Applied Mathematics & Information Sciences
An International Journal

http://dx.doi.org/10.18576/amis/200212

A Subclass of Bi-Univalent Functions Associated with the q-Wright Operator

Abdullah Alsoboh[1,*], *Ala Amourah*[2,3], *Yousef Al-Qudah*[3], *Amer Darweesh*[4] *and Ahmad Al Kasbi*[1]

[1]College of Applied and Health Sciences, A'Sharqiyah University, Post Box No. 42, Post Code No. 400, Ibra, Sultanate of Oman
[2]Mathematics Education Program, Faculty of Education and Arts, Sohar University,Sohar 311, Oman
[3]Department of Mathematics, Faculty of Arts and Science, Amman Arab University, Amman, 11953, Jordan
[4]Department of Mathematics and Statistics, Faculty of Science and Arts, Jordan University of Science and Technology, P.O. Box 3030, Irbid 22110, Jordan

Received: 12 Jan. 2026, Revised: 12 Feb. 2026, Accepted: 24 Feb. 2026
Published online: 1 Mar. 2026

Abstract: Motivated by the growing interaction between q-calculus and geometric function theory, this paper introduces a new subclass of bi-univalent functions defined through a convolution operator associated with the q-Wright function and the q-analogue of Fibonacci numbers. The proposed operator is constructed via the Hadamard convolution, enabling the analytic and combinatorial properties of q-special functions to be naturally embedded into the geometric framework of bi-univalent mappings. Using the fundamental principal of the subordination, we derive the upper bounds of the initial Taylor–Maclaurin coefficients $|\rho_2|, |\rho_3|$, and Fekete–Szegö inequalities. The analysis further demonstrates how the deformation parameter affects the associated coefficient q on the coefficient structure. The present results highlight the structural significance of q-special functions in the construction of convolution-type operators and contribute to the further development of coefficient theory for bi-univalent functions. Moreover, the proposed framework offers a basis for future investigations at the intersection of operator theory and geometric function theory.

Keywords: Analytic Functions, Fekete–Szegö inequality, Fibonacci sequence, quantum calculus , Univalent functions, Wright Functions

1 Introduction

In complex analysis, the theory of geometric functions on an open unit disk is dealing with analytic and univalent functions. We have seen the field move forward largely because of how we now use operators and subordination to categorize functions. The goal behind this is to have an actual tool to extract coefficient values and to study the geometric growth of analytic functions in the unit disk.

Over the past couple decades, we have seen the q-calculus evolve from a niche discrete version of standard calculus into a powerhouse for building q-extensions of classic operators. Its real power lies in its pliability; it allows us to craft generalized analytic operators that do more than just trituration numbers—they give us a much deeper look into the coefficient problems and geometric quirks of both univalent and bi-univalent functions.

What really sets q-calculus apart is how it blends discrete and continuous perspectives. This superposition is what makes it so easy to drag in difference operators and q-special function expansions when we are doing analytic work. It is this specific process that has given us all these new q-versions of standard analytic kernels. Because of that, we have seen a whole new wave of subclasses stand out, such as the q-starlike, q-convex, and q-bi-univalent groups. In conclusion, the q-calculus acts as a bridge; it connects special functions, combinatorics, and orthogonal polynomials into one cohesive framework for GFT. (see, for example, [5,6,7,8,12,13,14,15,16,17,18,19,20,21,27,28,29,30]), some other applications in algebra can be found in [22,23,24,25,26].

Within this setting, the q-Wright function occupies a significant position because of its broad structural form and its connections with other q-special functions, such as the q-Mittag–Leffler and q-hypergeometric functions. When combined with convolution and subordination

* Corresponding author e-mail: abdullah.alsoboh@asu.edu.om

techniques, it serves as an effective tool for defining new analytic operators and corresponding subclasses of bi-univalent functions.In recent work, Alsoboh et al. [4] explored the relationship between the q-calculus and the q-Fibonacci sequence by introducing an appropriate generating kernel, which extends the classical Fibonacci function into the analytic framework. This development underscores the versatility of q-operators in modeling subtle geometric features.

Motivated by recent progress in the area, we introduce a new class of bi-univalent functions defined through a q-Wright operator and subordinated to a kernel related to the q-Fibonacci sequence. A key contribution of this paper is the introduction of a q-operator based on Hadamard product with q-Wright kernels, which leads to a generalized class of bi-univalent functions defined through a q-Fibonacci type subordination condition.

Within this approach, we determine the upper bounds for $|\rho_2|$ and $|\rho_3|$ and derive the related Fekete–Szegö type estimates. The results presented here both recover and broaden several recent advances in the coefficient theory of bi-univalent functions. Moreover, the study underlines the importance of q-special functions in shaping operator-based constructions and advancing research within geometric function theory.

2 Preliminaries

We first introduce the standard class $\mathscr{A}$ consisting of functions that are analytic on the open unit disk

$$\mathscr{O} = \{z = \mathfrak{a} + i\mathfrak{b} \in \mathbb{C} : \mathfrak{a}, \mathfrak{b} \in \mathbb{R},\ |z| < 1\}.$$

From a geometric perspective, $\mathscr{O}$ represents the interior of the unit circle centered at the origin, excluding its boundary. Each function $f \in \mathscr{A}$ is normalized by the conditions

$$f(0) = 0 \qquad \text{and} \qquad f'(0) = 1,$$

which remove translational and dilational degrees of freedom, thereby fixing the mapping uniquely at the origin.

Every $f \in \mathscr{A}$ admits the Taylor–Maclaurin expansion

$$f(z) = z + \sum_{n=2}^{\infty} \rho_n z^n, \qquad z \in \mathscr{O}, \tag{1}$$

where the coefficients ρ_n describe the nonlinear contribution of the mapping.

A function f is called a *Schwarz function* if it is analytic in $\mathscr{O}$, satisfies $f(0) = 0$, and fulfills $|f(z)| < 1$ for all $z \in \mathscr{O}$.

For analytic functions $f_1, f_2 \in \mathscr{A}$, we say that f_1 is *subordinate* to f_2, written $f_1 \prec f_2$, if there exists a Schwarz function η such that

$$f_1(z) = f_2(\eta(z)), \qquad z \in \mathscr{O}.$$

The concept of subordination preserves analyticity and serves as a fundamental tool for comparing geometric inclusion properties of analytic mappings.

Let $\mathsf{S} \subset \mathscr{A}$ denote the family of univalent functions in $\mathscr{O}$. The Carathéodory class P consists of analytic functions p in $\mathscr{O}$ with positive real part. Each $\mathsf{p} \in \mathsf{P}$ can be expressed in the form

$$\mathsf{p}(z) = 1 + \sum_{n=1}^{\infty} \mathsf{p}_n z^n = 1 + \mathsf{p}_1 z + \mathsf{p}_2 z^2 + \cdots, \qquad z \in \mathscr{O}, \tag{2}$$

and satisfies the sharp coefficient bound according to quarter-Koebe theorem

$$|\mathsf{p}_n| \le 2, \qquad n \ge 1, \tag{3}$$

in accordance with the classical Carathéodory lemma [2]. Equivalently,

$$\mathsf{p}(z) \in \mathsf{P} \iff \mathsf{p}(z) \prec \frac{1+z}{1-z}, \qquad z \in \mathscr{O}.$$

For each $f \in \mathsf{S}$, the inverse function f^{-1} exists in a disk of radius at least $\mathsf{r}_0(f) \ge 0.25$ and satisfies

$$z = f^{-1}(f(z)), \qquad \xi = f(f^{-1}(\xi)), \qquad (|\xi| < \mathsf{r}_0(f)). \tag{4}$$

