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Theoretical Analysis of NTQ-NOT Quantum Gate Based on Many Qubits

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Abstract: Based on N+1 transmon qubits driven by a strong microwave field and resonator field this work introduces quantum logic gates via quantum circuit. The operation times do not increase with the growth of the qubit number. Due to the virtual excitations of the resonator, the scheme is insensitive to the decay of the resonator. Numerical analysis shows that the scheme can be implemented with high fidelity. Moreover, we propose a detailed procedure and analyze the experimental feasibility. So, our proposal can be experimentally realized in the range of current circuit QED techniques.

Keywords: Transmon qubit, NTQ-NOT gate, NiSWAP-NOT gate, $N\sqrt{i\text{ SWAP}}$-NOT gate, circuit QED.

1 Introduction

Quantum computation has attracted much attention since a quantum computer has an ability to solve hard computational problems with high efficiency compared to a classical computer [2,3,1]. Also, it is well known that it is difficult to simulate the behaviour of a quantum mechanical system with a classical computer. The difficulty arises because quantum systems are not confined to their eigenstates but can in general exist in any superposition of them, thus the vector space needed to describe the system is extremely large. Quantum logic gates are the basic building blocks of a quantum computer [4,5,6]. A universal quantum computer can be built on a series of two-qubit logic gates [7]. Up to now, a large number of theoretical proposals have been introduced to implement various gates, such as two-qubit Controlled-NOT gate [8], iSWAP gate [9], $\sqrt{i\text{SWAP}}$ gate [8] in different systems, such as cavity QED system [10] and circuit QED system [11]. The circuit QED is currently the promising candidate for future quantum computer.

In recent works, We have proposed on another type of multiqubit controlled gate that is a NTCP gate by using qubit-qubit interaction in a circuit QED [12]. Also, we have presented a method for realizing NTCP-NOT and NTQ-NOT gates with one control superconducting qubit simultaneously controlling N target qubits in a cavity QED [13]. In the present study, we will present and demonstrate a method for realizing N two-transmon-qubit quantum logic gates of one transmon qubit simultaneously controlling N target transmon qubits in circuit QED with nearest qubit–qubit interaction by adding a strong microwave field. These type of quantum logic gates are useful in quantum computation and quantum information processing. On the other hand, the qubit-qubit interaction could influence the evolution of the system used in ref. [14]. Moreover, the presence of both qubit-qubit interaction and Jaynes-Cummings make our system powerful.

2 Implementation of N Two-Qubit-NOT Quantum Logic Gates

Let us consider an $(N+1)$ transmon qubits which are capacitively coupled to a TLR driven by a strong microwave field [15] as depicted in Fig.1. A microwave field of frequency ω_d is applied to the input wire of the TLR. The $(N+1)$ transmon qubits each having two-level subspaces driven by a conventional field added, these transmon qubits are capacitively coupled to it. Moreover,

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the qubit-qubit interaction should be included in the circuit QED [16], where the direct interaction between the qubits, in circuit QED, is virtually realized in the dispersive regime [17]. The Hamiltonian of the whole system (assuming $\hbar = 1$) given by [18]

$$H = \omega_q \sum_{j=1}^{N+1} \sigma_{z,j} + \omega_r a^+ a + \Omega \sum_{j=1}^{N+1} (\sigma_j^+ e^{-i\omega_d t} + \sigma_j^- e^{i\omega_d t}) + \sum_{j=1}^{N+1} g_j (a^+ \sigma_j^- + a\sigma_j^+) + \sum_{j,k=1,\, j\neq k}^{N+1} \Gamma_{jk} \sigma_j^+ \sigma_k^-, \quad (1)$$

