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Optimization for PID Control Parameters on Pitch Control of Aircraft Dynamics Based on Tuning Methods

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Abstract: Today many aircraft control systems and process control industries are employing classical controller such as Proportional Integral Derivative Controller (PID) to improve the system characteristics and dynamic performance. To improve the stability analysis and system performance of an aircraft, PID controller is employed in this paper. The safety of flight envelope can be improved by tuning parameters of PID controller for pitch control dynamics of an aircraft. Designing the mathematical model is necessary and important to describe the longitudinal pitch control of general aviation aircraft system. PID controller is developed based on dynamic and mathematical modeling of an aircraft system. The various tuning methods such as Zeigler-Nichols method (ZN), Modified Zeigler-Nichols method, Tyreus- Luyben tuning and Astrom-Hagglund tuning methods are evaluated for general aviation aircraft system. The simulation results prove that PID controller parameters tuned by ZN method for general aviation aircraft dynamics is better compared to the other methods in improving the stability and performance of flight in all conditions such as climb, cruise and approach phase.

Keywords: Pitch Control, PID controller, Optimum Parameters, Tuning methods.

1 Introduction

Many inventions and thousands of experiments are performed in gliders in developing the first successful airplane. Wright brothers successful invention motivated many researchers in designing the dynamic characteristics of an aircraft. Aircrafts flying qualities is necessary and important for safe flight envelope. This can be achieved by evaluating the aircrafts dynamic performance such as stability analysis and control characteristics. In general, the poor flying qualities will make the airplane difficult to fly and could be dangerous in all conditions such as climb, cruise and approach phase. This paper focuses mainly in designing the optimum values of PID controller parameters for general aviation aircraft.

General aviation (GA) flights range from gliders and powered parachutes to corporate jet flights. General aviation covers a wide range of operations such as aviation for agricultural needs, clubs for flying, training pilots, maintaining and manufacturing of low weight aircrafts [1] and [2]. Many countries are included as representatives of all civil and general aviation, belonging to the International Civil Aviation Organization (ICAO). The main objective of the paper is to design PID controller parameter for longitudinal pitch control aircraft[3] and [4]. To obtain the optimum parameter value, various methods such as Zeigler-Nichols method (ZN), Modified Zeigler-Nichols method, Tyreus- Luyben tuning, Astrom-Hagglund tuning methods are employed.[5] and [6]. The ultimate gain constant and period of oscillation of flight control system is estimated by employing classical approach such as root locus method. The approach of the work illustrates time domain specifications of the system to obtain characteristics performance of an aircraft [7].

2 Dynamical Equation for Flight Vehicles

The aircraft motion problem consists of two coordinate systems. One coordinate system is fixed to the earth and the other coordinate fixed to the airplane called body coordinate system. The aerodynamic thrust and gravitational forces acting on an airplane can be resolved along fixed axis to the airplanes centre of gravity. The mathematical equations of motion are obtained from Newton's second law of motion. The forces, moments and

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velocity components in the body fixed coordinate of aircraft system can be described as shown in Figure 1. The X, Y and Z variables represent aerodynamic force. The component L, M and N denote aerodynamic moment. Variables u, v and w denote the velocity components. Variables p, q and r denote the angular rates. Variables x, y, z denote the coordinates, with origin at the center of mass of the vehicle [10]. The x-axis [11] points toward the nose of the flight. The x-axis and z-axis lie in the plane of symmetry. The z-axis is perpendicular to the x-axis, and pointing approximately down. The y-axis is pointing approximately out the right wing [12].