Its series representation takes the form

$$\begin{aligned} \chi(\xi) = f^{-1}(\xi) &= \xi - \rho_2\xi^2 + (2\rho_2^2 - \rho_3)\xi^3 \\ &\quad - (5\rho_2^3 - 5\rho_2\rho_3 + \rho_4)\xi^4 + \cdots. \end{aligned} \tag{5}$$

A function $f \in \mathsf{S}$ is termed *bi-univalent* if both f and its inverse f^{-1} are univalent in $\mathscr{O}$. The collection of all such functions is denoted by $\Sigma \subset \mathsf{S}$. Bi-univalent functions extend the classical notion of univalence by requiring injectivity to be preserved under inversion, and they naturally arise in problems related to coefficient estimates, subordination relations, and geometric transformations. Typical examples include mappings.

$$f(z) = \frac{z}{1+z},\ f(z) = -\log(1-z),\ f(z) = \frac{1}{2}\log\left(\frac{1+z}{1-z}\right),$$

each of which is univalent in $\mathscr{O}$ together with its analytic inverse.

In 2008, Shahed and Salem [1] proposed a q-Wright function, which is defined as follows:

$$\begin{aligned} \mathscr{W}_{\kappa,\vartheta}(z;q^{\tau}) &= \sum_{n=0}^{\infty} \frac{q^{\frac{n(n+1)}{2}} z^n}{\lceil n \rfloor_q !\, \Gamma_{q^{\tau}}(\kappa n + \vartheta)} = \frac{1}{\Gamma_{q^{\tau}}(\vartheta)} \\ &+ \frac{q\, z}{\Gamma_{q^{\tau}}(\kappa + \vartheta)} + \frac{q^3\, z^2}{\lceil 2 \rfloor_q !\, \Gamma_{q^{\tau}}(2\kappa + \vartheta)} + \cdots, \end{aligned} \tag{6}$$

where $\kappa = -\log(1-q)/1 - q^{\tau}$, $\vartheta \in \mathbb{C}$, $z \in \mathscr{O}$, $0 < q < 1$ and

$$\Gamma_{q^{\tau}}(\varkappa) = \frac{(q^{\tau}, q^{\tau})_{\infty}}{(q^{\tau\varkappa}, q^{\tau})_{\infty}} (1 - q^{\tau})^{1-\varkappa}, \qquad \varkappa \ne 0, -1, -2, \cdots.$$

The q-Wright function $\mathscr{W}_{\kappa,\vartheta}(z;q)$ is normalized to obtain

$$\mathbb{R}_{\kappa,\vartheta}(z) = z\Gamma_{q^\tau}(\vartheta)\mathscr{W}_{\kappa,\vartheta}(z;q)$$
$$= z + \sum_{n=2}^{\infty} \frac{q^{\frac{n(n-1)}{2}}\Gamma_{q^\tau}(\vartheta)}{\lceil n-1 \rfloor_q! \Gamma_{q^\tau}(\kappa(n-1)+\vartheta)} z^n. \quad (7)$$

The associated convolution operator $\mathscr{V}_{\kappa,\vartheta}$ that acts on $\mathscr{A}$ is defined by

$$\mathscr{V}_{\kappa,\vartheta} f(z) = \left(\mathbb{R}_{\kappa,\vartheta} * f\right)(z)$$
$$= z + \sum_{n=2}^{\infty} \frac{q^{\frac{n(n-1)}{2}}\Gamma_{q^\tau}(\vartheta)}{\lceil n-1 \rfloor_q! \Gamma_{q^\tau}(\kappa(n-1)+\vartheta)} \rho_n z^n, \quad (8)$$

where $\kappa = -\log(1-q)/1-q^\tau$, $\vartheta \in \mathbb{C}$, $z \in \mathscr{O}$, $0<q<1$.

This operator preserves analyticity in $\mathscr{O}$ and will be used to define new subclasses of bi-univalent functions. Following Alsoboh et al. [4], the q-Fibonacci generating function is defined as

$$\Omega(z;q) = \frac{1+q\lambda_q^2 z^2}{1-\lambda_q z - q\lambda_q^2 z^2}, \quad \lambda_q = \frac{1-\sqrt{4q+1}}{2q}. \quad (9)$$

Its coefficients satisfy the recurrence relation

$$\widehat{\mathsf{p}}_n = \begin{cases} \lambda_q, & n=1, \\ (2q+1)\lambda_q^2, & n=2, \\ (3q+1)\lambda_q^3, & n=3, \\ \left(\varphi_{n+1}(q) + q\varphi_{n-1}(q)\right)\lambda_q^n, & n \geq 4, \end{cases} \quad (10)$$

where the q-Fibonacci polynomials are defined by

$$\varphi_n(q) = \frac{(1-q\lambda_q)^n - \lambda_q^n}{\sqrt{4q+1}}, \quad (11)$$

where

$$\begin{cases} \varphi_0(q) = 0, \\ \varphi_1(q) = 1, \\ \varphi_2(q) = 1, \\ \varphi_3(q) = 1+q, \\ \quad \vdots \end{cases}$$

which degenerate into the classical Fibonacci numbers in the limit $q \to 1^-$. Detailed discussions and related geometric implications can be found in the extensive study [9, 10, 11].

3 Definition and Examples

Motivated by the analytic and geometric features associated with q-Fibonacci numbers, we introduce in this section a new subclass of bi-univalent functions defined through an operator related to the q-Wright function. The construction relies on a suitably formulated generalized q-differential operator, designed to unify and extend several previously studied subclasses through a common subordination framework with respect to an analytic generating function $\Omega(z;q)$.

This operator-based approach brings together different q-analytic components within a consistent setting and provides a systematic method for defining new families of bi-univalent functions. In this way, it broadens the scope of geometric function theory in the context of q-calculus. Furthermore, the proposed framework offers an effective basis for examining coefficient estimates, Fekete–Szegö type functionals, and related geometric properties associated with bi-univalent mappings.

Definition 1. *Let f be a bi-univalent function expressed in the form (1). Then $f \in \mathsf{R}_{\Sigma}^{\kappa,\vartheta}(\gamma;q)$ if and only if the following subordination relations are satisfied:*

$$(1-\gamma)\frac{\mathscr{V}_{\kappa,\vartheta} f(z)}{z} + \gamma \eth_q \langle \mathscr{V}_{\kappa,\vartheta} f(z) \rangle \prec \Omega(z;q), \quad (12)$$

and

$$(1-\gamma)\frac{\mathscr{V}_{\kappa,\vartheta} \chi(\xi)}{\xi} + \gamma \eth_q \langle \mathscr{V}_{\kappa,\vartheta} \chi(\xi) \rangle \prec \Omega(\xi;q), \quad (13)$$

where $\gamma \geq 0$, $\vartheta \in \mathbb{C}$, and $\chi = f^{-1}$ denotes the analytic inverse of f as defined in (5). The parameter λ_q and $\Omega(z;q)$, given by (9), serves as the generating function responsible for the underlying shell-like geometric structure of the domain.

The class $\mathsf{R}_{\Sigma}^{\kappa,\vartheta}(\gamma;q)$ thus serves as a unifying platform connecting the q-calculus and the theory of geometric functions. By tuning the parameters γ and $q \in (0,1)$, one can recover a variety of analytic subclasses that exhibit distinct geometric and structural behaviors within the open unit disk $\mathscr{O}$.

