The Non-equidistant New Information Optimizing MGM(1,n) Based on a Step by Step Optimum Constructing Background Value

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Abstract: Grey system is a theory which studies poor information specially, and it possesses wide suitability. Applying a step by step optimum new information modeling method to construct new information background value of multi-variable non-equidistance new information Grey model MGM(1,n), taking the mth component of each variable as initial value of response function, taking the mean relative error as objective function, and taking the modified values of response function initial value as design variables, the multi-variable non-equidistance new information Grey model MGM(1,n) was built. The proposed model can be used to build model in non-equal interval and equal interval time series. It enlarges the scope of application and has high precision and easy to use. Example validates the practicability and reliability of the proposed model.

Keywords: Multivariable, background value, non-equidistance sequence, a step by step optimum modeling, new information, optimizing, non-equidistance MGM(1,n), least square method.

1. Introduction

The theory of Grey system is the study of the grey system analysis, modeling, prediction, decision making and control theory. Gray model is an important content of the grey system theory, In the search for laws between data, it makes up for the lack of available data mining method, and provides a new scientific method for data mining.

Since professor Deng Ju Long bring about the grey system theory in 1982, gray model is widely applied in many areas [1,2]. Grey model has more types, mainly including GM (1,1), GM (1,N), MGM (1,N) etc. where GM (1,1) has been used widely and researched deeply , GM(1,N) Can only be used for the qualitative analysis and cannot be used to predict. Being an extension of GM (1,1) model in case of n variable, MGM(1,N) model is neither a simple combination of GM (1,1) model ,also differs from GM(1,n) model.There are n differential equations contained n elements In MGM(1,N) model ,but just a single first order differential equation contained n element in GM(1,N) model, then we can find their simultaneous solution , and parameters in model MGM (1,N) can reflect the interrelating and interacting relationship among multiple variables. Because Study on MGM (1,N) model is much less than that on GM (1,1) model so far, studying deeply on it has important theory significance and application value. Literature [1] have corrected and established optimization MGM(1,N) model that regard the first component of the sequence as the initial conditions of grey differential equation. According to the new information priority principle of grey system theory, Literature [4] have corrected and established multivariate variables new information optimization MGM (1,N) model that regard the nth component of the sequence X^{(1)} as the initial conditions of grey differential equation. Literature [4] have established multivariate new information MGM (1, N) model that regard the nth component of X^{(1)} as initial conditions of the grey differential equation, and made optimal correction for the initial value and background value coefficient(background value is introduced in the form of z_{i}^{(1)} = qx_{i}^{(1)} (k+1) + (1-q)x_{i}^{(1)} (k), (q \in [0,1]), but these MGM (1,N) model are equidistant model. Literature [6] built non equidistant multivariable MGM (1,N) by means of homogeneous ex-
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2. Non-equidistant multivariate new information optimization model based on a step by step optimization modeling to construct new information background values

Definition 1: Suppose the sequence

\[ X_i^{(0)} = [x_i^{(0)}(t_1), x_i^{(0)}(t_2), \ldots, x_i^{(0)}(t_m)], \]

where \( i = 1, 2, \ldots, n \). If

\[ \Delta t_j = t_j - t_{j-1} \neq \text{cost}, \quad (2 \leq j \leq m), \]

then \( X_i^{(0)} \) is called non-equidistant sequence.

Definition 2: Let \( X_i^{(1)} \) denoted by

\[ X_i^{(1)} = [x_i^{(1)}(t_1), x_i^{(1)}(t_2), \ldots, x_i^{(1)}(t_m)], \]

the sequence \( X_i^{(1)} \) is called the first-order accumulated generating operation of non equidistant sequence \( X_i^{(0)} \), if

\[ x_i^{(1)}(t_j) = x_i^{(0)}(t_1), x_i^{(1)}(t_j) = x_i^{(1)}(t_{j-1}) + x_i^{(0)}(t_j)\Delta t_j, \]

where \( i = 1, 2, \ldots, n, \quad j = 2, 3, \ldots, m, \quad \Delta t_j \) is same to Definition 1.

