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On the Magneto-Elastic Waves in Transversely Isotropic Plates

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Abstract: In this paper, the dispersion of plane harmonic waves in a thin flat homogeneous transversely isotropic plate of finite width and infinite length permeated by a constant magnetic field is examined. The frequency equations corresponding to the magnetized symmetric and anti-symmetric modes of vibration of the plates are obtained in the closed form, some limiting and special cases of the frequency equations are then discussed and exhibied graphically. .

Keywords: Magneto-elastic, transversely isotropic, frequency equation, symmetric, anti-symmetric

1 Introduction

The magneto-elastic field is an important subject for many structures and machines. In recent years numbers of investigations are considered for studying the magnetism magneto-elastic effect in engineering. Theoretical basics concerning models of continuum mechanics, which take account the effect of the magneto-elastic fields of diverse physical nature are studied and presented by Altenbach et al. [1], Ambarcumyan et al. [2], Dorfmann and Ogden[3]. Study of wave propagation with reference to the propagation of magneto-elastic waves is of vital importance in engineering and physical science and is the source of diverse phenomena, such as vibrations, noises, mechanical losses in magnetic circuits and other fields as geophysics, optics, acoustics and astrophysics. Magneto-elastic waves are the outcome of the interaction of the spin and elastic vibrations in ferromagnetic materials. Knopoff [4] studied the propagation of plane waves in magneto-elastics and revealed that uncoupled waves, which are plane polarized, their polarizations remain unchanged with wave propagation in the magnetic fields. A review on wave motion in magnetizable deformable solids is given in Maugin[5]. An analysis on the behavior of magneto-elastic waves in ferromagnetic plates and films is presented by authors Kaliski et al. [6] and Chadwick [7]. Buchwald and Davis [8] studied that approximate displacements in an infinite perfectly conducting isotropic elastic medium at large distances from a point source subjected to a uniform uni-axial magnetic field. Dunkin and Eringen [9] explained the coupling of electromagnetic and elastic waves in linearized electromagnetic theory. Purushotham [10] studied the Magneto-elastic coupling effects on the propagation of harmonic waves in an electrically conducting elastic plates. Abubakar [11]have considered the vibration of a perfectly conducting plate in a uniform magnetic field. Verma[12] and Verma and Hasebe [13] considered problems of wave propagations in transversely isotropic heat conducting plates with thermal relaxations. Extensive literature, elastic waves propagation in layered media are specified in Ewing [14]. In this paper propagation of plane harmonic wave in an infinite transversely isotropic plate of thickness $2d$ permeated by a uniform, static magnetic field is considered and discussed. The plate is assumed to be of an infinite conductivity. It is revealed that only very large applied magnetic fields produce appreciable effects, when it is possible for conical propagation to occur. A lower limit for the magnetic field is obtained above which conical propagation takes place.

2 Equations of motion

Elastic displacements are infinitesimal and the displacement fluxes are insignificant in compared with the conductivity currents according to Kaliski[6] assumption

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and the general form of the linearized magnetoelastic equations for a perfectly conducting homogeneous transversely isotropic body in the absence of body forces are:

$$\begin{aligned}&\nabla\times\mathbf{h}=\frac{4\pi}{c}\mathbf{J},\ \nabla\mathbf{.h}=0,\\&\nabla\times\mathbf{E}=-\frac{\mu_e}{c}\frac{\partial\mathbf{h}}{\partial t},\ \mathbf{E}=-\frac{\mu_e}{c}\left(\frac{\partial\mathbf{u}}{\partial t}\times\mathbf{H}\right),\end{aligned}\tag{1}$$

where $\mathbf{h}$. and $\mathbf{E}$ denote perturbations of the magnetic and electric fields respectively, $\mathbf{J}$ is the electric current density vector, $\mathbf{H}$ is the initial constant magnetic field, $\mathbf{u}$ is the mechanical displacement vector, μ_e is the magnetic permeability, and is the velocity of light. Equations governing the propagation of small elastic disturbances in a perfectly conducting anisotropic medium having electromagnetic force $\mathbf{J}\times\mathbf{B}$ (the Lorentz force, $\mathbf{J}$ being the electric current density and $\mathbf{B}$ being the magnetic induction vector) as the only body forces are

$$\begin{aligned}&\frac{\partial\sigma_{11}}{\partial x}+\frac{\partial\sigma_{12}}{\partial y}+\frac{\partial\sigma_{13}}{\partial z}+(\mathbf{J}\times\mathbf{B})_x=\rho\frac{\partial u}{\partial t^2},\\&\frac{\partial\sigma_{21}}{\partial x}+\frac{\partial\sigma_{22}}{\partial y}+\frac{\partial\sigma_{23}}{\partial z}+(\mathbf{J}\times\mathbf{B})_y=\rho\frac{\partial v}{\partial t^2},\\&\frac{\partial\sigma_{31}}{\partial x}+\frac{\partial\sigma_{32}}{\partial y}+\frac{\partial\sigma_{33}}{\partial z}+(\mathbf{J}\times\mathbf{B})_z=\rho\frac{\partial w}{\partial t^2}.\end{aligned}\tag{2}$$

The well-known Maxwell's equations governing the electromagnetic field are:

$$\begin{aligned}&\nabla\mathbf{.B}=0,\nabla\mathbf{.D}=\varepsilon\nabla\mathbf{.E}=\rho,\ \mathbf{D}=\varepsilon\mathbf{E}\\&\nabla\times\mathbf{E}=-\frac{\partial\mathbf{B}}{\partial t}=-\mu\frac{\partial\mathbf{H}}{\partial t},\ \mathbf{B}=\mu\mathbf{H}\\&\nabla\times\mathbf{H}=\mathbf{J}+\frac{\partial\mathbf{D}}{\partial t}=\sigma\mathbf{E}+\frac{\partial\varepsilon\mathbf{E}}{\partial t},\ \mathbf{J}=\sigma\mathbf{E}\\&\mathbf{B}=\mu_e\mathbf{H},\ \mathbf{J}=\sigma\left(\mathbf{E}+\frac{\partial\mathbf{u}}{\partial t}\right)\end{aligned}\tag{3}$$

where $\mathbf{E}$ is the induced electric field, $\mathbf{J}$ is the current density vector and magnetic field $\mathbf{H}$ includes both primary and induced magnetic fields and σ are the induced permeability and conduction coefficient respectively. Consider displacement vector $\mathbf{u}=(u,v,w)$, and the constant applied magnetic field $\mathbf{H}=(H_1,H_2,H_3)$ with respect to the rectangular Cartesian coordinate system. If $\mathbf{H}_2=0$, then the motion represented by equation (1) can be separated into purely horizontally polarized motion corresponding to the SH motion and corresponding to the P and SV type motion. Thus on considering $\mathbf{H}_2=0$ in equations (1)-(3) yields the two dimensional equation of motions and the heat equation in the $x-z$ plane as

$$\frac{\partial\sigma_{11}}{\partial x}+\frac{\partial\sigma_{13}}{\partial z}+(\mathbf{J}\times\mathbf{B})_x=\rho\frac{\partial u}{\partial t^2},\tag{4}$$

$$\frac{\partial\sigma_{31}}{\partial x}+\frac{\partial\sigma_{33}}{\partial z}+(\mathbf{J}\times\mathbf{B})_z=\rho\frac{\partial w}{\partial t^2}.\tag{5}$$

