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Notes on G-Algebra and its Derivations

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Abstract: In this paper we generate a G-algebra from a non-empty set and we obtain the quotient G-algebra via normal subalgebra. Furthermore, we prove a fundamental theorem on homomorphism for G-algebra. We prove that every G-algebra satisfying the associative law is a 2-group. We also show that every BP-algebra is a G-algebra and introduce a necessary condition for which the converse will be true. Finally, we introduce the notion of a left-right (resp. right-left) derivation of G-algebra. We show that the composition of two derivations is a derivation and we investigate some related properties.

Keywords: Derivations; G-algebra, p-semisimple

1 Introduction and preliminaries

The notion of G-algebra was introduced in [1], as a generalization of QS-algebra. It has been shown in [1] that every QS-algebra is a G-algebra but the converse need not, in general, be true. The notion of derivations as defined in rings and near-rings theory (see [2]) has been applied to BCI-algebra in 2004 by Jun and Xin. Then they introduced a new concept of derivations in BCI-algebra. As in [3], a map $d : X \longrightarrow X$ is said to be a left-right derivation (briefly (l,r)-derivation) of X if it satisfies the identity $d(x*y) = (d(x)*y) \wedge (x*d(y))$ for all $x,y \in X$. If d satisfies the identity $d(x*y) = (x*d(y)) \wedge (d(x)*y)$ for all $x,y \in X$, then d is said to be a right-left derivation (briefly (r,l)-derivation) of X. If d is both (l,r)-and (r,l)-derivation, then d is a derivation of X. Many researches have been done on derivations of BCI-algebra in different aspects. For example, (α,β)-derivations of BCI-algebra was introduced in [4] and some related properties are investigated. In [5], the notion of t-derivations of BCI-algebra was given and the study was extended to t-derivations in a p-semisimple BCI-algebra. A new kind of derivations in BCI-algebra has been introduced in [6]. Other authors applied the notion of derivations, as defined above, on different classes of abstract algebras: BCC-algebra, B-algebra and BCI-algebra and some related properties have been investigated ([7], [8], [9]).

Motivated by the work done on derivations of BCI-algebras and derivations of other algebra classes, we study in this paper derivations of G-algebra. We start in Section 1 by giving definitions and propositions needed. In Section 2, inspired by the work done on BG-algebras [10], we construct a G-algebra from a non-empty set; we use the notion of normal subalgebra to obtain the quotient G-algebra. As a consequence, we obtain the fundamental theorem of homomorphisms of G-algebra. Also we show that every BP-algebra is a G-algebra and we show that every G-algebra satisfying the identity $(z*x)*(z*y) = y*x$, is a BP-algebra. In the final section, we start with the concept of (l,r)- and (r,l)-derivations as defined in [3] then the concept is applied to G-algebra and a new form of the definition is introduced using the fact that $x \wedge y = x$ in G-algebra. We conclude the section with Theorem 3.2 and Theorem 3.3 where we show that the composition of two derivations d_1 and d_2 is also a derivation and that $d_1 \circ d_2 = d_2 \circ d_1$ when d_2 is a (l,r)-derivation and d_1 is a (r,l)- derivation.

We recall the following definitions and propositions related to QS-algebra, BP-algebra and G-algebra.

Definition 1.1.([11],[12]) Let X be a set with a binary operation $*$ and a constant 0. Then $(X,*,0)$ is called a QS-algebra, if it satisfies the following conditions:

(1) $x*0 = x$,
(2) $(z*x)*(z*y) = y*x$,

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(3) $x * x = 0$,
(4) $(x * y) * z = (x * z) * y$, for all $x, y, z \in X$.

Definition 1.2.([11], [13]) An algebra $(X, *, 0)$ is called a *BP*-algebra if it satisfies the following conditions:

(1) $x * x = 0$,
(2) $x * (x * y) = y$,
(3) $(x * z) * (y * z) = x * y$, for all $x, y, z \in X$.

