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Generalized Exponential Ratio Estimator of Population Mean Using Two Auxiliary Variables in Simple Random Sampling with an Application to Agricultural Data

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Abstract: By taking advantage of the correlation between the study variable and the auxiliary variable, ratio, product, and regression estimators are widely used to estimate the population mean of the study variable or other parameters of interest. In this paper an exponential ratio-type estimator using two auxiliary variables for estimation of a finite population mean has been proposed. Mean Square Error expression of the proposed estimator has been derived up to first degree of approximation. The efficiency conditions of the proposed ratio estimator have been studied. The theoretical result is illustrated with real data sets for numerical comparisons.
Keywords: Two Auxiliary variables, Efficiency comparison, Exponential ratio estimator, Mean square error, good seeds

1 Introduction

Time and expense constraints, make sample surveys frequently employed to collect primary data instead of census surveys. Use of the ratio, the product and the regression estimators are very common for estimating the mean of the variable of interest or other parameters of interest, by taking advantage of the correlation among study variable and auxiliary variable, either at the stage of estimation or at the designing stage or at both the stages. For judging the efficiency, these estimators are compared using their approximated mean squared errors. It has been seen that Mean Squared Error (MSE) of the ratio, product and the regression estimators are considerably reduced, with some modifications. In practice, there are a variety of scenarios where the population mean or total must be estimated and can be estimated with help of some auxiliary information. Various modifications of the ratio, product and regression type estimators have been proposed by various authors in literature, employing one or more auxiliary variables to estimate the finite population mean under various sampling techniques. The goal of this research is to create a modified exponential ratio estimator of population mean, investigate its properties in terms of mean square error, and compare it to existing ratio estimators in the literature.

2 Experimental Section

2.1 Literature Survey

[1] did major contribution in the explicit use of the auxiliary information. Ratio estimator was established by [2] by using the auxiliary information. [3] proposed a product type of estimator when the coefficient of correlation is negative. [4] estimated population mean by using multi-auxiliary info, positively correlated with the study variable, by employing a linear combination of ratio estimators based on each auxiliary variable individually. An expression of the product estimator was given by [5]. [6] has studied ratio estimators with two or more correlated variables. With the help of the provided auxiliary data, the exponential type estimators of population mean were investigated by [7]. [8] used transformed auxiliary variable for estimating the mean of the study character. There are numerous other contributions in the literature, and new estimators have surfaced; some of them being, [9], [10], [11], [12]. Further, [13] and [14] developed some better

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exponential type estimators for population mean based on information from two altered auxiliary variables as part of their effort. For detailed study of the topic one can refer to [15]. Also [16], [17], [18] and [24] have made some recent contributions to the study.

Consider a population $U = (U_1, U_2, \dots, U_N)$ that is finite and of N identifiable, distinct units. Suppose the variable of our study concern is Y, and two auxiliary variables X and Z, correlated with the Y are available. In order to estimate the population, mean $\bar{Y}$ we make use of these two auxiliary variables. A basic random sampling without replacement approach is used to select a sample of size n from the population. Let y_i, and (x_i, z_i) be the values of variable under study (y), and the auxiliary variables (x, z) respectively and let $\bar{y} = \frac{1}{n}\sum_i^n y_i$, $\bar{x} = \frac{1}{n}\sum_i^n x_i$ and $\bar{z} = \frac{1}{n}\sum_i^n z_i$ be their sample means. Assume that the auxiliary variables' population means $\bar{X} = \left(\frac{1}{N}\right)\sum_i^N x_i$ and $\bar{Z} = \left(\frac{1}{N}\right)\sum_i^N z_i$ are known. Further notations are defined below:

$$f = \frac{n}{N}$$

$$\theta = \frac{(1-f)}{N}$$

The Coefficients of Variation (C.V) of y, x and z are

$$C_y = \frac{S_y}{\bar{Y}}, \qquad C_x = \frac{S_x}{\bar{X}}, \qquad C_z = \frac{S_z}{\bar{Z}}$$

Population variances of y, x and z are:

$$S_y^2 = (N-1)^{-1}\sum_i^N (y_i - \bar{Y})^2$$

$$S_x^2 = (N-1)^{-1}\sum_i^N (x_i - \bar{X})^2$$

$$S_x^2 = (N-1)^{-1}\sum_i^N (z_i - \bar{Z})^2$$

Population Co-variances are:

$$S_{yx} = (N-1)^{-1}\sum_i^N (x_i - \bar{X})(y_i - \bar{Y})$$

$$S_{yz} = (N-1)^{-1}\sum_i^N (y_i - \bar{Y})(z_i - \bar{Z})$$

$$S_{xz} = (N-1)^{-1}\sum_i^N (x_i - \bar{X})(z_i - \bar{Z})$$

2.2 *Existing Ratio Estimators*

The two most widely used estimators of population mean are the ordinary sample mean $\bar{y}$ and the ratio estimator.

