Numerical Study of Heat Exchange in Human Body Through Clothes and Thermal Stability of Biological Tissues

Aijaz A. Najar and M. A. Khanday *

Department of Mathematics, University of Kashmir, Srinagar-190006, India.

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Abstract: The change in ambient temperature plays a key role in body thermoregulation. The use of protective layer (clothes) has not only ethical values but also acts as a shield to combat with the severe environmental conditions. A mathematical model on bioheat transfer has been formulated with appropriate diffusing and matching conditions at the dermal layers and the protective layer. The finite difference scheme has been employed to solve the model and the conditions for the thermal stability of tissue temperatures were illustrated using MATLAB and FlexPDE software’s. The results obtained are applicable to a wide range of problems to maintain body core temperature irrespective of the outside temperature.

Keywords: matching condition; mathematical model; diffusion.

Mathematical Subject Classification (2010): 92BXX; 92CXX; 92C35; 92C50; 46N60

1 Introduction

The metabolic and cellular processes in human body converts energy provided by the food into work and heat. To maintain the constant balance of temperature within the body and at the surface of the skin, an amount of heat is dissipated through skin to the outside environment. The thermoreceptors and hypothalamus are responsible for the homeostasis of the body through vasodilation, vasoconstriction, shivering, perspiration etc. Since, the body is usually covered with the protective layer by means of clothes, it influences the heat transfer from the skin, acting as an insulating layer or absorbant of moisture or heat exchanger. In human body diffusion has a vital role in many important processes like oxygen transfer, drug diffusion, homeostasis etc. Diffusion through multilayers is again an important process as the human body is made up of different materials with multiple properties. It has many applications in other branches of engineering [9,10,11], geological profiles [7] etc. Henry [1] developed one of the theories of coupled heat and moisture transfer through clothing considering accumulation effects. A steady-state model was studied by Ognewicz and Tien [2] by incorporating the convective and diffusive transport mechanisms along with phase change due to condensation and evaporation. Heat transfer in biological systems is relevant in many diagnostic and therapeutic applications that involve changes in temperature. For example, in hyperthermia, the temperature in tissue may rise to 42°C – 43°C. Khanday and his co-workers [13,14,15,16,17,18] also studied the diffusion of heat and mass in the biological tissues particularly in dermal regions and human head. Some recent works in this area have been studied by Khanday et al [17] through Variational Finite Element Methods. They studied the fluid distribution pattern in human dermal layers by taking into account similar type of mass diffusion equation. Investigation of thermal properties of skin ([5,6,8,12]) leading to thermal injuries are usually studied through the classical equation of Pennes’ bioheat equation [3]. The most recent work using an explicit form of finite difference method for estimating the temperature variation in human dermal regions has been studied by Khanday and Fida [18]. The present paper studies the temperature distribution in multilayered skin with protective layer. A finite difference scheme with

* Corresponding author e-mail: khanday@gmail.com

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Finally making the transformation, \( V = T e^{-\alpha t} \), equation-(4) reduces to the following standard form

\[
\frac{\partial T}{\partial t} = D \frac{\partial^2 T}{\partial x^2} \tag{5}
\]

The layerwise heat transfer through different dermal and insulating layers having different physiological properties, based on equation-(5) is given by

\[
\frac{\partial T_i}{\partial t} = D_i \frac{\partial^2 T_i}{\partial x^2}, \quad i = 1, 2, 3, 4 \tag{6}
\]

where \( T_i \) is the concentration of diffusing substance in layer \( i \) at time \( t \), \( D_i \) is the diffusivity of layer \( i \).

The humans and other mammals are warm blooded animals, therefore, the core temperature is maintained by the thermoregulatory system up to a large extent even in extreme environmental conditions. Moreover, the clothes are used for insulation to combat with the external climate interference to the thermal stability of the tissues. Thus the boundary conditions associated with the model are

\[
T = 37^{\circ}C \text{ for the internal core}
\]

\[
\frac{\partial T}{\partial t} = 0 \text{ for outer temperature surrounding the clothes} \tag{7}
\]

Due to the roughness of the materials in contact at the interfaces, the use of appropriate continuity conditions of the diffusing material are given by equations (8)-(10).

\[
T_i(x_i, t) = T_{i+1}(x_i, t) \tag{8}
\]

\[
k_i \frac{\partial T_i}{\partial x} = k_{i+1} \frac{\partial T_{i+1}}{\partial x} \tag{9}
\]

where, \( k_i = \rho_i c_i D_i \).