we get $\Omega = g_j \varepsilon / \delta$ where $\delta = \omega_r - \omega_d$ is the detuning between the resonance frequency of the TLR ω_r and the frequency of the external drive applied to the TLR ω_d, $\Gamma_{jk} = g_j g_k (\frac{1}{\Delta_j} + \frac{1}{\Delta_k})$ [17] which describes the effective interaction between qubits j and k, and $\Delta_j(\Delta_k)$ is large detuning as compared to the qubit-TLR coupling $g_j(g_k)$ [19]. $\sigma_{z,j}$, σ_j^-, and σ_j^- are the collective operators for the $(1,2,...,N+1)$ qubits, which are given by $\sigma_{z,j} = \frac{1}{2}(|0_j\rangle\langle 0_j| - |1_j\rangle\langle 1_j|)$, $\sigma_j^+ = |1_j\rangle\langle 0_j|$, $\sigma_j^- = |0_j\rangle\langle 1_j|$ where $|1_j\rangle(|0_j\rangle)$ is the excited state (ground state) of the transmon qubit, $\omega_r = 1/\sqrt{LC}$ is the resonance frequency of the TLR where the transmission line resonator can be modeled as a simple harmonic oscillator composed of the parallel combination of an inductor L and a capacitor C, ω_q is the transition frequency of the transmon qubit with $\omega_{q_1} = \omega_{q_2} = ... = \omega_{q_{N+1}}$, a^+, a are the creation and annihilation of the resonator mode. In the high E_J/E_C limit the transition frequency between the ground state $|0_j\rangle$ and excited state $|1_j\rangle$ is given by $\omega_q = \sqrt{8E_J E_C}/\hbar$, where $E_C = e^2/2C_\Sigma$ and $E_J(\Phi) = E_{J0}|\cos(\pi\Phi/\Phi_0)|$, $C_\Sigma = C_S + (C_g^{-1} + C_g'^{-1})^{-1}$. Here, C_g, C_g' are the gate capacitance, C_S is the additional capacitor, C_Σ is the effective total capacitance, Φ is the external magnetic flux applied to the SQUID loop, $\Phi_0 = h/2e$ is the flux quantum, $E_J(\Phi)$ is the effective Josephson coupling energy, E_C is the charging energy, and E_{J0} is the Josephson coupling energy, where the qubit work at the regime $E_J/E_C \gg 1$. It is obvious that the frequency of the transmon qubit ω_q can be tuned by external magnetic flux Φ. When we work with a large amplitude driving field, quantum fluctuations in the drive are very small with respect to the drive amplitude and the drive can be considered, for all practical purposes, as a classical field.

For convince, we assume that we can tuned the coupling constant g_j and the coupling strength Γ_{jk} at the same time to have $g_j = g_k = g$ and $\Gamma_{jk} = \Gamma = \frac{2g^2}{\Delta}$ (in the case $\Delta_j = \Delta_k = \Delta$). By assuming that $\omega_d = \omega_q$, we have the following Hamiltonian H_I in the interaction picture

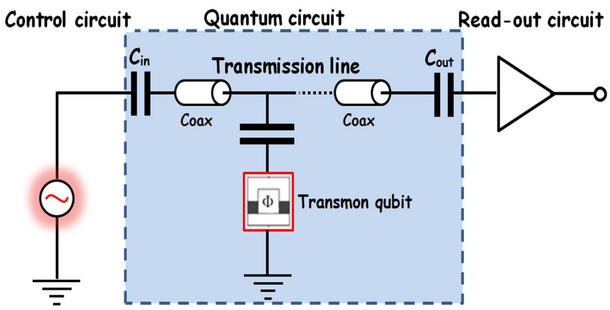

Fig. 1: Circuit quantum electrodynamics architecture for readout of a Cooper pair box. The Cooper pair box is coupled capacitively (C_g) to a transmission line resonator with capacitively coupled input (C_{in}) and output (C_{out}) ports. The input port is used to control the $(N+1)$ transmon qubits state using microwave pulses and to apply the readout microwave which, after transmission through the circuit, is detected at the readout port. In addition, the coupling between qubits i and j is via inductances coupling, where this coupling can be interpreted as an energy coupling between qubits.