The usual unbiased estimator for population mean is defined as:

$$\bar{y} = \left(\frac{1}{n}\right)\sum_i^n y_i$$

with variance is given as:

$$Var(\bar{y}) = \theta\, S_y^2 \;\; or\; \theta\, \bar{Y}^2 C_y^2 \tag{1}$$

The classical Ratio estimator of $\bar{Y}$, given by [2], is defined as:

$$\bar{y}_R = \bar{y}\frac{\bar{X}}{\bar{x}}$$

where population mean $\bar{X}$ of auxiliary/correlated variable is considered as known. We can write it as:

$$\bar{y}_R = \frac{\bar{y}}{\bar{x}}\bar{X} = \hat{R}\bar{X}$$

where x and y are positively correlated and $\hat{R} = \frac{\bar{y}}{\bar{x}} = \frac{y}{x}$ is the ratio of means or totals of variables y and x

The Bias and MSE of the ratio estimator $\bar{y}_R$ are defined as:

$$B(\bar{y}_R) \cong \theta\frac{1}{\bar{X}}\left[RS_x^2 - \rho_{yx}S_yS_x\right]$$

$$MSE(\bar{y}_R) \cong \theta\left[S_y^2 + R^2S_x^2 - 2R\rho_{yx}S_yS_x\right]$$

which can also be written as

$$MSE(\bar{y}_R) \cong \theta\bar{Y}^2\left[C_y^2 + C_x^2 - 2\rho_{yx}C_yC_x\right] \tag{2}$$

The product estimator delivers a better estimate when the correlation between Y and X is negative and large. It was given by [19] as:

$$\bar{y}_p = \bar{y}\frac{\bar{x}}{\bar{X}}$$

MSE of the product estimator is given as:

$$MSE(\bar{y}_P) \cong \theta\bar{Y}^2\left[C_y^2 + C_x^2 + 2\rho_{yx}C_yC_x\right]$$

[20] and [21] studied the efficiency of sample mean $\bar{y}$, the ratio estimator $\bar{y}_R$ and product estimator $\bar{y}_P$ and concluded that these estimators are most efficient when

$$-\frac{1}{2} \le \rho_{yx}\frac{C_y}{C_x} \le \frac{1}{2}$$

$$\rho_{yx}\frac{C_y}{C_x} > \frac{1}{2}$$

$$\rho_{yx}\frac{C_y}{C_x} < -\frac{1}{2}$$

respectively.

The conventional regression estimator for the population mean $\bar{Y}$ of the variable of interest y using one auxiliary variable x whose mean $\bar{X}$, is given as,

$$\bar{y}_{reg} = \bar{y} + \beta(\bar{X} - \bar{x})$$

where $\beta = S_{yx}/S_x^2$ is the regression coefficient for Y on X

The MSE of $\bar{y}_{reg}$ is

$$MSE(\bar{y}_{reg}) = \theta S_y^2\left[1 - \rho_{yx}^2\right]$$

[5] suggested a ratio estimator with two auxiliary variables X and Z as:

$$\bar{y}_{S_1} = \bar{y}\left(\frac{\bar{X}}{\bar{x}}\right)\left(\frac{\bar{Z}}{\bar{z}}\right)$$

The mean squared error of which is give as:

$$MSE(\bar{y}_{S_1}) = \theta\bar{Y}^2\left[C_y^2 + C_x^2 + C_z^2 - 2\rho_{yx}C_yC_x - 2\rho_{yz}C_yC_z + 2\rho_{xz}C_xC_z\right]$$

The exponential form of ratio estimator for the population mean was given by [7] as:

$$\bar{y}_{BT} = \bar{y}\exp\left[\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right]$$

Its Bias is given as

$$B(\bar{y}_{BT}) \cong \theta\bar{Y}\left[\frac{3}{8}C_x^2 - \frac{1}{2}\rho_{yx}C_yC_x\right]$$

and the MSE of the estimator is given by

$$MSE(\bar{y}_{BT}) \cong \theta\,\bar{Y}^2\left[C_y^2 + \frac{C_x^2}{4} - \rho_{yx}C_yC_x\right]$$

