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A New Generalization of Length Biased Prakaamy Distribution with Properties and its Applications to Real Life Data

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Abstract: In this paper, a new extension of Prakaamy distribution has been proposed. The new distribution is called Length Biased Prakaamy distribution (LBPD) for modeling real life-time data. The statistical properties of this distribution including survival function, hazard function, and moments, moment generating function, order statistics, entropies, Bonferroni and Lorenz curves have been derived. The maximum likelihood technique is used to estimate the parameters of the model. Finally, the helpfulness and application of the distribution has been demonstrated by real life-time data.

Keywords: Bonferroni and Lorenz curve; Entropies; Length Biased Prakaamy distribution; Maximum Likelihood Estimator; Prakaamy distribution; Reliability Analysis.

1 Introduction

Prakaamy distribution is a newly proposed lifetime model formulated by Shukla [1] which is a two-component mixture of exponential (θ) and gamma $(6,\theta)$ distributions for modeling real-life data with mixing proportion

$$p=\frac{\theta^5}{\theta^5+120}.$$

Shukla [1] was introduced behavior and properties of Prakaamy distribution and its applications for life time data from engineering and medical science. Its various statistical properties such as mean, variance, coefficient of variation, moments and moment generating function have been obtained. The applicability and the goodness of fit of the Length Biased Prakaamy distribution over Akash, Prakaamy, Pranav and Exponential distribution have been illustrated with real-life data set. The concept of weighted distributions was first introduced by Fisher [2] to model the ascertainment bias. Which were later developed by Rao [3] in a unifying manner. The Weighted models were formulated in such situations to record the observations according to someweighted function. The weighted distribution reduces to length biased distribution when the weight function considers only the length of the units.

The concept of length biased sampling was first introduced by Cox and Zelen [4], [5]. Generally, the size-biased distribution is when the sampling mechanism selects the units with probability which is proportional to some measure of the unit size. Different authors have reviewed and discussed the various weighted probability models and illustrated their applications in different fields. Weighted distributions are applied in various research areas related to biomedicine, reliability, ecology and branching processes. Statistical applications of weighted distributions related to human population and ecology was studied by Patil and Rao [6]. Sajish kumar and Subramanian [7] obtained a new extension of devya distribution with properties and applications in real life data. Rashid and Rajagopalan [8] discussed on length biased weighted new quasi Lindley distribution with its statistical properties and applications. Rajagopalan et al. [9] discussed length biased Aradhana distribution along with applications. Rather and Subramanian, [10] discussed length biased Sushila distribution with properties. Finally, in this paper the goodness of fit criteria is Akaike Information Criteria (AIC), Bayesian Information Criteria (BIC) and Akaike Information Criteria Corrected (AICC) are used to identify the best fitted

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distribution and also we found that Length Biased Prakaamy distribution is better fit than the Prakaamy, Akash, Pranav and Exponential distribution for the considered real life data.

2 Length Biased Prakaamy Distribution (LBPD)

The probability density function (pdf) of Prakaamy distribution is given by

$$f(x,\theta)=\frac{\theta^6}{\theta^5+120}\left(1+x^5\right)e^{-\theta x} \qquad x>0 \;\; \theta>0 \tag{1}$$

and its cumulative distribution function is given by

$$F(x,\theta)=1-\left[1+\frac{\theta x\theta^4x^4+5\theta^3x^3+20\theta^2x^2+60\,\theta x+120}{\theta^5+120}\right]e^{-\theta x} \qquad x>0, \quad \theta>0$$

Suppose X is a non-negative random variable with probability density function $f(x)$. Let $w(x)$ be the non-negative weight function, and then the probability density function of the weighted random variable X_w is given by

$$f_w(x)=\frac{w(x)f(x)}{E(w(x))} \tag{2}$$

Where $w(x)$ is a non-negative weight function and

$$E(w(x))=\int w(x)f(x)dx \qquad 0<E(w(x))<\infty$$

For different weighted models, we can have different choices of the weight function $w(x)$. In this paper, we will introduce Length Biased version of Prakaamy distribution, in weights we use c=1, $w(x)=x$, in order to get the Length Biased Prakaamy distribution and the probability density function (pdf) of Length Biased version is given as

