

An EOQ Model for Generalized Weibull Distribution Deterioration with Selling Price Sensitive Demand and Varying Holding Cost

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Abstract: In this paper, we present an inventory model for deteriorating items with non-linear price dependent demand rate. The deterioration rate of the proposed model is assumed as three parameter Weibull distribution deterioration. The demand rate is price sensitive that means considered as a quadratic selling price dependent demand rate. Shortages are allowed. Mathematical formulation with the solution procedure of the introduced model is developed. Some numerical examples are recommended to substantiate the solution procedure. Eventually, the sensitivity analysis of some parameters and conclusion of the proposed model are discussed.

Keywords: Inventory model, selling price dependent demand, three parameter Weibull deterioration, partial backlogging

1 Introduction

Supply chain management (SCM) [21] is the outline of, products, merchandise, raw materials, finance, information, and etc. from supplier to manufacturer, manufacturer to wholesaler, wholesaler to retailers, and retailers to consumers. The eventual focus of any intensive supply chain management system is to control the inventory problems. Inventory management is a part of supply chain management, adjusted the amount of supply merchandise. Inventory models depend on the minimizing the aggregate important stock expenses or boosting the aggregate benefits, i.e. minimizing the total relevant inventory costs or maximizing the total profits.

Deterioration is characterized as damage, harms, spoilage, or decay. Sustenance things, drugs, pharmaceuticals, electronic parts and radioactive substances are a few illustrations of things in which adequate deterioration might happen, amid the typical stockpiling period. This loss must be considered while analyzing the inventory problems of those products.

In the classical EOQ model, i.e. Harris [8] (1913) Model, deterioration rate was considered as constant. The fundamental EOQ model was extended by Ghare and

Schrader [6] (1963) considering the deterioration rate as non-constant. Then many authors work in deteriorating items say, [16]-[18], etc. Covert and Philip (1973) [5] further extended the Ghare and Schrader's [6] model and evolved an EOQ model for a variable rate assuming deterioration of a two-parameter Weibull distribution. The mathematical statement that portrays the model is given by

$$\frac{dI(t)}{dt} + g(t)I(t) = -d \quad (1)$$

where, d = demand rate, $I(t)$ = inventory level, and $g(t) = \alpha\beta t^{\beta}$; α = shape parameter, β = scale parameter.

The two-parameter Weibull distribution, is proper for those things which have diminished rate of deterioration, if just at the beginning, the rate of deterioration is to very high and comparable, the expanding rate of deterioration if just at the starting is roughly zero. However, said conditions are not satisfied in the case of numerous items, to eliminate this situation, Philip (1974) [10] expanded the model [6] by considering a three-parameter Weibull dissemination and supplant $g(t)$ with

$$g(t) = \alpha\beta(t - \gamma)^{\beta} \quad (2)$$

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where, α = shape parameter, β = scale parameter, and γ = location parameter of Weibull distribution deterioration. Next, Tadikamalla [14], Chakrabarty et al. [3] considered an inventory model with three parameter Weibull distribution rate. For more details one can refer to ([1], [7], [12], and the references therein.)

Offering the costs of a stock is a vital part in order to attend the shopkeepers in a business association. Generally, they pay reasonable costs of stock, whatever might be the costs, on the basis of its quality and lifespan. The price of any product is effected by its demand. The expanding costs show the diminishing demand and diminishing demands is the reason for expanding the cost. The first inventory model for price dependent demands was given by Whitin [20] in 1955. After that, many researcher works on these aspects such as [2], [4], Ladany and Sternlieb [9], [11], Urban [15], Wee [19], and the references therein.

Later, in the EOQ (Economic Order Quantity) models for price dependent demands the researchers have used two types of demand functions. First, in which where the demand is the linear decreasing function of price and in the other, demand rate is a negative power function of price. In 2011, Sana [13] developed an EOQ model for deteriorating items by considering the finite time horizon and demand is assumed as a quadratic decreasing function of price and divides the time horizon into finite (n) equal periods.

In this paper, we propose an inventory model for deteriorating items with non-linear price dependent demand rate. The deterioration rate of the proposed model is assumed as three parameter Weibull distribution deterioration. The demand rate is price sensitive that means considered as a quadratic selling price dependent demand rate. Shortages is allowed and which is partial backlogged.

The rest of the paper is organized as follows: assumptions and notation are given in section 2. The mathematical formulation is established in section 3. Section 4, presents the optimal solution procedure of the proposed model. Numerical examples are given in section 5. Finally, the conclusion of the introduced model is presented in section 6.