Set the original data matrix of multivariable

\[ X^{(0)} = [X_1^{(0)}, X_2^{(0)}, \ldots, X_n^{(0)}]^T \]

\[ = \begin{bmatrix} x_1^{(0)}(t_1) & x_1^{(0)}(t_2) & \cdots & x_1^{(0)}(t_m) \\ x_2^{(0)}(t_1) & x_2^{(0)}(t_2) & \cdots & x_2^{(0)}(t_m) \\ \vdots & \vdots & \ddots & \vdots \\ x_n^{(0)}(t_1) & x_n^{(0)}(t_2) & \cdots & x_n^{(0)}(t_m) \end{bmatrix}, \]

where \( x_i^{(0)}(t_j)(i = 1, 2, \ldots, n, j = 1, 2, \ldots, m) \) is the observed values of variable \( X_i^{(0)} \) in the moments of \( t_j \), \([x_i^{(0)}(t_1), x_i^{(0)}(t_2), \ldots, x_i^{(0)}(t_m)]\) is non equidistant sequence, namely, the distance of \( t_j - t_{j-1}(j = 2, 3, \ldots, m) \) is not constant.

In order to establish model, first taking the accumulated raw data, a new matrix can be obtained, namely

\[ X^{(1)} = [X_1^{(1)}, X_2^{(2)}, \ldots, X_n^{(n)}]^T \]

\[ = \begin{bmatrix} x_1^{(1)}(t_1) & x_1^{(1)}(t_2) & \cdots & x_1^{(1)}(t_m) \\ x_2^{(1)}(t_1) & x_2^{(1)}(t_2) & \cdots & x_2^{(1)}(t_m) \\ \vdots & \vdots & \ddots & \vdots \\ x_n^{(1)}(t_1) & x_n^{(1)}(t_2) & \cdots & x_n^{(1)}(t_m) \end{bmatrix}, \]

where \( x_i^{(1)}(t_j)(i = 1, 2, \ldots, n) \) meet the definition 2, namely,

\[ x_i^{(1)}(t_j) = \begin{cases} x_i^{(0)}(t_1) + \sum_{k=2}^{j} x_i^{(0)}(t_k)\Delta t_k, & (j = 2, \ldots, m), \\ x_i^{(0)}(t_1), & (j = 1). \end{cases} \]

Multivariable non equidistant MGM (1, n) model is first order differential equations containing \( n \) element

\[ \begin{align*}
\frac{dx_1^{(1)}}{dt} &= p_{11}x_1^{(1)} + p_{12}x_2^{(1)} + \cdots + p_{1n}x_n^{(1)} + q_1, \\
\frac{dx_2^{(1)}}{dt} &= p_{21}x_1^{(1)} + p_{22}x_2^{(1)} + \cdots + p_{2n}x_n^{(1)} + q_2, \\
&\vdots &
\frac{dx_n^{(1)}}{dt} &= p_{n1}x_1^{(1)} + p_{n2}x_2^{(1)} + \cdots + p_{nn}x_n^{(1)} + q_n.
\end{align*} \]

Note

\[ A = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1n} \\ p_{21} & p_{22} & \cdots & p_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ p_{n1} & p_{n2} & \cdots & p_{nn} \end{bmatrix}, \quad B = \begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q_n \end{bmatrix}. \]
then (4) may be written as
\[
\frac{dX^{(1)}(t)}{dt} = AX^{(1)}(t) + B.
\]
\[\text{(5)}\]

According to new information priority principle of grey system theory, we regard the first component of the sequence \(x_i^{(1)}(t_j)(j = 1, 2, \ldots, m)\) as initial conditions of the grey differential equation, causing that new information cannot be applied fully. If we regard mth component of the sequence \(x_i^{(1)}(t_j)(j = 1, 2, \ldots, m)\) as initial conditions of the grey differential equation, causing that new information can be applied fully, in which continuous time response of (5) is
\[
X^{(1)}(t) = e^{At}X^{(1)}(t_m) + A^{-1}(e^{At} - I)B,
\]
where \(e^{At} = I + \sum_{k=1}^{\infty} \frac{A^k}{k!}t^k, I\) is unit matrix.