Many authors have modified the ratio and exponential ratio estimator using some other available parameters such as coefficient of variation, kurtosis, skewness, median or correlation coefficient. [22] gave the following general form of estimator which can be used for any available information of X for estimating the mean $\bar{Y}$:

$$\bar{y}_{S_2} = \bar{y}\exp\left[\frac{(a\bar{X}+b)-(a\bar{x}+b)}{(a\bar{X}+b)+(a\bar{x}+b)}\right]$$

Where $a\ (\neq 0)$ and b are either real numbers or the functions of known parameters of X. The MSE of this estimator has been given as follows:

$$MSE(\bar{y}_{S_2}) \cong \theta\bar{Y}^2\left[C_y^2 + \tau^2\frac{C_x^2}{4} - \tau\rho_{yx}C_yC_x\right]$$

Where $\tau = a\bar{X}/(a\bar{X}+b)$

We can also write the expression $\left[\frac{(a\bar{X}+b)-(a\bar{x}+b)}{(a\bar{X}+b)+(a\bar{x}+b)}\right]$ as $\frac{a(\bar{X}-\bar{x})}{a(\bar{X}+\bar{x})+2b}$ as has been used by [18] in their estimator given as:

$$\bar{y}_{SY} = \left[\bar{y}+\hat{\beta}(\bar{X}-\bar{x})\right]\exp\left(\frac{a(\bar{X}-\bar{x})}{a(\bar{X}+\bar{x})+2b}\right)$$

The MSE of which is given as

$$MSE\ (\bar{y}_{SY}) \cong \theta\left[\tau_1^2\left(\frac{S_x^2}{4}\right) + S_y^2(1-\rho^2)\right]$$

where $\tau_1 = a\bar{Y}/(a\bar{X}+b)$

2.3 Proposed Estimator

Making the use of two auxiliary variables, we propose the following estimator:

$$\bar{y}_{SR} = \bar{y}\exp\left[\frac{a(\bar{X}-\bar{x})}{a(\bar{X}+\bar{x})+2b}\right]\left[\frac{\bar{Z}}{\bar{z}}\right]$$

Taylor's linearization approach by [23] can be used to find the mean squared error up to the first degree of approximation in the following steps:

Let the error terms be defined as

$$e_0 = \frac{\bar{y} - \bar{Y}}{\bar{Y}}, e_1 = \frac{\bar{x} - \bar{X}}{\bar{X}}, e_2 = \frac{\bar{z} - \bar{Z}}{\bar{Z}}$$

From the definition of e_0, e_1, e_2 we obtain

$$E(e_0) = E(e_1) = E(e_2) = 0$$

$$V(e_0) = E(e_0^2) = \frac{1-f}{n} C_y^2$$

$$V(e_1) = E(e_1^2) = \frac{1-f}{n} C_x^2$$

$$V(e_2) = E(e_2^2) = \frac{1-f}{n} C_z^2$$

$$Cov(e_0, e_1) = E(e_0 e_1) = \frac{1-f}{n} \rho_{yx} C_y C_x$$

$$Cov(e_0, e_2) = E(e_0 e_2) = \frac{1-f}{n} \rho_{yz} C_y C_z$$

$$Cov(e_1, e_2) = E(e_1 e_2) = \frac{1-f}{n} \rho_{xz} C_x C_z$$

where $\rho_{yx} = \frac{S_{yx}}{\sqrt{S_y^2 S_x^2}}$, $\rho_{yz} = \frac{S_{yz}}{\sqrt{S_y^2 S_z^2}}$, $\rho_{xz} = \frac{S_{xz}}{\sqrt{S_x^2 S_z^2}}$ are the correlation coefficients between yx, yz and xz respectively.

For finding the Bias and MSE of our proposed estimator, we express it in terms of e's (adjusted from the error terms mentioned above), as:

$$\bar{y}_{SR} = \bar{Y}(1 + e_0) \exp\left[\frac{a(\bar{X} - \bar{X} - \bar{X}e_1)}{a(\bar{X} + \bar{X} + \bar{X}e_1) + 2b}\right]\left[\frac{\bar{Z}}{\bar{Z}(1 + e_2)}\right]$$

i.e., we write we replace $\bar{y}, \bar{x}$ and $\bar{z}$ by the following terms

$$\bar{y} = \bar{Y}(1 + e_0), \bar{x} = \bar{X}\,(1 + e_1), \bar{z} = \bar{Z}(1 + e_2)$$

