Applied Mathematics & Information Sciences
An International Journal

Towards More Reliable Fixed Phase Quantum Search Algorithm

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Received: 122 Jun 2012; Revised 21 Sep. 2012 ; Accepted 25 Sep. 2012
Published online: 1 Jan. 2013

Abstract: Building quantum devices using fixed operators is a must to simplify hardware construction of a quantum computer. Quantum search engine is not an exception. In this paper, a fixed phase quantum search algorithm that searches for M matches in an unstructured list of size N will be proposed. Fixing phase shifts to 1.91684π in the standard amplitude amplification will make the minimum probability of success is 99.58% in $O\left(\sqrt{N/M}\right)$ for $0 < M \leq N$ better than any know fixed operator quantum search algorithm. The algorithm will be able to handle either a single match or multiple matches in the search space. The algorithm will find a match in $O\left(\sqrt{N/M}\right)$ whether the number of matches is known or not in advance.

Keywords: Quantum search algorithm, amplitude amplification, fixed phase.

1. Introduction

In 1996, Lov Grover [10] presented an algorithm that quantum mechanically searches an unstructured list assuming that a unique match exists in the list with quadratic speed-up over classical algorithms. To be able to define the target problem of this paper, we have to organize the efforts done by others in that field. The unstructured search problem targeted by Grover's original algorithm is deviated in the literature to the following four major problems:

– Unstructured list with a unique match.
– Unstructured list with one or more matches, where the number of matches is known
– Unstructured list with one or more matches, where the number of matches is unknown.
– Unstructured list with strictly multiple matches.

The efforts done in all the above cases, using Grover's original work, used quantum parallelism by preparing superposition that represents all the items in the list. The superposition could be uniform or arbitrary. The techniques used in most of the cases to amplify the amplitude(s) of the required state(s) have been generalized later to an amplitude amplification technique that iterates the operation $UR_s(\phi)U^{\dagger}R_t(\varphi)$, on $U|s\rangle$ where U is unitary operator, $R_s(\phi) = I-(1-e^{i\phi})|s\rangle\langle s|$, $R_t(\varphi) = I-(1-e^{i\varphi})|t\rangle\langle t|$, $|s\rangle$ is the initial state of the system, $|t\rangle$ represents the target state(s) and I is the identity operator.

Grover's algorithm used W instead of U, where W is the Walsh-Hadamard transform, prepares the superposition $W|0\rangle$ (uniform superposition) and iterates the operation $WR_s(\pi)WR_t(\pi)$ $O\left(\sqrt{N}\right)$ times, where N is the size of the list. Grover's original algorithm is shown be optimal to get the highest probability with the minimum number of iterations [24], such that there is only one match in the search space.

In [11,15,13,19,1], Grover's algorithm is generalized by showing that U can be replaced by almost any arbitrary superposition and the phase shifts ϕ and φ can be generalized to deal with the arbitrary superposition and/or to increase the probability of success even with a factor increase in the number of iterations to still run in $O(\sqrt{N})$. These give a larger class of algorithms for amplitude amplification using variable operators from which Grover's algorithm was shown to be a special case.

In another direction, work has been done trying to generalize Grover's algorithm with a uniform superposition

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for known number of multiple matches in the search space [3,7–9], where it was shown that the required number of iterations is approximately $\pi/4\sqrt{N/M}$ for small M/N, where M is the number of matches. The required number of iterations will increase for $M > N/2$, i.e. the problem will be harder where it might be excepted to be easier [17]. Another work has been done for known number of multiple matches with arbitrary superposition and phase shifts [16,2,5,14,18] where the same problem for multiple matches occurs. In [4,16,5], a hybrid algorithm was presented to deal with this problem by applying Grover's fixed operators algorithm for $\pi/4\sqrt{N/M}$ times then apply one more step using specific ϕ and φ according to the knowledge of the number of matches M to get the solution with probability close to certainty. Using this algorithm will increase the hardware cost since we have to build one more R_s and R_t for each particular M. For the sake of practicality, the operators should be fixed [6] for any given M and are able to handle the problem with high probability whether or not M is known in advance. In [21,22], Younes et al presented an algorithm that exploits entanglement and partial diffusion operator to do the search and can perform in case of either a single match or multiple matches where the number of matches is known or not [22] covering the whole possible range, i.e. $1 \leq M \leq N$.