The more general matching condition at the interfaces is given as

\[
\begin{aligned}
    k_i \frac{\partial T_i}{\partial x} &= H_i(T_{i+1} - T_i) \\
    k_{i+1} \frac{\partial T_{i+1}}{\partial x} &= H_i(T_{i+1} - T_i) \tag{10}
\end{aligned}
\]

where \( H_i \) is the heat transfer coefficient. If \( H_i \) is sufficiently large, then the contact between the layers is perfect for the smooth flow of the diffusing material.

### 3 Numerical Solution

To study the heat distribution in human dermal regions through three layers as shown in Figure-1, we consider the size of the three layers as

- **Subcutaneous Tissue** \((L_0 \leq x \leq L_1)\) of length \( l_1 \)
- **Dermis** \((L_1 \leq x \leq L_2)\) of length \( l_2 \) and
- **Epidermis** \((L_2 \leq x \leq L_3)\) of length \( l_3 \)
Fig. 1: Domain of study with different layers and the boundary conditions.

We now find the values of different nodal point temperatures shown in Figure-1 with the help of Taylor series as follows:

\[
\begin{align*}
T_{j-1} &\approx T_{b^-} - h_1 \frac{\partial T_{b^-}}{\partial x} + \frac{h_1^2}{2} \frac{\partial^2 T_{b^-}}{\partial x^2} \\
T_{j-2} &\approx T_{b^-} - (h_1 + h_0) \frac{\partial T_{b^-}}{\partial x} + \frac{(h_1 + h_0)^2}{2} \frac{\partial^2 T_{b^-}}{\partial x^2} \\
T_{j+1} &\approx T_{b^+} - h_2 \frac{\partial T_{b^+}}{\partial x} + \frac{h_2^2}{2} \frac{\partial^2 T_{b^+}}{\partial x^2} \\
T_{j+2} &\approx T_{b^+} - (h_2 + h_0) \frac{\partial T_{b^+}}{\partial x} + \frac{(h_2 + h_0)^2}{2} \frac{\partial^2 T_{b^+}}{\partial x^2} \\
T_{k-1} &\approx T_{c^-} - h_3 \frac{\partial T_{c^-}}{\partial x} + \frac{h_3^2}{2} \frac{\partial^2 T_{c^-}}{\partial x^2} \\
T_{k-2} &\approx T_{c^-} - (h_3 + h_0) \frac{\partial T_{c^-}}{\partial x} + \frac{(h_3 + h_0)^2}{2} \frac{\partial^2 T_{c^-}}{\partial x^2} \\
T_{k+1} &\approx T_{c^+} - h_4 \frac{\partial T_{c^+}}{\partial x} + \frac{h_4^2}{2} \frac{\partial^2 T_{c^+}}{\partial x^2} \\
T_{k+2} &\approx T_{c^+} - (h_4 + h_0) \frac{\partial T_{c^+}}{\partial x} + \frac{(h_4 + h_0)^2}{2} \frac{\partial^2 T_{c^+}}{\partial x^2}
\end{align*}
\] (12)

The two dimensional mesh of the domain generated using FlexPDE software [22] is given in Figure-2.

On employing central difference method, Taylor series method and the geometry of the dermal regions described in Figure-1, the following set of equations corresponding to each layer is given by

\[
\begin{align*}
\frac{\partial^2 T_{j-1}}{\partial x^2} &\approx \frac{h_0 T_{b^-} - (h_0 + h_1) T_{j-1} + h_1 T_{j-2}}{h_0 h_1 (h_0 + h_1)} \\
\frac{\partial^2 T_{j+1}}{\partial x^2} &\approx \frac{h_0 T_{b^+} - (h_2 + h_0) T_{j-1} + h_2 T_{j+2}}{h_0 h_2 (h_0 + h_2)} \\
\frac{\partial^2 T_{k-1}}{\partial x^2} &\approx \frac{h_0 T_{c^-} - (h_3 + h_0) T_{k-1} + h_3 T_{k-2}}{h_0 h_3 (h_0 + h_3)} \\
\frac{\partial^2 T_{k+1}}{\partial x^2} &\approx \frac{h_0 T_{c^+} - (h_4 + h_0) T_{k+1} + h_4 T_{k+2}}{h_0 h_4 (h_0 + h_4)}
\end{align*}
\] (11)