We get

$$\bar{y}_{SR} = \bar{Y}(1 + e_0) \exp\left[-\tfrac{\tau e_1}{2}\left(1 + \tfrac{\tau}{2} e_1\right)^{-1}\right][(1 + e_2)]^{-1} \tag{3}$$

where $\tau = \frac{a\bar{X}}{a\bar{X} + b}$

Expanding and simplifying the right-hand side of (3), and neglecting the higher powers of e's, we get:

$$\bar{y}_{SR} = \bar{Y}\left(1 + e_0 - e_2 - \frac{\tau e_1}{2}\right)$$

Or

$$(\bar{y}_{SR} - \overline{Y}) = \bar{Y}\left(e_0 - e_2 - \frac{\tau e_1}{2}\right) \tag{4}$$

Squaring both sides of (4) and taking expectation while neglecting the higher power terms, we derive the MSE term to the first degree of approximation as:

$$E(\bar{y}_{SR} - \bar{Y})^2 = \bar{Y}^2\left(e_0 - e_2 - \frac{\tau e_1}{2}\right)^2$$

After substituting the variance and co-variance terms we get:

$$MSE(\bar{y}_{SR}) = \bar{Y}^2\left(C_y^2 + C_z^2 + \frac{\tau C_x^2}{4} - \tau\rho_{yx}C_yC_x - 2\rho_{yz}C_yC_z + \tau\rho_{xz}C_xC_z\right) \tag{5}$$

2.4 Efficiency Comparison

The proposed estimator is compared with other existing estimators in terms of efficiencies.

The proposed estimator $\bar{y}_{SR}$ is more efficient than:

(i): $\bar{y}$

if,

$$MSE(\bar{y}_{SR}) < V(\bar{y})$$

$$\theta\bar{Y}^2\left(C_y^2 + C_z^2 + \frac{\tau C_x^2}{4} - \tau\rho_{yx}C_yC_x - 2\rho_{yz}C_yC_z + \tau\rho_{xz}C_xC_z\right) < \theta\,\bar{Y}^2C_y^2$$

or

$$C_z^2 + \frac{\tau C_x^2}{4} - \tau C_{yx} - 2C_{yz} + \tau C_{xz} < 0$$

(ii): $\bar{y}_R$

if,

$$MSE(\bar{y}_{SR}) < V(\bar{y}_R)$$

$$\theta\bar{Y}^2\left(C_y^2 + C_z^2 + \frac{\tau C_x^2}{4} - \tau\rho_{yx}C_yC_x - 2\rho_{yz}C_yC_z + \tau\rho_{xz}C_xC_z\right) < \theta\bar{Y}^2\left[C_y^2 + C_x^2 - 2\rho_{yx}C_yC_x\right]$$

or

$$C_z^2 + C_x^2\left(\frac{\tau}{4} - 1\right) - C_{yx}(\tau - 2) - 2C_{yz} + \tau C_{xz} < 0$$

(iii): $\bar{y}_P$

if,

$$MSE(\bar{y}_{SR}) < V(\bar{y}_P)$$

$$\theta\bar{Y}^2\left(C_y^2 + C_z^2 + \frac{\tau C_x^2}{4} - \tau\rho_{yx}C_yC_x - 2\rho_{yz}C_yC_z + \tau\rho_{xz}C_xC_z\right) < \theta\bar{Y}^2[C_y^2 + C_x^2 + 2\rho_{yx}C_yC_x]$$

or

$$C_z^2 + C_x^2\left(\frac{\tau}{4} - 1\right) - C_{yx}(\tau + 2) - 2C_{yz} + \tau C_{xz} < 0$$

(iv): $\bar{y}_{S_1}$

if,

$$MSE(\bar{y}_{SR}) < V(\bar{y}_{S_1})$$

$$\theta\bar{Y}^2\left(C_y^2 + C_z^2 + \frac{\tau C_x^2}{4} - \tau\rho_{yx}C_yC_x - 2\rho_{yz}C_yC_z + \tau\rho_{xz}C_xC_z\right) < \theta\bar{Y}^2[C_y^2 + C_x^2 + C_z^2 - 2\rho_{yx}C_yC_x - 2\rho_{yz}C_yC_z + 2\rho_{xz}C_xC_z]$$

