Mathematical Modelling and Robust PID Controller Design for Compressed Air Pressure Control Process

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Abstract: Industrial Compressed Air Pressure Control (ICAPC) process is one of the significant and demanding tasks in all the process industries. Due to the unpredictable leakages and nonlinear behavior of the pneumatic control valve, the automatic pressure control is very difficult with respect to its various preferable operating points. In this paper mathematical modeling, robust PID controller and novel parameter optimization technique have been proposed. This paper deals with various effects of disturbances and uncertainties affecting the process dynamics. The novel robust controller yields better set-point tracking capability and survives against the sudden regulation in the load parameter changes. For mathematical modelling and parameter optimization, the robust hybrid form of fminsearch and fmincon (FSFC) technique has been used, which provides the accurate transfer function model and the model validation has been analyzed with respect to real-time pressure process. This paper also proposed PID controller optimization techniques such as Pattern Search (PS) and Fsolve (FS) for the pressure process with split-range control scheme and the closed loop performances are compared with existing Ziegler-Nichols (Z-N) and Genetic Algorithm (GA) based PID controller. The simulation and real time closed loop response have been obtained based on the Servo-problem (set point change) and Regulatory-problem(load change) operating conditions.

Keywords: Robust control, PID Controller, Model Optimization, Controller Optimization, pressure Process, Optimization, Mathematical Model

1 Introduction and Related Mechanism

Pressure is one of the important parameter to be controlled in an industrial environment such as boilers, compressing unit, molding process, hydraulic and pneumatic systems. For the analysis of pressure process, the mathematical model is required. The obtained model is Integer First Order Transfer Function (IFOTF) model consists of System gain (K), time constant (T) and transportation lag (L) will gives the complete information about the system.

1.1 Controller

The data acquired from the pressure process is considered to be a process variable of the real-time system, the process variable must be regulated and sustained even in unusual environment. It is interesting to note that most of the industrial process are utilizes PID or modified PID controllers. Because most of PID controller parameters are adjusted in the real-time process, many types of tuning rules have been proposed in the literature. Using these tuning rules, delicate and fine tuning of PID controller can be made on-site. Also automatic tuning methods have been developed and some of the PID controllers may possess on-line automatic tuning capabilities, the sensitivity of the process parameter depends on the size of the pressure tank.

1.2 Challenges in the pressure process

The nonlinear components in the pressure process such as residual drift, transportation delay, dead zone and saturation occur in the process. These nonlinear components are due to the unpredictable leakages, turbulent variations in the centralized compressing unit and non linear variations in the field instruments installed in the real-time environment. Controlling of pressure process is difficult because of involvement of non linear components.

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1.3 Existing Control Schemes

Yadav (2016) considered a tank to be with very small size compared to the tank considered for this proposed work and also the new version of control scheme is followed for the pressure control process, it is based on the manipulating feedback control mechanism, and also this work deals with the anti-reset wind-up mechanism. This scheme is not required for such a mechanism, because the control scheme optimized PID controller parameters are selected so the performance of controller rejects such kind of errors. Wang (2017) pressure control is done for a hydraulic cylinder used for compression process, this platform is different from the proposed system even though the optimization techniques are followed for the tuning of PID controller used in the feedback loop control scheme i.e. evolutionary algorithm Particle Swarm Optimization (PSO) techniques and Genetic Algorithm (GA) are applied and the performances are compared, there are various types of Particle Swarm Optimization (PSO) techniques used in this paper, Stable Cross Probability For Particle Swarm algorithm (WPSO), Weight of Particle Swarm Algorithm (WPSO) and Adaptive Particle Swarm Algorithm (APSO) are used and the performances are analyzed. Kishore (2018) faced the difficulty due to the high non-linearity and sensitivity, the Two Degree Of Freedom Fractional Order PID (2DOF-FOPID) controller for real-time pressure process. Two types of PID controller configurations are executed such as parallel and series configurations for the closed loop control system and performances are compared. Sahaj work (2012) based on the simulation study of blood pressure dynamics; the dynamics are analyzed based of obtained model, this kind of dynamics needs highly sensitive control scheme so the Internal Model Control (IMC) structure is recommended. In IMC two different algorithms are designed, One-Degree of Freedom Proportional Integral (ODF-PI) and Two - Degree of Freedom Proportional Integral Derivative (TDF-PID) controller. Sudha (2016) show that the classical PID controller is implemented with Tyreus-Luyben and Astrom-Hagglund tuning methods for pitch and yaw control of aircraft and the proposed tuning methods are compared with the Z-N and modified Z-N method of tuning.