For unknown number of matches, an algorithm for estimating the number of matches (*quantum counting algorithm*) was presented [4,16]. In [3], another algorithm was presented to find a match even if the number of matches is unknown which will be able to work if M lies within the range $1 \leq M \leq 3N/4$ [22].

To simplify the hardware construction, it is necessary to have a fixed phase shifts search engine that returns the result with the highest possible probability of success. An attempt in this direction started by Grover [12] in an algorithm with phase shifts $\phi = \varphi = \pi/3$ with a behaviour similar to the classical algorithm in the worst case. In [23], an algorithm is proposed with phase shifts $\phi = \varphi = 1.825\pi$ to get a probability of success of 98% in $O(\sqrt{N/M})$. This result is enhanced in [20] to get a probability of success of 99.38% with a slightly increase in the required number of iterations using phase shifts of $\phi = \varphi = 0.1\pi$.

In this paper, an algorithm that runs in $O(\sqrt{N/M})$ will be proposed. This algorithm is able to handle the range $1 \leq M \leq N$ for both known and unknown number of matches more reliably than known fixed operator quantum search algorithms that target this case to get a probability of success of 99.58%.

The plan of the paper is as follows: Section 2 introduces the definition of the unstructured search problem. Section 3 presents the algorithm for both known and unknown number of matches. The paper will end up with a general conclusion in Section 4.

2. Unstructured Search Problem

Consider an unstructured list L of N items. For simplicity and without loss of generality we will assume that $N = 2^n$ for some positive integer n. Suppose the items in the list are labeled with the integers $\{0, 1, ..., N-1\}$, and consider a function (oracle) f which maps an item $i \in L$ to either 0 or 1 according to some properties this item should satisfy, i.e. $f : L \to \{0, 1\}$. The problem is to find any $i \in L$ such that $f(i) = 1$ assuming that such i exists in the list. In conventional computers, solving this problem needs $O\left(N/M\right)$ calls to the oracle (query),where M is the number of items that satisfy the oracle.

3. Fixed Phase Algorithm

3.1. Known Number of Matches

Assume that the system is initially in state $|s\rangle = |0\rangle$. Assume that $\sum_i{}'$ denotes a sum over i which are desired matches, and $\sum_i{}''$ denotes a sum over i which are undesired items in the list. So, Applying $U\,|s\rangle$ we get,

$$\left|\psi^{(0)}\right\rangle = U\,|s\rangle = \frac{1}{\sqrt{N}}\sum_{i=0}^{N-1}{}'\,|i\rangle + \frac{1}{\sqrt{N}}\sum_{i=0}^{N-1}{}''\,|i\rangle, \quad (1)$$

where $U = W$ and the superscript in $\left|\psi^{(0)}\right\rangle$ represents the iteration number.

Let M be the number of matches, $\sin(\theta) = \sqrt{M/N}$ and $0 < \theta \leq \pi/2$, then the system can be re-written as follows,

$$\left|\psi^{(0)}\right\rangle = \sin(\theta)\,|\psi_1\rangle + \cos(\theta)\,|\psi_0\rangle\,, \quad (2)$$

where $|\psi_1\rangle = |t\rangle$ represents the matches subspace and $|\psi_0\rangle$ represents the non-matches subspace.

Let $D = UR_s\left(\phi\right)U^{\dagger}R_t\left(\varphi\right)$, $R_s\left(\phi\right) = I - (1 - e^{i\phi})\,|s\rangle\,\langle s|$, $R_t\left(\varphi\right) = I - (1 - e^{i\varphi})\,|t\rangle\,\langle t|$ and set $\phi = \varphi$ as the best choice [14]. Applying D on $\left|\psi^{(0)}\right\rangle$ we get,

$$\left|\psi^{(1)}\right\rangle = D\left|\psi^{(0)}\right\rangle = a_1\,|\psi_1\rangle + b_1\,|\psi_0\rangle\,, \quad (3)$$

such that,

$$a_1 = \sin(\theta)(2\cos\left(\delta\right)e^{i\phi} + 1), \quad (4)$$

$$b_1 = e^{i\phi}\cos(\theta)(2\cos\left(\delta\right) + 1), \quad (5)$$

where $\cos\left(\delta\right) = 2\sin^2(\theta)\sin^2(\frac{\phi}{2}) - 1$.