[18]

$$H_I = \Omega \sum_{j=1}^{N+1} \left(\sigma_j^+ + \sigma_j^-\right) + g \sum_{j=1}^{N+1} \left(e^{i\delta t} a\sigma_j^+ + e^{-i\delta t} a^+ \sigma_j^-\right) + \Gamma \sum_{j,k=1, j\neq k}^{N+1} \sigma_j^+ \sigma_k^-, \quad (2)$$

where $\delta = \omega_r - \omega_q$ is the detuning between the frequency of the qubit ω_q and the frequency of the resonator ω_r. When $\Omega \gg \delta, g, \Gamma$ and $\delta \gg g, \Gamma$ we can get the Hamiltonian H_I in the interaction picture [20,21,22,23]

$$H_I = H_0 + H_{eff}, \quad (3)$$

with

$$H_0 = 2\Omega \sum_{j=1}^{N+1} \sigma_{x,j} \quad (4)$$

$$H_{eff} = \frac{g^2}{\delta} \left[\sum_{j=1}^{N+1} (\sigma_j^+ \sigma_j^- + a^+ a \sigma_{z,j}) + \sum_{j,k=1,\, j\neq k}^{N+1} \sigma_j^+ \sigma_k^- \right] + \Gamma \sum_{j,k=1, j\neq k}^{N+1} \left(\sigma_{x,j}\sigma_{x,k} + \frac{1}{4}\left(\sigma_j^+ \sigma_k^- + \sigma_j^- \sigma_k^+\right) \right), \quad (5)$$

where $\sigma_{x,j} = \frac{1}{2}\left(\sigma_j^+ + \sigma_j^-\right)$. From $Eq.$(3), we can easily obtain the corresponding evolution operator $U_I(t)$ as follows [20,22,24]

$$U_e(t) = \exp\{-i\frac{g^2}{\delta} t (\sum_{j=1}^{N+1} \sigma_j^+ \sigma_j^- + a^+ a \sigma_{z,j}) + \sum_{j,k=1,\, j\neq k}^{N+1} \sigma_j^+ \sigma_k^-) - i\Gamma t \sum_{j,k=1,\, j\neq k}^{N+1} (\sigma_{x,j}\sigma_{x,k} + \frac{1}{4}\left(\sigma_j^+ \sigma_k^- + \sigma_j^- \sigma_k^+\right) \sigma_j^+ \sigma_j^- + a^+ a \sigma_{z,j}) + \sum_{j,k=1,\, j\neq k}^{N+1} \sigma_j^+ \sigma_k^-) - i\Gamma t \sum_{j,k=1,\, j\neq k}^{N+1} (\sigma_{x,j}\sigma_{x,k} + \frac{1}{4}\left(\sigma_j^+ \sigma_k^- + \sigma_j^- \sigma_k^+\right)\} \quad (6)$$

For a charge qubit 1 coupled to the resonator, assumed $\delta \gg |g|$. Then, the corresponding evolution operator can be expressed as [14,25]

$$U_{e1}(t) = \exp\left[-i\frac{g^2}{\delta}t(\sigma_1^+\sigma_1^- + a^+a\sigma_{z,1})\right]. \quad (7)$$

We now show how to utilize the above model ($Eq.$(8)) to implement an NTQ-NOT, NiSWAP-NOT and $N\sqrt{i\mathrm{SWAP}}$-NOT gates. We note that: (i) for each one of qubits $1,2,...,N+1$ of each step of the operations, the dc gate voltage $V_g^{dc} = e/C_g$, such that $E_z = 0$ for each qubit, and (ii) the resonator mode frequency ω_r is fixed during the entire operation. We now consider two special cases: $\delta > 0$ as well as $\delta < 0$. The results from the unitary evolution, obtained for these two special cases, will be employed below for the gate implementation. The detailed procedure is given as follows:

Step 1. Let us begin with the case $\delta > 0$, set $V_g^{ac} = V_0\cos(\omega t)$ for each qubit, adjust the transition frequencies of $(N+1)$ qubits by external flux Φ_j, then $\omega_q = \omega_d$ is satisfied, with the pulse Rabi frequency $\Omega = g\varepsilon/\delta$ of the amplitude V_0 where the detuning $\delta = \omega_r - \omega_q > 0$. So, after a period of evolution time $\tau_1 = 2\pi/\delta$, the evolution operator for the qubit system corresponding to this step would be the $U_I(t)$ of $Eq.$(8).