$$f_l(x)=\frac{x\,f(x)}{E(x)} \qquad c>0 \tag{3}$$

Where

$$E(x)=\int_0^\infty x\,f(x)dx$$

$$=\int_0^\infty x\left(\frac{\theta^6}{\theta^5+120}\right)(1+x^5)e^{-\theta x}dx$$

$$=\left(\frac{\theta^6}{\theta^5+120}\right)\int_0^\infty x(1+x^5)e^{\theta x}dx$$

$$=\left(\frac{\theta^6}{\theta^5+120}\right)\int_0^\infty x\,e^{\theta x}dx\int_0^\infty x^6\,e^{\theta x}dx$$

$$=\left(\frac{\theta^6}{\theta^5+120}\right)\int_0^\infty x^{2-1}e^{\theta x}\,dx\int_0^\infty x^{7-1}\,e^{\theta x}\,dx \tag{4}$$

Using gamma function to equation (4), we get

$$E(x^c)=\frac{\theta^5+720}{\theta^c(\theta^5+120)} \tag{5}$$

Substituting equation (1) and equation (5) in equation (3) we will get probability density function (pdf) of Length Biased Prakaamy Distribution.

$$f_l(x;\theta) = \left(\frac{x\,\theta^7(1+x^5)e^{-\theta x}}{\theta^5+720}\right) \tag{6}$$

Now, the cumulative distribution function (cdf) of the Length Biased Prakaamy distribution is obtained as

$$F_l(x;\theta) = \int_0^x f_l(x)\,dx$$

$$= \frac{\theta^7}{\theta^5+720}\int_0^x (x+x^6)e^{-\theta x}\,dx$$

$$= \frac{\theta^7}{\theta^5+720}\left[\int_0^x xe^{-\theta x}\,dx + \int_0^x x^6 e^{-\theta x}\,dx\right]$$

Put $\theta x = z \;\Rightarrow\; x = \dfrac{z}{\theta} \;\Rightarrow\; \theta dx = dz \;\Rightarrow\; dx = \dfrac{dz}{\theta}$

When $x \to x \;\Rightarrow z \to \theta x, \qquad x \to 0 \;\Rightarrow z \to 0$

$$= \frac{\theta^7}{\theta^5+720}\left[\left(\frac{1}{\theta}\right)^2 \int_0^{\theta x}\left(z^{2-1}\right)e^{-z}\,dz + \left(\frac{1}{\theta}\right)^7 \int_0^{\theta x}\left(z^{7-1}\right)e^{-z}\,dz\right]$$

$$F_l(x,\theta) = \left(\frac{\theta^5\gamma(2,\theta x)+\gamma(7,\theta x)}{\theta^5+720}\right) \tag{7}$$