or

$$C_x^2\left(\frac{\tau}{4} - 1\right) - (\tau - 2)(C_{yx} - C_{xz}) < 0$$

(v) $\bar{y}_{BT}$

if,

$$MSE(\bar{y}_{SR}) < V(\bar{y}_{BT})$$

$$\theta\bar{Y}^2\left(C_y^2 + C_z^2 + \frac{\tau C_x^2}{4} - \tau\rho_{yx}C_yC_x - 2\rho_{yz}C_yC_z + \tau\rho_{xz}C_xC_z\right) < \theta\,\bar{Y}^2\left[C_y^2 + \frac{C_x^2}{4} - \rho_{yx}C_yC_x\right]$$

or

$$C_z^2 + (\tau - 1)(\frac{C_x^2}{4} - C_{yx}) - 2C_{yz} + \tau C_{xz} < 0$$

(vi): $\bar{y}_{S_2}$

if,

$$MSE(\bar{y}_{SR}) < V(\bar{y}_{S_2})$$

$$\theta\bar{Y}^2\left(C_y^2 + C_z^2 + \frac{\tau C_x^2}{4} - \tau\rho_{yx}C_yC_x - 2\rho_{yz}C_yC_z + \tau\rho_{xz}C_xC_z\right) < \theta\bar{Y}^2\left[C_y^2 + \tau^2\frac{C_x^2}{4} - \tau\rho_{yx}C_yC_x\right]$$

or

$$C_z^2 + (\tau - 1)(\frac{C_x^2}{4} - C_{yx}) - 2C_{yz} + \tau C_{xz} < 0$$

(vii): $\bar{y}_{SY}$

if,

$$MSE(\bar{y}_{SR}) < V(\bar{y}_{SY})$$

$$\theta\bar{Y}^2\left(C_y^2 + C_z^2 + \frac{\tau C_x^2}{4} - \tau\rho_{yx}C_yC_x - 2\rho_{yz}C_yC_z + \tau\rho_{xz}C_xC_z\right) < \theta\left[\tau_1^2\left(\frac{S_x^2}{4}\right) + S_y^2(1-\rho^2)\right]$$

Where $\tau_1 = a\bar{Y}/(a\bar{X} + b)$

or

$$\bar{Y}^2\left(C_z^2 - C_y^2\rho_{yx}^2\right) + \frac{C_x^2}{4}(\bar{Y}^2\tau - \bar{X}^2\tau_1^2) - \tau C_{yx} - 2C_{yz} + \tau C_{xz} < 0$$

2.5 Numerical Illustration

For a numerical comparison of different estimators, we utilise the following data set.

Data: [Source: [25]]

Y = Cotton Output

X = Area of Plant

Z = Proportion of Good seeds

The data statistics are:

$$N = 180, n = 70, \bar{Y} = 13.9951, \bar{X} = 27.3981, \bar{Z} = 38.7167, C_y = 0.4180, C_x = 0.4254, C_z = 0.3339$$

$$\rho_{yx} = 0.5630, \rho_{yz} = 0.5273, \rho_{xz} = 0.2589, \beta_2(x) = 4.2724)$$

Table 1: Estimators and their MSE values.

S.no	Estimator	MSE	MSE value
1	Sample Mean $(\bar{y})$	$\theta\,\bar{Y}^2 C_y^2$	0.298
2	Classical Ratio Estimator $\bar{y}_R = \frac{\bar{y}}{\bar{x}}\bar{X}$	$\theta\bar{Y}^2\left[C_y^2 + C_x^2 - 2\rho_{yx}C_yC_x\right]$	0.265
3	Classical Product Estimator $\bar{y}_p = \bar{y}\frac{\bar{x}}{\bar{X}}$	$\theta\bar{Y}^2\left[C_y^2 + C_x^2 + 2\rho_{yx}C_yC_x\right]$	0.950
4	$\bar{y}_{S_1} = \bar{y}\left(\frac{\bar{X}}{\bar{x}}\right)\left(\frac{\bar{Z}}{\bar{z}}\right)$	$\theta\bar{Y}^2\left[C_y^2 + C_x^2 + C_z^2 - 2\rho_{yx}C_yC_x - 2\rho_{yz}C_yC_z + 2\rho_{xz}C_xC_z\right]$	0.330
5	$\bar{y}_{SY} = \left[\bar{y} + \hat{\beta}(\bar{X} - \bar{x})\right]\exp\left(\frac{a(\bar{X} - \bar{x})}{a(\bar{X} + \bar{x}) + 2b}\right)$	$\theta\left[\tau_1^2\left(\frac{S_x^2}{4}\right) + S_y^2(1 - \rho^2)\right]$	0.285
6	**Proposed estimator** $\boldsymbol{\bar{y}_{SR} = \bar{y}\ \mathrm{exp}\left[\frac{a(\bar{X} - \bar{x})}{a(\bar{X} + \bar{x}) + 2b}\right]\left[\frac{\bar{Z}}{\bar{z}}\right]}$	$\boldsymbol{\theta\bar{Y}^2\left(C_y^2 + C_z^2 + \frac{\tau C_x^2}{4} - \tau\rho_{yx}C_yC_x - 2\rho_{yz}C_yC_z + \tau\rho_{xz}C_xC_z\right)}$	**0.206**
For s.no. 5 & 6 estimators, values are for a=1, b=0			