On solving the above system of equations along with the matching conditions given in equation-(10), the values of the unknowns $T_{b^-}, T_{b^+}, T_{c^-}$ and $T_{c^+}$ obtained are given by the following equations:
The mixed boundary condition at $T_{n+1}$ is given by

\[
T_{n+1} = \frac{1}{\Delta t} \left\{ \begin{array}{c}
[-(h_0+h_2)(h_0+k_1+h_2+k_0) + h_2k_1] T_{n+2} \\
+ [(h_0+h_1)(h_1+k_0+h_2) + (h_0+h_2)k_1] T_{n+1} \\
+ [(h_0+h_1)(h_1+k_0+h_2) + (h_0+h_2)k_1] T_{n+1} \\
+ (h_0+h_1)(h_1+k_0+h_2)k_1 T_{n+1} \\
+ (h_0+h_1)(h_1+k_0+h_2)k_1 T_{n+1} \\
+ (h_0+h_1)(h_1+k_0+h_2)k_1 T_{n+1} \\
\end{array} \right\}
\]

The steady state solution and $V_i(x,t)$ is the transient part of solution with initial condition $T_i(x,0) = f_i(x) = 0$. Therefore, the solution is given by

\[
T_i(x,t) = U_i(x) + \sum_{k=1}^{\infty} \alpha_k e^{-\lambda_k^2 x} X_{i,k}(x)
\]

The steady state solutions $U_1(x)$, $U_2(x)$ and $U_3(x)$ corresponding to the three layers shown in Figure-1 are given by

\[
U_1(x) = 37 - \frac{37k_1}{k_3} H_1 \left[ x - x_1 \right] + N_1(x)
\]

\[
U_2(x) = 37 - \frac{37k_2}{k_3} H_1 \left[ x - x_2 \right] + N_2(x)
\]

\[
U_3(x) = 37 - \frac{37k_3}{k_3} H_1 \left[ x - x_3 \right] + N_3(x)
\]

where, $N_1 = l_1 H_1 + k_1$ and $N_2 = l_2 H_2 + k_2$.

Also, \( \alpha_k = \frac{1}{\sum_{i=1}^{3} \rho_i c_i \int_{x_{i-1}}^{x_i} g_i(x) X_{i,k}(x) \, dx} \)

where \( g_i(x) = f_i(x) - x_i(x) \), \( i = 1, 2, 3 \) and

\[
X_{i,k}(x) = J_{i,k} \sin \left( \frac{\lambda_k}{d_i} (x - x_{i-1}) \right) + K_{i,k} \cos \left( \frac{\lambda_k}{d_i} (x - x_{i-1}) \right)
\]

where \( J_{1,k} = 1 \), \( K_{1,k} = 0 \) and

\[
J_{2,k} = \frac{k_1 \sqrt{D_2}}{\sqrt{D_1} k_2} \cos \left( \frac{\lambda_k}{\sqrt{D_1}} \right) - \sin \left( \frac{\lambda_k}{\sqrt{D_1}} \right)
\]

\[
J_{3,k} = \frac{k_2 \sqrt{D_2}}{\sqrt{D_3} k_3} \left[ J_{2,k} \cos \left( \frac{\lambda_k}{\sqrt{D_1}} \right) + \frac{k_2 \lambda_k}{\sqrt{D_2}} \cos \left( \frac{\lambda_k}{\sqrt{D_2}} \right) \right]
\]

\[
K_{3,k} = J_{2,k} \sin \left( \frac{\lambda_k}{\sqrt{D_2}} \right) + \frac{k_2 \lambda_k}{\sqrt{D_2}} \sin \left( \frac{\lambda_k}{\sqrt{D_2}} \right)
\]

The eigenvalues $\lambda_k$ are defined by the following equation

\[
J_{3,k} \left[ \cos \left( \frac{\lambda_k}{\sqrt{D_2}} \right) - \frac{k_2 \lambda_k}{\sqrt{D_2}} \sin \left( \frac{\lambda_k}{\sqrt{D_2}} \right) \right] - K_{3,k} \left[ \sin \left( \frac{\lambda_k}{\sqrt{D_2}} \right) + \frac{k_2 \lambda_k}{\sqrt{D_2}} \cos \left( \frac{\lambda_k}{\sqrt{D_2}} \right) \right] = 0
\]

4 Analytical Solution

To solve equation-(6), the model consisting of bio-heat equation given in equation-(1) and boundary, matching and interface conditions given in (7), (8), (10) and (11), we invoke variables separable technique.

Define the solution of equation-(6) in the form $T_i(x,t) = U_i(x) + V_i(x,t)$; \( i = 1, 2, 3 \) where $U_i(x)$ is the...
The basic Pennes’ bio-heat equation has been suitably reformulated in which the role of protective layer at the skin surface is incorporated. The appropriate interface, matching and boundary conditions have been suitably defined to form a boundary value problem. The model has been transformed into the standard heat transfer model with the help of certain transformations. The solution obtained to the transformed equation-(6) is then substituted back to get the solution of the original model equation-(1). The variation of temperature profiles across the defined layers in presence of different protective layers of clothes at various climatic conditions were calculated at the nodal points of the layered skin with the help of Taylor series and finite difference method.