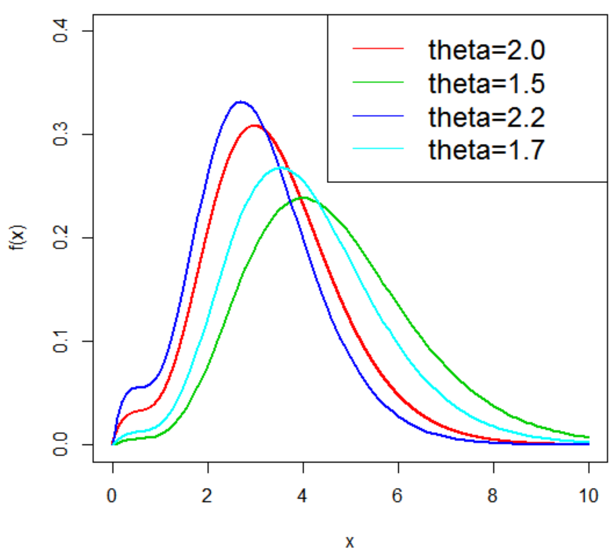

Fig.1 pdf plot of Length biased Prakaamy distribution

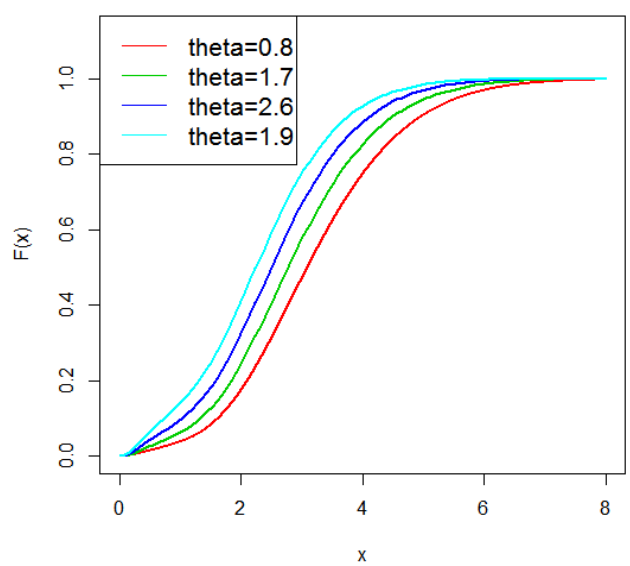

Fig.2 cdf plot of Length biased Prakaamy distribution

Pdf and Cdf Plots of Length Biased Prakaamy Distribution

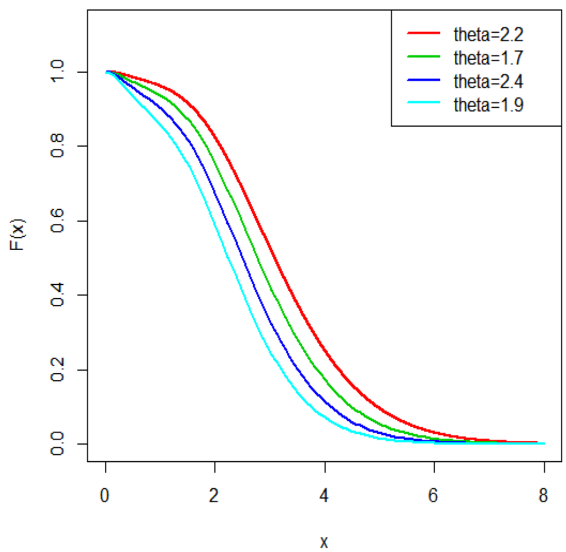

Fig.3 survival plot of Length biased Prakaamy distribution

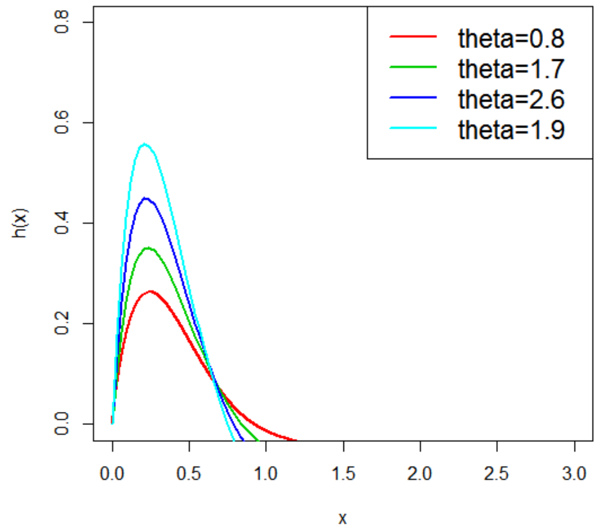

Fig.4 hazard plot of Length biased Prakaamy distribution

Survival and Hazard Plots of Length Biased Prakaamy Distribution

3 Reliability Analysis

In this section, we will discuss about the survival function, hazard function and reverse hazard function of the Length Biased Prakaamy distribution. The survival function or the reliability function of the Length biased Prakaamy distribution is given by

$$S_l(x) = 1 - F(x)$$

$$S_l(x) = \left(\frac{\theta^5 + 720 - \theta^5 \gamma(2, \theta x) + \gamma(7, \theta x)}{\theta^5 + 720} \right) \tag{8}$$

The corresponding hazard function or failure rate is given by

$$h_l(x;\theta) = \frac{f_l(x;\theta)}{S_l(x;\theta)}$$

$$h_l(x) = \frac{x\theta^7(1+x^5)e^{-\theta x}}{\theta^5 + 720 - \theta^5\gamma(2,\theta x) + \gamma(7,\theta x)} \tag{9}$$

The Reverse hazard rate of length biased Prakaamy can be calculated as

$$h_r(x,\theta) = \frac{f_l(x;\theta)}{F_l(x;\theta)}$$

$$h_r(x,c,\theta) = \frac{x\,\theta^7 (1 + x^5)\, e^{-\theta x}}{\theta^5 \gamma(2, \theta x) + \gamma(7, \theta x)}$$