2 Notation and Assumptions

The notations which are used throughout the paper, given as below:

Notation

p	the selling price per unit
$D(p)$	the selling price dependent demand rate
$\theta(t)$	the time dependent three parameter Weibull deterioration rate

A	the ordering cost per unit
h	the holding cost per unit
c	the deterioration cost per unit
I_0	the initial inventory level
T	the fixed time length of inventory cycle when the inventory level reaches zero

Assumptions

- 1.The deterioration rate is considered as a time dependent, i.e., $\theta(t) = \alpha\beta(t - \gamma)^{(\beta-1)}$.
- 2.Demand rate $D(p)$ is considered as a price sensitive, i.e. $D(p) = a - bp - cp^2$.
- 3.The replenishment rate is instantaneous, and time horizon is infinite.
- 4.Lead time is considered as zero.
- 5.Shortages are not allowed.
- 6.Holding cost $h(t)$ per unit is assumed as time dependent, i.e. $h(t) = h + \alpha t$, where $\alpha; h > 0$.

3 Mathematical Formulation

The demand rate [13] decreases quadratically with respect to price p , such that

$$D(p) = a - bp - cp^2; \text{ where } a > 0, b > 0, \text{ and } c > 0$$

Assume that, the supplier, purchase the seasonal goods with I_0 units and has the authority to change the prices as mark up and mark down on selling price p . In the time period $[0, T]$ the inventory level decreases due to demand and deterioration both. Thus, the differential equation of inventory level during time period $[0, T]$ is given as below :

$$\frac{dI(t)}{dt} + \alpha\beta(t - \gamma)^{(\beta-1)}I(t) = -D(p); \quad 0 \leq t \leq T. \quad (3)$$

Now, we solve the above equation with boundary condition $I(0) = I_0$, and $I(T) = 0$. We have

$$I(t) = e^{(-\alpha(t-\gamma)^\beta)} \left[D(p) \left((T-t) + \frac{\alpha}{(\beta+1)} \right) \left((T-\gamma)^{(\beta+1)} - (t-\gamma)^{(\beta+1)} \right) \right]. \quad (4)$$

$$I_0 = e^{(-\alpha(-\gamma)^\beta)} \left[D(p) \left(T + \frac{\alpha}{(\beta+1)} \right) \left((T-\gamma)^{(\beta+1)} - (-\gamma)^{(\beta+1)} \right) \right]. \quad (5)$$

Now, to find the total relevant inventory cost, we calculate the following costs:

$$1. \text{Ordering cost (OC)} = \frac{A}{T}.$$

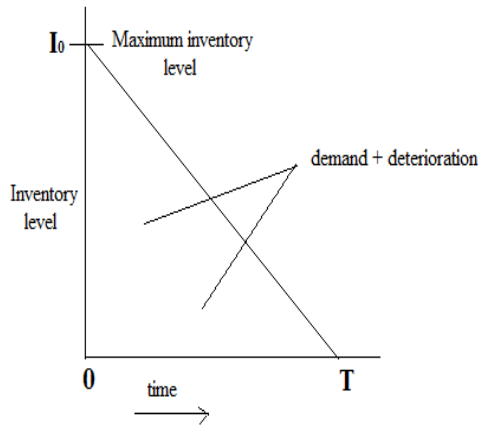


Fig. 1: Inventory System

2. Holding Cost (HC) -

$$\begin{aligned}
 HC &= \frac{h}{T} \left(\int_0^T I(t) dt \right) \\
 &= \frac{h}{T} \left(\int_0^T e^{-\alpha(t-\gamma)^\beta} \right. \\
 &\quad \left[D(p) \left((T-t) + \frac{\alpha}{(\beta+1)} ((T-\gamma)^{(\beta+1)} - (t-\gamma)^{(\beta+1)}) \right) \right] dt \right). \tag{6}
 \end{aligned}$$

3. Deterioration Cost (DC) -

$$\begin{aligned}
 DC &= \frac{cI_0}{T} = \frac{ce^{-\alpha(-\gamma)^\beta}}{T} \\
 &\quad \left[D(p) \left(T + \frac{\alpha}{(\beta+1)} ((T-\gamma)^{(\beta+1)} - (-\gamma)^{(\beta+1)}) \right) \right]. \tag{7}
 \end{aligned}$$