$$U_e(\tau_1) = \exp\{-i\frac{g^2}{\delta}\tau(\sum_{j=1}^{N+1}(\sigma_j^+\sigma_j^- + a^+a\sigma_{z,j}) + \sum_{j,k=1,\ j\neq k}^{N+1}\sigma_j^+\sigma_k^-) \\ -i\Gamma\tau\sum_{j,k=1,\ j\neq k}^{N+1}(\sigma_{x,j}\sigma_{x,k} + \frac{1}{4}(\sigma_j^+\sigma_k^- + \sigma_j^-\sigma_k^+))\}. \quad (8)$$

Step 2. Let us now consider the case $\delta < 0$, set $V_g^{ac} = 0$ for qubit 1, adjust the level spacings of qubit 1 such that the resonator mode is coupled to qubits $(2,3,...,N+1)$, so $\omega_q = \omega_d$ is satisfied, with the pulse Rabi frequency is Ω' of the amplitude V_0', where the detuning $\delta' = \omega_r - \omega_q' = -\delta > 0$. Then, after a period of time $\tau_2 = 2\pi/\delta'$, the evolution operator of the system corresponding to this step can be described by

$$U_e(\tau_2) = \exp\{-i\frac{g^2}{\delta}\tau(\sum_{j=2}^{N+1}(\sigma_j^+\sigma_j^- + a^+a\sigma_{z,j}) + \sum_{j,k=2,\ j\neq k}^{N+1}\sigma_j^+\sigma_k^-) \\ -i\Gamma\tau\sum_{j,k=2,\ j\neq k}^{N+1}(\sigma_{x,j}\sigma_{x,k} + \frac{1}{4}(\sigma_j^+\sigma_k^- + \sigma_j^-\sigma_k^+))\}. \quad (9)$$

Step 3. We adjust the transition frequency of qubits $2,3,...,N+1$ such that the resonator mode is largely decoupled from each qubit, but we leave the transition frequency unchanged for qubit 1 such that $g^2/|\delta'| \ll |g|$ (i.e. $|\delta'| \gg |g|$ where $\delta' < 0$) and $\Omega' = g\varepsilon/\delta'$. When the condition $\delta' = -\delta$ is satisfied, the time evolution operato after a period of time τ is [14,25]

$$U_{e1}(\tau_3) = \exp\left[i\frac{g^2}{\delta}\tau(\sigma_1^+\sigma_1^- + a^+a\sigma_{z,1})\right]. \quad (10)$$

After the three-step process, a phase flip (i.e., $|1\rangle \to -|1\rangle$) on the state $|1\rangle$ of each target transmon qubit is achieved when the control transmon qubit 1 is initially in the state $|1\rangle$, but nothing happens to the states $|0\rangle$ and $|1\rangle$ of each target transmon qubit when the control transmon qubit 1 is initially in the state $|0\rangle$. When the conditions $\frac{\Gamma}{2} + \frac{g^2}{\delta} = \gamma$ and $\Gamma = 4\chi$, the evolution operator of the system, after the above three steps operation, is given by

$$U_{es}(\tau) = \prod_{j=2}^{N+1} U_{gate}(1,j), \quad (11)$$

where $U_{gate}(1,j) = \exp[-i\chi\tau 4\sigma_{x,1}\sigma_{x,j} - i\gamma\tau(\sigma_1^+\sigma_j^- + \sigma_1^-\sigma_j^+)]$, $j = 2,3,...,N+1$. According to the evolution operator $U_{gate}(1,j)$ above on the basis $(|0_1\rangle, |1_1\rangle)$ for qubit 1, so the basis $(|0_j\rangle, |1_j\rangle)$ for qubit $(2,3,....N+1)$, we can obtain following evolutions