Here $\sigma_{ij}=C_{ijkl}\varepsilon_{kl}$. where C_{ijkl} are the second order elastic constants and ε_{kl} are the strain tensor defined as $\varepsilon_{kl}=\frac{1}{2}\left(\frac{\partial u_k}{\partial x_l}+\frac{\partial u_l}{\partial x_k}\right)$. The governing field equations for displacement vector $\mathbf{u}(x,z,t)=(u,0,w)$ in the absence of the body forces equations (4) and (5) become

$$c_{11}u_{,xx}+c_{44}u_{,zz}+(c_{13}+c_{44})w_{,xz}+(\mathbf{J}\times\mathbf{B})_{,x}=\rho\ddot{u}\tag{6}$$

$$(c_{13}+c_{44})u_{,xz}+c_{44}w_{,xx}+c_{33}w_{,zz}+(\mathbf{J}\times\mathbf{B})_{,z}=\rho\ddot{w},\tag{7}$$

where C_{ij} are the elastic parameters. The superposed dot denotes time differentiation and the comma notation is used for spatial derivatives. Equations (6) and(7) with (3) can be written as

$$\begin{aligned}&(c_{13}+\frac{1}{2}c_{44})u_{,xz}+\frac{1}{2}c_{44}w_{,xx}+c_{33}w_{,zz}+\frac{H_1^2}{4\pi}(w_{,xx}+w_{,zz})\\&-\frac{H_1H_3}{4\pi}(u_{,xx}+u_{,zz})=\rho\ddot{w}\end{aligned}\tag{8}$$

$$\begin{aligned}&(c_{13}+\frac{1}{2}c_{44})u_{,xz}+\frac{1}{2}c_{44}w_{,xx}+c_{33}w_{,zz}+\frac{H_1^2}{4\pi}(w_{,xx}+w_{,zz})\\&-\frac{H_1H_3}{4\pi}(u_{,xx}+u_{,zz})=\rho\ddot{w}\end{aligned}\tag{9}$$

For a plane harmonic wave travelling in the x-direction, the solutions u,w. of equations (8) and (9) take the form

$$u=f(z)\exp[ik(x-Vt)],\tag{10}$$

$$w=g(z)\exp[ik(x-Vt)].\tag{11}$$

where $V\,(=\omega/k)$ and k are phase velocity and wave number respectively;ω is the circular frequency, and $\imath=\sqrt{-1}$. Substituting for from equations (10) and (11) into equations (8) and (9), we have

$$\begin{aligned}&\left[(c_{44}+F_3)D^2-\left(c_{11}+F_3-\rho V^2\right)k^2\right]f\\&+\left[ik\,(c_{13}+c_{44})D-F_2\left(D^2-k^2\right)\right]g=0\end{aligned}\tag{12}$$

$$\begin{aligned}&\left[ik\,(c_{13}+c_{44})D-F_2\left(D^2-k^2\right)\right]f\\&+\left[(c_{33}+F_1)D^2-\left(c_{44}+F_1-\rho V^2\right)kc_2k^2\right]g=0,\end{aligned}\tag{13}$$

where

$$\begin{aligned}&F_1=\frac{H_1^2}{4\pi},F_2=\frac{H_1H_3}{4\pi},F_3=\frac{H_3^2}{4\pi},\\&D=\frac{d}{dz},D^2=\frac{d^2}{dz^2}\end{aligned}\tag{14}$$

Let

$$(f(z),g(z))=(L,M))\,e^{ksz}.\tag{15}$$

where L and M are constants. Substituting (15) into (12) and (13) gives

$$\begin{aligned}&\left[(c_{44}+F_3)s^2-\left(c_{11}+F_3-\rho V^2\right)\right]L\\&+\left[is(c_{13}+c_{44})D-F_2\left(s^2-1\right)\right]M=0\end{aligned} \tag{16}$$

$$\begin{aligned}&\left[i(c_{13}+c_{44})s-F_2\left(s^2-1\right)\right]L\\&+\left[(c_{33}+F_1)s^2-\left(c_{44}+F_1-\rho V^2\right)\right]M=0\end{aligned} \tag{17}$$

In the following sections two cases of the applied magnetic field are examined separately.

3 Particular Cases

Firstly consider the applied magnetic field $\mathbf{H}=(0,0,H_3)$ implies that $F_1=0$, consequently $F_2=0$ and therefore (16) and (17) become

$$\begin{aligned}&\left[(c_{44}+F_3)s^2-\left(c_{11}+F_3-\rho V^2\right)\right]L\\&+\left[is(c_{13}+c_{44})D\right]M=0\end{aligned} \tag{18}$$

$$\left[i(c_{13}+c_{44})s\right]L+\left[c_{33}s^2-\left(c_{44}-\rho V^2\right)\right]M=0. \tag{19}$$

Eliminating L and M between (18) and (19) gives

$$A_0s^4+A_1s^2+A_2=0 \tag{20}$$

where

$$\begin{aligned}&A_0=(F_3+c_{44})c_{33}\\&A_1=F_3\left(\rho V^2-c_{33}-c_{44}\right)\rho V^2+\{(c_{13}+c_{44})^2\\&-c_{33}\left(c_{11}-\rho V^2\right)-c_{44}\left(c_{44}-\rho V^2\right)\}\\&A_2=\left(c_{11}+F_3-\rho V^2\right)\left(c_{44}-\rho V^2\right).\end{aligned} \tag{21}$$

Let s_1^2 and s_2^2 be the roots of the equation (20). They are given by

$$s_1^2,s_2^2=\left(-A_1\pm\sqrt{A_1{}^2-4A_0A_2}\right)/(2A_0). \tag{22}$$

From (18). we have

$$m_j=M/L=\frac{(c_{44}+F_3)s_j^2-(c_{11}+F_3-\rho V^2)}{-i(c_{13}+c_{44})s_j},\ j=1,2. \tag{23}$$

Thus the corresponding expressions for the displacements in the plate are

$$\begin{aligned}u=&[P_1\exp(-ks_1z)+P_2\exp(-ks_2z)\\&+Q_1exp(ks_1z)+Q_2exp(ks_2z)]exp[ik(x-Vt)],\\w=&[m_1P_1\exp(-ks_1z)+m_2P_2\exp(-ks_2z)\\&-m_1Q_1\exp(ks_1z)-m_2Q_2\exp(ks_2z)]\exp[ik(x-Vt)].\end{aligned} \tag{24}$$

The components of Maxwell's stresses in the elastic medium are τ_{xz}^* and $\tau_{zz}^*$and in vacuum are given by

$$\tau_{xz}^*=\frac{H_3h_1^*}{4\pi},\tau_{zz}^*=\frac{H_3h_3^*}{4\pi}. \tag{25}$$