Let q represents the required number of iterations to get a match with the highest possible probability. After q applications of D on $\left|\psi^{(0)}\right\rangle$ we get,

$$\left|\psi^{(q)}\right\rangle = D^q\left|\psi^{(0)}\right\rangle = a_q\,|\psi_1\rangle + b_q\,|\psi_0\rangle\,, \quad (6)$$

such that,

$$a_q = \sin(\theta)\left(e^{iq\phi}U_q(y) + e^{i(q-1)\phi}U_{q-1}(y)\right), \quad (7)$$

$$b_q = \cos(\theta)e^{i(q-1)\phi}\left(U_q(y) + U_{q-1}(y)\right), \quad (8)$$

where $y = cos(\delta)$ and U_q is the Chebyshev polynomial of the second kind defined as follows,

$$U_q(y) = \frac{\sin((q+1)\delta)}{\sin(\delta)}. \quad (9)$$

Let P_s^q represents the probability of success to get a match after q iterations and P_{ns}^q is the probability not to get a match after applying measurement, so $P_s^q = |a_q|^2$ and $P_{ns}^q = |b_q|^2$ such that $P_s^q + P_{ns}^q = 1$. To calculate the required number of iterations q to get a match with certainty, one of two approaches might be followed. The first approach is to equate P_s^q to 1 or P_{ns}^q to 0 and then find an algebraic formula that represents the required number of iterations, as well as, the phase shifts ϕ and φ in terms on M. Using this approach is not possible for the case that the phase shifts should be fixed for an arbitrary M such that $1 \le M \le N$. To prove this, let D be an amplitude amplification operator such that $D = UR_s(\phi)U^\dagger R_t(\varphi)$, where U is unitary operator, $R_s(\phi) = I - (1 - e^{i\phi})|s\rangle\langle s|$, $R_t(\varphi) = I - (1 - e^{i\varphi})|t\rangle\langle t|$, $|s\rangle$ is the initial state of the system, $|t\rangle$ represents the target state(s) and I is the identity operator. Let D performs on a system initially set to $U|s\rangle$. If the phase shifts ϕ and φ should be fixed, then iterating D an arbitrary number of times will not find a match with certainty for an arbitrary known number of matches M such that $1 \le M \le N$.

To show this, start with $P_s^q = 1$ or $P_{ns}^q = 0$ and calculate the required number of iterations q. Since $P_s^q = |a_q|^2$ and from "Equation(7), we can re-write P_s^q as follows setting $\phi = \varphi$ as the best choice [14],

$$P_s^q = \frac{\sin^2(\theta)}{\sin^2(\delta)}(1 - \cos(\delta)\cos((2q+1)\delta) + 2\cos(\phi)\sin((q+1)\delta)\sin(q\delta)). \quad (10)$$

Setting $P_s^q = 1$ and using simple trigonometric identities we get, $q = \frac{-1}{2}$, i.e. the required number of iterations is independent of M, ϕ and φ, and represents an impossible value for a required number of iterations.

The other approach, which is used in this paper, is to empirically assume an algebraic form for the required number of iterations that satisfy the quadratic speed-up of the known quantum search algorithms then finds the best phase shift ϕ that satisfy the condition,

$$\max\left(\min\left(P_s^q(\phi)\right)\right) \; such \; that \; 0 \le \phi \le 2\pi \; and, \; 1 \le M \le N, \quad (11)$$

i.e. find the value of ϕ that maximize the minimum value of P_s^q over the range $1 \le M \le N$.

Assume that $q = \left\lfloor \frac{\phi}{\sin(\theta)} \right\rfloor = O\left(\sqrt{\frac{N}{M}}\right)$.