*Example 1.*For $\gamma = 1$ in Definition 1, the class $\mathsf{R}_{\Sigma}^{\kappa,\vartheta}(1;q)$ is characterized by the subordinations

$$\eth_q \langle \mathscr{V}_{\kappa,\vartheta} f(z) \rangle \prec \Omega(z;q), \qquad \eth_q \langle \mathscr{V}_{\kappa,\vartheta} \chi(\xi) \rangle \prec \Omega(\xi;q),$$

which correspond to a purely q-differential subclass whose geometric structure depends exclusively on the first q-derivative of the transformed function.

*Example 2.*For $\gamma = 0$ in Definition 1, the class $\mathsf{R}_{\Sigma}^{\kappa,\vartheta}(0;q)$ satisfies

$$\frac{\mathscr{V}_{\kappa,\vartheta} f(z)}{z} \prec \Omega(z;q), \qquad \frac{\mathscr{V}_{\kappa,\vartheta} \chi(\xi)}{\xi} \prec \Omega(\xi;q),$$

representing the fundamental subclass in which the characterization is governed entirely by the operator quotient, devoid of any q-differential influence.

*Example 3.*As $q \to 1^-$, the Definition 1 reduces to the classical operator-based subclass

$$(1-\gamma)\frac{\mathscr{V}_{\kappa,\vartheta}f(z)}{z} + \gamma\left(\mathscr{V}_{\kappa,\vartheta}f(z)\right)' \prec \frac{1+\lambda^2 z^2}{1-\lambda z-\lambda^2 z^2},$$

along with the corresponding subordination for $\chi = f^{-1}$. This limit demonstrates that the newly proposed q-framework generalizes well-established analytic subclasses in the classical setting.

4 Coefficient Estimates of the Class $\mathsf{R}_{\Sigma}^{\kappa,\vartheta}(\gamma;q)$

In this section, we derive coefficient estimates for functions belonging to the class $\mathsf{R}_{\Sigma}^{\kappa,\vartheta}(\gamma;q)$ introduced in Definition 1. The main goal is to obtain explicit upper bounds for the initial Taylor–Maclaurin coefficients, specifically $|\rho_2|$ and $|\rho_3|$, corresponding to this newly defined family of bi-univalent functions. These estimates play a fundamental role in understanding the analytic structure and geometric behavior of the class under consideration.

We first consider an analytic function of the form

$$\mathsf{p}(z) = 1 + \mathsf{p}_1 z + \mathsf{p}_2 z^2 + \mathsf{p}_3 z^3 + \cdots,$$

which is assumed to satisfy the subordination relation

$$\mathsf{p}(z) \prec \Omega(z;q).$$

That is, there exists a Schwarz function η such that $\mathsf{p}(z) = \Omega(\eta(z);q)$ for all $z \in \mathscr{O}$.

Consequently, there exists a Schwarz function $\varphi \in \mathsf{P}$ such that

$$|\varphi(z)| < 1 \quad (z \in \mathscr{O}) \qquad \text{and} \qquad \mathsf{p}(z) = \Omega(\varphi(z);q).$$

Following the classical representation, we define

$$h(z) = \frac{1+\varphi(z)}{1-\varphi(z)} = 1 + \varepsilon_1 z + \varepsilon_2 z^2 + \varepsilon_3 z^3 + \cdots, \qquad z \in \mathscr{O}, \tag{14}$$

where $h \in \mathsf{P}$ denotes the Carathéodory function associated with φ.

From the above, the analytic function $\varphi(z)$, subordinate to $\Omega(z;q)$, admits the following power-series expansion:

$$\varphi(z) = \frac{\varepsilon_1 z}{2} + \frac{1}{2}\left(\varepsilon_2 - \frac{\varepsilon_1^2}{2}\right) z^2 + \frac{1}{2}\left(\varepsilon_3 - \varepsilon_1\varepsilon_2 - \frac{\varepsilon_1^3}{4}\right) z^3 + \cdots. \tag{15}$$

Substituting this expansion into $\Omega(\varphi(z);q)$, we obtain the following.

$$\Omega(\varphi(z);q) = 1 + \frac{\widehat{\mathsf{p}_1}\varepsilon_1}{2} z + \frac{1}{2}\left[\left(\varepsilon_2 - \frac{\varepsilon_1^2}{2}\right)\widehat{\mathsf{p}_1} + \frac{\varepsilon_1^2}{2}\widehat{\mathsf{p}_2}\right] z^2 + \frac{1}{2}\left[\left(\varepsilon_3 - \varepsilon_1\varepsilon_2 + \frac{\varepsilon_1^3}{4}\right)\widehat{\mathsf{p}_1} + \varepsilon_1\left(\varepsilon_2 - \frac{\varepsilon_1^2}{2}\right)\widehat{\mathsf{p}_2} + \frac{\varepsilon_1^3}{4}\widehat{\mathsf{p}_3}\right] z^3 + \cdots \tag{16}$$

In a similar fashion, corresponding to the inverse mapping $\chi = f^{-1}$, there exists a Schwarz function ν analytic in $\mathscr{O}$ with $|\nu(\xi)| < 1$ for all $\xi \in \mathscr{O}$ such that

$$\mathsf{p}(\xi) = \Omega(\nu(\xi);q).$$

Thus, the function associated with the inverse is also subordinate to the generating kernel $\Omega(\cdot;q)$ through the Schwarz function ν.

To proceed, we introduce the corresponding function Carathéodory.

$$\kappa(\xi) = \frac{1+\nu(\xi)}{1-\nu(\xi)} = 1 + \theta_1\xi + \theta_2\xi^2 + \theta_3\xi^3 + \cdots, \quad \xi \in \mathscr{O}, \tag{17}$$

where $\kappa \in \mathsf{P}$. Consequently, the Schwarz function ν admits the representation

$$\nu(\xi) = \frac{\theta_1}{2}\xi + \frac{1}{2}\left(\theta_2 - \frac{\theta_1^2}{2}\right)\xi^2 + \frac{1}{2}\left(\theta_3 - \theta_1\theta_2 - \frac{\theta_1^3}{4}\right)\xi^3 + \cdots. \tag{18}$$

Substituting this expansion into the generating function $\Omega(\cdot;q)$, we obtain the series representation

$$\Omega(\nu(\xi);q) = 1 + \frac{\widehat{\mathsf{p}_1}\theta_1}{2}\xi + \frac{1}{2}\left[\left(\theta_2 - \frac{\theta_1^2}{2}\right)\widehat{\mathsf{p}_1} + \frac{\theta_1^2}{2}\widehat{\mathsf{p}_2}\right]\xi^2 + \frac{1}{2}\left[\left(\theta_3 - \theta_1\theta_2 + \frac{\theta_1^3}{4}\right)\widehat{\mathsf{p}_1} + \theta_1\left(\theta_2 - \frac{\theta_1^2}{2}\right)\widehat{\mathsf{p}_2} + \frac{\theta_1^3}{4}\widehat{\mathsf{p}_3}\right]\xi^3 + \cdots. \tag{19}$$

With the above expansions at hand, we are now in a position to obtain sharp estimates for the initial Taylor–Maclaurin coefficients of functions belonging to the class $\mathsf{R}_{\Sigma}^{\kappa,\vartheta}(\gamma;q)$. These limits explicitly reflect how the behavior of the coefficient is influenced by the q-deformation and the geometric parameter γ. Consequently, the resulting inequalities offer a clear description of the analytic features and geometric flexibility exhibited by the associated bi-univalent mappings.