Let the faces of the plate be at $z=\mp d$. Conditions that satisfied at the plate vacuum interfaces are: the continuity of the tangential component of the electric field, the vector field and the total normal and shear stresses across the interfaces. The total stress T_{ij} is stress is defined as the sum of the elastic stresses σ_{ij} , and the electromagnetic stress $\tau_{ij}=\frac{\mu_e\left(H_ih_j+H_jh_i-H_kh_k\delta_{ij}\right)}{4\pi}$ on neglecting second order quantities. The stresses in the plate relevant to the problem are

$$T_{zz}=\sigma_{zz}+\frac{H_3h_3}{4\pi} \tag{26}$$

$$T_{zx}=\sigma_{zx}+\frac{H_3h_2}{4\pi} \tag{27}$$

where

$$\sigma_{zz}=c_{13}u_{,x}+c_{33}w_{,z}, \tag{28}$$

$$\sigma_{zx}=c_{44}(u_{,z}+w_{,x}) \tag{29}$$

4 Boundary Conditions

The boundary conditions involved in determining the frequency vibrations of the plate are the continuity of the normal and shear stresses across the plate-vacuum interface. Further from equations (26)-(29) , it can be understood that the continuity of the magnetic field vector across the interface reduces the continuity of the stress to that of the vanishing of σ_{zz} on the faces of the plate. Hence at $z=\mp d$, we have

$$\sigma_{zz}=\sigma_{xz}=0 \tag{30}$$

Substitution of expressions (24 for the displacement components into equations (28), and (29), and using boundary conditions (30, in the resulting stresses, yield four equations involving the arbitrary constants $P1,P2$ and $Q1,Q2$ as

$$\begin{bmatrix}D_{11}E_1^- & D_{11}E_1^+ & D_{12}E_2^- & D_{12}E_2^+\\ D_{11}E_1^+ & D_{11}E_1^- & D_{12}E_2^+ & D_{12}E_2^-\\ D_{21}E_1^- & -D_{21}E_1^+ & D_{22}E_2^- & -D_{22}E_2^+\\ D_{21}E_1^+ & -D_{21}E_1^- & D_{22}E_2^+ & -D_{22}E_2^-\end{bmatrix}\mathrm{I}\begin{bmatrix}P_1\\P_2\\Q_1\\Q_2\end{bmatrix}=0 \tag{31}$$

where

$$\begin{aligned}&E_j^-=\exp(-kds_j),\ E_j^+=\exp(kds_j)\\&D_{1j}=ic_{13}-c_{33}m_js_j,D_{2j}=im_j-s_j\end{aligned} \tag{32}$$

5 Frequency Equation when $\mathbf{H} = (0,0,H_3)$

The consistency of (31),requires that the determinant of the coefficients of the arbitrary constants in simultaneously four equations (31),must vanish. This provides the frequency equation of the plate vibrations. The obtained frequency equation is further factorize into two factors, each of which yields the equations

$$D_{11}D_{22}sh(ks_2z)ch(ks_1z) = D_{12}D_{21}sh(ks_1z)ch(ks_2z), \tag{33}$$

$$D_{11}D_{22}sh(ks_1z)ch(ks_2z) = D_{12}D_{21}sh(ks_2z)ch(ks_1z). \tag{34}$$

These are the period equations which correspond to the symmetric and antisymmetric motion of the plate with respect to the medial plane $z = 0$. Evidently (33),corresponds to the symmetric motion and (34),corresponds to the antisymmetric motion. Thus the expression for the displacements in the symmetric vibration to the plate are given by

$$\begin{aligned} u &= [W_1\cosh(ks_1d) + W_2\cosh(ks_2d)]\exp[ik(x-Vt)], \\ w &= -[m_1W_1\sinh(ks_1d) + m_2W_2\sinh(ks_2d)]\exp[ik(x-Vt)] \end{aligned} \tag{35}$$

and in the antisymmetric motion by

$$\begin{aligned} u &= [W_1\sinh(ks_1d) + W_2\sinh(ks_2d)]\exp[ik(x-Vt)], \\ w &= -[m_1W_1\cosh(ks_1d) + m_2W_2\cosh(ks_2d)]\exp[ik(x-Vt)] \end{aligned} \tag{36}$$

If the applied magnetic field is zero then $F_3 = 0$ and (33),) and (34),reduce, respectively, to the equations studied by Iya Abubakar [11]. The discussion of transcendental equations (33),) and (34),general is difficult; we therefore, consider the results for limiting cases.

5.1 Magnetoelastic Symmetric Waves

For waves **long** compared with the thickness $2d$ of the plate, kd is small and consequently $kds_j, (j = 1,\ 2)$ may be assumed small as long as V is finite. Hence the hyperbolic function can be replaced by their arguments [Ewing et. al,[14], Abubakar ,[11],] and from equation (33) we then obtain

$$\frac{ikd}{M_3}(s_1^2 - s_2^2)[\left(M_5 - \rho V^2\right)M_2 - \left(M_1 + \rho V^2\right)M_4] = 0. \tag{37}$$

Hence either

$$s_1^2 - s_2^2 = 0, \tag{38}$$

or

$$\left(M_5 - \rho V^2\right)M_2 - \left(M_1 + \rho V^2\right)M_4 = 0 \tag{39}$$

where

$$M_1 = -c_{33}F_3 - c_{11}c_{33} - {c_{13}}^2 - c_{13}c_{44},$$

$$M_2 = c_{33}(F_3 + c_{44}),\ M_3 = (c_{13} + c_{44}),$$
$$M_4 = F_3 - c_{13} - 2c_{44},\ M_5 = F_3 + c_{11}.$$

If $s_1^2 = s_2^2$, the form of the original solution assumed, cannot satisfy the boundary conditions. Hence equation (39) holds. This equation gives the phase velocity of long compressional or plate waves in magneto-elasticity.

$$V = \left[\frac{(c_{13} + c_{33})F_3 + 2c_{13}c_{44} + c_{11}c_{33} + {c_{13}}^2}{\rho c_{33}}\right]^{\frac{1}{2}}. \tag{40}$$

On putting $F_3 = 0$

$$V = \left[\frac{2c_{13}c_{44} + c_{11}c_{33} + {c_{13}}^2}{\rho c_{33}}\right]^{\frac{1}{2}}. \tag{41}$$

If we take

$$\begin{aligned} c_{11} = c_{33} &= \lambda + 2\mu, \\ c_{44} = \mu, c_{13} &= \lambda \end{aligned} \tag{42}$$

On using the isotropic relations (42) expression (40) reduces to

$$V = \left[\frac{2(\lambda + \mu)(\lambda + 2\mu + F_3)}{\rho(\lambda + 2\mu)}\right]^{\frac{1}{2}} \tag{43}$$

$$\begin{aligned} V &= \left[\frac{2(\lambda + \mu)}{\rho}\right]^{\frac{1}{2}} = 4\beta\left(\frac{\alpha^2}{\beta^2} - 1\right)^{\frac{1}{2}}, \\ \alpha^2 &= \frac{(\lambda + 2\mu)}{\rho},\ \beta^2 = \frac{2\mu}{\rho} \end{aligned} \tag{44}$$

