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Modeling of the Selecting Optimum Cross Section of Open Canals

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Abstract: we discuss the problem of selecting optimum form of the cross section of open canals with the aim of the maximum transportation at the permanent flow of viscous biphasic medium. Semielliptical, semicircular, parabolical and trapezoidal cross sections have been considered. Comparing the consumptions for various cross sections among themselves in the equal areas, we define the optimum section with maximum consumption, that is with the maximum transportation of the flow.

Keywords: Permanent flow, biphasic medium, incondensible mediums, real density of the phase, reduced density of the phase, phase concentration, phase speed, phase viscosity coefficient, interaction coefficient between phases, free surface, canal cross section.

1 Introduction

In recent years, particular advances in the field of researches of flows dynamics in the open canals have been achieved in hydromechanics [1]. However, these models cover process physics incompletely, as the water, used for irrigation,is non-homogeneous and contains definite quantity of solid particles [2]. From the variety of multiphase mediums, one can distinguish the disperse flows, having comparatively regular character and presenting a mixture of several phases, one of which is various inclusions (drops, bubbles, solid particles), that is aerosols, fogs, bubble liquids, suspensions and so on. The appearance of the small quantity of solid particles in the flow, as it is known, essentially changes the character and the structure of the processes [3,4]. Meanwhile, new macroscopic parameters occur, in particular, reduced densities, interacting forces between phases as well as other mechanical characteristics. These parameters of the flow break the main conservation law of mixture components, the components interact that causes speed redistribution, concentrations of separate component and change mixture consumption. Therefore the problem of selecting optimum form of the cross section of open canals with the aim of the most transportation differs from the transportation of pure water, considered in the work [1]. We consider the case of permanent single speed two-dimensional flow of viscous biphasic mediums (water + solid particles). Even for this case, the comparative analysis of various forms of cross section of the canal with the aim of the maximum transportation of the mixture has not yet been made. The aim of the article is to fill up this gap.

2 Problem settlement

Defining optimum cross section of the open canal with the maximum consumption, that is with the maximum transportation of the flow for the permanent flow of viscous biphasic mediums. Meanwhile semielliptical, semicircular, parabolical and trapezoidal cross sections are being considered, such comparative analysis for the biphasic medium are made for the first time.

Solution methods. The numerical method of Mathcad medium has been used for solving the received system of the differential equations in partial derivatives with the appropriate boundary conditions.

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3 The main part

The main point of the permanent single speed flow of the biphasic mediums consists in the fact that we ignore the vertical and cross components of phases speeds. Meanwhile, one can assume that the concentrations of the first and second phases are constant, that is $f1 = const$, consequently and $f2 = const$. This means that in the given case in any volume unit, the quantity of the first and second phase remains constant and doesn't depend on the coordinates and time. As, incompressible mediums are considered, it follows from the concentrations constancy that the reduced densities are

$$\rho 1 = const, \rho 2 = const.$$

As it is known, the flow in open canals is free-flow, then for the considered case, the 'interpenetrating' model of the flow of biphasic medium, based on the following assumptions: interphase transition is absent; for each phase, summands, characterizing the interaction between phases are included into the equation; each phase is taken as the separate continuous medium and described by separable equations and takes the form [5,6,7,8]

$$\begin{cases} \frac{\mu_1}{\rho_{1i}} \frac{\partial^2 u_1}{\partial y^2} + \frac{\mu_1}{\rho_{1i}} \frac{\partial^2 u_1}{\partial z^2} + \frac{k}{\rho_{1i} f_1} (u_2 - u_1) = 0, \\ \frac{\mu_2}{\rho_{2i}} \frac{\partial^2 u_2}{\partial y^2} + \frac{\mu_2}{\rho_{2i}} \frac{\partial^2 u_2}{\partial z^2} + \frac{k}{\rho_{2i} f_2} (u_1 - u_2) = 0 \end{cases} \quad (1)$$

equations of nonseparability and

$$\frac{\partial u_1}{\partial x} = 0, \frac{\partial u_2}{\partial x} = 0$$

consequently, the free surface

$$H = H_0 = const,$$

where ρ_{ni} -the real density of n phase (n=1,2); u_n-the longitudinal component of the speed of n phase; μ_n-viscosity coefficient of n phase; k-interaction coefficient between phases; x, y, z-longitudinal, vertical, cross coordinate axes. Let's consider the boundary conditions for the received system of equations (1). At the bottom and side walls of the canal for the phases speeds, we use sticking condition, that is $u_1 = 0, u_2 = 0$ $y = 0$, $u_1 = 0, u_2 = 0$ $z = \pm\varphi(y)$, where $\varphi(y)$-the function, describing the side walls of the canal. We ignore the frictional force of the air for each phase on the free surface

$$\frac{\partial u_1}{\partial y} = 0, \frac{\partial u_2}{\partial y} = 0,$$

with $y = H_0$.