Theorem 1.*Let $f \in \mathsf{R}_{\Sigma}^{\kappa,\vartheta}(\gamma;q)$. Then*

$$|\rho_2| \le \min\left\{\frac{|\lambda_q|\,\Gamma_{q^\tau}(\kappa+\vartheta)}{q\left|1+q\gamma\right|\Gamma_{q^\tau}(\vartheta)}, L\right\}, \tag{20}$$

and

$$|\rho_3| \le \frac{\lambda_q^2 \Gamma_{q^\tau}^2(\kappa+\vartheta)}{q^2(1+q\gamma)^2\Gamma_{q^\tau}^2(\vartheta)} + \left|\frac{\lambda_q \lceil 2\rfloor_q! \,\Gamma_{q^\tau}(2\kappa+\vartheta)}{q^3(1+q\lceil 2\rfloor_q\gamma)\Gamma_{q^\tau}(\vartheta)}\right|. \tag{21}$$

where

$$L = \frac{|\lambda_q|\Gamma_{q^\tau}(\kappa+\vartheta)\sqrt{\lceil 2\rfloor_q \Gamma_{q^\tau}(2\kappa+\vartheta)}}{q\sqrt{\left|\begin{array}{l} q\lambda_q\Gamma_{q^\tau}(\vartheta)\Gamma_{q^\tau}^2(\kappa+\vartheta)\left(1+q\lceil 2\rfloor_q\gamma\right) \\ \quad -\lceil 2\rfloor_q\Gamma_{q^\tau}(2\kappa+\vartheta)\left((2q+1)\lambda_q-1\right) \\ \quad \times\Gamma_{q^\tau}^2(\vartheta)(1+q\gamma)^2 \end{array}\right|}}$$

Proof. Let $f \in \mathsf{R}_{\Sigma}^{\kappa,\vartheta}(\gamma;q)$ and $\xi = f^{-1}$. Taking into account (12) and (13), we have

$$(1-\gamma)\frac{\mathscr{V}_{\kappa,\vartheta}f(z)}{z} + \gamma\eth_q\langle\mathscr{V}_{\kappa,\vartheta}f(z)\rangle = \Omega(\varphi(z);q), \ (z\in\mathscr{O}), \tag{22}$$

and

$$(1-\gamma)\frac{\mathscr{V}_{\kappa,\vartheta}\chi(\xi)}{\xi} + \gamma\eth_q\langle\mathscr{V}_{\kappa,\vartheta}\chi(\xi)\rangle = \Omega(\nu(\xi);q), \ (\xi\in\mathscr{O}). \tag{23}$$

Upon substituting the operator $\mathscr{V}_{\kappa,\vartheta}f(z)$ defined in (8) into equation (22), the left-hand side transforms into

$$\begin{aligned}(1-\gamma)&\frac{\mathscr{V}_{\kappa,\vartheta}f(z)}{z} + \gamma\eth_q\langle\mathscr{V}_{\kappa,\vartheta}f(z)\rangle \\ &= 1 + \frac{q(1+q\gamma)\Gamma_{q^\tau}(\vartheta)}{\Gamma_{q^\tau}(\kappa+\vartheta)}\rho_2 z \\ &+ \frac{q^3(1+q\lceil 2\rfloor_q\gamma)\Gamma_{q^\tau}(\vartheta)}{\Gamma_{q^\tau}(\kappa+\vartheta)}\rho_3 z^2 + \cdots.\end{aligned} \tag{24}$$

Similarly for equation (23), the left-hand side transforms into

$$(1-\gamma)\frac{\mathscr{V}_{\kappa,\vartheta}\chi(\xi)}{\xi} + \gamma\eth_q\langle\mathscr{V}_{\kappa,\vartheta}\chi(\xi)\rangle = 1 - \frac{q(1+q\gamma)\Gamma_{q^\tau}(\vartheta)}{\Gamma_{q^\tau}(\kappa+\vartheta)}\rho_2\xi + \frac{q^3(1+q\lceil 2\rfloor_q\gamma)\Gamma_{q^\tau}(\vartheta)}{\Gamma_{q^\tau}(\kappa+\vartheta)}\left(2\rho_2^2-\rho_3\right)\xi^2 + \cdots \tag{25}$$

Substituting (16) and (24) into (23) produces

$$\begin{aligned}&\frac{q(1+q\gamma)\Gamma_{q^\tau}(\vartheta)}{\Gamma_{q^\tau}(\kappa+\vartheta)}\rho_2 z + \frac{q^3(1+q\lceil 2\rfloor_q\gamma)\Gamma_{q^\tau}(\vartheta)}{\Gamma_{q^\tau}(\kappa+\vartheta)}\rho_3 z^2 + \cdots \\ &= \frac{\widehat{\mathsf{p}}_1\varepsilon_1}{2}z + \frac{1}{2}\left[\left(\varepsilon_2 - \frac{\varepsilon_1^2}{2}\right)\widehat{\mathsf{p}}_1 + \frac{\varepsilon_1^2}{2}\widehat{\mathsf{p}}_2\right]z^2 + \cdots.\end{aligned} \tag{26}$$

In addition, substituting (19) and (25) into (24) yields

$$\begin{aligned}&-\frac{q(1+q\gamma)\Gamma_{q^\tau}(\vartheta)}{\Gamma_{q^\tau}(\kappa+\vartheta)}\rho_2\xi + \\ &\frac{q^3(1+q\lceil 2\rfloor_q\gamma)\Gamma_{q^\tau}(\vartheta)}{\Gamma_{q^\tau}(\kappa+\vartheta)}(2\rho_2^2-\rho_3)\xi^2 + \cdots \\ &= \frac{\widehat{\mathsf{p}}_1\theta_1}{2}\xi + \frac{1}{2}\left[\left(\theta_2 - \frac{\theta_1^2}{2}\right)\widehat{\mathsf{p}}_1 + \frac{\theta_1^2}{2}\widehat{\mathsf{p}}_2\right]\xi^2 + \cdots.\end{aligned} \tag{27}$$

Equating the pertinent coefficient in (26) and (27), we obtain the following.

$$\frac{q(1+q\gamma)\Gamma_{q^\tau}(\vartheta)}{\Gamma_{q^\tau}(\kappa+\vartheta)}\rho_2 = \frac{\widehat{\mathsf{p}}_1\varepsilon_1}{2} \tag{28}$$

$$-\frac{q(1+q\gamma)\Gamma_{q^\tau}(\vartheta)}{\Gamma_{q^\tau}(\kappa+\vartheta)}\rho_2 = \frac{\widehat{\mathsf{p}}_1\theta_1}{2} \tag{29}$$

$$\frac{q^3(1+q\lceil 2\rfloor_q\gamma)\Gamma_{q^\tau}(\vartheta)}{\lceil 2\rfloor_q!\,\Gamma_{q^\tau}(2\kappa+\vartheta)}\rho_3 = \frac{1}{2}\left[\left(\varepsilon_2 - \frac{\varepsilon_1^2}{2}\right)\widehat{\mathsf{p}}_1 + \frac{\varepsilon_1^2}{2}\widehat{\mathsf{p}}_2\right] \tag{30}$$

and

$$\frac{q^3(1+q\lceil 2\rfloor_q\gamma)\Gamma_{q^\tau}(\vartheta)}{\lceil 2\rfloor_q!\,\Gamma_{q^\tau}(2\kappa+\vartheta)}(2\rho_2^2-\rho_3) = \frac{1}{2}\left[\left(\theta_2 - \frac{\theta_1^2}{2}\right)\widehat{\mathsf{p}}_1 + \frac{\theta_1^2}{2}\widehat{\mathsf{p}}_2\right] \tag{31}$$