Definition 1.3.([1, Definition 2.1]) A G-algebra is a non-empty set X with a constant 0 and a binary operation $*$ satisfying the axioms:

(1) $x * x = 0$,
(2) $x * (x * y) = y$, for all $x, y \in X$.

Proposition 1.1.([1, Proposition 2.1]) If $(X, *, 0)$ is a G-algebra, then the following conditions hold:

(1) $x * 0 = x$,
(2) $0 * (0 * x) = x$, for any $x \in X$.

Proposition 1.2.([1, Proposition 2.2]) Let $(X, *, 0)$ be a G-algebra. Then, the following conditions hold for any $x, y \in X$,

(1) $(x * (x * y)) * y = 0$,
(2) $x * y = 0 \Longrightarrow x = y$,
(3) $0 * x = 0 * y \Longrightarrow x = y$.

Lemma 1.1.([1, Lemma 2.1]) Let $(X, *, 0)$ be a G-algebra. Then $a * x = a * y$ implies $x = y$ for any $a, x, y \in X$.

Definition 1.4.([10]) A non-empty subset N of X is said to be normal of X if:

$$(x * a) * (y * b) \in N \text{ for any } x * y, a * b \in N.$$

Definition 1.5.([1]) For any G-algebra X, consider the set $B(X) = \{x \in X \mid 0 * x = 0\}$. If $B(X) = \{0\}$ then G-algebra is said to be p-semisimple.

2 G-algebra

In this section, we construct a G-algebra from a given binary operation, we also create a G-algebra from a given one using normal subalgebra.

In any G-algebra, we can define a partial order $\leq$ by putting $x \leq y$ if and only if $y * x = 0$. This means that a G-algebra can be considered as a partially ordered set with $x \leq x$ and $x \leq y * (y * x)$. Denote $y * (y * x)$ by $x \wedge y$ for all $x, y \in X$. Thus using Definition 1.3(2), we have $x \wedge y = x$.

Proposition 2.1. Let $(X, *, 0)$ be a G-algebra. Then for any $x, y, z \in X$, we have:

(1) For $x \neq y$, $x \wedge y \neq y \wedge x$,
(2) $x \wedge (y \wedge z) = (x \wedge y) \wedge z$,
(3) $x \wedge 0 = x$ and $0 \wedge x = 0$,
(4) For $x \neq 0$, $x \wedge (y * z) \neq (x \wedge y) * (x \wedge z)$.

Proof. Direct to prove.

Next we define a binary operation $*$ on X as follows:

Definition 2.1. Let X be a set with $0 \in X$. Define a binary operation $*$ on X for all $x, y \in X$ by:

$$x * y := \begin{cases} x & if \quad y = 0, \\ 0 & if \quad y \neq 0, x = y, \\ y & if \quad y \neq 0, x \neq y. \end{cases}$$

Theorem 2.1. If we define a binary operation $*$ on X as in Definition 2.1, then $(X, *, 0)$ is a G-algebra.

Proof. We will show that $x * (x * y) = y$, in the three cases as follows:

If $x \neq y$ then $x * (x * y) = x * y = y$.
If $x = y$ then it is easy to see that $x * (x * y) = x * (x * x) = x * 0 = x = y$.
If $y = 0$, we have $x * (x * y) = x * (x * 0) = x * x = 0 = y$.

The following G-algebra is constructed using Theorem 2.1.

Example 2.1. Let $X = \{0, 1, 2, 3, 4\}$ in which $*$ is defined by the Cayley table (Table 1). Then $(X, *, 0)$ is a G-algebra.

Table 1: Cayley table

$*$	0	1	2	3	4
0	0	1	2	3	4
1	1	0	2	3	4
2	2	1	0	3	4
3	3	1	2	0	4
4	4	1	2	3	0

Definition 2.2. A non-empty subset S of a G-algebra X is called a G-subalgebra if for all $x, y \in S$ implies $x * y \in S$.

Theorem 2.2. Let $(X, *, 0)$ be a G-algebra. Then every normal subset N of X is a subalgebra of X but the converse is not true.

Proof. Let $x, y \in N$, then by Proposition 2.1(1), $x * 0$, $y * 0 \in N$. Since N is normal, we have $(x * y) * (0 * 0) \in N$.