Hence, the total cost is given as,

$$\begin{aligned}
 TC &= OC + HC + DC \\
 &= \frac{A}{T} + \frac{h}{T} \left(\int_0^T e^{-\alpha(t-\gamma)^\beta} \right. \\
 &\quad \left[D(p) \left((T-t) + \frac{\alpha}{(\beta+1)} ((T-\gamma)^{(\beta+1)} - (t-\gamma)^{(\beta+1)}) \right) \right] dt \right) \\
 &\quad + \frac{ce^{-\alpha(-\gamma)^\beta}}{T} \left[D(p) \left(T + \frac{\alpha}{(\beta+1)} \right. \right. \\
 &\quad \left. \left. ((T-\gamma)^{(\beta+1)} - (-\gamma)^{(\beta+1)}) \right) \right]. \tag{8}
 \end{aligned}$$

Next, to find the total profit (TP), we calculate the sales revenue, i.e.,

$$\text{Sales Revenue} = p(\alpha - \beta p - \gamma p^2).$$

Thus, the total profit function (TP) is given as below

$$\begin{aligned}
 TP &= \text{Sales Revenue} - \text{total cost} \\
 TP &= p(\alpha - \beta p - \gamma p^2) - TC \\
 &= p(\alpha - \beta p - \gamma p^2) - \frac{A}{T} - \frac{h}{T} \left(\int_0^T e^{-\alpha(t-\gamma)^\beta} \right. \\
 &\quad \left[D(p) \left((T-t) + \frac{\alpha}{(\beta+1)} ((T-\gamma)^{(\beta+1)} - (t-\gamma)^{(\beta+1)}) \right) \right] dt \right) \\
 &\quad - \frac{ce^{-\alpha(-\gamma)^\beta}}{T} \left[D(p) \left(T + \frac{\alpha}{(\beta+1)} \right. \right. \\
 &\quad \left. \left. ((T-\gamma)^{(\beta+1)} - (-\gamma)^{(\beta+1)}) \right) \right]. \tag{9}
 \end{aligned}$$

The above Equation (9) is a function of two variables T and p, thus for the necessary condition, we differentiate TP with respect to T and p and equate it to be 0, i.e.,

$$\frac{\partial TP}{\partial T} = 0 \text{ and } \frac{\partial TP}{\partial p} = 0 \tag{10}$$

and finds the values of T and p. For the convexity of TC(T, p), we must have

$$\frac{\partial^2 TP}{\partial T^2} < 0, \quad \frac{\partial^2 TP}{\partial p^2} < 0 \quad \text{and} \quad \frac{\partial^2 TP}{\partial T^2} \frac{\partial^2 TP}{\partial p^2} - \left(\frac{\partial TP}{\partial T} \frac{\partial TP}{\partial p} \right)^2 > 0.$$

4 Numerical Example

Example To verify the proposed model we substitute the appropriate values of the parameter, i.e., $h = 1, c = 25, \alpha = 0.002, \beta = 2, \gamma = 3, \alpha_1 = 0.001$ per units per years; $a = 100; b = 1.8; c = 0.25; A = 150$, in Equation (10), and find the optimal values of T and p. Then, the optimum values are $T = 19.79$ weeks and $p = \$39.42$ per unit per weeks and total profit is $TP = \$5998.01$ per unit per weeks. Thus, the global (maximum) optimality are illustrated in Figure (2), Figure (3), and Figure (4).

5 Sensitivity Analysis

The impact of sensitivity analysis of some significant parameters is given in Table (1). Taking into account the given after effect of Table (1), we give the sensitivity of above model as below:

1. Increase in the value of the scale parameter, i.e., α , we observe the cycle time T increases with high sensitivity, and the total profit function TP and selling price p decrease with high sensitivity for scale parameter.
2. Increase in the value of the shape parameter, i.e., β , we observe the cycle time T, the total profit functions TP, and the selling price p decreases with high sensitivity for scale parameter.

