 \gamma \left[\begin{array}{l} C_y^{\ 2} + \alpha^2 C_x^{\ 2} + \theta^2 C_x^{\ 2} \\ -2\alpha\theta C_x^{\ 2} + 2\alpha\rho C_x C_y \\ -2\theta\rho C_x C_y \end{array}\right]. \tag{15}$$

Differentiating $MSE(t^*)$ we get α_{opt},

$$\alpha_{opt} = \frac{(\theta C_x - \rho C_y)}{C_x}.$$

Using α_{opt} in (15), we get the $MSE_{\min}(t^*)$ given as

$$MSE_{\min}(t^*) = \gamma \bar{Y}^2 C_y^{\ 2}(1-\rho^2) \quad . \tag{16}$$

One can see the estimator t^* at its optimum condition coincides with the linear regression estimator.

3 Efficiency Comparison

The conditions for which the estimator t^* is an improvement over $\bar{y}$, T_R, T_P, T_E and t are given as,

$$\left[Var(\bar{y}) - MSE_{\min}(t^*)\right] = \gamma \bar{Y}^2 \rho^2 > 0.$$

$$\left[MSE(T_R) - MSE_{\min}(t^*)\right] = \gamma \bar{Y}^2 (C_x - \rho C_y)^2 > 0.$$

$$\left[MSE(T_P) - MSE_{\min}(t^*)\right] = \gamma \bar{Y}^2 (C_x + \rho C_y)^2 > 0.$$

$$\left[MSE(T_E) - MSE_{\min}(t^*)\right] = \gamma \bar{Y}^2 \left(\frac{C_x}{2} - \rho C_y\right)^2 > 0.$$

$$\left[MSE(t) - MSE_{\min}(t^*)\right] = \gamma \bar{Y}^2 (C_x \theta - \rho C_y)^2 > 0.$$

It is perceived that the recommended estimator performs well than all other estimators, because the above conditions are satisfied.

4 Numerical Illustrations

To demonstrate the relevance of the estimator, the data is given in table 2.

Table 1: Some Members of the Suggested Estimator.

Some Members of Suggested estimator t^*		
Values of		Estimators
a	b	
1	1	$t_1^* = \bar{y}\left(\frac{\bar{x}}{\bar{X}}\right)^{\alpha} \exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right)$
1	$\beta_2(x)$	$t_2^* = \bar{y}\left(\frac{\bar{x}}{\bar{X}}\right)^{\alpha} \exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}+2\beta_2(x)}\right)$
1	C_x	$t_3^* = \bar{y}\left(\frac{\bar{x}}{\bar{X}}\right)^{\alpha} \exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}+2C_x}\right)$
1	ρ_{yx}	$t_4^* = \bar{y}\left(\frac{\bar{x}}{\bar{X}}\right)^{\alpha} \exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}+2\rho_{yx}}\right)$
$\beta_2(x)$	C_x	$t_5^* = \bar{y}\left(\frac{\bar{x}}{\bar{X}}\right)^{\alpha} \exp\left(\frac{\beta_2(x)(\bar{X}-\bar{x})}{\beta_2(x)(\bar{X}+\bar{x})+2C_x}\right)$
C_x	$\beta_2(x)$	$t_6^* = \bar{y}\left(\frac{\bar{x}}{\bar{X}}\right)^{\alpha} \exp\left(\frac{C_x(\bar{X}-\bar{x})}{C_x(\bar{X}+\bar{x})+2\beta_2(x)}\right)$
C_x	ρ_{yx}	$t_7^* = \bar{y}\left(\frac{\bar{x}}{\bar{X}}\right)^{\alpha} \exp\left(\frac{C_x(\bar{X}-\bar{x})}{C_x(\bar{X}+\bar{x})+2\rho_{yx}}\right)$
ρ_{yx}	C_x	$t_8^* = \bar{y}\left(\frac{\bar{x}}{\bar{X}}\right)^{\alpha} \exp\left(\frac{\rho_{yx}(\bar{X}-\bar{x})}{\rho_{yx}(\bar{X}+\bar{x})+2C_x}\right)$
$\beta_2(x)$	ρ_{yx}	$t_9^* = \bar{y}\left(\frac{\bar{x}}{\bar{X}}\right)^{\alpha} \exp\left(\frac{\beta_2(x)(\bar{X}-\bar{x})}{\beta_2(x)(\bar{X}+\bar{x})+2\rho_{yx}}\right)$
ρ_{yx}	$\beta_2(x)$	$t_{10}^* = \bar{y}\left(\frac{\bar{x}}{\bar{X}}\right)^{\alpha} \exp\left(\frac{\rho_{yx}(\bar{X}-\bar{x})}{\rho_{yx}(\bar{X}+\bar{x})+2\beta_2(x)}\right)$

The source of the data statistics in Table 1 are taken from:

Population 1: Source [5]: Y is the fixed capital and X is the output of the 80 factories.