This equations (43) and (44) give the phase velocity of long compressional or plate waves in magnetoelastic medium and in the absence of magnetic field. For very **short** waves and V such that s_1, s_2 and are real, kd is large and the hyperbolic functions tend to unity. Hence equation (33) becomes

$$\begin{aligned} &\left\{c_{33}\left(s_1^2 - 1\right)\left(s_2^2 - 1\right)\right\}(s_1 - s_2)F_3^2 \\ &+ \left\{\left(-c_{33}c_{11} - S_1\right)\left(s_1^2 + s_2^2\right) + s_1^2s_2^2S_8 + s_1s_2S_7 + S_2\right\}(s_1 - s_2)F_3 \\ &+ c_{33}V^4\rho^2 + \left(S_3 + S_1V^2\rho\right)(s_1 - s_2)\left(s_1^2 + s_2^2\right) \\ &- \left(s_1s_2S_0 + {c_{13}}^2 + S_2\right)(s_1 - s_2)V^2\rho \\ &+ \left(s_1^2s_2^2S_4 + s_1s_2S_5 + S_6\right)(s_1 - s_2) = 0. \end{aligned} \tag{45}$$

Evidently $(s_1 - s_2)$ is a factor. Therefore, from equation (45) we obtain

$$\begin{aligned} &\left\{c_{33}\left(s_1^2 - 1\right)\left(s_2^2 - 1\right)\right\}F_3^2 \\ &+ \left\{\left(-c_{33}c_{11} - S_1\right)\left(s_1^2 + s_2^2\right) + s_1^2s_2^2S_8 + s_1s_2S_7 + S_2\right\}F_3 \\ &+ c_{33}V^4\rho^2 + \left(S_3 + S_1V^2\rho\right)\left(s_1^2 + s_2^2\right) \\ &- \left(s_1s_2S_0 + {c_{13}}^2 + S_2\right)V^2\rho \\ &+ s_1^2s_2^2S_4 + s_1s_2S_5 + S_6 = 0. \end{aligned} \tag{46}$$

where $(s_1 - s_2)$ are roots of equation (20) and are given by

$$
\begin{aligned}
&S_0 = c_{33}(c_{13}+c_{44}),\ S_1 = c_{33}c_{44},\\
&S_2 = c_{13}c_{44}+2c_{33}c_{11}+c_{13}{}^2,\\
&S_3 = -c_{11}c_{33}c_{44},\\
&S_4 = c_{33}c_{44}(c_{13}+2c_{44}),\\
&S_6 = c_{11}(c_{33}c_{11}+c_{13}c_{44}+c_{13}{}^2),\\
&S_5 = c_{11}c_{33}(c_{13}+c_{44})+(3c_{44}+1)c_{13}{}^2\\
&+2c_{13}c_{44}{}^2,\\
&S_7 = (c_{13}+c_{44})(c_{13}+c_{33}),\\
&S_8 = (c_{13}+3c_{44})c_{33}.
\end{aligned}
\tag{47}
$$

Equation (46) can be identified as the phase velocity equation for Rayleigh waves in transversely isotropic half-space. If $F_3 = 0$ then (46) reduces to

$$
\begin{aligned}
&c_{33}V^4\rho^2 - \left(s_1s_2S_0 + c_{13}{}^2 + S_2\right)V^2\rho\\
&+\left(S_3+S_1V^2\rho\right)\left(s_1^2+s_2^2\right)+s_1^2s_2^2S_4\\
&+s_1s_2S_5+S_6 = 0.
\end{aligned}
\tag{48}
$$

On using the isotropic relations (42) expression (46) can be identified as the phase velocity equation for Rayleigh waves in isotropic half space, which is in agreement with the corresponding result of Nayfeh and Nasser,[15].

5.2 Magnetoelastic Anti-symmetric Wavess

For **waves long compared**with thickness of the plate s_1 and s_2 are real and then we may replace the hyperbolic functions by the approximation

$$
\tanh(\theta) \cong \theta - \frac{\theta^3}{3} \tag{49}
$$

After some algebraic transformation and reductions, and neglecting $O\left[(kd)^3\right]$ we obtain

$$
\begin{aligned}
&L_{10}\rho^2V^4 + L_{20}\rho V^2 + L_{30}\frac{\left(s_1{}^2-s_2{}^2\right)}{s_1s_2M_3^2}\left(\frac{1}{3}ikd\right)\\
&+(kd)^2\frac{\left(s_1{}^2-s_2{}^2\right)}{s_1s_2M_3^2}\left(\frac{1}{3}ikd\right)\left\{L_0\rho^3V^6+L_1\rho^2V^4+L_2\rho V^2+L_3\right\}=0
\end{aligned}
\tag{50}
$$

This implies that either

$$
\frac{\left(s_1{}^2-s_2{}^2\right)}{s_1s_2M_3^2} = 0. \tag{51}
$$

Or

$$
\begin{aligned}
&L_{10}\rho^2V^4 + L_{20}\rho V^2 + L_{30}\\
&+(kd)^2\left(L_0\rho^3V^6+L_1\rho^2V^4+L_2\rho V^2+L_3\right)=0
\end{aligned}
\tag{52}
$$

where

$$
\begin{aligned}
&L_0 = N_1(N_1 - M_4 + c_{33}) - c_{33}M_4 - M_2,\\
&L_1 = (2N_1 - M_4 + c_{33})N_0 - N_1{}^2M_5\\
&+N_1(M_5M_4 + c_{44}M_4 + M_1)\\
&+c_{44}(M_2 + c_{33}M_4) + M_4(c_{33}M_5 - M_1)\\
&+4M_5M_2,\\
&L_2 = [N_0 + (c_{44}+M_5)M_4 - M_5N_1]M_1\\
&+\left(-4M_5c_{44} - 3M_5{}^2\right)M_2\\
&+(M_5N_0 - c_{44}M_5N_1 - c_{44}c_{33}M_5 + c_{44}N_0)\times\\
&M_4 - 2M_5N_1N_0 + N_0{}^2 + M_5N_1,\\
&L_3 = 3c_{44}M_5{}^2M_4 - M_5N_0{}^2 - c_{44}M_4\\
&\times(M_5N_0 + M_5M_1) + M_5N_0(1-M_1),
\end{aligned}
\tag{53}
$$

$$
\begin{aligned}
&L_{10} = -3M_1N_1 + 3M_2M_4 - 3c_{33}M_2\\
&L_{20} = 3M_2(-N_0 + M_5N_1 - M_4M_5\\
&-M_1 - c_{44}M_4)\\
&L_{30} = 3M_2M_5(M_1 + N_0 + c_{44}M_4)
\end{aligned}
\tag{54}
$$

$$
\begin{aligned}
&N_0 = (c_{33}+c_{44})F_3 - c_{13}{}^2 - 2c_{13}c_{44} + c_{11}c_{33},\\
&N_1 = -(c_{33}+c_{44}+F_3),\\
&M_1 = -c_{33}(c_{11}+F_3) - c_{13}(c_{13}+c_{44}),\\
&M_2 = c_{33}(c_{44}+F_3),M_3 = (c_{13}+c_{44}),\\
&M_4 = -(c_{13}+2c_{44}+F_3),\ M_5 = (c_{11}+F_3).
\end{aligned}
\tag{55}
$$