In order to obtain A and B, we integral the both side on the interval \([t_{j-1}, t_j]\),
\[
x_i^{(0)}(t_j)\Delta t_j = \sum_{k=1}^{n} p_k \int_{t_{j-1}}^{t_j} x_i^{(1)}(t) dt + q_i \Delta t_j,
\]
\[\text{(7)}\]
\[
x_i^{(0)}(t_j) = \sum_{k=1}^{n} p_k \int_{t_{j-1}}^{t_j} x_i^{(1)}(t) dt + q_i.
\]
\[\text{(8)}\]

Let
\[
x_i^{(1)}(t_j) = \frac{\int_{t_{j-1}}^{t_j} x_i^{(1)}(t) dt}{\Delta t_j},
\]
\[\text{The traditional background value calculation formula actually use trapezoidal area}{z_i^{(1)}(t_j)\Delta t_j}, \text{thus, there is big error. But we obtain Parameter matrix } A\text{ and } B\text{ by means of regarding } z_i^{(1)}(t_j) = \int_{t_{j-1}}^{t_j} x_i^{(1)}(t) dt \text{ as background value calculation} x_i^{(1)}\text{ in the interval } [t_{j-1}, t_j]\text{ that is more suitable for whitening equation (4). According to exponential rule of gray forecast model and modeling theory and method [10] of a step by step optimization new information non-equidistant GM (1, 1) model, we take } x_i^{(1)}(t) = a_ie^{bt} + c_i, \text{ where } a_i, b_i, c_i\text{ are the undetermined coefficient. The literature [10] analyzed thought and method of a step by step optimization new information non-equidistant GM (1, 1) model and the key of modeling is whitening grey derivative } \frac{dx_i^{(1)}(t)}{dt}. \text{If we choose reasonable whitening grey derivative, accuracy of modeling will be improved. The method to obtain whitening grey derivative with the most intuitionist and easy to understand is a differential quotient instead of derivative.}
\[
\frac{dx_i^{(1)}(t_j+1)}{dt} \approx \frac{x_i^{(1)}(t_j+1) - x_i^{(1)}(t_j)}{t_j+1 - t_j},
\]
\[\text{(9)}\]
\[
\frac{dx_i^{(1)}(t_j)}{dt} \approx \frac{x_i^{(1)}(t_j+1) - x_i^{(1)}(t_j)}{t_j+1 - t_j}.
\]
\[\text{(10)}\]

In fact, on the premise of \(X(t)\) being a derivative, the Lagrange mean value theorem shows, \(\frac{x_i^{(1)}(t_{j+1}) - x_i^{(1)}(t_{j})}{t_{j+1} - t_j}\) is a derivative value in a point of the interval \([t_k, t_{k+1}]\), that is , derivative value can be thought as being known value, the corresponding variable values are interval gray numbers \((t_k, t_{k+1})\). For the introduction of grey derivative correction coefficient correction of grey derivative, we adopt grey derivative correction coefficient \(\rho\) and \(\xi\) to correct grey derivative and construct modeling \(\rho_i x_i^{(1)}(t_{j+1}) - x_i^{(1)}(t_j)\).

Actually, we don’t know \(\xi_i(t_j)\) and \(\rho_i(t_j)\). We take step by step optimization method, and, the steps are as follows:
1) Obtain original data \(x_i^{(0)}\).
2) Take initial value of the iteration step number repeatedly \(s = 0, a_i(s) = a_i(0) = 0, \xi\),
\[
\xi_i(s) = \frac{e^{a_i(s)}}{e^{a_i(s)} - 1} = \frac{1}{a_i(s)} = \frac{1}{2},
\]
\[
\rho_i(s) = \frac{a_i(s)(1 + e^{-a_i(s)})}{2(1 - e^{-a_i(s)})} = 1.
\]