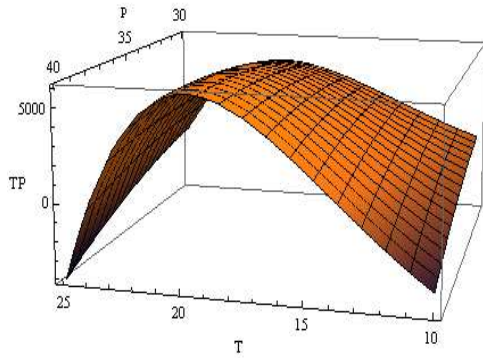


Fig. 2: Concavity of $TP|(T, p)$

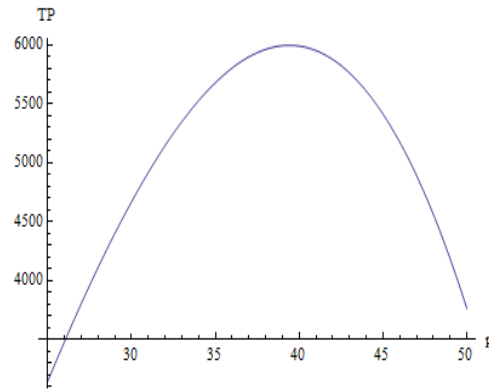


Fig. 4: Concavity of $TP|(p)$

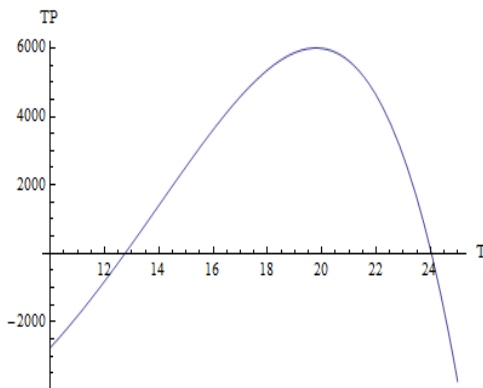


Fig. 3: Concavity of $TP|(T)$

Table 1: Effect of changes in the parameters of Example

parameters	value	T^*	p^*	TP
α	0.01	25.3796	43.21	8436.64
	0.02	19.7867	39.42	5998.01
	0.03	17.0643	35.88	4148.23
	0.04	15.2953	32.38	2689.35
β	1	100.329	61.23	2770.5
	2	57.78	39.23	2823.9
	3	34.89	11.554	2899.7
	4	0.3592	1.861	2941.9
γ	1	17.9533	45.20	9919
	2	18.953	43.01	8297.02
	3	19.786	39.42	5998.01
	4	20.43	34.65	3596.56
a	50	19.786	37.928	6870.34
	75	19.786	38.685	6424.86
	100	19.786	39.42	5998.01
	125	19.78	40.128	5589.2
b	1.4	19.7867	39.68	5735.76
	1.6	19.7867	39.55	5866.58
	1.8	19.7867	39.418	5998.01
	2.0	19.7867	39.288	6130.04
c	0.15	19.7867	40.46	3414.04
	0.20	19.7867	39.83	4702.27
	0.25	19.7867	39.42	5998.01
	0.30	19.7867	39.13	7297.92
A	50	19.78	39.42	6003.07
	150	19.78	39.42	5998.01
	200	19.78	39.42	5995.49
	250	19.78	39.42	5992.96
α_1	0.0005	19.82	39.47	6028.3
	0.0010	19.786	39.42	5998.01
	0.0125	19.099	38.40	5450.51
	0.0150	18.974	38.28	5360.46

3. Increase in the value of location parameter, i.e., γ , we observe the cycle time T increases slowly and the total profit function TP decreases fast. Also, the selling price p decreases moderately.
4. Increase in the value of a , b , and c , we observe the cycle time T is unchanged, i.e. insensible, but, the selling price p and the total profit function TP are moderately sensitive.
5. When the value of ordering cost, i.e. A is changing, the selling price p , cycle time T is unchanged, i.e. insensitive, but the total profit function TP decreases slowly, that is the total profit is lowly sensitive.
6. For α_1 , the cycle time T , the selling price p , and the total profit function TP are less changeable, i.e. lowly sensitive.

6 Conclusion

In the present business world, the selling price assumes a vital part to expanding or diminishing the demand rates of

any items. Therefore, the demands of numerous items are subject to offering cost. For the results the researchers are established, many inventory models with price dependent demand rates. The holding cost is assumed as a time dependent, i.e. varying with time. The researchers

established the demand is the linear decreasing function of price or the demand rate is a negative power function of price. Sana [13] developed a quadratic type price dependent demand rates. In this proposed model we assumed as quadratic type price depending demand rates with varying holding cost dependent on times. For future research, the model will be reached out by considering the finite rate of replenishment, time dependent deterioration, warehouse effects, trade credit or permissible delay in payments, inflation rate, quantity discounts, based inventory models, etc.

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