From (28) and (29), we have

$$\varepsilon_1 = -\theta_1 \iff \varepsilon_1^2 = \theta_1^2, \tag{32}$$

and

$$\rho_2^2 = \frac{\lambda_q^2\Gamma_{q^\tau}^2(\kappa+\vartheta)}{8q^2(1+q\gamma)^2\Gamma_{q^\tau}^2(\vartheta)}\left(\varepsilon_1^2+\theta_1^2\right). \tag{33}$$

Using (3), we have

$$|\rho_2| \le \frac{|\lambda_q|\,\Gamma_{q^\tau}(\kappa+\vartheta)}{q|1+q\gamma|\Gamma_{q^\tau}(\vartheta)}. \tag{34}$$

Rearranging the terms in (33), we obtain the equivalent relation

$$\varepsilon_1^2+\theta_1^2 = \frac{8q^2(1+q\gamma)^2\Gamma_{q^\tau}^2(\vartheta)}{\lambda_q^2\Gamma_{q^\tau}^2(\kappa+\vartheta)}\rho_2^2. \tag{35}$$

Now, by summing (30) and (31), we obtain

$$\begin{aligned}&\frac{2q^3(1+q\lceil 2\rfloor_q\gamma)\Gamma_{q^\tau}(\vartheta)}{\lceil 2\rfloor_q\,\Gamma_{q^\tau}(2\kappa+\vartheta)}\rho_2^2 \\ &= \frac{(\varepsilon_2+\theta_2)\lambda_q}{2} + \left[\frac{(2q+1)\lambda_q^2}{4} - \frac{\lambda_q}{4}\right]\left(\varepsilon_1^2+\theta_1^2\right).\end{aligned} \tag{36}$$

By substituting the expression of $(\varepsilon_1^2+\theta_1^2)$ from (35) into the equation (36) and simplifying the resulting identity, we arrive at the following explicit representation for ρ_2^2:

$$\rho_2^2 = \frac{(\varepsilon_2+\theta_2)\lceil 2\rfloor_q \Gamma_{q^\tau}(2\kappa+\vartheta)\lambda_q^2\Gamma_{q^\tau}^2(\kappa+\vartheta)}{4q^2\left\{\begin{array}{l} q\lambda_q\Gamma_{q^\tau}(\vartheta)\Gamma_{q^\tau}^2(\kappa+\vartheta)\left(1+q\lceil 2\rfloor_q\gamma\right) \\ -\lceil 2\rfloor_q\Gamma_{q^\tau}(2\kappa+\vartheta)\left((2q+1)\lambda_q-1\right) \\ \times\Gamma_{q^\tau}^2(\vartheta)\left(1+q\gamma\right)^2 \end{array}\right\}}. \tag{37}$$

Using (3) for (37), we have

$$|\rho_2| \le \frac{|\lambda_q|\Gamma_{q^\tau}(\kappa+\vartheta)\sqrt{\lceil 2\rfloor_q\Gamma_{q^\tau}(2\kappa+\vartheta)}}{q\sqrt{\left|\begin{array}{l} q\lambda_q\Gamma_{q^\tau}(\vartheta)\Gamma_{q^\tau}^2(\kappa+\vartheta)\left(1+q\lceil 2\rfloor_q\gamma\right) \\ -\lceil 2\rfloor_q\Gamma_{q^\tau}(2\kappa+\vartheta)\left((2q+1)\lambda_q-1\right) \\ \times\Gamma_{q^\tau}^2(\vartheta)\left(1+q\gamma\right)^2 \end{array}\right|}}. \tag{38}$$

Now, so as to find the bound on $|\rho_3|$, let us subtract from (30) and (31) along (33), we obtain the following.

$$\rho_3 = \rho_2^2 + \frac{\lambda_q\lceil 2\rfloor_q!\,\Gamma_{q^\tau}(2\kappa+\vartheta)}{4q^3\left(1+q\lceil 2\rfloor_q\gamma\right)\Gamma_{q^\tau}(\vartheta)}(\varepsilon_2-\theta_2). \tag{39}$$

Hence, we get

$$|\rho_3| \le |\rho_2|^2 + \left|\frac{\lambda_q\lceil 2\rfloor_q!\,\Gamma_{q^\tau}(2\kappa+\vartheta)}{q^3\left(1+q\lceil 2\rfloor_q\gamma\right)\Gamma_{q^\tau}(\vartheta)}\right|. \tag{40}$$

Then, in view of (34), we obtain

$$|\rho_3| \le \frac{\lambda_q^2\Gamma_{q^\tau}^2(\kappa+\vartheta)}{q^2(1+q\gamma)^2\Gamma_{q^\tau}^2(\vartheta)} + \left|\frac{\lambda_q\lceil 2\rfloor_q!\,\Gamma_{q^\tau}(2\kappa+\vartheta)}{q^3\left(1+q\lceil 2\rfloor_q\gamma\right)\Gamma_{q^\tau}(\vartheta)}\right|. \tag{41}$$

This proves (21).

Theorem 2. *For $\alpha \in \mathbb{C}^*$, let $f \in \mathrm{R}_\Sigma^{\kappa,\vartheta}(\gamma;q)$. Then* $\left|\rho_3-\alpha\rho_2^2\right| \le$

$$\begin{cases} \dfrac{|\lambda_q|\lceil 2\rfloor_q\Gamma_{q^\tau}(2\kappa+\vartheta)}{q^3\Gamma_{q^\tau}(\vartheta)\left|1+q\lceil 2\rfloor_q\gamma\right|}, & 0\le|\mathscr{L}(\alpha)|\le\dfrac{1}{q\left(1+q\lceil 2\rfloor_q\gamma\right)} \\[2ex] \dfrac{|\lambda_q|\lceil 2\rfloor_q\Gamma_{q^\tau}(2\kappa+\vartheta)}{q^2\Gamma_{q^\tau}(\vartheta)}|\mathscr{L}(\alpha)|, & |\mathscr{L}(\alpha)|\ge\dfrac{1}{q\left(1+q\lceil 2\rfloor_q\gamma\right)} \end{cases} \tag{42}$$

where

$$\mathscr{L}(\alpha) = \frac{(1-\alpha)\lambda_q\Gamma_{q^\tau}^2(\kappa+\vartheta)}{\left\{\begin{array}{l} q\lambda_q\Gamma_{q^\tau}^2(\kappa+\vartheta)\left(1+q\lceil 2\rfloor_q\gamma\right) - \\ \lceil 2\rfloor_q\Gamma_{q^\tau}(\vartheta)\Gamma_{q^\tau}(2\kappa+\vartheta)\left((2q+1)\lambda_q-1\right)(1+q\gamma)^2 \end{array}\right\}}$$