Table 2: Cayley table

$*$	0	1	2
0	0	1	2
1	1	0	2
2	2	1	0

That is, $x*y \in N$. Therefore, N is a subalgebra of X. The converse is proved using the following example.

Example 2.2. Consider the G-algebra given by the Cayley table (Table 2). It is easy to see that $N = \{0,2\}$ is a subalgebra of X but is not normal of X as $0*2$, $1*2 \in N$ but $(0*1)*(2*2) = 1*0 = 1 \notin N$.

Lemma 2.1. Let $(X,*,0)$ be a G-algebra and N a normal subalgebra of X. The relation R on X defined by: xRy if and only if $x*y \in N$, where $x,y \in X$, is an equivalence relation on X.

Proof. Let $x \in X$. Since N is normal, we have $0 \in N$. Thus, $x*x \in N$. Hence, R is reflexive. To prove that R is symmetry, consider $y*y \in N$ and $x*y \in N$. As N is normal, we have $(y*x)*(y*y) = (y*x)*0 = y*x \in N$. For transitivity, suppose that $x*y \in N$ and $y*z \in N$. Then, $x*y \in N$ and $z*y \in N$. As N is normal, we have, $x*z \in N$. This proves that R is an equivalence relation on X.

Definition 2.3. For each element $x \in X$, the equivalence class of x denoted by $[x]_N$ is the set of all elements $y \in Y$ such that x is related to y. That is, $[x]_N = \{y \in X \mid xRy\}$.

Consider the set of all equivalence classes $\{[x]_N \mid x \in X\} =: X/N$. We show in the next theorem that X/N the quotient of G-algebra X by N is a G-algebra as well. Furthermore, we obtain the fundamental theorem of homomorphisms of G-algebras as a consequence.

Theorem 2.3. Let $(X,*,0)$ be a G-algebra and N be a normal subalgebra of X. Then X/N is a G-algebra.

Proof. Define the operation $*$ by $[x]_N * [y]_N := [x*y]_N$. The operation is well defined as if xRp and yRq then $x*p \in N$ and $y*q \in N$. From the definition of normality, we have $(x*y)*(p*q) \in N$, that is, $(x*y)R(p*q)$. Thus, $[x*y]_N = [p*q]_N$. To check that X/N is a G-algebra, take $[x]_N \in X/N$. Then $[x]_N * [x]_N = [x*x]_N = [0]_N = \{x \in X \mid xR0\} = \{x \in X \mid x*0 \in N\} = \{x \in X \mid x \in N\} = N$, (where N is the zero element in X/N). Hence the first condition holds. Also,

$$[x]_N * ([x]_N * [y]_N) = [x*(x*y)]_N = [y]_N.$$

Therefore, X/N is a G-algebra.

In the next part, known notions are applied to G-algebras.

Definition 2.4. Let X and Y be G-algebras. A mapping ϕ: $X \longrightarrow Y$ is called a homomorphism if $\phi(x*y) = \phi(x)*\phi(y)$, $\forall\, x,y \in X$. The homomorphism ϕ is said to be a monomorphism (resp., an epimorphism) if it is injective (resp., surjective). If the map ϕ is both injective and surjective then X and Y are said to be isomorphic, written $X \cong Y$. For any homomorphism ϕ: $X \longrightarrow Y$, the set $\{x \in X \mid \phi(x) = 0_Y\}$ is called the kernel of ϕ and denoted by $Ker\phi$.

Lemma 2.2. Let ϕ: $(X,*,0_X) \longrightarrow (Y,\circ,0_Y)$ be a homomorphism of G-algebras, then we have the following:
(i) $\phi(0_X) = 0_Y$,
(ii) $Ker\phi$ is a normal G-subalgebra of X,
(iii) $Im\phi = \{y \in Y \mid y = \phi(x),$ for some $x \in X\}$ is a G-subalgebra.