4 Moments and associated Measures

Let X_l denotes the random variable following length biased Prakaamy distribution then the r^{th} order moment, that is, $E(x^r)$ can be obtained as

$$\mu_r' = E(x^r) = \int_0^\infty x^r f_w(x)dx$$

$$= \int_0^\infty x^{r+1} \frac{\theta^7 (1+x^5)e^{-\theta x}}{\theta^5 + 720} dx$$

$$= \frac{\theta^7 (1+x^5)e^{-\theta x}}{\theta^5 + 720} \int_0^\infty x^{r+1}(1+x^5)e^{-\theta x} dx$$

On simplification, we get

$$\mu_r' = \left(\frac{\theta^5 (r+1)! + (r+6)!}{\theta^r (\theta^5 + 720)} \right) \tag{10}$$

Putting r=1 in equation (10), we will get the mean of Length biased Prakaamy distribution which is given by

$$\mu_1' = E(X)$$

$$\mu_1' = \frac{2\theta^5 + 5040}{\theta(\theta^5 + 720)}$$

putting r=2, we get the second moment of Length biased Prakaamy distribution as

$$\mu_2' = \frac{6\theta^5 + 40320}{\theta^2(\theta^5 + 720)}$$

and putting r=3, we get the third moment of Length biased Prakaamy distribution as

$$\mu_3' = \frac{24\theta^5 + 9!}{\theta^3(\theta^5 + 720)}$$

Variance $\mu_2 = \mu_2' - (\mu_1')^2$

$$\mu_2 = \frac{6\theta^5 + 40320}{\theta^2(\theta^5 + 720)} - \left(\frac{2\theta^5 + 5040}{\theta(\theta^5 + 720)} \right)^2$$

Standard deviation $$\sigma = \sqrt{\frac{6\theta^5 + 40320}{\theta^2(\theta^5 + 720)} - \left(\frac{2\theta^5 + 5040}{\theta(\theta^5 + 720)} \right)^2}$$

5 Moment generating function and Characteristic function

Let **X** follows weighted Prakaamy distribution, and then the moment generating function (mgf) of X is given by

$$M_X(t) = E(e^{tx}) = \int_0^\infty e^{tx} f_w(x, c, \theta)dx$$

Using Taylor's series expansion

$$= \int_0^\infty \left(1 + tx + \frac{(tx)^2}{2!} + \ldots \right) f_w(x)dx$$

$$=\int_0^\infty \sum_{r=0}^\infty \frac{(tx)^r}{r!} f_w(x)dx$$

$$=\sum_{r=0}^\infty \frac{t^r}{r!}\int_0^\infty x^r f_w(x)dx$$

$$=\sum_{r=0}^\infty \frac{t^r}{r!}\mu_r'$$

$$M_X(t)=\sum_{r=0}^\infty \frac{t^r}{r!}\left(\frac{\theta^5(r+1)!+(r+6)!}{\theta^r(\theta^5+720)}\right)$$

Similarly we can get the characteristic function of Length Biased Prakaamy distribution.

$$\phi_X(t)=E(e^{itx})=\int_0^\infty e^{itx}f(x)dx$$

$$\phi_X(it)=\sum_{r=0}^\infty \frac{(it)^r}{r!}\left(\frac{\theta^5(c+r)!+(c+r+5)!}{\theta^r(\theta^5(c!)+(c+5)!}\right)$$

6 Entropies

The concept of entropy is important in different areas such as probability, statistics, physics, communication theory and economics. Entropies quantify the diversity, uncertainty, or randomness of a system. Entropy of a random variable X is a measure of variation of the uncertainty.

6.1 Renyi Entropy

The Renyi entropy is important in ecology and statistics as an index of diversity. It is also important in quantum information, where it can be used as a measure of entanglement. For a given probability distribution, Renyi entropy is given by

$$R(\alpha)=\frac{1}{\alpha-1}\log\int_0^\infty \left(f_w^\alpha(x)\right)dx \qquad \alpha>0,$$

$$R(\alpha)=\frac{1}{\alpha-1}\log\int_0^\infty \left(\frac{x\theta^7(1+x^5)e^{-\theta x}}{\theta^5+720}\right)^\alpha dx$$

$$=\frac{1}{\alpha-1}\log\left(\frac{\theta^7}{\theta^5+720}\right)^\alpha \int_0^\infty x^\alpha(1+x^5)^\alpha e^{-\theta\alpha x}dx \tag{11}$$