Equation (51) not satisfying the boundary conditions. Hence equation (52) holds, and it is the dispersion equation of long flexural waves and it can be seen that the phase velocity decreases as the wavelength increases in magneto-elasticity as the phase velocity tends to zero as the wavelength increases. Using the isotropic relations(42) equation (52) reduces to isotropic case [15].

$$
\left(Q_{10}\rho^3V^6 + Q_{20}\rho^2V^4 + Q_{30}\rho V^2 + Q_{40}\right)(s_1 - s_2) \tag{56}
$$

where

$$
\begin{aligned}
&Q_{10} = M_2N_1{}^2 + (-2M_2M_4 + 2M_2c_{33})N_1 - M_2{}^2\\
&+(M_2 - M_4{}^2)c_{33}{}^2 - 4M_2M_4c_{33} + M_2M_4{}^2
\end{aligned}
\tag{57}
$$

$$
\begin{aligned}
&Q_{20} = \left[-2M_1c_{33} + \left(c_{33}{}^2 - 2M_2\right)c_{44} - M_5M_2\right]M_4{}^2\\
&+[4M_2c_{33}c_{44} + (c_{44}+2M_5)2M_2N_1 + (4c_{33}M_5 - 4M_1 - 2N_0)M_2]M_4\\
&+2(\mathrm{c}_{33}+N_1)\mathrm{M}_2N_0 - (c_{33}{}^2 + 2N_1{}^2)M_5M_2 + (M_1 - c_{33}M_5)2M_2N_1\\
&+(c_{44}+2M_5)M_2{}^2 + 2c_{33}M_1M_2,
\end{aligned}
\tag{58}
$$

$$
\begin{aligned}
&Q_{30} = M_2N_0{}^2 + (c_{44}M_4 + M_1 - M_5N_1 + M_4M_5 - c_{33}M_4)2M_2N_0\\
&-(M_1 + c_{44}M_4)2N_1M_5M_2 + M_4{}^2\mathrm{M}_2c_{44}{}^2\\
&+(M_1M_4 - 2M_5M_2)2M_4c_{33}c_{44} + 2M_5M_2M_4{}^2c_{44}\\
&-2M_5M_2{}^2c_{44} + 4M_2M_4M_1c_{44} + 4M_1M_2M_4M_5\\
&-2c_{33}M_1M_2M_5 - (M_1M_4)^2 - (M_2M_5)^2 + M_2M_1{}^2,
\end{aligned}
\tag{59}
$$

$$Q_{40}=\left[(M_1M_4)^2+(M_2M_5)^2-M_2M_4M_5(2N_0+4M_1)\right]c_{44}$$
$$-\left(M_1{}^2-2N_0M_1-N_0{}^2\right)M_2M_5-c_{44}{}^2M_5M_2M_4{}^2 \tag{60}$$

Equation (56) Rayleigh equation and the propagation degenerates to Rayleigh waves associated with both surfaces of the plate. If $F_3=0$ then via equations (55) and (54-60) equations (50) and(52) reduce to equations obtained by Iya Abubakar [11].

6 Frequency Equation when $\mathbf{H}=(H_1,0,0)$

Consider the applied field $\mathbf{H}=(H_1,0,0)$, in this case $F_2=0$ and consequently $F_3=0$, the consistency of (31) require that the determinant of the coefficients of the arbitrary constants in simultaneously four equations (31) must vanish. This gives an equation for the frequency of the plate oscillations. The frequency equation is found to factorize into two factors, each of which yields the equations this implies that $F_3=0$ and consequently $F_2=0$ and therefore (16) and (17) become

$$\left[c_{44}s^2-\left(c_{11}-\rho V^2\right)\right]L+\left[is\left(c_{13}+c_{44}\right)\right]M=0. \tag{61}$$

$$\left[i\left(c_{13}+c_{44}\right)s\right]L+\left[\left(c_{33}+F_1\right)s^2-\left(c_{44}+F_1-\rho V^2\right)\right]M=0. \tag{62}$$

Eliminating L and M between (61) and(62) gives

$$A_0{}'s'^4+A_1{}'s'^2+A_2{}'=0, \tag{63}$$

Where

$$\begin{aligned}
&A_0{}'=\left(F_1+c_{33}\right)c_{44},\\
&A_1{}'=F_1\left(\rho V^2-c_{11}-c_{44}\right)\rho V^2+\\
&\{\left(c_{13}+c_{44}\right)^2-c_{33}\left(c_{11}-\rho V^2\right)-c_{44}\left(c_{44}-\rho V^2\right)\},\\
&A_2{}'=\left(c_{44}+F_1-\rho V^2\right)\left(c_{11}-\rho V^2\right).
\end{aligned} \tag{64}$$

Let s'^2_1 and s'^2_2 be the roots of the equation (20). They are given by

$$s'^2_1,s'^2_2=\left(-A_1\pm\sqrt{A_1{}^2-4A_0A_2}\right)/(2A_0) \tag{65}$$

From (18) we have

$$m_j{}'=M/L=\frac{c_{44}s'^2_j-\left(c_{11}-\rho V^2\right)}{-i(c_{13}+c_{44})s'_j},$$
$$m_j{}'=M/L=\frac{-i(c_{13}+c_{44})s'_j}{F_1\left(s'^2_j-1\right)+c_{33}s'^2_j-\left(c_{44}-\rho V^2\right)}.j=1,2.$$

The boundary conditions are the same as for the case I.

$$\begin{bmatrix} D_{11}{}'E_1^- & D_{11}{}'E_1^+ & D_{12}{}'E_2^- & D_{12}{}'E_2^+\\ D_{11}{}'E_1^+ & D_{11}{}'E_1^- & D_{12}{}'E_2^+ & D_{12}{}'E_2^-\\ D_{21}{}'E_1^- & -D_{21}{}'E_1^+ & D_{22}{}'E_2^- & -D_{22}{}'E_2^+\\ D_{21}{}'E_1^+ & -D_{21}{}'E_1^- & D_{22}{}'E_2^+ & -D_{22}{}'E_2^- \end{bmatrix}\begin{bmatrix}P'_1\\P'_2\\Q'_1\\Q'_2\end{bmatrix}=0. \tag{66}$$

Thus following the above procedure as in case I we obtain the frequency equation decoupled as:

$$D_{11}{}'D_{22}{}'sh(ks'_2z)ch(ks'_1z)=D_{12}{}'D_{21}{}'sh(ks'_1z)ch(ks'_2z], \tag{67}$$

$$D_{11}{}'D_{22}{}'sh(ks'_1z)ch(ks'_2z)=D_{12}{}'D_{21}{}'sh(ks'_2z)ch(ks'_1z)., \tag{68}$$

where

$$\begin{aligned}
&D_{1j}{}'=ic_{13}{}'-c_{33}m_j{}'s_j{}',D_{2j}{}'=im_j{}'-s_j{}'\\
&E'^-_j=\exp(-kds'_j),\ E'^+_j=\exp(kds'_j)
\end{aligned} \tag{69}$$

These are the period equations (67) and (68) correspond to the symmetric and antisymmetric motion of the plate with respect to the medial plane $z=0$. In the absence of the applied magnetic field $F_1=0$ equations (67) and (68) correspond to the symmetric and antisymmetric motion of the plate with respect to the medial plane for the freely vibrating unmagnified transversely isotropic plate.