Taking linear regression to whitening values \(x_i^{(1)}(t_j) = ([1 - \xi_i(t_j) x_i^{(1)}(t_j) + \xi_i(t_j)] x_i^{(1)}(t_j+1)\).
We obtain the whitening values of parameter:
\[
\hat{a}_i(s+1) = -\rho_i(s) \frac{S_i(s)_{xy}}{S_i(s)_{xx}},
\]
\[\text{(11)}\]
where
\[
x_i^{(1)}(t_j) = [1 - \xi_i(s)x_i^{(1)}(t_j)] + \xi_i(s)x_i^{(1)}(t_j+1),
\]
\[
x_i^{(1)}(t_j) = \frac{1}{m-1} \sum_{j=1}^{m-1} x_i^{(1)}(t_j),
\]
\[
y_i(s)(t_j) = \frac{x_i^{(1)}(t_j+1) - x_i^{(1)}(t_j)}{t_{j+1} - t_j},
\]
\[
\bar{y}_i(s) = \frac{1}{m-1} \sum_{j=1}^{m-1} y_i(s)(t_j),
\]
\[
S_i(s)_{xx} = \sum_{j=1}^{m-1} [x_i^{(1)}(t_j) - \bar{x}_i^{(1)}]^2,
\]
\[
S_i(s)_{xy} = \sum_{j=1}^{m-1} [x_i^{(1)}(t_j) - \bar{x}_i^{(1)}] [y_i(t_j) - \bar{y}_i(s)].
\]

Then, we construct the linear regression to get index model \(M_{s+1}^1\):
\[
x_i^{(1)}(t_j+1) = \tilde{c}_i(s+1)e^{-\tilde{a}_i(s+1)(t_j-t_m)} + \tilde{b}_i(s+1),
\]
\[\text{(12)}\]
where
\[
\hat{c}_i(s+1) = \frac{1}{m} \sum_{j=1}^{m} \left[ e^{-\hat{a}_i(s+1) \Delta t} - \hat{a}_i(s+1) \Delta t \right]^2
\]
\times \left\{ \frac{1}{m} \sum_{j=1}^{m} \left[ e^{-\hat{a}_i(s+1) \Delta t} - \hat{a}_i(s+1) \Delta t \right] \right\}
\times [x^{(1)}_{i(s)}(t_j) - \frac{1}{m} \sum_{j=1}^{m} x^{(1)}_{i(s)}(t_j)] \right\},
\]
\[
\hat{b}_i(s+1) = \frac{1}{m} \sum_{j=1}^{m} x^{(1)}_{i(s)}(t_j) - \hat{c}_i(s+1) \frac{1}{m} \sum_{j=1}^{m} e^{-\hat{a}_i(s+1) \Delta t}.
\]

After finding out \( \hat{a}_i, \hat{b}_i, \hat{c}_i, z^{(1)}_i(t_j) \) can be solved, and then get
\[
z^{(1)}_i(t_j) = \frac{f_{t_j}^{t_{j+1}} x^{(1)}_i(t_j) dt}{\Delta t} = x^{(1)}_i(t_j).
\]

Set \( p_i = (p_{i1}, p_{i2}, \ldots, p_{im}, q_i)^T, \quad (i = 1, 2, \ldots, n) \), \( \hat{p}_i \) can be achieved the value \( \hat{p}_i \) by the least square method.
\[
\hat{p}_i = (\hat{p}_{i1}, \hat{p}_{i2}, \ldots, \hat{p}_{im}, \hat{q}_i)^T = (L^T L)^{-1} L^T Y_i, \quad (i = 1, 2, \ldots, n),
\]
where,
\[
L = \begin{bmatrix}
\hat{z}^{(1)}(t_2) & \hat{z}^{(1)}(t_3) & \cdots & \hat{z}^{(1)}(t_m) \\
\hat{z}^{(1)}(t_3) & \hat{z}^{(1)}(t_4) & \cdots & \hat{z}^{(1)}(t_m) \\
\vdots & \vdots & \ddots & \vdots \\
\hat{z}^{(1)}(t_m) & \hat{z}^{(1)}(t_1) & \cdots & \hat{z}^{(1)}(t_{m-1})
\end{bmatrix},
\]
\[
Y_i = [x^{(0)}_i(t_2), x^{(0)}_i(t_3), \ldots, x^{(0)}_i(t_m)]^T.
\]