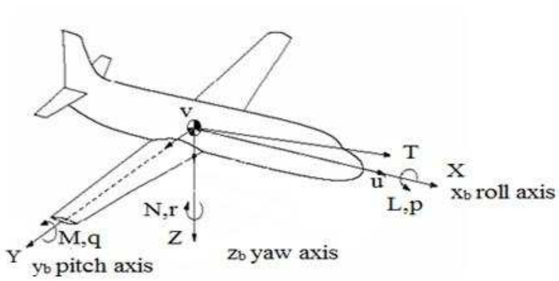

Fig. 1: Force, moments, and velocity components

The kinematic and dynamic equations can be expressed as a function of all the motion variables as force and moment equations. Where g acceleration due to gravity, q perturbed pitch rate, w perturbed velocity along Z.

$$X = mgsin\theta + m(\dot{u} + qw - rv), \quad (1)$$
$$Y = -mgcos\theta cos\phi + m(\dot{w} + pv - qu). \quad (2)$$

Moment Equation:

$$M = I\dot{q} + rq(I_x - I_z) + I_{xz}(p^2 - q^2) \quad (3)$$

Equations (1),(2) and (3) completely describe the longitudinal motion of a flight vehicle, subject to the prescribed aerodynamic (and propulsive) forces and moments. The equations are linearized by using small disturbance theory about an equilibrium flight condition. The linearized longitudinal equations can be formed by assuming small deviations about steady flight conditions [13]. This theory is difficult to be applied to the problems in which large amplitude motions are to be expected. The large amplitude deviation is due to spinning or stalled flight. Yaw is causing the accidents as a stall and spin. The improper use of rudder is the main cause of yaw. This can be avoided by proper yaw control mechanisms. However in many cases the small disturbance theory yields sufficient accuracy for practical engineering problems. Hence, the small disturbance theory is good in all flight conditions provided with suitable yaw control mechanisms.

These equations are nonlinear and coupled, and generally can be solved only numerically, yielding relatively little insight into the dependence of the stability and controllability of the vehicle on basic aerodynamic parameters of the vehicle. The complete set of linearized equations of motion is represented in Equations in (4),(5) and (6). Where M_q dimensional variation of pitching moment with pitch rate, M_u dimensional variation of pitching moment with speed,M_α dimensional variation of pitching moment with angle of attack,$M_{\dot{\alpha}}$ dimensional variation of pitching moment with rate of change of angle of attack,S reference wing area,T thrust,u perturbed velocity along X,U_o component of steady state velocity along X,X_q dimensional variation of X force with pitch rate,X_{tu} dimensional variation of X force due to thrust with speed,X_u dimensional variation of X force with speed,X_α dimensional variation of X force with angle of attack,Z_q dimensional variation of Z force with pitch rate,Z_u dimensional variation of Z force with speed, Z_α dimensional variation of Z force with angle of attack,$Z_{\dot{\alpha}}$ dimensional variation of Z force with rate of change angle of attack, α perturbed angle of attack,ζ_p damping ratio the phugoid,ζ_{sp} damping ratio the short period,θ disturbed pitch attitude angle,θ_1 steady state pitch attitude angle,ρ air density.

$$\left[\frac{d}{dt} - X_u\right]\Delta u + (g_0 cos\theta_0)\Delta\theta - X_w \Delta w$$
$$= X_{\delta e}\Delta\delta_e + X_{\delta T}\Delta\delta_T \quad (4)$$
$$-Z_u\Delta u + \left[(1 - Z_{\dot{w}})\frac{d}{dt} - Z_w\right]\Delta w - \left\{[u_0 + Z_q]\frac{d}{dt} - g_0 sin\theta_0\right\}\Delta\theta$$
$$= Z_{\delta e}\Delta\delta_e + Z_{\delta T}\Delta\delta_T \quad (5)$$
$$-M_u\Delta u - \left[(M\dot{w})\frac{d}{dt} - M_w\right]\Delta w - \left[\frac{d^2}{dt^2} - M_q\frac{d}{dt}\right]\Delta\theta$$
$$= M_{\delta e}\Delta\delta_e + M_{\delta T}\Delta\delta_T \quad (6)$$

where $\Delta\delta_e$ and $\Delta\delta_T$ are the aerodynamic and propulsive controls respectively. The linearized equations give valuable information of dynamic characteristics of airplane motion.