Proof. Let $f \in \mathrm{R}_\Sigma^{\kappa,\vartheta}(\gamma;q)$, from (37) and (39) we have

$$\begin{aligned}\rho_3-\alpha\rho_2^2 &= \frac{1}{q\left(1+q\lceil 2\rfloor_q\gamma\right)}(\varepsilon_2-\theta_2) \\ &+ \frac{(1-\alpha)\lambda_q\Gamma_{q^\tau}^2(\kappa+\vartheta)(\varepsilon_2+\theta_2)}{\left\{\begin{array}{l} q\lambda_q\Gamma_{q^\tau}^2(\kappa+\vartheta)\left(1+q\lceil 2\rfloor_q\gamma\right) \\ -\lceil 2\rfloor_q\Gamma_{q^\tau}(\vartheta)\Gamma_{q^\tau}(2\kappa+\vartheta)\left((2q+1)\lambda_q-1\right)(1+q\gamma)^2 \end{array}\right\}} \\ &= \frac{\lambda_q\lceil 2\rfloor_q\Gamma_{q^\tau}(2\kappa+\vartheta)}{4q^2\Gamma_{q^\tau}(\vartheta)}\left[\begin{array}{l}\left(\mathscr{L}(\alpha)+\dfrac{1}{q\left(1+q\lceil 2\rfloor_q\gamma\right)}\right)\varepsilon_2 \\ +\left(\mathscr{L}(\alpha)-\dfrac{1}{q\left(1+q\lceil 2\rfloor_q\gamma\right)}\right)\theta_2\end{array}\right],\end{aligned} \tag{43}$$

where

$$\mathscr{L}(\alpha) = \frac{(1-\alpha)\lambda_q\Gamma_{q^\tau}^2(\kappa+\vartheta)}{\left\{\begin{array}{l} q\lambda_q\Gamma_{q^\tau}^2(\kappa+\vartheta)\left(1+q\lceil 2\rfloor_q\gamma\right) \\ -\lceil 2\rfloor_q\Gamma_{q^\tau}(\vartheta)\Gamma_{q^\tau}(2\kappa+\vartheta)\left((2q+1)\lambda_q-1\right)(1+q\gamma)^2 \end{array}\right\}}$$

Consequently, taking the modulus on both sides of (43), we arrive at the desired result stated in (42).

5 Corollaries

The coefficient estimates established in Theorems 1 and 2 admit several meaningful specializations when particular choices of the parameters γ and q are considered. By fixing these parameters appropriately, the general bounds reduce to more explicit and tractable forms, thereby recovering a number of significant subclasses as immediate consequences.

The corollaries presented below illustrate these reductions and emphasize notable instances of the class $\mathrm{R}_\Sigma^{\kappa,\vartheta}(\gamma;q)$ that arise directly from the principal results. These special cases further demonstrate the flexibility of the proposed framework and its capacity to unify diverse families of bi-univalent functions within a common analytic setting.

Corollary 1. *Let $\alpha \in \mathbb{C}^*$ and $f \in \mathrm{R}_\Sigma^{\kappa,\vartheta}(1;q)$ (as in Example 1). Then the initial coefficients satisfy*

$$|\rho_2| \le \min\left\{\frac{|\lambda_q|\Gamma_{q^\tau}(\kappa+\vartheta)}{q|1+q|\Gamma_{q^\tau}(\vartheta)}, R\right\},$$

and

$$|\rho_3| \le \frac{\lambda_q^2\Gamma_{q^\tau}^2(\kappa+\vartheta)}{q^2(1+q)^2\Gamma_{q^\tau}^2(\vartheta)} + \left|\frac{\lambda_q\lceil 2\rfloor_q!\,\Gamma_{q^\tau}(2\kappa+\vartheta)}{q^3\left(1+q\lceil 2\rfloor_q\right)\Gamma_{q^\tau}(\vartheta)}\right|.$$

Moreover, for any $\alpha \in \mathbb{C}^*$, $\left|\rho_3-\alpha\rho_2^2\right| \le$

$$\begin{cases} \dfrac{|\lambda_q|\lceil 2\rfloor_q\Gamma_{q^\tau}(2\kappa+\vartheta)}{q^3\Gamma_{q^\tau}(\vartheta)\left|1+q\lceil 2\rfloor_q\right|}, & |\mathscr{L}(\alpha)|\le\dfrac{1}{q\left(1+q\lceil 2\rfloor_q\right)}, \\[2ex] \dfrac{|\lambda_q|\lceil 2\rfloor_q\Gamma_{q^\tau}(2\kappa+\vartheta)|\mathscr{L}(\alpha)|}{q^2\Gamma_{q^\tau}(\vartheta)}, & |\mathscr{L}(\alpha)|\ge\dfrac{1}{q\left(1+q\lceil 2\rfloor_q\right)}, \end{cases}$$

where

$$R=\frac{|\lambda_q|\,\Gamma_{q^\tau}(\kappa+\vartheta)\sqrt{\lceil 2\rfloor_q\,\Gamma_{q^\tau}(2\kappa+\vartheta)}}{q\sqrt{\left|\begin{array}{l} q\lambda_q\Gamma_{q^\tau}(\vartheta)\Gamma^2_{q^\tau}(\kappa+\vartheta)\left(1+q\lceil 2\rfloor_q\right)\\ -\left(\lceil 2\rfloor_q\Gamma_{q^\tau}(2\kappa+\vartheta)\left((2q+1)\lambda_q-1\right)\right.\\ \left.\times\Gamma^2_{q^\tau}(\vartheta)\left(1+q\right)^2\right)\end{array}\right|}}$$

$$\mathscr{L}(\alpha)=\frac{(1-\alpha)\,\lambda_q\Gamma^2_{q^\tau}(\kappa+\vartheta)}{\left\{\begin{array}{l} q\lambda_q\Gamma^2_{q^\tau}(\kappa+\vartheta)\left(1+q\lceil 2\rfloor_q\right)\\ -\left(\lceil 2\rfloor_q\Gamma_{q^\tau}(\vartheta)\Gamma_{q^\tau}(2\kappa+\vartheta)\right.\\ \left.\times\left((2q+1)\lambda_q-1\right)\left(1+q\right)^2\right)\end{array}\right\}}.$$

Corollary 2. *Let $\alpha\in\mathbb{C}^*$ and suppose $f\in\mathsf{R}^{\kappa,\vartheta}_{\Sigma}(0;q)$. (Equivalently, by Example 2. Then the initial coefficients satisfy*

$$|\rho_2|\;\leq\;\min\left\{\frac{|\lambda_q|\,\Gamma_{q^\tau}(\kappa+\vartheta)}{q\,\Gamma_{q^\tau}(\vartheta)},\,T\right\},$$

and

$$|\rho_3|\;\leq\;\frac{\lambda_q^2\,\Gamma^2_{q^\tau}(\kappa+\vartheta)}{q^2\,\Gamma^2_{q^\tau}(\vartheta)}\;+\;\left|\frac{\lambda_q\,\lceil 2\rfloor_q!\,\Gamma_{q^\tau}(2\kappa+\vartheta)}{q^3\,\Gamma_{q^\tau}(\vartheta)}\right|.$$