Thus, we solve the equations system (1) with the boundary conditions

$$\begin{cases} u_1 = 0,\ u_2 = 0 \quad when \quad y = 0, \\ \frac{\partial u_1}{\partial y} = 0, \frac{\partial u_2}{\partial y} = 0, \quad when \quad y = H_0, \\ u_1 = 0, u_2 = 0 \quad when \quad z = -\varphi(y), \\ u_1 = 0, u_2 = 0 \quad when \quad z = \varphi(y) \end{cases} \quad (2)$$

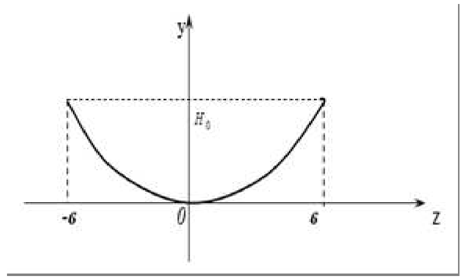

Fig. 1: Semielliptical cross section

Further let's consider the following forms of the cross sections of the open canals:

$$\frac{z^2}{b^2} + \frac{(y - H_0)^2}{H_0^2} = 1,$$

from which

$$z = \pm b\sqrt{1 - \frac{(y - H_0)^2}{(H_0)^2}}$$

consequently,

$$\varphi(y) = b\sqrt{1 - \frac{(y - H_0)^2}{(H_0)^2}},$$

where b - ellipse semi-axle in the direction of z axis.
Semicircular cross section:

$$z^2 + (y - H_0)^2 = H_0{}^2$$

,

from which
$z = \pm\sqrt{H_0{}^2 - (y - H_0)^2}$,
consequently, $\varphi(y) = \sqrt{H_0{}^2 - (y - H_0)^2}$.
Parabolical cross section:

$$z^2 = \frac{a^2}{H_0} y,$$

from which

$$z = \pm a\sqrt{\frac{y}{H_0}},$$

consequently, $\varphi(y) = a\sqrt{\frac{y}{H_0}}$, where a - parabola parameter.

Trapezoidal cross section: $z = \pm(c + my)$, consequently, $\varphi(y) = c + my$, where c- half- width of the canal bottom; m- slope base coefficient. For distinguishing the optimum form of the cross section with the aim of the maximum transportation of the flow, we solve the problem (1), (2) and calculate the mixture consumption for the various form of the cross sections:

$$Q_{sm}^{pe} = \int_0^{H_0} \int_{-\varphi(y)}^{\varphi(y)} (f_1 u_1 + f_2 u_2) dydz$$

where *pe* - semielliptical;

$$\varphi(y) = b\sqrt{1 - \frac{(y - H_0)^2}{H_0}}$$

$$Q_{sm}^{pk} = \int_0^{H_0} \int_{-\varphi(y)}^{\varphi(y)} (f_1 u_1 + f_2 u_2) dydz$$

where *pk* -semicircle;

$$\varphi(y) = \sqrt{H_0^2 - (y - H_0)^2}.$$

$$Q_{sm}^{pb} = \int_0^{H_0} \int_{-\varphi(y)}^{\varphi(y)} (f_1 u_1 + f_2 u_2) dydz$$

where *pb*-parabola;

$$\varphi(y) = a\sqrt{\frac{y}{H_0}}.$$

$$Q_{sm}^{tr} = \int_0^{H_0} \int_{-\varphi(y)}^{\varphi(y)} (f_1 u_1 + f_2 u_2) dydz$$

where *tr* - trapezoid;

$$\varphi(y) = c + my.$$

Further, comparing $Q_{sm}^{pe}, Q_{sm}^{pk}, Q_{sm}^{pb}, Q_{sm}^{tr}$, among themselves in the equal areas, we define the optimum section with the maximum consumption, that is with the maximum transportation of the flow. The problem (1), (2) has been solved in Mathcad medium [9] in the following original data:

$\rho_{1i} = 100kg * s^2/m^4, \rho_{2i} = 250kg * s^2/m^4, k = 100kg * s/m^4, f_1 = 0,7, f_2 = 0,3, b = 2m, H_0 = 0,5m$ (semielliptical section)$H_0 = 1m$ (semicircular section), $a = 0,75m, H_0 = 1,57m$ (parabolical section), $m = 1, c = 0,57m, H_0 = 1m$ (trapezoidal section). At these data, the areas of the various forms of cross sections will be equal.

4 Conclusions

The comparison of the calculation results for the canals of semielliptical, semicircular, parabolical and trapezoidal forms of the cross sections in the equal areas has shown that the optimum form, providing the maximum flow capacity is the canal of the semicircular form, further of semielliptical, parabolical and trapezoidal ones. The speeds of the both phases according to the receding from the canal bottom increase and achieve the maximum value on the free surface, besides the speed of the first phase (of water) is more than the speed of the second phase (of solid phase) (Fig.2). Across the canal width the phases speeds achieve maximum values in the middle of the canal width and here $u_n, m/s$ also retarding of solid particles is observed in comparison with water (Fig.3). These results are qualitatively in keeping with the experimental data [10].

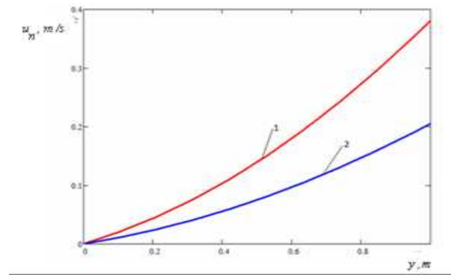

Fig. 2: Change of the phases speeds $u_n(n = 1,2)$ in depth of the flow for the semicircular form at $z = 0,8m$
1 - the speed of the first phase; 2 - the speed of the second phase

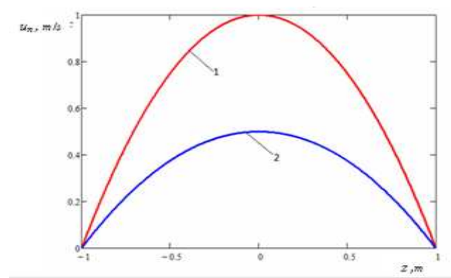

Fig. 3: Change of the phases speeds $u_n(n = 1,2)$ across the width of flow for the semicircular form at $y = 1m$.
1 - the speed of the first phase; 2 - the speed of the second phase

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