Table 2: Special cases of proposed estimator.

a	b	Estimator using different values of a & b	MSE of Proposed Est
$\boldsymbol{C_x}$	$\boldsymbol{\beta_2(x)}$	$\boldsymbol{\bar{y}_{SR1} = \bar{y}\, exp\left[\frac{C_x(\bar{X}-\bar{x})}{C_x(\bar{X}+\bar{x})+2\beta_2(x)}\right]\left[\frac{\bar{Z}}{\bar{z}}\right]}$	**0.1998**
ρ_{yx}	$\beta_2(x)$	$\bar{y}_{SR2} = \bar{y}\, exp\left[\frac{\rho_{yx}(\bar{X}-\bar{x})}{\rho_{yx}(\bar{X}+\bar{x})+2\beta_2(x)}\right]\left[\frac{\bar{Z}}{\bar{z}}\right]$	0.2003
1	$\beta_2(x)$	$\bar{y}_{SR3} = \bar{y}\, exp\left[\frac{(\bar{X}-\bar{x})}{(\bar{X}+\bar{x})+2\beta_2(x)}\right]\left[\frac{\bar{Z}}{\bar{z}}\right]$	0.2019
C_x	ρ_{yx}	$\bar{y}_{SR4} = \bar{y}\, exp\left[\frac{C_x(\bar{X}-\bar{x})}{C_x(\bar{X}+\bar{x})+2\rho_{yx}}\right]\left[\frac{\bar{Z}}{\bar{z}}\right]$	0.2048
ρ_{yx}	C_x	$\bar{y}_{SR5} = \bar{y}\, exp\left[\frac{\rho_{yx}(\bar{X}-\bar{x})}{\rho_{yx}(\bar{X}+\bar{x})+2C_x}\right]\left[\frac{\bar{Z}}{\bar{z}}\right]$	0.2055
1	ρ_{yx}	$\bar{y}_{SR6} = \bar{y}\, exp\left[\frac{(\bar{X}-\bar{x})}{(\bar{X}+\bar{x})+2\rho_{yx}}\right]\left[\frac{\bar{Z}}{\bar{z}}\right]$	0.2058
1	C_x	$\bar{y}_{SR7} = \bar{y}\, exp\left[\frac{(\bar{X}-\bar{x})}{(\bar{X}+\bar{x})+2C_x}\right]\left[\frac{\bar{Z}}{\bar{z}}\right]$	0.2060
$\beta_2(x)$	ρ_{yx}	$\bar{y}_{SR8} = \bar{y}\, exp\left[\frac{\beta_2(x)(\bar{X}-\bar{x})}{\beta_2(x)(\bar{X}+\bar{x})+2\rho_{yx}}\right]\left[\frac{\bar{Z}}{\bar{z}}\right]$	0.2065
$\beta_2(x)$	C_x	$\bar{y}_{SR9} = \bar{y}\, exp\left[\frac{\beta_2(x)(\bar{X}-\bar{x})}{\beta_2(x)(\bar{X}+\bar{x})+2C_x}\right]\left[\frac{\bar{Z}}{\bar{z}}\right]$	0.2066
1	0	$\bar{y}_{SR10} = \bar{y}\, \exp\left[\frac{(\bar{X}-\bar{x})}{(\bar{X}+\bar{x})}\right]\left[\frac{\bar{Z}}{\bar{z}}\right]$	0.2067

3 Results and Discussion

Based on the information gained from two auxiliary variables, the study suggested a family of new ratio estimators of finite population mean. The results clearly show that the ratio estimator gives less mean square error than the regular estimator (Table 1). In comparison to the sample mean, ratio estimator, and product estimator, exponential ratio estimators perform better. The proposed estimator performs best when the kurtosis of the auxiliary variable X and coefficient of variation of auxiliary variable Z is used (Table 2).

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