$$\begin{aligned} U_{gate}(1,j)|0_1\rangle|0_j\rangle &= e^{-i\eta\pi}[\cos\chi\tau|0_1\rangle|0_j\rangle - i\sin\chi\tau|1_1\rangle|1_j\rangle] \\ U_{gate}(1,j)|0_1\rangle|1_j\rangle &= e^{i\eta\pi}[\cos(\gamma+\chi)\tau|0_1\rangle|1_j\rangle \\ &\quad -i\sin(\gamma+\chi)\tau|1_1\rangle|0_j\rangle] \\ U_{gate}(1,j)|1_1\rangle|0_j\rangle &= e^{i\eta\pi}[\cos(\gamma+\chi)\tau|1_1\rangle|0_j\rangle \\ &\quad -i\sin(\gamma+\chi)\tau|0_1\rangle|1_j\rangle] \\ U_{gate}(1,j)|1_1\rangle|1_j\rangle &= e^{-i\eta\pi}[\cos\chi\tau|1_1\rangle|1_j\rangle - i\sin\chi\tau|0_1\rangle|0_j\rangle], \end{aligned} \quad (12)$$

A phase factor $2\eta\pi$ in the previous evolutions can be produced by several different proposals [26] and the overall phase factor $e^{-i\eta\pi}$ is omitted. By setting $\chi\tau = (2k+\frac{1}{2})\pi$ (with k being an integer), $\eta = 2m+\frac{3}{2}$ (with m being an integer) and $\gamma\tau = (2n+1)\pi$ (with n being an integer), we can easily we obtain N-two-qubit iSWAP-NOT operations as (from $Eq.$(12))

$$\begin{aligned} U_{NOT}(1,j)|0_1\rangle|0_j\rangle &= |1_1\rangle|1_j\rangle \\ U_{NOT}(1,j)|0_1\rangle|1_j\rangle &= |1_1\rangle|0_j\rangle \\ U_{NOT}(1,j)|1_1\rangle|0_j\rangle &= |0_1\rangle|1_j\rangle \\ U_{NOT}(1,j)|1_1\rangle|1_j\rangle &= |0_1\rangle|0_j\rangle. \end{aligned} \quad (13)$$

By setting $\chi\tau = \left(2k+\frac{3}{2}\right)\pi$ (with k being an integer), $\eta = 2m+\frac{1}{2}$ (with m being an integer) and $\gamma\tau = \left(2n+\frac{1}{2}\right)\pi$ (with n being an integer), we obtain N-two-qubit iSWAP-NOT operations as (from $Eq.$(12))

$$\begin{aligned} U_{iSWAPN}(1,j)|0_1\rangle|0_j\rangle &= |1_1\rangle|1_j\rangle \\ U_{iSWAPN}(1,j)|0_1\rangle|1_j\rangle &= i|0_1\rangle|1_j\rangle \\ U_{iSWAPN}(1,j)|1_1\rangle|0_j\rangle &= i|1_1\rangle|0_j\rangle \\ U_{iSWAPN}(1,j)|1_1\rangle|1_j\rangle &= |0_1\rangle|0_j\rangle. \end{aligned} \quad (14)$$

By setting $\chi\tau = \left(2k+\frac{1}{2}\right)\pi$ (with k being an integer), $\eta = 2m+\frac{1}{2}$ (with m being an integer) and

$\gamma\tau = \left(2n - \frac{1}{4}\right)\pi$ (with n being an integer), we obtain N-two-qubit $\sqrt{iSWAP}$-NOT operations as

$$\begin{aligned} U_{\sqrt{iSWAP}N}(1,j)|0_1\rangle|0_j\rangle &= |1_1\rangle|1_j\rangle \\ U_{\sqrt{iSWAP}N}(1,j)|0_1\rangle|1_j\rangle &= \frac{1}{\sqrt{2}}[|1_1\rangle|0_j\rangle + i|0_1\rangle|1_j\rangle] \\ U_{\sqrt{iSWAP}N}(1,j)|1_1\rangle|0_j\rangle &= \frac{1}{\sqrt{2}}[|0_1\rangle|1_j\rangle + i|1_1\rangle|0_j\rangle] \\ U_{\sqrt{iSWAP}N}(1,j)|1_1\rangle|1_j\rangle &= |0_1\rangle|0_j\rangle. \end{aligned} \quad (15)$$