Proof.
(i) Using Definition 1.3(1), $\phi(0_X) = \phi(0_X * 0_X) = \phi(0_X)*\phi(0_X) = 0_Y$.
(ii) Obviously, $Ker\phi \neq \emptyset$, as $0_X \in Ker\phi$. Let $x*y$, $a*b \in Ker\phi$. Then, $\phi(x)*\phi(y) = \phi(a)*\phi(b) = 0_Y$. From Proposition 1.2(2) we get that $\phi(x) = \phi(y)$ and $\phi(a) = \phi(b)$. It follows that, $\phi((x*a)*(y*b)) = \phi(x*a)*\phi(y*b) = (\phi(x)*\phi(a))*(\phi(x)*\phi(a)) = 0_Y$. Consequently, $(x*a)*(y*b) \in Ker\phi$ and so $Ker\phi$ is a normal G-subalgebra of X.
(iii) Direct to prove.

Theorem 2.4. Let ϕ: $(X,*,0_X) \longrightarrow (Y,\circ,0_Y)$ be an epimorphism of G-algebras and $N = Ker\phi$, then we have $X/N \cong Y$.

Proof. Define a mapping $\psi : X/Ker\phi \longrightarrow Y$ by

$$\psi([x]_{Ker\phi}) = \phi(x), \forall x \in X.$$

Suppose $[x]_{Ker\phi} = [y]_{Ker\phi}$, then $x*y \in Ker\phi$. It follows that, $\phi(x*y) = 0_Y$. From Proposition 1.2(2), $\phi(x) = \phi(y)$. That is, $\psi([x]_{Ker\phi}) = \psi([y]_{Ker\phi})$. Hence, ψ is well defined. The mapping ψ is a homomorphism, since $\psi([x]_{Ker\phi} * [y]_{Ker\phi}) = \psi([x*y]_{Ker\phi}) = \phi(x*y) = \phi(x)*\phi(y) = \psi([x]_{Ker\phi}) * \psi([y]_{Ker\phi})$. To check that ψ is injective, let $[x]_{Ker\phi}$, $[y]_{Ker\phi} \in X/Ker\phi$ with $[x]_{Ker\phi} \neq [y]_{Ker\phi}$. Then $x*y \notin Ker\phi$. It follows that $\phi(x*y) = \phi(x)*\phi(y) \neq 0_Y$ i.e. $\phi(x) \neq \phi(y)$. Hence, $\psi([x]_{Ker\phi}) \neq \psi([y]_{Ker\phi})$. The mapping ψ is surjective as for any $y \in Y$, there exists $x \in X$ such that $y = \psi([x]_{Ker\phi})$. Thus ψ is an isomorphism. This completes the proof.

Observe that G-algebra may not satisfy the associative law as in Example 2.2. If the associative law holds then we have the following:

Theorem 2.5. Let $(X,*,0_X)$ be a G-algebra. Then the following conditions are equivalent:
(i) The associative law holds on X.
(ii) X is a 2-group.

Proof. Consider the associative law $(x*y)*z = x*(y*z)$. Put $x = y = z$, we get $0*x = x*0$. From Proposition 2.1(1), we will have $0*x = x*0 = x$. Thus, the element 0 is the identity of X. From Definition 1.3(1), we have $x*x = 0$, this means that every element x has an inverse which is the element x its self. Therefore, $(X,*,0)$ is a group. Moreover, having $x*x = 0$ makes $(X,*,0)$ a 2-group. The converse is obvious.

In the next part we investigate the relation between G-algebra and BP-algebra.

Theorem 2.6.
(i) Every BP-algebra is a G-algebra.
(ii) Every G-algebra satisfying $(z*x)*(z*y) = y*x$ is a BP-algebra.

Proof. Easy and direct to prove.

Definition 2.5. We say that an element $x \in X$ is minimal if $x*y = 0$ implies $y = x$.

Theorem 2.7. Let $(X,*,0_X)$ be a G-algebra. Then:
(i) X is p-semisimple.
(ii) Every element in X is a minimal element.