Using Binomial expansion to above equation (11) and we have

$$=\frac{1}{\alpha-1}\log\left(\frac{\theta^7}{\theta^5+720}\right)^\alpha \sum_{i=0}^\infty \binom{\alpha}{i}\int_0^\infty x^{\alpha+5i}e^{-\theta\alpha x}dx$$

On simplification, we get

$$=\frac{1}{\alpha-1}\log\left(\frac{\theta^7}{\theta^5+720}\right)^\alpha \sum_{i=0}^\infty \binom{\alpha}{i}\left(\frac{\Gamma\alpha+5i+1}{(\theta\alpha)^{\alpha+5i+1}}\right)$$

6.2 Tsallis Entropy

A generalization of Boltzmann-Gibbs (B-G) statistical mechanics initiated by Tsallis has attracted a great deal of attention. This generalization of (B-G) statistics was proposed firstly by introducing the mathematical expression of Tsallis entropy [11] for a continuous random variable which is defined as follow

$$S(\lambda)=\frac{1}{1-\lambda}\left[1-\int_0^\infty f^{\lambda}(x)dx\right]$$

$$S(\lambda)=\frac{1}{1-\lambda}\left[1-\int_0^\infty\left(\frac{x\theta^7(1+x^5)e^{-\theta x}}{\theta^5+720}\right)^{\lambda}dx\right]$$

$$S(\lambda)=\frac{1}{1-\lambda}\left[1-\left(\frac{\theta^7}{\theta^5+720}\right)^{\lambda}\int_0^\infty x^{\lambda}\left(1+x^5\right)^{\lambda}e^{-\lambda\theta x}dx\right] \tag{12}$$

Using Binomial expansion on equation (12) and we obtain

$$S(\lambda)=\frac{1}{1-\lambda}\left[1-\left(\frac{\theta^7}{\theta^5+720}\right)^{\lambda}\sum_{i=0}^{\infty}\binom{\lambda}{i}\int_0^\infty x^{\lambda+5i}e^{-\lambda\theta x}dx\right]$$

On simplification, we get

7 Bonferrni and Lorenz Curves

The Bonferroni and Lorenz curves [12] have applications not only in economics to study income and poverty, but also in other fields like reliability, demography, insurance and medicine. The Bonferroni and Lorenz curves are defined as

$$B(p)=\frac{1}{\mu_1' p}\int_0^q xf(x)dx$$

Where $q=F^{-1}(p)$

$$\mu_1'=\frac{2\theta^5+5040}{(\theta^5+720)}$$

$$B(p)=\left(\frac{1}{\mu_1' p}\right)\left(\frac{\theta^7}{\theta^5+720}\right)\int_0^q x^2\left(1+x^5\right)e^{-\theta x}dx \quad B(p)=\left(\frac{1}{\mu_1' p}\right)\left(\frac{\theta^7}{\theta^5+720}\right)\int_0^q x^2e^{-\theta x}dx+\int_0^q x^7e^{-\theta x}dx$$

Put $\theta x=z$ $\quad x=\frac{z}{\theta}\quad dx=\frac{dz}{\theta}$

When $z\to\theta q\quad x\to q\quad x\to 0\quad z\to 0$

$$B(p)=\left(\frac{1}{\mu_1' p}\right)\left(\frac{\theta^7}{\theta^5+720}\right)\int_0^{\theta q}\left(\frac{z}{\theta}\right)^2e^{-z}\frac{dz}{\theta}+\int_0^{\theta q}\left(\frac{z}{\theta}\right)^7e^{-z}\frac{dz}{\theta}$$

$$B(p)=\left(\frac{1}{\mu_1' p}\right)\left(\frac{\theta^7}{\theta^5+720}\right)\left(\frac{1}{\theta}\right)^3\int_0^{\theta q}z^{3-1}e^{-z}dz+\left(\frac{1}{\theta}\right)^8\int_0^{\theta q}z^{8-1}e^{-z}dz$$

$$B(p)=\left(\frac{1}{\mu_1' p}\right)\left(\frac{\theta^5\gamma(3,\theta q)+\gamma(8,\theta q)}{\theta(\theta^5+720)}\right)$$

Similarly we can obtain the Lorenz curve of the Length Biased Prakaamy distribution

$$L(p)=p\,(B(p))$$

$$L(p)=\frac{1}{\mu_1'}\int_0^q xf(x)dx$$

$$L(p)=\left(\frac{\theta^5\gamma(3,\theta q)+\gamma(8,\theta q)}{\theta(\theta^5+720)\,\mu_1'}\right)$$