6.1 *Symmetric Waves when* $\mathbf{H}=(H_1,0,0)$

Applying and following the same technique adopted in case I it is found that for waves long compared with the thickness $2d$ of the plate, kd is small, then from (67) , we obtain

$$\frac{ikd}{(c_{13}+c_{44})}(s_1^2-s_2^2)[c_{13}\left(c_{13}+2c_{44}\right)-\left(c_{11}-\rho V^2\right)c_{33}]=0. \tag{70}$$

This is the phase velocity of the plate waves and is identical with the plate wave velocity for the unmagnified transversely isotropic plate. Thus the applied magnet field has no effect on the phase velocity of the plate wave. If waves are short compared with the thickness of the plate, that is $\xi d\to\infty$, then (67) reduces approximately to

$$\left(s_1-s_2\right)\left[C_{10}\rho^4V^8+C_{20}\rho^3V^6+C_{30}\rho^2V^4+C_{40}\rho V^2+C_{50}\right]=0, \tag{71}$$

where
$C_{10}=C_1\left(c_{44}-c_{33}-F_1\right)c_{33},\ C_1=c_{13}c_{44}\left(c_{13}+c_{44}\right)^2,$

$$C_{20}=C_1F_1(2c_{13}c_{44}-c_{33}c_{44}+3c_{11}c_{33}+2c_{13}^2+c_{33}^2+F_1c_{33})$$
$$-c_{33}C_1\left(2c_{11}c_{44}-c_{33}c_{44}-3c_{11}c_{33}-2c_{13}^2+4c_{44}^2\right),$$

$$C_{30} = -(2c_{13}c_{33}c_{44} - c_{13}^2c_{44} - c_{33}^2c_{44} + 2c_{13}^2c_{33} + 2c_{11}c_{33}^2)c_{44}(c_{13}+c_{44})^2F_1^2 - 4c_{44}^3\left(c_{13}^2 - c_{33}^2\right)(c_{13}+c_{44})^2F_1 - (4-2c_{33})c_{13}^2c_{44}^2(c_{13}+c_{44})^2F_1 - (4c_{13}-c_{33})c_{11}c_{33}c_{44}^2(c_{13}+c_{44})^2F - \left(3c_{11}+3c_{11}^2+2c_{13}^2\right)c_{33}^2c_{44}(c_{13}+c_{44})^2F_1]$$

$$C_{40} = c_{44}(c_{13}+c_{44})^2F_1{}^2P_1 + c_{44}(c_{13}+c_{44})^2F_1P_2 + c_{44}(c_{13}+c_{44})^2P_3 \tag{72}$$

where

$$P_1 = 3c_{11}{}^2c_{33}{}^2 + c_{13}^4 + 4c_{11}c_{13}c_{33}c_{44} - 2c_{11}c_{33}^2c_{44} + 4c_{13}^2c_{44}^2 + 4c_{13}^3c_{44} - 2c_{11}c_{13}^2c_{44} + 4c_{11}c_{13}^2c_{33}$$

$$P_2 = c_{11}{}^2c_{33}{}^2 - 4c_{11}c_{13}{}^2c_{33}c_{44} + c_{11}c_{13}^4 + c_{11}{}^2c_{33}{}^2c_{44} + c_{13}^4c_{33} + 4c_{13}{}^3c_{44}{}^2 + 2c_{11}{}^2c_{13}^2c_{33} + 4c_{11}(c_{13}c_{44})^2 + 4c_{13}{}^2c_{44}{}^3 + 2c_{11}{}^2c_{13}c_{33}c_{44} + c_{13}{}^4c_{44} - 8c_{11}c_{33}{}^2c_{44}{}^2 + 4c_{11}c_{13}^2c_{33}{}^2 + 4c_{13}^3c_{44}c_{33} + 4c_{11}c_{13}^2c_{44} + 4(c_{13}c_{44})^2c_{33} + 3c_{11}{}^2c_{33}{}^2$$

$$P_3 = c_{11}{}^2c_{33}{}^2 + 4(c_{13}c_{44})^2c_{33} + c_{13}^3c_{44}c_{33} + 3c_{11}{}^2c_{33}{}^2c_{44} - 4c_{11}c_{13}{}^2c_{33}{}^2c_{44} - 8c_{11}c_{33}{}^2c_{44}{}^2 - 4c_{11}{}^2c_{33}{}^2c_{44}{}^2 + 4c_{11}c_{13}{}^2c_{33}c_{44}{}^2 + c_{11}c_{13}{}^4c_{33} - 8c_{11}c_{13}c_{33}{}^2c_{44}{}^2 + 2c_{11}{}^2c_{13}^2c_{33}{}^2 + 4c_{11}c_{13}{}^2c_{44}c_{33}$$

$$C_{50} = \left(c_{13}^4 - 4c_{13}^3c_{44} - 2c_{11}c_{33}\right)c_{44}c_{11}(c_{13}+c_{44})^2F_1{}^2 + c_{11}c_{13}^2c_{44}^2(c_{11}-4c_{44})(c_{13}+c_{44})^2F_1{}^2 - 2c_{11}^2c_{13}c_{33}c_{44}^2(c_{13}+c_{44})^2F_1{}^2 + (c_{44}-c_{11})c_{11}^2c_{33}^2c_{44}^2(c_{13}+c_{44})^2F_1{}^2 + -4c_{11}c_{13}^2c_{44}^4(c_{13}+c_{44})^2F_1 + 4c_{11}c_{44}^3\left(c_{11}{}^2c_{33} - c_{33}c_{13}^2 - c_{13}^3\right)(c_{13}+c_{44})^2F_1 + c_{11}c_{44}^2\left(-c_{11}{}^2c_{33}{}^2 - c_{13}^4\right)(c_{13}+c_{44})^2F_1 + 2c_{11}c_{33}\left(c_{11}c_{13}^2 - 2c_{13}^3\right)(c_{13}+c_{44})^2F_1 - c_{33}c_{44}c_{11}\left(c_{11}{}^2c_{33} + 2c_{11}c_{13}^2c_{33} + c_{13}^4\right)(c_{13}+c_{44})^2F_1 - c_{11}^3c_{13}^2c_{33}^4c_{44}^3(c_{13}+c_{44})^2 + \left(c_{13}^2 + 2c_{13}c_{44} + 2c_{44}^2\right)c_{11}^2c_{13}^2c_{33}^3c_{44}^3(c_{13}+c_{44})^2 - c_{44}c_{11}\left(c_{13}^2 + 4c_{13}c_{44} + 4c_{44}^2\right)(c_{13}+c_{44})^2$$

This is the phase velocity equation for Rayleigh waves associated with both the faces of the plate. On putting $F_1 = 0$, it reduces to (53) which is the Rayleigh wave equation in unmagnified plate.