Using this form for q and taking $\phi = 6.02193 \approx 1.91684\pi$, the minimum probability of success will be at least 99.58% higher than that of known fixed phase quantum search algorithms as shown in Table 1. To prove these results, using $\phi = 1.91684\pi$, the lower bound for the probability of success is as follows as shown in (1).

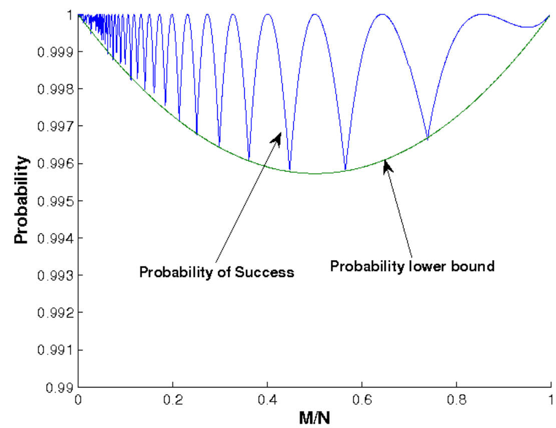

Figure 1 The probability of success of the proposed algorithm after the required number of iterations.

$$\begin{aligned} P_s^q &= \frac{\sin^2(\theta)}{\sin^2(\delta)}(1 - \cos(\delta)\cos((2q+1)\delta) \\ &\quad + 2\cos(\phi)\sin((q+1)\delta)\sin(q\delta)) \\ &= \frac{\sin^2(\theta)}{\sin^2(\delta)}(1 - \cos(\delta)\cos((2q+1)\delta) \\ &\quad + \cos(\phi)\cos(\delta) - \cos(\phi)\cos((2q+1)\delta)) \\ &\ge \frac{\sin^2(\theta)}{\sin^2(\delta)}(1 + \cos^2(\delta) + 2\cos(\phi)\cos(\delta)) \ge 0.9958. \end{aligned} \quad (12)$$

where, $\cos(\delta) = 2\sin^2(\theta)\sin^2(\frac{\phi}{2}) - 1$, $0 < \theta \le \pi/2$, and $\cos((2q+1)\delta) \le -cos(\delta)$.

(2) shows a comparison between the proposed algorithm and the top two fixed phase algorithms [20] and [23], where it shows the probability of success after the required number of iterations for $0 < M \le N$ (left) and a zoom to the same results for small M/N (right). Table 1 compares the required number of iterations and the minimum probability of success between the proposed algorithm and the known fixed-phase quantum search algorithms.

3.2. Unknown Number of Matches

In case we do not know the number of matches M in advance, we can apply the algorithm shown in [3] for $1 \le M \le N$ by replacing Grover's step with the proposed algorithm. The algorithm can be summarized as follows,

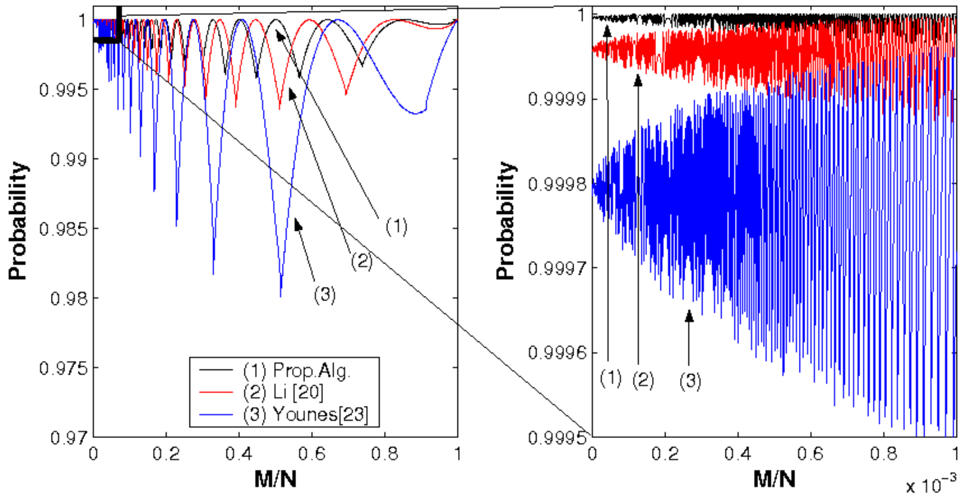

Figure 2 Comparing the probability of success of the proposed algorithm, Li [20] and Younes [23] algorithms after the required number of iterations for $0 < M \leq N$ (left) and for small M/N (right).