Proof. (i) We have $B(X) = \{x \in X \mid 0*x = 0\}$. It follows from Proposition 1.2(2) that $B(X) = \{x \in X \mid x = 0\} = \{0\}$. Hence, every G-algebra is a p-semisimple.
(ii) Let x be an element in X such that $y \le x$ for some $y \in X$. Hence, $x*y = 0$ and so, from Proposition 1.2(2), $x = y$. Thus x is a minimal element in X.

3 Derivations of G-algebra

In this section we start by introducing the notation of a derivation of a G-algebra.

Definition 3.1. Let X be a G-algebra and d a self-map of X. We say that

d is (l,r)-derivation of X if $d(x*y) = (d(x)*y) \wedge (x*d(y))$,
d is (r,l)-derivation of X if $d(x*y) = (x*d(y)) \wedge (d(x)*y)$.
If d is both (l,r)-derivation and (r,l)-derivation of X then we say that d is a derivation of X.

Remark 3.1. In G-algebra, we have defined $x \wedge y = x$. Thus, to check that d is (l,r)-derivation of X, it is enough to check that $d(x*y) = d(x)*y$. Similarly, if $d(x*y) = x*d(y)$ then d is (r,l)-derivation of X.

Hence we can re-write Definition 3.1 as follows:

Definition 3.2. Let X be a G-algebra and d a self-map of X. We say that d is a derivation of X if d is (l,r)-derivation of X and (r,l)-derivation of X. That is, for all $x,y \in X$:

Table 3: Cayley table

$*$	0	1	2	3
0	0	3	2	1
1	1	0	3	2
2	2	1	0	3
3	3	2	1	0

$d(x*y) = d(x)*y$ and
$d(x*y) = x*d(y)$, respectively.

Example 3.1. Consider the G-algebra given by Cayley table (Table 2). Define a map $d : X \longrightarrow X$ by:

$$d(x) = \begin{cases} 1 & if \quad x \in \{0,2\}, \\ 0 & if \quad \text{otherwise}. \end{cases}$$

Then by direct calculations we have $d(2*2) = d(0) = 1$ and $d(2)*2 = 1*2 = 2$. As $d(2*2) \neq d(2)*2$ then d is not (l,r)-derivation of X. Similarly, d is not (r,l)-derivation of X as $d(1*2) \neq 1*d(2)$. Hence, d is not a derivation of X.

Example 3.2. Let $X = \{0,1,2,3\}$ in which $*$ is defined by (Table 3). Define a map $d : X \longrightarrow X$ by:

$$d(x) = \begin{cases} 2 & if \quad x = 0, \\ 3 & if \quad x = 1, \\ 0 & if \quad x = 2, \\ 1 & if \quad x = 3. \end{cases}$$

Then it is straight forward to check that d is a derivation of X.

Theorem 3.1. In G-algebras X, the identity map is a derivation on X.

Proof. We need to consider two cases to prove that d is a derivation of X. We will show that d is a (l,r)-derivation of X and it can be shown similarly that d is a (r,l)-derivation of X.
Let $x,y \in X$. If $x = y$ then $d(x*x) = d(0) = 0$. On the other hand, $d(x)*x = x*x = 0$.
If $x \neq y$ then either $d(x*y) = d(x)$ or $d(x*y) = d(y)$. If $d(x*y) = d(x)$ then on one hand, $d(x*y) = x$. On the other hand, $d(x*y) = d(x)*y$. Thus, $x = x*y$ and so, from Proposition 2.1(1), $y = 0$. Therefore, $d(x)*y = x$. Consider the case $d(x*y) = d(y)$. Then $d(x*y) = y$. We have, $d(x*y) = d(x)*y = x*y$. Therefore, $y = x*y$. Thus, $d(x*y) = d(x*(x*y)) = d(y) = y$. Hence, d is a (l,r)-derivation of X.

Definition 3.3. A derivation d of a G-algebra is said to be regular if $d(0) = 0$.

Lemma 3.1. If d is a regular derivation of a G-algebra X, then d is the identity map on X.

Proof. If d is a regular derivation of a G-algebra X then $d(0) = 0$. From Definition 1.3(1) we know that $x*x = 0$ and so $d(x*x) = d(0)$. Hence, $d(x)*x = 0$. Therefore, from Proposition 1.2(2), we have $d(x) = x$.