2 Mathematical Modeling of Pressure Process

For implementation of best control schemes, the preliminary requirement is optimized transfer function model. Improving and understanding the pressure process operation is a major overall objective for developing a dynamic process model. Feedback control systems are used to maintain process variable at desirable value. For complex systems, it is necessary to base the control system design on a process model.

2.1 First-Principle Model

The pressure process tank contains of air inlet flow and outlet flow, the mathematical model is obtained by writing a mass balance equation. The gas equation for the pressure ‘P’ is given by

\[ pV = mR_0T \]

Here V denotes the volume of the container, T is the temperature, m is the mass of the gas in the container and \( R_0 \) is the gas constant, here gas is only the atmospheric air, derivative of the gas equation is

\[ \frac{dp}{dt} = \frac{Q_v - Q_o}{Kd} \]

where

\[ K = \frac{V}{R_0T} \]

\[ K^p = F(p_v, p, h) - F(p, p_0, h) \]

\[ T \Delta p = \Delta p + k_1 \Delta h - k_2 \Delta h_0 \]

where

\[ \Delta p = p - p_0 \]
\[ \Delta h = h - h_0 \]
\[ \Delta h_0 = h_{02} - h_{01} \]

\[ T = \frac{KW_1W_2}{W_1 + W_2} \]
\[ k_1 = \frac{W_1W_2c_1}{W_1 + W_2} \]
\[ k_2 = \frac{W_1W_2c_2}{W_1 + W_2} \]
The inlet stream has flow $Q_i(t)$ and pressure $P_1$. The outlet stream contains a control valve which is connected to a controller, which has a pressure $P_2$ and outlet flow $Q_O(t)$. 

The transfer function for the control valve can be expressed as:

$$G(s) = \frac{k_1 e^{-Lt s}}{1 + Ts}$$  \hfill (12)$$

$$G_d(s) = \frac{k_2}{1 + Ts}$$  \hfill (13)$$

$k_1, k_2$ - Gain  
L - Time constant  
T - Lag

The table shows the different set of values of $k_1$, $k_2$, T and L for the different optimization algorithms.

### 2.2 Open-Loop Test

The open-loop test is conducted by applying various step input to the final control element. Here in this process the pneumatic control valve acts as a final control element. (3-15) PSI is applied to the pneumatic control value from the MATLAB with the help of all the necessary interfacing and converting elements and we observe the dynamics behavior of the process variable and we repeat the test for various levels of input signal by keeping the disturbance as constant and tabulating the input and output values with respect to time. Graph is plotted, and also system parameters such as time constant, system gain and transportation lags.

$$G_o(s) = \frac{0.9666e^{-2.2t}}{32.25s + 1}$$  \hfill (14)$$

### 2.3 Optimization for the selection of best model

In this selection process the following optimization algorithms are used:
- Genetic Algorithm
- Fminsearch
- Fmincon
- Open loop response

Genetic Algorithm: Genetic Algorithm is a method of solving constrained and unconstrained optimization problems based on the vision of natural life and bio-life evolution. Genetic Algorithm (GA) begin with an underlying "populace" of conceivable arrangements (people) to a given issue (nature) where every individual is spoken to utilize some type of encoding called as a "chromosome". These chromosomes are assessed somehow or another for their wellness Using their wellness as a model, certain chromosomes in the populace are chosen for proliferation (survival of the fittest).

$$G_{ga}(s) = \frac{0.9282e^{-3.7831t s}}{(29.6173s + 1)}$$  \hfill (15)$$

Fminsearch: it is the algorithm which is used to find the minimum of unconstrained multivariable function using derivative-free complimentary method. The Syntax is as follow:

```matlab
y = fminsearch(p, y0)

y = fminsearch(p, y0, options)

[y, fval] = fminsearch(...)

[y, fval, exitflag] = fminsearch(...)
```

Fminsearch is used to obtain the minimum of scalar functions of several variables, starting at an initial estimate. This is commonly referred to as unconstrained nonlinear optimization.

$$x = fminsearch(p, x_0)$$

begins at the point $y_0$ and finds a local minimum $y$ of the function described as ‘p’ which can be scalar, vector or matrix. ‘P’ is a function processed.

$$y = fminsearch(p, y_0, options)$$

minimizes with the optimization variable specified in the structure options.

$$[x, f_{val}] = fminsearch(...)$$

returns in $fval$ the value of the objective function $fun$ at the solution $x$.

$$G_{F_{fminsearch}}(s) = \frac{0.928e^{-29.587ts}}{(3.791s + 1)}$$  \hfill (16)$$