8 Order Statistics

Order statistics has a wider application in several aspects of statistical theory and practice. Let $X_{(1)},X_{(2)},X_{(3)},....X_{(n)}$ denotes the order statistics of a random sample $X_1,X_2,X_3,....X_n$ from a continuous population with cumulative distribution function $F_X(x)$ and probability density function $f_X(x)$, and then the pdf of p^{th} order statistics $X_{(p)}$ is given by

$$f_{X(p)}(x)=\frac{n!}{(p-1)!(n-p)!}\,f(x)\,(F(x))^{p-1}\left[1-F(x)\right]^{n-p} \tag{13}$$

Substituting (6) and (7) of Length Biased Prakaamy distribution in equation (13), then we obtain the probability density function of the pth order statistics of the Length Biased Prakaamy distribution as

$$f_{X(p)}(x)=\frac{n!}{(p-1)!(n-p)!}\left(\frac{x\theta^7(1+x^5)e^{-\theta x}}{\theta^5+720}\right)\left[\left(\frac{\theta^5\gamma(2,\theta x)+\gamma(7,\theta x)}{\theta^5+720}\right)\right]^{p-1}$$

$$\left[1-\left(\frac{\theta^5\gamma(2,\theta x)+\gamma(7,\theta x)}{\theta^5+720}\right)\right]^{n-p}$$

The probability density function of first order statistic can be obtained as

$$f_{X(1)}(x)=n\left(\frac{x\theta^7(1+x^5)e^{-\theta x}}{\theta^5+720}\right)\left[1-\left(\frac{\theta^5\gamma(2,\theta x)+\gamma(7,\theta x)}{\theta^5+720}\right)\right]^{p-1}$$

The probability density function of higher order statistic can be obtained as

$$f_{X(n)}(x)=n\left(\frac{x\theta^7(1+x^5)e^{-\theta x}}{\theta^5+720}\right)\left[\frac{\theta^5\gamma(c+1,\theta x)+\gamma(c+6,\theta x)}{\theta^5 c!+(c+5)!}\right]^{p-1}$$

$$S(\lambda)=\frac{1}{1-\lambda}\left[1-\left(\frac{\theta^7}{\theta^5+720}\right)^{\lambda}\sum_{i=0}^{\infty}\binom{\lambda}{i}\left(\frac{\Gamma\lambda+5i+1}{(\theta\lambda)^{\lambda+5i+1}}\right)\right]$$

9 Maximum Likelihood Estimation

In this section, we will discuss the maximum likelihood of the parameters of Length Biased Prakaamy distribution. Consider $(x_1,x_2,x_3.......x_n)$ to be a random sample of size n from Length Biased Prakaamy distribution. Then the likelihood function is given by

$$L(x_i,\theta)=\prod_{i=1}^{n}f(x_i,\theta)$$

$$L(x_i,\theta)=\prod_{i=1}^{n}\frac{x_i\,\theta^7(1+x_i{}^5)e^{-\theta x_i}}{\theta^5+720}$$

$$=\left(\frac{\theta^7}{\theta^5+720}\right)^{n}\prod_{i=1}^{n}x_i(1+x_i^5)e^{-\theta x_i} \tag{14}$$

The log-likelihood is defined by equation (13) as

$$\log L = n\{7\log\theta - 5\log\theta - \log 720\} + \sum_{i=1}^{n}\log x_i + \sum_{i=1}^{n}(1 + x^5{}_i) - \theta\sum_{i=1}^{n}x_i \qquad (15)$$

The maximum likelihood estimates of θ and α can be obtained by differentiating equation (13) with respect to θ and α must satisfy the normal equations.

$$\frac{\partial \log L}{\partial\theta} = \frac{7n}{\theta} - \frac{5n}{\theta} - \sum_{i=1}^{n}x_i$$

$$7n - 5n - \theta\sum_{i=1}^{n}x_i = 0$$

$$\hat{\theta} = \frac{2n}{\sum_{i=1}^{n}x_i} \qquad (16)$$

Because of the complicated form of likelihood equation (16), it is very difficult to solve the system of nonlinear equations. Therefore, we use R and Wolfram Mathematica for estimating the required parameter of the length biased Prakaamy distribution.