We can get discrimination value of A and B:
\[
\hat{A} = \begin{bmatrix}
\hat{p}_{i1} & \hat{p}_{i2} & \cdots & \hat{p}_{im} \\
\hat{p}_{i2} & \hat{p}_{i3} & \cdots & \hat{p}_{im} \\
\vdots & \vdots & \ddots & \vdots \\
\hat{p}_{im} & \hat{p}_{i1} & \cdots & \hat{p}_{im}
\end{bmatrix}, \quad \hat{B} = \begin{bmatrix}
\hat{q}_1 \\
\hat{q}_2 \\
\vdots \\
\hat{q}_n
\end{bmatrix}.
\]

Computation value of new information MGM (1, n) mode is
\[
\hat{X}^{(1)}_i(t_j) = e^{\hat{A}(t_{j} - t_m)} \hat{X}^{(1)}_i(t_m) + \hat{A}^{-1} (e^{\hat{A}(t_{j} - t_m)} - I) \hat{B}.
\]

We use the nth component of the data as initial value of the grey differential equation and modify initial value, namely, \( X^{(0)}(t_m) + \beta \) instead of \( X^{(0)}(t_m) \), which \( \beta \) is vector \( \beta = [\beta_1, \beta_2, \ldots, \beta_n]^T \). The fitting values of the original data are obtained.
\[
\hat{X}^{(0)}_i(t_j) = \left\{ \begin{array}{ll}
\lim_{\Delta t \to 0} \frac{X^{(1)}_i(t_j) - X^{(1)}_i(t_j - \Delta t)}{\Delta t}, & (j = 1), \\
\frac{X^{(1)}_i(t_j) - X^{(1)}_i(t_{j-1})}{t_j - t_{j-1}}, & (j = 2, 3, \ldots, m).
\end{array} \right.
\]

The absolute error of ith variable is \( e_i(t_j) = x^{(0)}_i(t_j) - x^{(0)}_i(t_j) \). Relative error of ith variables is
\[
e_i(t_j) = \frac{\hat{x}^{(0)}_i(t_j) - x^{(0)}_i(t_j)}{x^{(0)}_i(t_j)} \times 100.
\]

The mean value of relative error of ith variable is
\[
\bar{e} = \frac{1}{m} \sum_{j=1}^{m} | e_i(t_j) |.
\]

Taking the average error \( f \) as objective function and as design variables, using the optimization function of Matlab 7.5 optimization method or other method for solving, all parameters are obtained.

3. Precision inspecting for the model

The inspecting means contain residual analysis, correlation degree analysis, and post-error analysis [7, 12, 13]. The displacement relative degree,

the speed related degree, the acceleration degree, and the total related degree are calculated simultaneously. These kinds of related degrees are called related degrees of C-type [14], which can be used to both of the whole analysis and the dynamic analysis. The following related degree inspection of C-type is employed in this paper.

1) To calculate the three-layer related degrees.

Displacement related degree \( d^{(0)}(t_j) \):
\[
d^{(0)}(t_j) = \frac{x^{(0)}(t_j)}{\hat{x}^{(0)}(t_j)}, \quad (j = 1, 2, \ldots, m).
\]

Speed related degree \( d^{(1)}(t_j) \):
\[
d^{(1)}(t_j) = \frac{x^{(0)}(t_{j+1}) - x^{(0)}(t_j)}{\hat{x}^{(0)}(t_{j+1}) - \hat{x}^{(0)}(t_j)}, \quad (j = 1, 2, \ldots, m - 1).
\]
Acceleration related degree $d^{(2)}(t_j)$:

$$d^{(2)}(t_j) = \frac{\hat{x}^{(0)}(t_{j+1}) - 2x^{(0)}(t_j) + \hat{x}^{(0)}(t_{j-1})}{\hat{x}^{(0)}(t_{j+1}) - 2\hat{x}^{(0)}(t_j) + \hat{x}^{(0)}(t_{j-1})},$$  \hspace{1cm} (22)

where $j = 1, 2, \cdots, m - 1$.