Algorithm	Req. no of iterations	Minimum Probability of Success			
		$0 < M \leq N$	$\frac{M}{N} \leq 10^{-2}$	$\frac{M}{N} \leq 10^{-3}$	$\frac{M}{N} \leq 10^{-4}$
Prop. Alg	$\left\lfloor 1.916\pi\sqrt{\frac{N}{M}} \right\rfloor$	99.58%	99.98%	100.0%	100.0%
Li[20]	$\left\lfloor 1.591\pi\sqrt{\frac{N}{M}} \right\rfloor$	99.38%	99.95%	99.99%	99.99%
Younes[23]	$\left\lfloor 0.9125\pi\sqrt{\frac{N}{M}} \right\rfloor$	98%	99.83%	99.95%	99.97%
Younes et al[22]	$\left\lfloor \frac{\pi}{2\sqrt{2}}\sqrt{\frac{N}{M}} \right\rfloor$	87.88%	99.57%	99.95%	100.0%
Grover[10]	$\left\lfloor \frac{\pi}{4}\sqrt{\frac{N}{M}} \right\rfloor$	50%	99.04%	99.90%	99.90

Table 1 Comparing the required number of iterations and the minimum probability of success for known fixed-phase quantum search algorithms.

1-Initialize $m = 1$ and $\lambda = 8/7$. (where λ can take any value between 1 and $4/3$)
2-Pick an integer j between 0 and $m - 1$ in a uniform random manner.
3-Run j iterations of the proposed algorithm on the state $\left|\psi^{(0)}\right\rangle$:

$$\left|\psi^{(j)}\right\rangle = D^j \left|\psi^{(0)}\right\rangle. \tag{13}$$

4-Measure the register $\left|\psi^{(j)}\right\rangle$ and assume i is the output.
5-If $f(i) = 1$, then we found a solution and exit.
6-Set $m = min\left(\lambda m, \sqrt{N}\right)$ and go to step 2.

where m represents the range of random numbers (step 2), j represents the random number of iterations (step3), and λ is a factor used to increase the range of random numbers after each trial (step 6).

Following the same style of analysis used in [3], the following lemmas are required before starting the analysis.

Lemma 1. *For any positive integer m and real numbers θ, δ such that*

$\cos(\delta) = c\sin^2(\theta) - 1$, $0 < \theta \leq \pi/2$ *where* $c = 2\sin^2(\frac{\phi}{2})$ *is a constant,*

$$\sum_{q=0}^{m-1} \sin^2((q+1)\delta) + \sin^2(q\delta) = m - \frac{\cos(\delta)\sin(2m\delta)}{2\sin(\delta)}.$$

*Proof.*By mathematical induction.

Lemma 2.*For any positive integer m and real numbers θ, δ such that* $\cos(\delta) = c\sin^2(\theta) - 1$, $0 < \theta \leq \pi/2$ *where* $c = 2\sin^2(\frac{\phi}{2})$ *is a constant,*

$$\sum_{q=0}^{m-1} \sin((q+1)\delta)\sin(q\delta) = \frac{m}{2}\cos(\delta) - \frac{\sin(2m\delta)}{4\sin(\delta)}.$$

*Proof.*By mathematical induction.