3 PID Controller Parameters

In general, PID controller measures the value called error value which is considered as the difference in value between the output of the system and required reference value. The PID controller accomplishes to reduce the value of error by regulating the pitch control inputs. The PID controller parameters are called three-term control such as the Proportional, the Integral and Derivative parameters depicted as K_P, K_I, and K_D. Tuning the P, I, and D parameters by a procedural steps of algorithm, the PID controller can provide control action developed for specific flight requirements [15],[16] and [17]. The controller output can be represented in terms of error value of the controller, the degree to which deviates the controller reference value, and the measure of oscillation of the vehicle. The structure is also known as parallel form and is represented by,

$$G(s) = K_P + K_I \frac{1}{s} + K_D s = K_P \left[1 + \frac{1}{TS} + T_D S \right] \quad (7)$$

where K_P is proportional gain, K_I is integral gain, K_D derivative gain; T_I is integral time constant and T_D is derivative time constant.

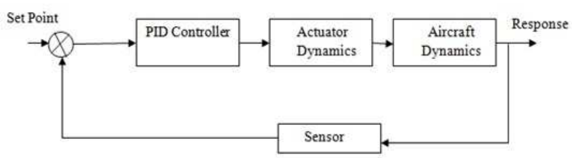

Fig. 2: Block diagram of Aircraft system with PID Controller

The block diagram of general aviation aircraft with actuator dynamics and PID controller is shown in Figure 2. The proportional term provides the error signal through the constant gain factor indices. The integral term helps to reduce steady-state error and the derivative term helps to improve transient response of the aircraft system. The effect of variation of controller parameter for closed loop response is given in Table 1. The PID controller performs better compared to independent operations of P, I and D term. The selection of gains for the PID controllers can be obtained by a different closed loop tuning methods.

4 Aircraft Dynamics without Controller Effect

In general, the non linear aircraft model is complex, and the complexity arises from the mathematical model of dynamics. Considering the general aviation aircraft, the rate of change in the pitch value to the rate of change in the angle of elevator deflection is given in the Equation (**??**).

$$\frac{\Delta q(s)}{\Delta \delta_e(s)} = \frac{-\left(M_{\delta_e} + \frac{M\dot{\alpha} Z_{\delta_e}}{u_0}\right) s - \left(\frac{M_\alpha Z_{\delta_e}}{u_0} - \frac{Z_\alpha M_{\delta_e}}{u_0}\right)}{s^2 - \left(M_q + M_{\dot{\alpha}} + \frac{Z_\alpha}{u_0}\right) + s + \left(\frac{Z_\alpha M_q}{u_0} - M_\alpha\right)} \quad (8)$$

For simplicity and to reduce complexity in computational analysis of an aircraft system, a first order model of an actuator is employed with the transfer function as given in Equation (9), and time constant $\tau = 1$ second is employed.

$$H(s) = \frac{10}{\tau s + 10} \quad (9)$$

The transfer function of longitudinal dynamics of general aviation aircraft and actuator dynamics is given in Equation (10),

$$G(s) = \frac{110s + 243.8}{s^4 + 12.7s^3 + 43.64s^2 + 127.94s} \quad (10)$$

Figure 3 shows step responses of system without controller. The rise time is 1.23 seconds and settling time is high in the range of 9.12 seconds. The simulation is carried out using Matlab-R2012a in Intel core processor i5-3210M, 2.5GHz speed, 4GB RAM. Though overshoot is less but the response leads to oscillation for longer period [14]. This leads the aircraft difficult to fly and make the performance unstable in nature. Table 2 shows the values of parameters of dynamic response of aircraft without PID controller.

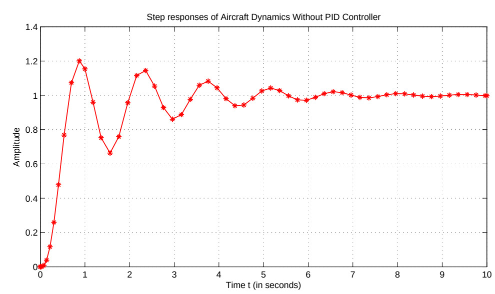

Fig. 3: Step responses of aircraft dynamics without PID controller

Stability of the vehicle can be improved by reducing the oscillations and it can be analyzed by tuning the parameters of PID controller.