![Open loop test for Pressure process](image-url)
Table 1: System parameters of different optimization

<table>
<thead>
<tr>
<th>OPTIMIZATION</th>
<th>K</th>
<th>T</th>
<th>a</th>
<th>L</th>
</tr>
</thead>
<tbody>
<tr>
<td>Open loop test</td>
<td>0.9666</td>
<td>32.25</td>
<td>NA</td>
<td>2</td>
</tr>
<tr>
<td>Fminsearch Unconstrained Integer Order</td>
<td>0.874</td>
<td>49.17</td>
<td>1.137</td>
<td>1.275</td>
</tr>
<tr>
<td>Fmin search Unconstrained Fractional Order</td>
<td>0.928</td>
<td>29.587</td>
<td>NA</td>
<td>3.791</td>
</tr>
<tr>
<td>GA</td>
<td>0.9282</td>
<td>29.6173</td>
<td>NA</td>
<td>3.7831</td>
</tr>
<tr>
<td>Fmin constrained nonlinear minimization</td>
<td>0.928</td>
<td>29.5872</td>
<td>NA</td>
<td>3.7906</td>
</tr>
</tbody>
</table>

Fmincon: The optimization technique is employed to find minimum of constrained nonlinear multivariable function MinP(x) The Syntax is as follow:

\[ c(x) \leq 0 \]  \hspace{1cm} (17)
\[ c_{eq}(x) = 0 \] \hspace{1cm} (18)
\[ A.x \leq b \] \hspace{1cm} (19)
\[ Aeq.x = beq \] \hspace{1cm} (20)
\[ lb \leq x \geq ub \] \hspace{1cm} (21)
\[ 2.4 \text{ Comparison of different models based on the optimization techniques.} \]

Based on the process, parameters tabulated in the Table.1 for different models are obtained. The model dynamics are compared and shown in fig.4

3.1 Split range control scheme

The pressure process station with advanced control scheme is shown in the fig.5. In this process station split range controller is implemented, the split range controller consisting of single manipulated variable with two final control elements i.e two valves are provide one valve is for regulating the inlet flow of the compressed air and other valve for regulating the outlet flow air.

3.2 Data Acquisition

In this process for the data acquisition, the pressure transmitter is employed with the flowing specifications, it is a series type of pressure transmitter, span limit of 24-2004KPH, output signal in the range of 4-20mA and the power supply in the angle of 10.5 to 42v DC. The transmitter is able to convert (0-90) psi pressure range into the current signal of (4-20) mA, current-to-voltage
converter is employed to convert the (4-20) mA to voltage signal of (0-5) V, the data is interfaced to personal computer with MATLAB software by using DAQ card. The EASYIO platform is the USB version of the HILINK platform and RAPCON platform. It is a complete and low-cost real-time development package for both educational and industrial applications that do not require high sampling rates. The platform achieves real-time operation with sampling rates up to 1024 Hz. Inlet valve is a Normally Closed (NC) type is implemented and outlet Normally Open (NO) type is implemented.

3.3 Data Generation

In this generation process MATLAB generates the control signal based on the deviation, control signal is sent to the final control element, the final control element used in this process is pneumatic control valve with the following specifications: Body: 2/4, Trim: 1/4, Flange: ANSI150RF, Rating: 150CV2, Body Matl: A216WCD, Trim Matl: SS316, plug char: EQUAL

4 Robust PID Controller Design

In control system principles, robust control is a method to controller design that is explicitly designed for overcoming the uncertainty. Robust controller are designed to control the parameter even though the process affected by a large disturbances. The following optimization techniques are employed to set optimum controller parameters such as proportional gain 'Kp', Integral Constant 'Ki' and Derivative gain 'Kd'. The following optimization techniques are employed to tune the PID controller parameters of the pressure process systems. Pattern Search (PS), Fsolve, Genetic algorithm (GA)

4.1 Fsolve(Nonlinear equation solving)

Fsolve(Nonlinear equation solving): fsolve finds a root (zero) of a system of nonlinear equations. The Fsolve optimization techniques consists of three types 1. Trusted region dogleg technique 2. trusted region reflective technique 3. Levenberg Marquardt technique. For the optimization of controller Levenberg Marquardt technique is suitable for this controller design problem and other two techniques are not applicable for other optimization problem. The Syntax is as follow:

\[ x = \text{patternsearch}(\text{fun},x0,A,b,Aeq,beq,LB,UB) \]

defines a set of lower and upper bounds on the design variables, \( x \), so that a solution is found in the range \( LB \leq x \leq UB \). (Set Aeq=[] and beq=[]) if no linear equalities exist.)