Then, the N two-transmon qubit-NOT, N two-transmon-qubit iSWAP-NOT and N two-transmon-qubit $\sqrt{iSWAP}$-NOT operations are simultaneously performed on N qubit pairs $(1,2)$, $(1,3)$, ..., $(1,N+1)$. Each gate operation (NOT, iSWAP and $\sqrt{iSWAP}$ gates) includes the same control qubit and a different target qubit. Thus, the NTQ-NOT, NiSWAP and $N\sqrt{iSWAP}$ gates are implemented simultaneously between the first qubit and the N qubits.

3 Fidelity and Discussion

Let us now study the fidelity of the gate operation. In order to check the validity of our proposal, we define the following fidelity to characterize the deviation of how much the output states $|\Psi(t)\rangle$ deviate in amplitude and phase from the ideal logical gate transformation for the different input states [24]

$$\begin{aligned} F &= |\langle\Psi(t)|U(t)|\Psi(0)\rangle|^2 \\ &= 1 - \left(\frac{\sqrt{2}g}{\Omega}\right)^2 \sin^2(\frac{\Omega}{2g}\pi), \end{aligned} \quad (16)$$

where $|\Psi(t)\rangle$ represents the final state the whole system after the gate operations that the initial state $|\Psi(0)\rangle$ followed by an ideal phase operation, and $U(t)$ describers the overall evolution operator of the system are performed in a real situation. We numerically simulate the relationship between the fidelity of the system and the ratio $\frac{\Omega}{g}$. Our numerical calculation shows that a high fidelity can be achieved when $\Omega > 10g$, , which is in good agreement with the approximation made by neglecting the terms oscillating fast (see Fig. 2). It should be noted that the estimation of Fidelity is obtained in the interaction picture.

Finally, we give a brief discussion about our proposal. We notice that δ satisfies the equation $(g/\delta_1)^2 = n + \frac{1}{2}$ of the NTQ-NOT gate, $(g/\delta_2)^2 = n + \frac{3}{4}$ of the NiSWAP gate and the equation $(g/\delta_3)^2 = n + \frac{5}{8}$ of the $\sqrt{iSWAP}$ gate, where n is an integer. So, when $n = 0$, δ takes maximum $\delta_1 = \sqrt{2}g$, $\delta_2 = \frac{2\sqrt{3}}{3}g$ and $\delta_3 = \frac{2\sqrt{10}}{5}g$. Then, the operations times $t_{op_1} = 2\pi \times \frac{3\sqrt{2}}{2g}$ (NTQ-NOT gate), $t_{op_2} = 3\tau = 2\pi \times \frac{3\sqrt{3}}{2g}$ (NiSWAP gate) and $t_{op3} = 3\tau = 2\pi \times \frac{3\sqrt{10}}{4g}$ ($N\sqrt{iSWAP}$ gate) are independents of the number of target qubits N. Then, the direct calculation shows that the operations times required to implement the NTQ-NOT gate, NiSWAP gate and $N\sqrt{iSWAP}$ gate with transmon qubits will be $t_{op_1} = 10.61ns$, $t_{op_2} = 13ns$ and $t_{op_3} = 11.86ns$.

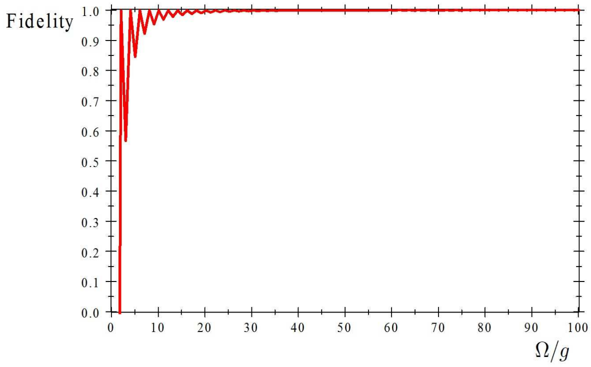

Fig. 2: Numerical results for fidelity of the gates operations versus the Ω/g for the implementation of NTQ-NOT gate. Here we consider the errors introduced by the Stark Shifts.