Moreover,
$|\rho_3-\alpha\rho_2^2|\;\leq$

$$\begin{cases}\dfrac{|\lambda_q|\,\lceil 2\rfloor_q\Gamma_{q^\tau}(2\kappa+\vartheta)}{q^3\,\Gamma_{q^\tau}(\vartheta)}, & 0\leq|\mathscr{L}_0(\alpha)|\leq\dfrac{1}{q},\\[2ex] \dfrac{|\lambda_q|\,\lceil 2\rfloor_q\Gamma_{q^\tau}(2\kappa+\vartheta)}{q^2\,\Gamma_{q^\tau}(\vartheta)}\,|\mathscr{L}_0(\alpha)|, & |\mathscr{L}_0(\alpha)|\geq\dfrac{1}{q},\end{cases}$$

where

$$T=\frac{|\lambda_q|\,\Gamma_{q^\tau}(\kappa+\vartheta)\sqrt{\lceil 2\rfloor_q\,\Gamma_{q^\tau}(2\kappa+\vartheta)}}{q\sqrt{\left|\begin{array}{l} q\lambda_q\Gamma_{q^\tau}(\vartheta)\Gamma^2_{q^\tau}(\kappa+\vartheta)\\ -\lceil 2\rfloor_q\Gamma_{q^\tau}(2\kappa+\vartheta)\left((2q+1)\lambda_q-1\right)\Gamma^2_{q^\tau}(\vartheta)\end{array}\right|}}$$

$$\mathscr{L}_0(\alpha)=\frac{(1-\alpha)\,\lambda_q\Gamma^2_{q^\tau}(\kappa+\vartheta)}{\left(\begin{array}{l} q\lambda_q\Gamma^2_{q^\tau}(\kappa+\vartheta)-\left(\lceil 2\rfloor_q\Gamma_{q^\tau}(\vartheta)\right.\\ \left.\times\Gamma_{q^\tau}(2\kappa+\vartheta)\left((2q+1)\lambda_q-1\right)\Gamma_{q^\tau}(\vartheta)\right)\end{array}\right)}.$$

Corollary 3. *Let $\alpha\in\mathbb{C}^*$ and suppose $f\in\mathsf{R}^{\kappa,\vartheta}_{\Sigma}(\gamma)$ as in Example 3. Then, the initial coefficients satisfy*

$$|\rho_2|\;\leq\;\min\left\{\frac{|\lambda|\,\Gamma(\kappa+\vartheta)}{|1+\gamma|\,\Gamma(\vartheta)},\,M\right\},$$

and

$$|\rho_3|\;\leq\;\frac{\lambda^2\,\Gamma^2(\kappa+\vartheta)}{(1+\gamma)^2\,\Gamma^2(\vartheta)}\;+\;\left|\frac{2\lambda\,\Gamma(2\kappa+\vartheta)}{(1+2\gamma)\,\Gamma(\vartheta)}\right|.$$

Moreover,
$|\rho_3-\alpha\rho_2^2|\;\leq$

$$\begin{cases}\dfrac{2|\lambda|\,\Gamma(2\kappa+\vartheta)}{\Gamma(\vartheta)\,|1+2\gamma|}, & 0\leq|\mathscr{L}_{\rm cl}(\alpha)|\leq\dfrac{1}{1+2\gamma},\\[2ex] \dfrac{2|\lambda|\,\Gamma(2\kappa+\vartheta)}{\Gamma(\vartheta)}\,|\mathscr{L}_{\rm cl}(\alpha)|, & |\mathscr{L}_{\rm cl}(\alpha)|\geq\dfrac{1}{1+2\gamma},\end{cases}$$

where

$$M=\frac{|\lambda|\,\Gamma(\kappa+\vartheta)\sqrt{2\Gamma(2\kappa+\vartheta)}}{\sqrt{\left|\begin{array}{c}\lambda\Gamma(\vartheta)\Gamma^2(\kappa+\vartheta)\left(1+2\gamma\right)\\ -2\Gamma(2\kappa+\vartheta)\left(3\lambda-1\right)\Gamma^2(\vartheta)\left(1+\gamma\right)^2\end{array}\right|}}$$

$$\mathscr{L}_{\rm cl}(\alpha)=\frac{(1-\alpha)\,\lambda\,\Gamma^2(\kappa+\vartheta)}{\left(\begin{array}{c}\lambda\Gamma^2(\kappa+\vartheta)\left(1+2\gamma\right)\\ -2\Gamma(\vartheta)\Gamma(2\kappa+\vartheta)\left(3\lambda-1\right)\left(1+\gamma\right)^2\end{array}\right)}.$$

6 Conclusion and Future Work

In this work, we have proposed and analyzed a new subclass of bi-univalent functions associated with shell-type image domains generated by the q-Wright function and structured through the q-analogue of Fibonacci numbers. By introducing a generalized q-differential operator defined via Hadamard convolution with kernels involving the q-Wright function, we established a coherent analytic setting that integrates tools from q-calculus into the framework of geometric function theory.

Applying the method of subordination, we derived explicit bounds for the initial Taylor–Maclaurin coefficients $|\rho_2|$ and $|\rho_3|$, together with the corresponding Fekete–Szegö-type inequalities for the newly defined class. The results obtained herein extend several previously known subclasses as particular cases and demonstrate the effectiveness of q-special functions in generating refined geometric configurations and enriched analytic behavior within the theory of bi-univalent functions.

Future Work. The framework developed in this study opens several directions for future research. A natural continuation would be to examine analogous q-operators in the context of multivalent and quasi-biunivalent functions, with the objective of deriving corresponding coefficient estimates and distortion results. Further generalizations can be obtained by replacing the q-Wright kernel with alternative q-special functions, such as the q-Bessel, q–Mittag-Leffler or q-Rabotnov functions, thus generating broader operator-defined subclasses with enriched analytic structures.

Acknowledgement

The authors thank the anonymous referee for a careful review of the details and for helpful comments that improved this paper.