2) To calculate the three-layer related comprehensive degree at $t_j$:  

$$D(t_1) = \frac{d^{(1)}(t_1) + d^{(0)}(t_1)}{2}, \ D(t_m) = d^{(0)}(t_m),$$

$$D(t_j) = \frac{d^{(1)}(t_j) + d^{(1)}(t_j) + d^{(2)}(t_j)}{4},$$  \hspace{1cm} (24)

where $j = 2, 3, \cdots, m - 1$.

3) To calculate the total related degree of the model $\hat{x}^{(0)}(t_j)$:

$$D = \frac{1}{m} \sum_{j=1}^{m} D(t_j), j = 1, 2, \cdots, m.$$

When $0.6 < D < \frac{5}{6}$, the precision of the model is "good". When $0.30 \leq D \leq 0.60$, the precision of the model is "better". When $D < 0.30$ or $D > \frac{5}{6}$, the precision of the model is "bad" [14].

4. Model applications

Example 1: On the contacting strength calculation, principal curvature function $F(\rho)$ and the coefficient $m_a, m_b$ , with the point contacting ellipse length and short radius $a, b$. Parameters are generally processed by consulting table . The data extracted from the table 1 [11].

Table 1 The values of $F(\rho), m_a$, and $m_b$

<table>
<thead>
<tr>
<th>No.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F(\rho)$</td>
<td>0.9995</td>
<td>0.9990</td>
<td>0.9980</td>
<td>0.9970</td>
<td>0.9960</td>
</tr>
<tr>
<td>$m_a$</td>
<td>23.95</td>
<td>18.53</td>
<td>14.25</td>
<td>12.26</td>
<td>11.02</td>
</tr>
<tr>
<td>$m_b$</td>
<td>0.163</td>
<td>0.185</td>
<td>0.212</td>
<td>0.228</td>
<td>0.241</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>No.</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F(\rho)$</td>
<td>0.9950</td>
<td>0.9940</td>
<td>0.9930</td>
<td>0.9920</td>
<td>0.9910</td>
</tr>
<tr>
<td>$m_a$</td>
<td>10.15</td>
<td>9.46</td>
<td>8.92</td>
<td>8.47</td>
<td>8.10</td>
</tr>
<tr>
<td>$m_b$</td>
<td>0.251</td>
<td>0.260</td>
<td>0.268</td>
<td>0.275</td>
<td>0.281</td>
</tr>
</tbody>
</table>

The coefficient $m_b$ of the oval short radius $b$ is noted as $t_j$, principal curvature function $F(\rho)$ is noted as $X_1$, the coefficient $m_a$ of the oval short radius $a$ is noted as $X_2$. Establishing non-equidistance new information optimum GM(1,2) model with the proposed method, the model parameters are as follows:

$$A = \begin{bmatrix} -0.3787 & 0.0243 \\ 16.4555 & -8.1528 \end{bmatrix}, \quad B = \begin{bmatrix} 0.7945 \\ 196.8118 \end{bmatrix},$$

$$\beta = \begin{bmatrix} -0.0025205 \\ 0.070706 \end{bmatrix}.$$

Principal curvature function fitting value is

$$\hat{F}(\rho) = \begin{bmatrix} 1.00020 \\ 1.00040 \\ 0.99990 \\ 0.99837 \\ 0.99682 \\ 0.99534 \\ 0.99396 \\ 0.99262 \\ 0.99136 \\ 0.99022 \\ 0.98912 \\ 0.98808 \\ 0.98710 \\ 0.98621 \\ 0.98539 \\ 0.98446 \\ 0.98351 \\ 0.98276 \\ 0.98200 \\ 0.98112 \\ 0.98034 \\ 0.97955 \\ 0.97875 \\ 0.97806 \end{bmatrix}.$$

The absolute error of principal curvature function is

$$\frac{1}{N} \sum_{i=1}^{N} \left( \hat{F}(\rho)_i - F(\rho)_i \right)^2 = 0.06772, \quad 1.4183, \quad 1.8995, \quad 1.3741, \quad 0.8205, \quad 0.3375, \quad -0.0406, \quad -0.3793, \quad -0.6374, \quad -0.7824, \quad -0.8821, \quad -0.9236, \quad -0.8975, \quad -0.7944, \quad -0.6079, \quad -0.5405, \quad -0.4899, \quad -0.2399, \quad -0.0003, \quad 0.1195, \quad 0.3396, \quad 0.5505, \quad 0.7532, \quad 1.0631].$$