Lemma 3.*Assume M is the unknown number of matches such that $1 \leq M \leq N$. Let θ, δ be real numbers such that $\cos(\delta) = 2\sin^2(\theta)\sin^2(\frac{\phi}{2}) - 1$, $\sin^2(\theta) = M/N$, $\phi = 1.91684\pi$ and $0 < \theta \leq \pi/2$. Let m be any positive integer. Let q be any integer picked in a uniform random manner between 0 and $m-1$. Measuring the register after applying q iterations of the proposed algorithm starting from the initial state, the probability P_m of finding a solution is as follows,*

$$P_m = \frac{1}{c(1-\cos(\delta))}(1 + \cos(\delta)\cos(\phi) - \frac{(\cos(\delta)+\cos(\phi))\sin(2m\delta)}{2m\sin(\delta)}),$$

where $c = 2\sin^2(\frac{\phi}{2})$, then $P_m \geq 1/4$ for $m \geq 1/\sin(\delta)$ and small M/N.

*Proof.*The average probability of success when applying q iterations of the proposed algorithm when $0 \leq q \leq m$ is picked in a uniform random manner is as follows,

$$\begin{aligned} P_m &= \frac{1}{m}\sum_{q=0}^{m-1} P_s^q \\ &= \frac{\sin^2(\theta)}{m\sin^2(\delta)}\sum_{q=0}^{m-1}(\sin^2((q+1)\delta) + \sin^2(q\delta) \\ &\qquad + 2\cos(\phi)\sin((q+1)\delta)\sin(q\delta)) \\ &= \frac{\sin^2(\theta)}{m\sin^2(\delta)}(m - \frac{\cos(\delta)\sin(2m\delta)}{2\sin(\delta)} + \cos(\phi)\cos(\delta) \\ &\qquad - \frac{\cos(\phi)\sin(2m\delta)}{2\sin(\delta)}) \\ &= \frac{1}{c(1-\cos(\delta))}(1 + \cos(\delta)\cos(\phi) - \frac{(\cos(\delta)+\cos(\phi))\sin(2m\delta)}{2m\sin(\delta)}), \end{aligned}$$

If $m \geq 1/\sin(\delta)$ and $M \ll N$ then $\cos(\delta) \approx -1$, so,

$$\begin{aligned} P_m &\geq \frac{1}{2c}\left(1 - \cos(\phi) - \frac{(\cos(\phi)-1)\sin(2m\delta)}{2}\right) \\ &\geq \frac{1}{2c}\left(1 - \cos(\phi) - \frac{(1-\cos(\phi))}{2}\right) = \frac{1}{4} \end{aligned}$$

where $-1 \leq \sin(2m\delta) \leq 1$ for $0 < \theta \leq \pi/2$.

We calculate the total expected number of iterations as done in Theorem 3 in [3]. Assume that $m_q \geq 1/\sin(\delta)$, and $v_q = \lceil \log_\lambda m_q \rceil$. Notice that, $m_q = O\left(\sqrt{N/M}\right)$ for $1 \leq M \leq N$, then:

1-The total expected number of iterations to reach the critical stage, i.e. when $m \geq m_q$:

$$\frac{1}{2}\sum_{v=1}^{v_q} \lambda^{v-1} \leq \frac{1}{2(\lambda-1)} m_q = 3.5 m_q. \quad (14)$$

2-The total expected number of iterations after reaching the critical stage:

$$\frac{1}{2}\sum_{u=0}^{\infty}\left(\frac{3}{4}\right)^u \lambda^{v_q+u} = \frac{1}{2(1-0.75\lambda)} m_q = 3.5 m_q. \quad (15)$$

The total expected number of iterations whether we reach to the critical stage or not is $7m_q$, i.e. $O(\sqrt{N/M})$ for $1 \leq M \leq N$.

4. Conclusion

To be able to build a practical, reliable and stable quantum search engine, the hardware should be constructed from fixed operators that can handle the whole possible range of the search problem, i.e. whether a single match or multiple matches exist in the search space. It should also be able to handle the case where the number of matches is unknown. The engine should perform with the highest possible probability after performing the required number of iterations.

In this paper, a fixed phase quantum search algorithm is presented. It was shown that selecting the phase shifts to 1.91684π will enhance the searching process so as to get a solution with probability at least 99.58%. The algorithm still achieves the quadratic speed up of Grover's original algorithm.

In that sense, the proposed algorithm can perform efficiently in all the possible classes of the unstructured search problem.

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