6.2 Antisymmetric Waves when $\mathbf{H} = (H_1, 0, 0)$

For waves long compared with the plate thickness such that kd is small, accordingly equation (68) approximately becomes

$$\left({}^{-1}/_3W_5\rho^2V^4 + W_3F_1 + W_1F_1{}^2\right)(kd)^2 + \left(c_{13}F_1{}^2 + W_9 + W_4F_1\right)(kd)^2\rho V^2 + W_{10}(kd)^2W_6F_1 + W_2F_1{}^2 - (W_5F_1 - W_8)\rho V^2 + W_7 = 0, \tag{73}$$

where

$$\begin{aligned} &W_1 = c_{13}{}^2 - c_{11}c_{13} - c_{33}c_{44} + c_{13}c_{44}, \\ &W_2 = 3c_{33}c_{44} + 3c_{13}c_{44}, \\ &W_3 = c_{13}{}^2 + \left(2c_{13}{}^2 + 2c_{13}c_{44} - 2c_{44}{}^2\right)c_{33} + 3c_{44}c_{13}{}^2 + 2c_{44}{}^2c_{13} - (3c_{13}+c_{44})c_{11}c_{33}, \\ &W_4 = (3c_{13}+c_{44})c_{33} - c_{13}{}^2 - c_{13}c_{44}, \\ &W_5 = 3c_{33}c_{44},\ W_6 = 3c_{33}{}^2c_{44} + 3\left(3c_{13}c_{44} + 2c_{44}{}^2\right), \\ &W_7 = 6\left(c_{13}c_{44} + c_{44}{}^2\right)c_{33}{}^2,\ W_8 = 3c_{33}{}^2c_{44}, \\ &W_9 = (2c_{13}+c_{44})c_{33}{}^2 - \left(2c_{13}{}^2 + 2c_{13}c_{44} - 2c_{44}{}^2\right)c_{33}, \\ &W_{10} = \left(2c_{13}{}^2 + 6c_{44}{}^2c_{13} + 7c_{44}c_{13}{}^2\right)c_{33} - (2c_{13}+c_{44})c_{11}c_{33}{}^2. \end{aligned} \tag{74}$$

$$\begin{aligned} &-2c_{44}\left(3c_{13}+\rho V^2\right)\frac{\left(s_1{}^2-s_2{}^2\right)}{s_1s_2c_{13}}\left(\frac{1}{3}ikd\right)\left(c_{11}-\rho V^2\right) \\ &+c_{33}\left(c_{11}-\rho V^2\right)\frac{\left(s_1{}^2-s_2{}^2\right)}{s_1s_2c_{13}}\left(\frac{1}{3}ikd\right)\left(c_{11}-\rho V^2\right)(kd)^2 \\ &+\rho V^2\left(2c_{13}+\rho V^2\right)\frac{\left(s_1{}^2-s_2{}^2\right)}{s_1s_2c_{13}}\left(\frac{1}{3}ikd\right)\left(c_{11}-\rho V^2\right)(kd)^2 \\ &-7c_{13}(kd)^2\frac{\left(s_1{}^2-s_2{}^2\right)}{s_1s_2}\left(\frac{1}{3}ikd\right)\left(c_{11}-\rho V^2\right) \\ &+2\left(c_{11}-\rho V^2\right)c_{33}\frac{\left(s_1{}^2-s_2{}^2\right)}{s_1s_2c_{44}}\left(\frac{1}{3}ikd\right)\left(c_{11}-\rho V^2\right) \\ &-2c_{13}\left(c_{13}-\rho V^2\right)\frac{\left(s_1{}^2-s_2{}^2\right)}{s_1s_2c_{44}}\left(\frac{1}{3}ikd\right)\left(c_{11}-\rho V^2\right) \\ &-3c_{13}c_{44}\left(2c_44+2c_{13}-\rho V^2\right)=0 \end{aligned} \tag{75}$$

whereas in the unmagnified plate the phase velocity tends to zero as the wavelength increases to infinity, under the applied magnetic field the phase velocity tends For waves short compared with the thickness of the plate equation (75) reduces to Rayleigh's equation (71).

$$C_{10}\rho^4V^8 + C_{20}\rho^3V^6 + C_{30}\rho^2V^4 + C_{40}\rho V^2 + C_{50} = 0.] \tag{76}$$

7 Result and Discussion

In the sections 5 and 6, analytic results obtained in equations (32) and (66) for the frequency equations corresponding to the applied magnetic $\mathbf{H} = (0,0,H_3)$ and $\mathbf{H} = (H_1,0,0)$ are transcendental. Therefore, the general discussion of waves in transversely isotropic plates in the presence of applied magnet field is slight taxing. Therefore the long and short waves are considered to find numerical solution of the equations. Computation for the symmetric and antisymmetric wave modes in the presence of applied magnetic fields $\mathbf{H} = (0,0,H_3)$ and $\mathbf{H} = (H_1,0,0)$ have been carried out for a transversely isotropic plate whose physical data is given as:

$$c_{11} = 3.07\times10^{11}Nm^{-2}, c_{13} = 1.027\times10^{11}Nm^{-2},$$
$$c_{33} = 3.581\times10^{11}Nm^{-2}, c_{44} = 1.510\times10^{11}Nm^{-2},$$
$$\rho = 8.636\times10^{3}Kgm^{-3}$$

7.1 Symmetric waves when applied magnetic field is $\mathbf{H} = (0,0,H_3)$

For the waves **long** compared with the thickness $2d$ of the plate, obtained equation (40) in this case is clearly depend on the applied magnetic field, and observed that at the lower values of the applied magnetic field there is negligible impact on the phase velocity, whereas at higher values it increases with the applied magnetic field as exhibited in figure 1. In the absences of applied magnetic field the phase velocity is $V = 6920.329$ m/sec, which is the phase velocity of long compressional or plate waves.