10 Data Analysis

Data Set 1

The data set represents 40 patients suffering from blood cancer (leukemia) from one of ministry of Health Hospitals in Saudi Arabia, Abouammah et al. [13].The ordered lifetimes (in years) is given in table 1.

Table 1: Life times (in years) of 40 Blood Cancer (leukemia) patients

0.315	0.496	0.616	1.145	1.208	1.263	1.414	2.025	2.036	2.162
2.211	2.37	2.532	2.693	2.805	2.91	2.912	2.192	3.263	3.348
3.348	3.427	3.499	3.534	3.767	3.751	3.858	3.986	4.049	4.244
4.323	4.381	4.392	4.397	4.647	4.753	4.929	4.973	5.074	5.381

Data set 2

In an experiment at Florida state university to study the effect of methyl mercury poisoning on the life lengths of fish goldfish were subjected to various dosages of methyl mercury Kochar et al. [14].

Table 2: At one dosage level the ordered times to death in week is given by

6.000	6.143	7.286	8.714	9.429	9.857	10.143	11.571	11.714	11.714

In order to compare the Length Biased Prakaamy distribution with other distributions, we consider the criteria like Bayesian Information Criteria (BIC), Akaike Information Criteria (AIC), Akaike Information Criteria Corrected (AICC) and -2 logL. The better distribution is one corresponding to lesser values of AIC, BIC, AICC and -2 logL. AIC, BIC, AICC and -2logL can be evaluated by using the formula as follows:

$$AIC = 2k - 2\log L, \qquad BIC = k\log n - 2\log L, \qquad AICC = AIC + \frac{2k(k+1)}{(n-k-1)}$$

Where, k is the number of parameters, n is the sample size and -2logL is the maximized value of log likelihood function, these values are shown in Table 3 and table 4.

Table 3: Performance of Distributions for Dataset 1

Distributions	MLE	-2logL	AIC	BIC	AICC
Length Biased Prakaamy distribution	$\hat{\theta}$ =2.1503441 (0.1193869)	139.9152	141.9152	143.6041	142.0204
Prakaamy distribution	$\hat{\theta}$ =1.73827088 (0.09720255)	140.9196	142.9196	144.6085	143.0248
Akash distribution	$\hat{\theta}$ = 0.8055956 (0.0706564)	152.3956	154.3956	156.0845	154.5008
Exponential distribution	$\hat{\theta}$ =0.32095363 (0.05074673)	170.9169	172.9169	174.6058	173.0221
Pranav distribution	$\hat{\alpha} = 0.2135109$ (0.2209115) $\hat{\theta}$ =1.2146005 (0.1067916)	143.8637	147.8637	151.2414	148.1880

Table 4: Performance of Distributions for Dataset 2

Distributions	MLE	-2logL	AIC	BIC	AICC
Length Biased Prakaamy distribution	$\hat{\theta}$ =0.75599160 (0.09031392)	46.99704	48.99704	49.29962	49.40704
Prakaamy distribution	$\hat{\theta}$ =64763931 (0.08347796)	48.03604	50.03604	50.33863	50.53604
Akash distribution	$\hat{\theta}$ = 0.3139305 (0.0565190)	54.27783	56.27783	56.58042	56.77783
Exponential distribution	$\hat{\theta}$ =0.10802828 (0.03415861)	64.50782	66.50782	66.8104	67.00782
Pranav distribution	$\hat{\theta}$ =0.4320991 (0.0310256)	51.10229	55.10229	55.70746	55.60229

From Table 3 and table 4 we noted that the Length Biased Prakaamy distribution has a lesser AIC, BIC, AICC and -2logL values as compared to other distributions. Hence, we conclude that the Length Biased Prakaamy distribution leads to better fit than the other considered distribution.

11 Conclusion

In this study, Length Biased Prakaamy distribution has been proposed. The proposed distribution is generated by using the length biased technique to the baseline distribution. Some statistical properties along with reliability measures have been discussed. The parameters of the new distribution have been obtained by using maximum likelihood technique. Goodness of fit criteria illustrating the usefulness of the new distribution to real-life data sets was performed. The AIC, BIC, and AICC criteria suggested that proposed distribution provided better fit to the data as compared over Akash, Prakaamy, Pranav and Exponential distribution.

Conflict of interest

The authors declare that there is no conflict regarding the publication of this paper.

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