4.2 Pattern Search

In pattern search the following classifications are analysed for the controller optimization problem such as Naive Pattern Searching, KMP Algorithm, Rabin-Karp Algorithm and Finite Automata algorithm The Syntax is as follow:

\[ x = \text{ga}(\text{fitnessfcn,nvars,A,b,Aeq,beq,LB,UB}) \]

defines a set of lower and upper bounds on the design variables, \( x \), so that a solution is found in the range \( LB \leq x \leq UB \). (Set Aeq=[] and beq=[]) if no linear equalities exist.)

4.3 Genetic Algorithm

In the controller optimization problem, the following syntax is employed to reduce the integral time error.

\[ x = \text{ga}(\text{fitnessfcn,nvars,A,b,Aeq,beq,LB,UB}) \]

defines a set of lower and upper bounds on the design variables, \( x \), so that a solution is found in the range \( LB \leq x \leq UB \). (Set Aeq=[] and beq=[]) if no linear equalities exist.)

4.4 ZN Method of Tuning

The ZN method is a standard tuning technique, it was developed by John G. Ziegler and Nathaniel B. Nichols. the proportional gain 'Kc' of the PID controller is increased to get sustained oscillation by keeping the integral gain 'Ki' and derivative gain 'Kd' equal to zero that the value of 'Kc' is nothing but ultimate gain 'Ku' and the oscillation period 'Tu'. 'Ku' and 'Tu' is used to set the PI and D gains.

<table>
<thead>
<tr>
<th>Tuning Rules</th>
<th>Kp</th>
<th>Ki</th>
<th>Kd</th>
</tr>
</thead>
<tbody>
<tr>
<td>Classical Rule (ZN-C)</td>
<td>0.6*Ku</td>
<td>2Kp/Tu</td>
<td>Kp*Tu/8</td>
</tr>
<tr>
<td>Pessen Integral rule</td>
<td>0.7*Ku</td>
<td>2.5 Kp/Tu</td>
<td>0.15<em>Kp</em>Tu</td>
</tr>
<tr>
<td>Some overshoot (ZN-SO)</td>
<td>0.33*Ku</td>
<td>2Kp/Tu</td>
<td>Kp*Tu/3</td>
</tr>
<tr>
<td>No overshoot (ZN-NO)</td>
<td>0.2*Ku</td>
<td>2Kp/Tu</td>
<td>Kp*Tu/3</td>
</tr>
</tbody>
</table>

Arguments. The structure consists of PID controller mentioned in the closed loop control system and the condition, which is provided, is to minimize the Integral absolute time error.
5 Results and Discussions

5.1 Controller Performance

The controller outputs of different tuning methods are shown in Fig.9. While analyzing the output of the controllers, the genetic algorithm and pattern searching optimization technique based PID controllers respond quickly for the load (regulatory problem) and set point (servo problem) changes for the pressure process. The PID controllers are tuned with various optimization algorithms such as pattern search algorithm, Fsolve algorithm and Genetic Algorithm (GA), and good combinations of controller parameters are selected. The closed loop responses with all the optimized PID controllers are comparatively shown in Fig.8. The response of closed loop pressure process loop with pattern search optimization and genetic algorithm-based tuning techniques are very effective. While analyzing the output of controllers, the genetic algorithm and pattern searching optimization technique-based PID controllers responds quickly for the load and set point changes. The time domain specifications such as maximum overshoot, rise time and settling time of different optimized PID controllers are tabulated in the Table.3

6 Conclusion

The proposed optimization method like genetic algorithm, fminsearch, fmincon gives the exact models for the pressure process station. The hybrid fminsearch and fmincon (FSFC) and Genetic Algorithm (GA) based transfer function model gives the proper dynamic behavior such as a real time pressure process and the proposed controller optimization techniques generates the optimum parameter values of the PID controller with minimum value of IAE and ISE for various disturbance and set point changes, once compared to the Ziegler-Nichols (Z-N) tuning techniques. Fsolve based PID controller gives the closed response without peak overshoot but the high settling time. The Pattern Search (PS) PID controller perform better than other PID controllers which give low overshoot and less settling time (ts) hence it is recommended for this closed loop pressure control process. The conclusion of this proposed optimization process of closed loop control system gives very good solution to the industrialist to reduce the man power and improved energy efficiency of the pressure process and indirectly this optimization process would help the industrialist in the economical aspects.

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References


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