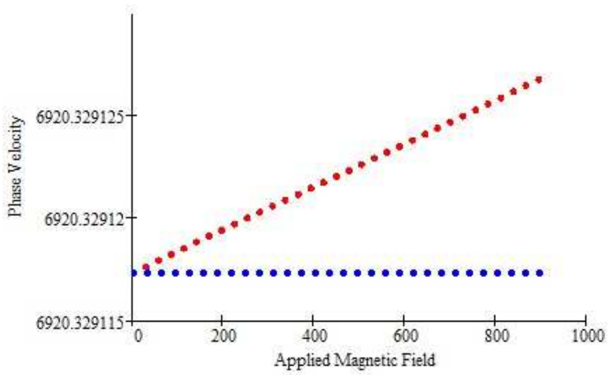

Fig. 1: Variation of phase velocity of long wave modes, compared with the thickness of the plate Vs applied magnetic field H_3.

Table 1: Longitudinal and Transverse wave modes Vs H_3

H_3	V_L(m/sec) (Longitudinal mode)	V_T(m/sec) Transverse Mode1	V_T(m/sec) Transverse Mode 2
0	6570.93	6093.52	6093.52
1	10110.31	10110.31	6516.56
2	12887.14	12887.14	6467.01
3	18710.65	11447.88	6422.50
5	27822.57	9989.21	6382.85
6	31177.10	9374.48	6347.69
7	34173.76	9015.16	6316.53

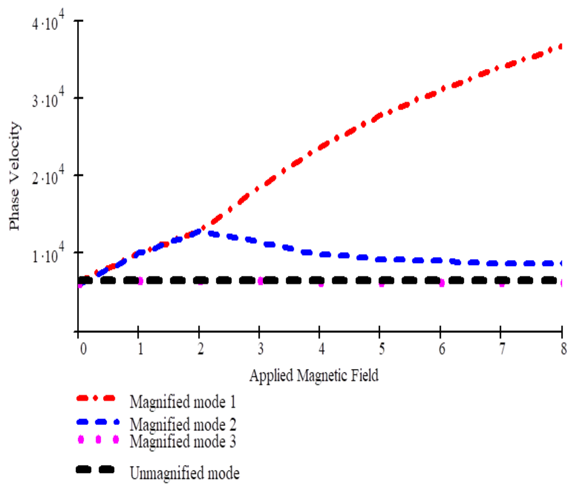

Fig. 2: Variation of phase velocity of short wave modes compared with of the thickness Vs applied magnetic field H_3 in the case of symmetric waves

7.2 Antisymmetric waves when applied magnetic field is $\mathbf{H} = (0,0,H_3)$

In this case for waves **long** compared with the thickness $2d$ of the plate, using equation (51) the results obtained for the variations of phase velocity with wave number in the presence and absence of applied magnetic field are exhibited in the figure 2. It is observed that at lower and higher limits of wave number there is no effect of applied magnetic field. Also at large applied magnetic field phase velocity decreases.

In this case for waves **short** compared with the thickness $2d$ of the plate, results obtained in equation (53) reduces to Rayleigh's equation and the propagation degenerates to waves associated with faces of the plate just as for the symmetric vibration and is $V_R = 6353.72$m/sec. It has also been observed that for

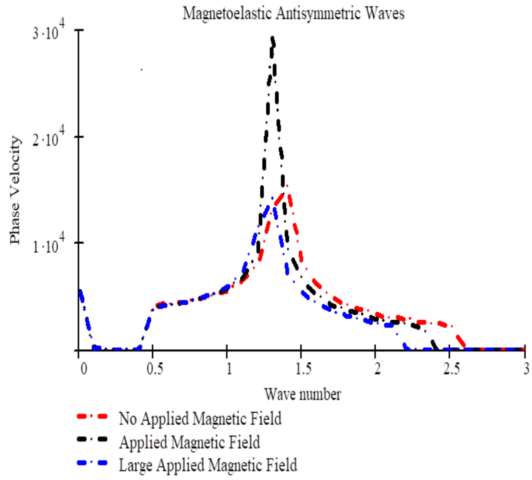

Fig. 3: Variation of phase velocity of long wave modes compared with the thickness of the plate Vs applied magnetic field H_3 in the case of antisymmetric waves

small values of applied magnetic fields Rayleigh waves speed is not affected whereas at very large values of applied magnetic fields Rayleigh waves speed is affected and increases with increase of applied magnetic field.

7.3 Symmetric waves when applied magnetic field is $\mathbf{H} = (H_1, 0, 0)$

For waves **long** compared with the thickness $2d$ of the plate, from equation (70) simplifies to the phase velocity of the plate waves which is $V_P = 5110$m/sec and it is alike with the plate wave velocity for the unmagnified transversely isotropic plate. Thus the applied magnetic field has no effect on the phase velocity of the plate wave.

For waves **short** compared with the thickness $2d$ of the plate, from equation (71) simplified and reduces to the phase velocity equation for Rayleigh waves associated with faces of the transversely isotropic plate and also effected by the case in the presence of the applied magnetic field as in the previous case..

7.4 Antisymmetric waves when applied magnetic field is $\mathbf{H} = (H_1, 0, 0)$

For waves long compared with the thickness $2d$ of the plate, equation (73) reduces to the dispersion equation for the long flexural waves in the transversely isotropic plate in the magnified field. Phase velocity Vs wavenumber dispersion curves in the magnified and unmagnified are displayed in the figure 4. It is evident from the figure that phase velocity is remain lower in the magnified than the unmagnified transversely isotropic plate at low values of wavenumber and converges at the higher values of wavenumber.

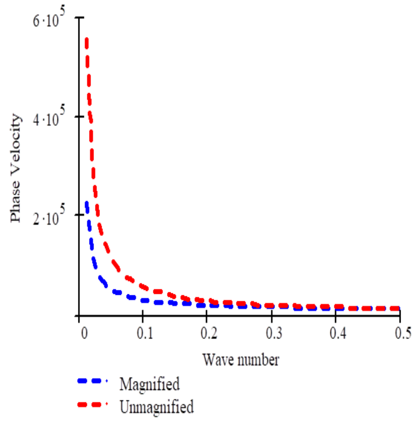

Fig. 4: Variation of phase velocity of long wave modes compared with the thickness of the plate Vs applied magnetic field H_1 in the case of antisymmetric waves.

In the case of waves **short**compared with the thickness $2d$ of the plate, again equation (76) reduces to Rayleigh's equation, which is also discussed in the symmetric case.

8 Conclusion

Two special cases $\mathbf{H} = (0, 0, H_3)$ and $\mathbf{H} = (H_1, 0, 0)$ of the constant applied magnetic field $\mathbf{H} = (H_1, H_2, H_3)$ are considered in studying the Magneto-elastic waves in transversely isotropic plates. On employing the boundary conditions, continuity of the normal and shear stresses across the plate-vacuum interface for both the cases, frequency equations are obtained, which are further factorized into the magnetized symmetric and antisymmetric vibration modes equations of the plate. It is found that the velocities of the long flexural wave in the plate and that of the Rayleigh waves associated with the free faces of the plate are affected by the magnetic field. The velocity of the long compressional waves is only affected when the applied magnetic field is normal to the direction of the propagation of the wave. Antisymmetric short waves compared with the thickness of the plate equation reduced to the Rayleigh's equation.

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