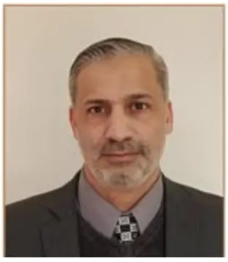
References

[1] M. El-Shahed and A. Salem, q-Analogue of Wright Function, Abstract and Applied Analysis **2008**, Article ID 962849 (2008).
[2] P. L. Duren, Univalent Functions, Springer, New York, 1983.
[3] W. Ma and D. Minda, A Unified Treatment of Some Special Classes of Univalent Functions, in: Proc. Conf. Complex Analysis, Tianjin, China, **1992**, 157–169.
[4] A. Alsoboh, A. Amourah, O. Alnajar, M. Ahmed, and T. M. Seoudy, Exploring q-Fibonacci Numbers in Geometric Function Theory: Univalence and Shell-like Starlike Curves, Mathematics **13**, 1294 (2025).
[5] A. Alsoboh and G. Oros, A Class of Bi-Univalent Functions in a Leaf-Like Domain Defined through Subordination via q-Calculus, Mathematics **2024**, 2366–2380 (2024).
[6] A. Amourah, A. Alsoboh, D. Breaz, and S. M. El-Deeb, A Bi-Starlike Class in a Leaf-Like Domain Defined through Subordination via q-Calculus, Mathematics **12**, 1735 (2024).
[7] A. Alsoboh, M. Çağlar, and M. Buyankara, Fekete-Szegő Inequality for a Subclass of Bi-Univalent Functions Linked to q-Ultraspherical Polynomials, Contemp. Math. **2024**, 2366–2380 (2024).
[8] A. Alsoboh, A. Amourah, M. Darus, and C. A. Rudder, Studying the Harmonic Functions Associated with Quantum Calculus, Mathematics **11**(10), 2220 (2023).
[9] H. Güney, G. Murugusundaramoorthy, and J. Sokół, Subclasses of Bi-Univalent Functions Related to Shell-Like Curves Connected with Fibonacci Numbers, Acta Univ. Sapient. Math. **10**(1), 70–84 (2018).
[10] J. Dziok, R. K. Raina, and J. Sokół, Certain Results for a Class of Convex Functions Related to a Shell-Like Curve Connected with Fibonacci Numbers, Comput. Math. Appl. **61**, 2605–2613 (2011).
[11] J. Dziok, R. K. Raina, and J. Sokół, On λ_q-Convex Functions Related to a Shell-Like Curve Connected with Fibonacci Numbers, Appl. Math. Comput. **218**, 996–1002 (2011).
[12] M. Ahmed, A. Alsoboh, A. Amourah, and J. Salah, On the Fractional q-Differintegral Operator for Subclasses of Bi-Univalent Functions Subordinate to q-Ultraspherical Polynomials, Eur. J. Pure Appl. Math. **18**(3), 6586 (2025).
[13] A. Alsoboh and M. Darus, New Subclass of Analytic Functions Defined by q-Differential Operator with Respect to k-Symmetric Points, Int. J. Math. Comput. Sci. **14**, 761–773 (2019).
[14] O. Alnajar, M. Darus, A. Amourah, and A. Alsoboh, Touchard Polynomials and the Identified New Subclass of Analytic Functions, International Journal of Analysis and Applications **24**, 46 (2026).
[15] A. Alsoboh, A. Amourah, W. G. Atshan, J. Salah, and T. Sasa, A Study of Bi-Univalent Class in Leaf-Like Domains Using Quantum Calculus Through Subordination, International Journal of Analysis and Applications **23**, 293 (2025).
[16] A. Alsoboh, A. Amourah, F. M. Sakar, O. Ogilat, G. M. Gharib, and N. Zomot, Coefficient Estimation Utilizing the Faber Polynomial for a Subfamily of Bi-Univalent Functions, Axioms **12**(6), 512 (2023).
[17] A. Alsoboh, A. Amourah, K. Al Mashrafi, and T. Sasa, Bi-Starlike and Bi-Convex Function Classes Connected to Shell-Like Curves and the q-Analogue of Fibonacci Numbers, International Journal of Analysis and Applications **23**, 201 (2025).
[18] T. Al-Hawary, A. Amourah, A. Alsoboh, O. Ogilat, I. Harny, and M. Darus, Applications of q-Ultraspherical Polynomials to Bi-Univalent Functions Defined by q-Saigo's Fractional Integral Operators, AIMS Math. **9**(7), 17063–17075 (2024).
[19] F. Yousef, A. A. Amourah, and M. Darus, Differential Sandwich Theorems for p-Valent Functions Associated with a Certain Generalized Differential Operator and Integral Operator, Ital. J. Pure Appl. Math. **36**, 543–556 (2016).
[20] A. Alsoboh, A. Amourah, and J. Salah, A Study of Bi-Convex Classes in Leaf-Like Domains Using Quantum Calculus through Subordination, Results Nonlinear Anal. **8**(3), 210–221 (2025).
[21] M. A. Ahmed and A. J. Pryde, Semigroup Crossed Products and the Induced Algebras of Lattice-Ordered Groups, J. Math. Anal. Appl. **364**(2), 498–507 (2010).
[22] M. Ahmed, Universal Covariant Representations and Positive Elements, AZJM **15**(1), 44–52 (2025).
[23] M. Ahmed and F. Mohd, The Graded Annihilating Submodule Graph, AKCE Int. J. Graphs Comb. (2025), 1–9.
[24] F. Moh'd and M. Ahmed, Ring Graph Induced by a Ring Endomorphism, Open Math. **23**(1), 20250197 (2025).
[25] M. Ahmed, Commutator Ideals in C*-Crossed Products by Hereditary Subsemigroups, Ann. Funct. Anal. **10**(3), 370–380 (2019).
[26] M. Ahmed, C*-Algebras Generated by Isometries and True Representations, Bol. Soc. Paran. Mat. (3s.) **38**(5), 215–232 (2020).
[27] Alsoboh, A., and Darus, M., New subclass of analytic functions defined by q-differential operator with respect to k-symmetric points, *Int. J. Math. Comput. Sci.* **14**, 761–773 (2019).
[28] A. Amourah, A. Alsoboh, J. Salah, and K. Al Kalbani, Bounds on Initial Coefficients for Bi-Univalent Functions Linked to q-Analog of Le Roy-Type Mittag-Leffler Function, WSEAS Trans. Math. **23**, 714–722 (2024).
[29] M. Illafe, A. Hussen, M. H. Mohd, and F. Yousef, On a Subclass of Bi-Univalent Functions Affiliated with Bell and Gegenbauer Polynomials, Bol. Soc. Parana. Mat. **43**(3), 1–10 (2025).
[30] A. Alsoboh, A. S. Tayyah, A. Amourah, A. A. Al-Maqbali, K. Al Mashraf, and T. Sasa, Hankel Determinant Estimates for Bi-Bazilevič-Type Functions Involving q-Fibonacci Numbers, Eur. J. Pure Appl. Math. **18**(3), 6698 (2025).

Abdullah Alsoboh received the Ph.D. degree in Mathematics from Universiti Kebangsaan Malaysia (UKM), Malaysia. He is currently serving as an Assistant Professor of Mathematics in the Faculty of Applied and Health Sciences, A'Sharqiyah University, Sultanate of Oman. His research interests lie in geometric function theory, analytic and bi-univalent functions, q-calculus, and special functions, with a particular focus on the study of operator-based subclasses and their analytic properties. He has authored and co-authored several research papers in well-established international journals of pure and applied mathematics. Dr. Alsoboh is an active referee and editorial board member for a number of reputable mathematical journals.

Ala Amourah received a Ph.D. degree in Mathematics from Universiti Kebangsaan Malaysia (UKM), Malaysia. He is currently serving as an Associate Professor in the Mathematics Education Program, Faculty of Education and Arts, Sohar University, Sohar 311, Sultanate of Oman. His research interests lie in geometric function theory, analytic and bi-univalent functions, q-calculus, and special functions, with a particular focus on the study of operator-based subclasses and their analytic properties. He has authored and co-authored several research papers in well-established international journals of pure and applied mathematics.

Yousef Al-Qudah received a Ph.D. degree in Mathematics Education from Universiti Kebangsaan Malaysia (UKM), Malaysia. He is currently serving as an Associate Professor in the Mathematics Department, Faculty of Science, Amman Arab University, Jordan. His research interests include mathematical problem-solving strategies. Yousef Al-Qudah has published numerous research articles in international journals of mathematics and education and actively participates in academic conferences and collaborative projects.



Amer Darweesh is a professor of mathematics at Jordan University of Science and Technology where he is currently the chairman of Mathematics and Statistics department. In 2015, he received a Ph.D. degree in Mathematics from Louisiana State University, Baton Rouge, USA. He works in harmonic analysis of functions on unit disk. On the other hand he works in applied mathematics and he published several papers in finding analytic or approximate solutions for differential or integro differential equations.

Ahmed Al Kasbi is an Assistant Professor of Mathematics in the Department of Basic and Applied Sciences, Faculty of Applied and Health Sciences, A'Sharqiyah University, Sultanate of Oman. His research focuses on mathematical modeling of coastal and environmental systems, with particular interest in analytical approaches to effluent dispersion, pollutant transport, and tidal processes in marine environments. His work addresses key issues related to fluid dynamics, discharge mechanisms, and the optimization of water distribution systems. Dr. Al Kasbi has more than twenty-two years of academic experience in Oman. He has taught at A'Sharqiyah University, Sultan Qaboos University, the University of Technology and Applied Sciences–Ibra, and has also served within the Ministry of Education. His teaching experience spans a wide range of undergraduate mathematics courses. In addition to teaching, he has been actively involved in academic service, including supervising mathematics teachers, mentoring students, and participating in various institutional committees. He has presented his research at international conferences and has published in peer-reviewed journals, contributing to the advancement of environmental fluid mechanics and applied mathematical modeling.