The finite difference method discussed in this paper has been compared with the theoretical analytical solution of equation-(6) described by equation-(17). The series in equation-(17) is truncated at \( k = 40 \). Equation-(17) is then plotted using MATLAB software along with the numerical solution computed through the system of differential equations.

### Table 1: Physiological parameters and their numerical values.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thickness of subcutaneous tissue ((l_1))</td>
<td>(\mu m)</td>
<td>1800</td>
</tr>
<tr>
<td>Thickness of Dermis ((l_2))</td>
<td>(\mu m)</td>
<td>2000</td>
</tr>
<tr>
<td>Thickness of epidermis ((l_3))</td>
<td>(\mu m)</td>
<td>80</td>
</tr>
<tr>
<td>Thermal conductivity of subcutaneous tissue ((k_1))</td>
<td>(Wm^{-1} 0°C^{-1})</td>
<td>0.19</td>
</tr>
<tr>
<td>Thermal conductivity of dermis ((k_2))</td>
<td>(Wm^{-1} 0°C^{-1})</td>
<td>0.45</td>
</tr>
<tr>
<td>Thermal conductivity of epidermis ((k_3))</td>
<td>(Wm^{-1} 0°C^{-1})</td>
<td>0.23</td>
</tr>
<tr>
<td>Heat transfer coefficient ((h))</td>
<td>(Cal.s^{-1}m^{-2} 0°C^{-1})</td>
<td>0.70</td>
</tr>
<tr>
<td>Specific heat of subcutaneous tissue ((c_1))</td>
<td>(Jkg^{-1} 0°C^{-1})</td>
<td>2675</td>
</tr>
<tr>
<td>Specific heat of dermis ((c_2))</td>
<td>(Jkg^{-1} 0°C^{-1})</td>
<td>3300</td>
</tr>
<tr>
<td>Specific heat of epidermis ((c_1))</td>
<td>(Jkg^{-1} 0°C^{-1})</td>
<td>3590</td>
</tr>
<tr>
<td>Density of subcutaneous tissue ((\rho_1))</td>
<td>(Jkg^{-3})</td>
<td>1000</td>
</tr>
<tr>
<td>Density of dermis ((\rho_2))</td>
<td>(Jkg^{-3})</td>
<td>1200</td>
</tr>
<tr>
<td>Density of epidermis ((\rho_3))</td>
<td>(Jkg^{-3})</td>
<td>1200</td>
</tr>
<tr>
<td>Diffusivity of subcutaneous tissue ((D_1))</td>
<td>(m^2min^{-1})</td>
<td>(204 \times 10^{-9})</td>
</tr>
<tr>
<td>Diffusivity of dermis ((D_2))</td>
<td>(m^2min^{-1})</td>
<td>(203 \times 10^{-9})</td>
</tr>
<tr>
<td>Diffusivity of epidermis ((D_3))</td>
<td>(m^2min^{-1})</td>
<td>(2 \times 10^{-9})</td>
</tr>
</tbody>
</table>

**Fig. 3:** Temperature profile across the four layers when the heat exchanges through the protective layers when the ambient temperature is 25°C, \(H_1 = 0.6\) and \(H_2 = 0.8\).
Fig. 4: 3-D view of the temperature profile variation across the layers.

Fig. 5: Variation of temperature with distance through the four layers when the heat exchanges through the protective layers at the ambient temperature of $10^0C$ and $H_1 = H_2 = 20$.

Fig. 6: Variation of temperature with distance through the four layers when the heat exchanges through the protective layers at the ambient temperature of $20^0C$ and $H_1 = H_2 = 20$.

Fig. 7: Variation of temperature with distance through the four layers when the heat exchanges through the protective layers at the ambient temperature of $37^0C$ and $H_1 = H_2 = 20$.

Fig. 8: Variation of temperature at different locations (shown on the right side of the graph) of the four layers when the heat exchanges through the protective layer at the ambient temperature of $10^0C$, seconds and $H_1 = H_2 = 20$.

Fig. 9: Variation of temperature at different locations (shown on the right side of the graph) of the four layers when the heat exchanges through the protective layers at the ambient temperature of $20^0C$ and $H_1 = H_2 = 20$. 
Fig. 10: Variation of temperature at different locations (shown on the right side of the graph) of the four layers when the heat exchanges through the protective layers at the ambient temperature of 25°C and $H_1 = H_2 = 20$.

Fig. 11: Temperature variation and its surface across the four layers of the domain of study when $H_1 = H_2 = 50$.