The relative error (percent) of principal curvature function is:

$$E = \begin{bmatrix} 0.067753, \quad 0.141972, \quad 0.190329, \quad 0.137826, \quad 0.082379, \quad 0.033929, \quad -0.00480, \quad -0.038197, \quad -0.064259, \quad -0.078950, \quad -0.089106, \quad -0.093386, \quad -0.090839, \quad -0.080488, \quad -0.061560, \quad -0.054871, \quad -0.049785, \quad -0.024408, \quad -0.000026, \quad 0.012178, \quad 0.034655, \quad 0.056231, \quad 0.070718, \quad 0.108810 \end{bmatrix}.$$

The average value of relative error is 0.069713%. The precision of the model is "Good". The average value of relative error of without optimization for new information model is 0.14657%. So, optimization model has very high precision.

Example 2: Refer to reference [7] of data for water absorption rate affecting the pure PA66 mechanics performance, according to mechanics performance test on PA66 samples with different water absorption rate, get the bending strength and flexural modulus of PA66 and the experimental data of the tensile strength changing with the water absorption rate. $t_j$ is water absorption rate. $X_1^{(0)}$ is the bending strength (Mpa), $X_2^{(0)}$ is the flexural modulus (Gpa), $X_3^{(0)}$ is the tensile strength (Mpa). The data is shown in table 2.

Table 2 Absorption rate’s influence for mechanics performance of pure PA66
According to the method mentioned in this paper, set up the non-equidistant model MGM (1,2). The model parameters are shown as follows:

\[
A = 10^4 \times \begin{bmatrix}
0.1155 & -4.2174 & 0.0172 \\
0.0046 & -0.1662 & 0.0007 \\
-0.0291 & 1.0613 & -0.0043
\end{bmatrix},
\]

\[
B = \begin{bmatrix}
185.1780 \\
6.1876 \\
97.9652
\end{bmatrix},
\]

\[
\rho = \begin{bmatrix}
-1.16073 \\
-0.03880 \\
1.4395
\end{bmatrix}.
\]

The fitted value of \(X_3^{(0)}\) is

\[
\hat{X}_3^{(0)} = \begin{bmatrix}
87.3769 \\
82.7884 \\
74.7615
\end{bmatrix},
\]

\[
\begin{bmatrix}
85.6582 \\
81.5031 \\
70.7947
\end{bmatrix},
\]

\[
\begin{bmatrix}
84.1890 \\
78.2813 \\
67.0939
\end{bmatrix}.
\]

The absolute error of \(X_3^{(0)}\) is

\[
q = [3.1769, \ 1.2852, \ -2.1110, \ -1.5116, \ 0.2031, \ 3.3813, \ -0.9385, \ -2.4053, \ 0.1939].
\]

The relative error (percent) of \(X_3^{(0)}\) is

\[
e = [3.7730, \ 1.5228, \ -2.4461, \ -1.7931, \ 0.2499, \ 4.5144, \ -1.2398, \ -3.2850, \ 0.2809].
\]

The average value of relative error is 2.1239%.

The precision of the model is "Good". The average value of relative error of without optimization for new information model is 3.6153%. So, optimization model has very high precision.

5. Conclusion

In view of multivariable non-equidistance sequence that multiple variables affect restrict mutually, we construct the multivariable non-equidistance new information optimization grey model MGM(1,n). In modeling, applying a step by step optimum new information modeling method to construct new information background value of multi-variable non-equidistance new information Grey model MGM(1,n), regarding mth component of the data as initial conditions of the grey differential equation, taking the minimum relative error as objective function, and taking revising correction values of the initial value as design variables. The new model is not only suitable for equidistance modeling, also suitable for non-equidistant model. It enlarge the scope of application of the grey model and has high precision, easy to use and so on. Actual examples show that the model is practical and reliable with important practical and theoretical significance. It is worth using widely.

References


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