5 Different Tuning Methods

The selection of gains for PID controller can be determined by various tuning methods [18] and [19]. The gains are determined in terms of two parameters, $k_p u$, called the ultimate gain, and T_u,, the period of the oscillation that occurs at the ultimate gain. From Figure 4, the ultimate gain can be obtained as 1.82 and the period of oscillation can be determined as 1.2.

Table 1: Parameters effecting system dynamics.

Closed loop Responses	Proportional Gain (K_p)	Integral Gain (K_I)	Derivative Gain (K_D)
Rise Time	Decrease in value	Decrease in value	Small change in value
Overshoot	Increase in value	Increase in value	Decrease in value
Settling time	Small change	Increase	Decrease
Steady-state error	Decrease in value	Eliminate	No change

Table 2: Effect of closed loop response without controller

Parameters	Rise Time(t_r)in Seconds	Settling Time(t_s)in Seconds	Delay Time(t_d)in Seconds	Overshoot(M_p)in percentage	Transient Behavior
Response values	1.23	9.12	0.48	23	Oscillation

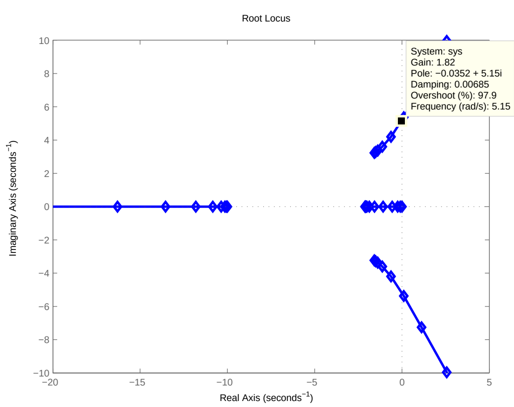

Fig. 4: Root locus of an aircraft system

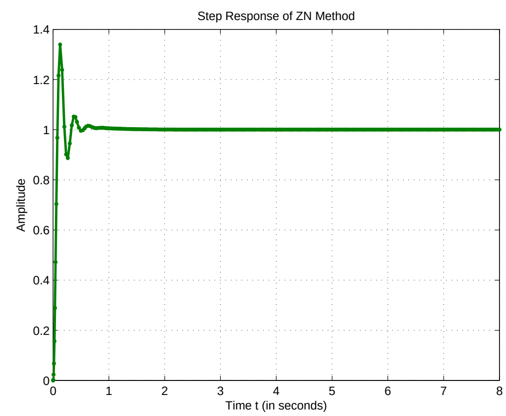

Fig. 5: Step Responses of ZN Method

5.1 Ziegler-Nichols (ZN) Method

In 1942, Ziegler and Nichols first proposed a trial and error tuning method. This method most widely used method for tuning of PID controllers[8]. ZN method can also be called as continuous cycling method or ultimate gain tuning method based on sustained oscillations. The gain of the controller is gradually reduced or increased until the system response oscillates continuously after a small external disturbance or step change. A main design criterion is considered as the decay of oscillation to one-fourth of its initial value. The parameters of the controllers can be evaluated using the ultimate gain and frequency values as listed under Table 3. This method is applicable for closed loop flight control systems. The values of gain K_P, K_I, and K_D can be determined as 2.91, 1.13 and 1.37. The step response of aircraft dynamics using ZN method is shown in Figure 5.

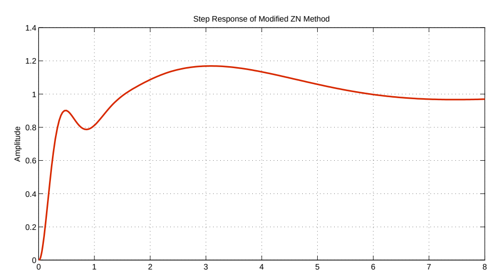

Fig. 6: Step Response of Modified ZN Method

5.2 Modified Ziegler-Nichols (ZN) Method

This is similar to ZN method but has modified setting values for ultimate gain and frequency values as listed under Table 3. In closed loop system, to reduce the amplitude of oscillation by a value of one-fourth its decay ratio forces the peak overshoot to high value and making the system undesirable [9]. In such cases, therefore are other some different methods like modified ZN settings can be employed to reduce the peak overshoot value. The values of K_P, K_I, and K_D can be calculated as 0.7, 0.68 and 0.46. The step response of aircraft dynamics using modified ZN method is shown in Figure 6.