Population 2: Source [6]: where Y is the apple production and X is the number of trees.

Population 3: Source [5]: here X is the number of workers and Y is the output for 80 factories.

Population 4: Source [2]: X is the number of rooms and Y is the number of persons.

The PREs based on populations 1-4 are given in Table 2.

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Table 2: Data Statistics.

Parameters	Population 1	Population 2	Population 3	Population 4
N	80	104	80	10
n	20	20	20	4
$\bar{Y}$	11.264	6.254	51.826	5.920
$\bar{X}$	51.826	13931.680	2.851	3.590
ρ	0.941	0.860	0.915	1.860
C_y	0.750	1.860	0.354	0.144
C_x	0.354	1.650	0.948	0.128
$\beta_2(x)$	0.063	17.516	1.300	0.381

Table 3: PRE of the Estimators.

Estimators	Population 1	Population 2	Population 3	Population 4
$\bar{y}$	100.000	100.000	100.000	100.000
T_R	298.972	382.945	30.586	158.823
T_P	47.369	30.186	7.651	34.089
T_E	163.521	230.504	292.078	161.440
t	104.054	323.244	319.840	8.197
t^*	873.218	384.025	614.345	173.784

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5 Conclusions

The PRE of the recommended estimator is minimum from all other estimators, given in table 3. It is detected that t requires the supplementary information, whereas in the recommended estimator t^*, the MSE can be attained without auxiliary information.

References

[1] Bahl, S. & Tuteja, R. K. (1991). Ratio and product type exponential estimator. Journal of Information and Optimization Sciences, doi:10.1080/02522667.1991.10699058. **12(1)**, 15163(1991).

[2] Cochran, W. G. (1977). Sampling techniques (3rd ed.). Wiley Eastern Limited.

[3] Enang, E. I., Uket, J. O. and Ekpenyong, E. J. (2017). "A modified class of exponential-type estimator of population mean in simple random sampling" International Journal of Advanced Statistics and Probability., **5(2)**, 95-103(2017).

[4] Khan, H. and Khan, M. A. (2017) "New Exponential-Ratio Type Estimators of Population Mean in Two- Phase Sampling Using No Information Case on Auxiliary Variables" Journal of Reliability and Statistical Studies., **10(2)**, 95-103(2017).

[5] Murthy, M. N. (1967). Sampling theory and methods. Calcutta, India: Statistical Publishing Society.

[6] Shabbir, J., Haq, A. & Gupta, S. (2014). A new difference-cum-exponential type estimator of finite population mean in simple random sampling. RevistaColombiana de Estadística, doi:10.15446/rce.v37n1.44366., **37(1)**, 197-209(2014).

[7] Singh, H. P. & Tailor, R. (2003). Use of known correlation coefficient in estimating the finite population mean. Statistics in Transition., **6(4)**, 555-560(2003).

[8] Singh, H. P. & Tailor, R. (2005). Estimation of finite population mean with coefficient of variation of an auxiliary character. Statistica, doi:10.6092/issn.1973-2201/1299., **65(3)**, 407-413(2005).

[9] Singh, H. P., Tailor, R., Tailor, R. & Kakran, M. S. (2004). An improved estimator of population mean using power transformation. Journal of the Indian Society Agricultural Statistics., **58(2)**, 223-230(2004).

[10] Singh, R., Chauhan, P., Sawan, N. & Smarandache, F. (2009). Improvement in estimating the population mean using exponential estimator in simple random sampling. International Journal of Statistics and Economics, **3(A09)**, 13-18(2009).

[11] Singh, R., Chauhan, P., Sawan, N. & Smarandache, F. (2011). Improved exponential estimator for population variance using two auxiliary variables. Italian Journal of Pure and Applied Mathematics., **28**, 101-109(2011).

[12] Sisodia, B. V. S. & Dwivedi, V. K. (1981). A modified ratio estimator using coefficient of variation of auxiliary variable. Journal of the Indian SocietyAgricultural Statistics., **33(2)**, 13-18(1981).

[13] Upadhyaya, L. N. & Singh, H. P. (1999). Use of transformed auxiliary variable in estimating the finite population mean. BiometricalJournal,doi:10.1002/(SICI)1521., **41(5)**, 627-636(1999).

[14] Yadav, S. K. & Kadilar, C. (2013a). Improved Class of Ratio and Product Estimators. Applied Mathematics and Computation., **219 (22)**, 10726-10731(2013a).

[15] Yadav, S. K. & Kadilar, C. (2013b). Improved exponential type ratio estimator of population variance. Revista Colombiana de Estadística., **36(1)**, 145-152(2013b).

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