For this method to work, we discuss some issues which are relevant for future experimental implementation of our proposal. Compared with the usual charge qubit, the transmon qubit is immune to the $1/f$ charge noise and it has much longer dephasing time as a result of the transmon qubits chosen in the system. For the sake of definiteness, let us consider the experimental parameters of the transmon qubits as $C_g = 1aF$, $C_{J0} = 300aF$, $E_c = 2\pi \times 2GHz$ and $E_J \simeq 2\pi \times 100GHz$ [20]. The transmon qubits with these parameters are available at present. Thus, the coupling three qubits with a transmission line resonator has been experimentally demonstrated [27]. Our scheme may have potential applications in multipartite entanglement. The charge fluctuations are principal only in low-frequency region and can be reduced by the echo technique [28] and by controlling the gate voltage to the degeneracy point, but an effective technique for suppressing charge fluctuations and keep the state coherent for a longer time is highly desired. So our proposal is realizable with presently available circuit QED techniques.

In a recent experiments, it was shown that the decoherence times T_1 and dephasing time T_2 can be made to be on the order of $20 - 100\ \mu s$ for transmon qubits (when $E_J/E_C = 50$) [29,30]. In addition, the coupling strength is $g = 2\pi \times 200MHz$ [17], which is experimentally available. Furthermore, the operation time required to implement the NTQ-NOT, NiSWAP-NOT and $N\sqrt{iSWAP}$-NOT gates with transmon qubits is independent of the number of target qubits N. It is clear from the above calculation that the total operation time

(t_{op_1}, t_{op_2} and t_{op_3}) of the quantum logic gates (NTQ-NOT, NiSWAP-NOT and $N\sqrt{i\text{SWAP}}$-NOT gates) which is much shorter than the decoherence times T_1 and dephasing time T_2, which satisfies our experimental requirement. Also, we note that the decoherence time of field state inside a resonator depends on the initial field state. However, the gate operation is independent of the initial state of the resonator because of the operator U_{gate} does not incluse the photon operator a and a^+ of the circuit QED.

4 Perspective

In Summary, We have used the system in which the transmon qubits are capacitively coupled to a TLR driven by a strong microwave field. As shown above, we have presented and demonstrated a method for realizing an NTQ-NOT, NiSWAP-NOT and $N\sqrt{i\text{SWAP}}$-NOT gates in circuit QED by introducing the qubit-qubit interaction. The operation time is independent of the number of qubits involved in the scheme, and the gates operations are insensitive to the initial state of the resonator. The system requires no disagreement between the qubits and the resonator. In addition, the operation time is only dependent of the detuning and the time can be controlled by adjusting the frequency between the $|0_j\rangle$ and $|1_j\rangle$. However, we have calculated an evolution operator in the case of the qubit-qubit interaction. Finally, we have applied the overall evolution operator to working basis of the qubit 1 and the qubits j ($j = 2, ..., N+1$) for find the NTQ-NOT, NiSWAP-NOT and $N\sqrt{i\text{SWAP}}$-NOT gates. In the our system, when we choose suitable detuning, the quantum logic gates can be realized in a time much shorter than decoherence time, and it will be more immune to the $1/f$ charge noise and have longer dephasing time as a result of the transmon qubits chosen in the system. In addition, numerical simulation of the gate operations shows that the scheme could be achieved with high fidelity under current state-of-the-art technology. Therefore, the present scheme might be realizable using the presently available techniques, and the experimental implementation of the present scheme would be a important step toward more complex quantum logic gates, serving to show the power of the transmon-qubit system for quantum information processing.

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