5.3 Tyreus-Luyben Method

Tyreus-Luyben's method is based on the Ziegler-Nichols tuning method [20]. The gain of the controller is varied until the system response continuously changed based on sustained oscillations. The parameters of the controllers can be evaluated with modified settings using the ultimate gain and frequency values as listed under Table 3. This is one of the most conservative tuning methods for the controller parameters to obtain better stability. This method only proposes settings for PI and PID controllers. The values of K_P, K_I, and K_D can be calculated as 0.95, 2.84 and 0.23. The step response of aircraft dynamics using Tyreus-Luyben method is shown in Figure 7.

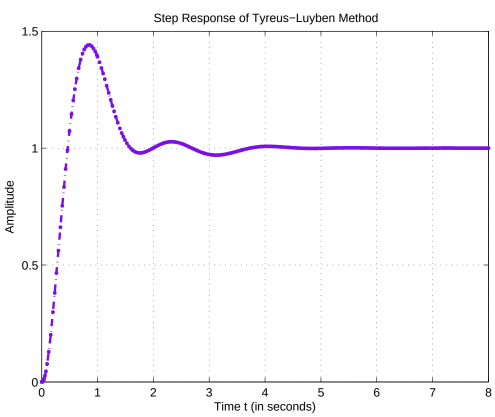

Fig. 7: Step Response of Tyreus-Luyben Method

5.4 Astrom and Hagglund Method

This method is proposed by Astrom and Hagglund and they used non linear relay feedback [21]. The ultimate gain and period of oscillation can be obtained from the limit cycle oscillation of the system. The advantage of Astrom and Hagglund method will not drive system to instable condition because of the good estimation of the ultimate gain. Astrom method is easy to automate and its procedure avoids a time consuming trial and error method for obtaining the ultimate gain values. In general, ZN tuning method has only one point in Nyquist curve. In this method, by varying limit cycles of the relay relies on generating several points on the Nyquist curve. Better tuning of the plant can be obtained by generating more points.

The values of K_P, K_I, and K_D can be estimated as 0.784, 1.4 and 0. The step response of aircraft dynamics using Astrom-Hagglund method is shown in Figure 8. The value of K_P, K_I, and K_D for different tuning methods are shown in Table 4.

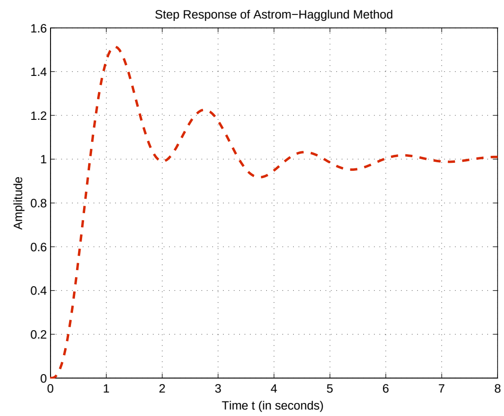

Fig. 8: Step Response of Tyreus-Luyben Method

6 Results and Discussion

The different tuning methods are compared and the results are shown in Table 5. The step response of different tuning methods is illustrated in Figure 9.