In the next part we define the composition of two derivations then we study some related properties.

Definition 3.4. Let X be a G-algebra and d_1, d_2 be two self-maps of X. We define $d_1 \circ d_2$: $X \longrightarrow X$, as $(d_1 \circ d_2)(x) = d_1(d_2(x))$, for all $x \in X$.

Theorem 3.2. Let X be a G-algebra and d_1, d_2 be two derivations of X. Then $d_1 \circ d_2$ is a derivation of X.

Proof. We will prove that $d_1 \circ d_2$ is a (l,r)-derivation using the assumption that d_1, d_2 are (l,r)-derivations. Let $x, y \in X$. Then $(d_1 \circ d_2)(x*y) = d_1(d_2(x*y)) = d_1(d_2(x)*y) = d_1(d_2(x))*y = ((d_1 \circ d_2)(x)*y)$.

We can prove similarly that $d_1 \circ d_2$ is a (r,l)-derivation of X and thus $d_1 \circ d_2$ is a derivation of X.

Lemma 3.2. Let X be a G-algebra and d_1, d_2 are either two (l,r)-derivations or (r,l)-derivations of X. Then the composition of the derivations is not necessary commutative.

Proof. Suppose that d_1, d_2 are (l,r)-derivations, then $(d_1 \circ d_2)(x*y) = d_1(d_2(x*y)) = d_1(d_2(x)*y) = d_1d_2(x)*y$. On the other hand, $(d_2 \circ d_1)(x*y) = d_2(d_1(x*y)) = d_2(d_1(x)*y) = d_2d_1(x)*y$. Hence, $(d_1 \circ d_2)(x*y) \neq (d_2 \circ d_1)(x*y)$.

Similarly, we can show that if d_1, d_2 are (r,l)-derivations, then $(d_1 \circ d_2)(x*y) = x*d_1d_2(y)$ and $(d_2 \circ d_1)(x*y) = x*d_2d_1(y)$. Therefore, $(d_1 \circ d_2)(x*y) \neq (d_2 \circ d_1)(x*y)$ in this case as well.

In the next theorem we give the assumptions needed for d_1, d_2 to assure that the composition between them is commutative.

Theorem 3.3. Let X be a G-algebra and suppose that d_2 is a (l,r)-derivation and d_1 is a (r,l)-derivation of X. Then the composition of the derivations is commutative.

Proof. We will show that $d_1 \circ d_2 = d_2 \circ d_1$ for all $x, y \in X$. To start, suppose that d_2 is a (l,r)-derivation, then $(d_1 \circ d_2)(x*y) = d_1(d_2(x*y)) = d_1(d_2(x)*y) = (d_2(x)*d_1(y))$ as d_1 is a (r,l)-derivation. Thus,

$$(d_1 \circ d_2)(x*y) = d_2(x)*d_1(y). \tag{1}$$

Similarly, suppose that d_1 is a (r,l)-derivation, then $(d_2 \circ d_1)(x*y) = d_2(d_1(x*y)) = d_2(x*d_1(y)) = (d_2(x)*d_1(y))$ as d_2 is a (l,r)-derivation. Hence,

$$(d_2 \circ d_1)(x*y) = d_2(x)*d_1(y). \tag{2}$$

From (1) and (2), we have $(d_1 \circ d_2)(x*y) = (d_2 \circ d_1)(x*y)$. Let $y = 0$, then $(d_1 \circ d_2)(x) = (d_2 \circ d_1)(x)$ as $x*0 = x$. Therefore, $d_1 \circ d_2 = d_2 \circ d_1$.

4 Conclusion

We have used the notion of normal subalgebra to obtain the quotient G-algebra. We proved that every G-algebra satisfying the associative law is a 2-group. We also showed that every BP-algebra is a G-algebra and introduce a necessary condition for which the converse will be true. Finally, we introduced the notion of a left-right (resp. right-left) derivation of G-algebra and we investigated some related properties.

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