Fig. 12: Temperature variation across the four layers of the domain of study when $H_1 = 0.5, H_2 = 0.5$.

Fig. 13: Comparison of exact, FDM and FlexPDE solutions across the four layers of the domain of study at different times (in seconds) and $H_1 = H_2 = 0.5$.

Fig. 14: Temperature variation across the four layers when the temperature outside the protective layer of clothes (Briefs, trousers, suit jacket) is 15°C.

Fig. 15: Temperature variation across the four layers when the temperature outside the protective layer of clothes (Briefs, shirt, trouser) is 45°C.

equations in (18),(13),(14) and the results were illustrated in Figure-13. The graphs clearly show the efficiency of the finite difference technique invoked in the paper.
The insulation of the clothes play a vital role in maintaining a suitable temperature at the skin surface. For this purpose, we took different protecting clothes listed in Table-1 at different atmospheric temperatures and plotted their graphs as shown in Figures- 14, 15, 16. It is clear from the graphs that the clothing layer maintains a suitable temperature at the skin surface irrespective of the outside temperature.

Finite difference schemes are helpful in exploring various research problems pertaining to the materials diffusing through the different layers having different physiological properties. The finite difference schemes are applicable to a wide range of problems involving multiple layers [5, 19, 20, 21] where numerical integration techniques are often used. The technique of using the finite difference scheme, outlined in this paper, is very easy to implement for obtaining good approximating results to certain degree of accepted errors. The analytical and numerical results obtained in this study have a capability to be applicable to many practical bioheat transfer problems than a few existing analytical solutions.

5.1 Role of Clothes

The sensible heat transfer from the skin surface to the clothing layer \( Q_s \) can be obtained using [24]

\[
Q_s = \frac{1}{I_0 + I_{cl}} A_{cl} (T_{skin} - T_{cl})
\]

(22)

where \( T_{skin} \) is the temperature at the skin surface, \( I_0 \) is the insulation of air, \( I_{cl} = \sum I_{clu,i} \) is the total effective insulation provided by the number of garments worn and \( A_{cl} \) is the area of the body covered by the clothes. The unit of insulation is taken as \( clo \) (1 \( clo \) = 0.1555m\(^2\)K/W). The average value of mass and insulation of different garments are listed in Table-2. The insulation of the clothing layer and the air in between skin surface and worn clothes may be effected by the air velocity and tissue movement. So, \( A_{cl} = A_{skin} f_{cl} \) with \( f_{cl} = 1 + 0.31I_{cl} \) [24]. The clothes play a vital role in maintaining a suitable temperature at the skin surface. For this purpose, we took different protecting clothes listed in table-1 at different atmospheric temperatures and plotted their graphs as shown in Figures- 14, 15, 16.

<table>
<thead>
<tr>
<th>Garment</th>
<th>Mass(Kg)</th>
<th>( l_{clu}(clo) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Briefs</td>
<td>0.065</td>
<td>0.04</td>
</tr>
<tr>
<td>Shirt</td>
<td>0.196</td>
<td>0.28</td>
</tr>
<tr>
<td>Trousers</td>
<td>0.459</td>
<td>0.24</td>
</tr>
<tr>
<td>Socks</td>
<td>0.049</td>
<td>0.03</td>
</tr>
<tr>
<td>Suit Jacket</td>
<td>0.652</td>
<td>0.04</td>
</tr>
</tbody>
</table>

Table 2: Mass and insulation coefficients of some garments [24]

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References


Aijaz Ahmad Najar has done Ph. D. in Mathematical Biology under the supervision of Dr. M.A. Khanday from the University of Kashmir, Srinagar, J&K, India in 2016. The author has qualified National and State Level exams conducted in the subject of Mathematics by the republic of India. His areas of interest are ODE and PDE, Numerical Analysis, Mathematical Biology etc. He is presently working in Islamic University of Science and Technology, Awantipora, India as an Assistant Professor.

Mukhtar Ahmad Khanday is a faculty member at the Department of Mathematics, University of Kashmir, Srinagar, J&K, India since last thirteen years. Born on March 27, 1979, he did his masters in pure mathematics in 2002 and later completed his PhD in Mathematical Biology in 2009. The primary research work of Dr. Khanday is based on heat and mass transport in biological tissues. He supervised many doctoral students and has published a good number of research papers in International Journals of repute with main focus on heat, drug and oxygen diffusion problems in biological systems. He is presently running three major research projects on various problems of mathematical biology funded by UGC, New Delhi; SERB-DST, Govt. of India; and National Board for Higher Mathematics-NBHM, Govt. of India.