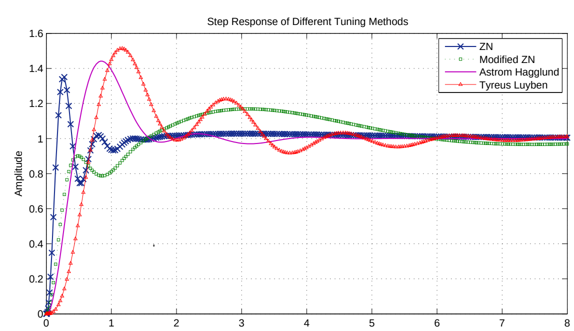

Fig. 9: Step responses of aircraft dynamics without PID controller

The delay and rise time give a measure of how fast the system responds to a step input. Rise time is less in Modified ZN method compared to other methods. The

Table 3: Different Tuning Methods

Tuning Methods	Proportional Gain K_P	Integral Gain K_I	Derivative Gain K_D
Z-N Method(Ziegler and Nichols 1942)	0.6 $K_(pu)$	(2) $\frac{K_(pu)}{T_u}$	$\frac{(K_(pu))(T_u)}{8}$
Modified Z-N Method (Hang et al 1991)	(0.33) $K_(pu)$	$\frac{T_u}{2}$	$\frac{T_u}{3}$
Tyreus Luyben Method (Luyben and Luyben 1997)	0.45 $K_(pu)$	2.2 T_u	$\frac{T_u}{6.3}$
Astrom Hagglund Method (Astrom and Hagglund 1994)	0.32 $K_(pu)$	0.94	0

Table 4: Values of K_P, K_I and K_D and for Different Tuning Methods.

Sl.No	Tuning Methods	K_P	K_I	K_D
1	ZN Method	2.91	1.13	1.37
2	Modified ZN	0.7	0.68	0.46
3	Tyreus-Luyben	0.95	2.84	0.23
4	Astrom-Hagglund	0.784	1.4	0

settling time is less in ZN method. Peak overshoot is less in Modified ZN method. Astrom-Hagglund method response is oscillatory. This leads to instable dynamics of aircraft. Compared to all methods Modified ZN method shows good in time response characteristics. Though Modified ZN method is better but it has high settling time as 5.5 seconds. Rise time is 1.134 seconds [22] and the value is high compared to other methods. Compared to other methods rise time is high but the response of aircraft settles at low value of settling time [23] and [24].

From the standpoint of aircraft control system design, the required characteristic is that the system has to respond rapidly for any change in input. This helps the flight to fly in safe envelope. By considering the response, ZN method gives optimal gain values of PID controller parameters. The results shows ZN method respond rapidly for any change in input and the response of an aircraft settles down to the steady state value quickly. The tuned controller parameters values can effectively eliminate the dangerous oscillations and provide smooth operation by settling fast for any sudden change in the environment. This optimum value works efficiently for longitudinal dynamics of pitch control aircraft where safety is high priority.

7 Conclusions

The lack of control and stability leaves issues resulting in uncertainties related to vehicle performance, flight safety, and cost. To achieve fast response and good stability, tuning the parameters of the controller is essential. The flight control system parameters may change from its equilibrium steady state value due to sudden change in the flight conditions. The changing environment conditions vary the parameter value and it may have the tendency to affect the desired performance of a control system. The designing of PID controller to obtain optimum values for general aviation aircraft are carried out in this paper. Various tuning methods such as Zeigler-Nichols method (ZN), Modified Zeigler-Nichols method, Tyreus- Luyben tuning and Astrom-Hagglund tuning methods are employed to obtain optimum parameters values. This ensures flight safety and improves characteristics performance of an aircraft. The knowledge about the controller parameter of an aircraft system is extremely important from the standpoint of improved system design, protection, and fault tolerant control to ensure safety flying conditions.

Table 5: Comparison of different tuning methods

Sl.No	Rise Time(tr)in Seconds	Settling Time(ts)in Seconds	Delay Time (td)in Seconds	Overshoot(Mp)in percentage	Transient Behavior
ZN Method	0.3	0.6	0.71	37	Smooth
Modified ZN Method	0.2	0.4	5.5	18	Smooth
Tyreus-Luyben Tuning	0.45	0.9	3.8	46	Smooth
Astrom-Hagglund Tuning	0.62	1	5.8	50	Oscillatory
Wahid et al (2011)	0.24	2.72	1.1	0	Smooth
Kada and Ghazzawi (2011)	0.56	1.134	1.472	0	Smooth
Nurbaiti and Nurhaffizah (2012)	0.24	2.831	0.727	0	Smooth

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