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Fuzzy Multi-Objective Capacitated Transportation Problem with Mixed Constraints

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Abstract: In this article a capacitated transportation problem is considered which is formulated as a multi objective capacitated transportation problem with mixed constraints. To determine the optimum compromise solution of multi objective capacitated transportation problem (MOCTP) with mixed constraints a Fuzzy multi objective programming approach has been applied in which we use three different forms of membership functions viz. linear, exponential and hyperbolic. A numerical illustration has been provided to illustrate the solution procedure.

Keywords: Capacitated Transportation problem; Compromise Solution; Fuzzy multi objective programming; Mixed constraints; Multi-Objective programming.

1 Introduction

The Transportation problem (TP) deals with a situation in which a single product is to be transported from several sources (also called origin, supply or capacity centers) to several sinks (also called destination, demand or requirement centers). Hitchcock (1941) developed the basic transportation problem. It has been seen that much effort has been concentrated on transportation problems (TP) with equality constraints such as fuzzy programming approach with linear membership function was applied by Bit et al. (1992) to solve multi objective transportation problem, Verma et al. (1997) and Li et al. (2000) presented fuzzy approach to the MOTP, Gupta et al. (2013) apply GP approach in transportation problem with equality constraints etc. In real life, however, most problems have mixed constraints. A literature search revealed no systematic method for finding an optimal solution for transportation problems with mixed constraints. Recently some authors discuss TPs with mixed constraints such as Adlakha et al. (2006), Mondal and Hossain (2012) etc.

Tanaka et al. (1974) proposed the concept of fuzzy mathematical programming not only on a general level but also on a more practical level. a relatively practical introduction of fuzzy set theory (Zadeh 1965) into conventional multi objective linear programming problems was first presented by Zimmermann (1978) and further studied by Leberling (1981) and Hannan (1981). Following the fuzzy decision or the minimum operator proposed by Bellman and Zadeh (1970) together with linear, hyperbolic, or piecewise linear membership functions respectively, they proved that there exist equivalent linear programming problems.

In this article, a capacitated transportation problem has been considered and formulated as multi objective capacitated transportation problem with mixed constraints in section 2. Section 3 describes solution procedure to solve MOCTP i.e. Fuzzy multi objective programming method with three different membership functions viz. linear, exponential and hyperbolic. In section 4 a numerical illustration is presented for demonstrating the computational procedure of the method and section 5 conclude and summarize the work.

2 Formulation of the problem

Let us consider m sources (origins) O_i $(i=1,2,\dots,m)$ and n destinations D_j $(j=1,2,\dots,n)$. At each source O_i $(i=1,2,\dots,m)$, let a_i be the amount of product to be shipped to the n destinations D_j in order to satisfy the demand b_j $(j=$

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$1,2,\ldots,n)$ there. Then the mathematical model for the multi objective capacitated transportation problem with mixed constraints is as follows:

$$\left.\begin{aligned} &Min\ Z^k = \textstyle\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{k} x_{ij}, \quad k = 1,2,\ldots,K \\ &Subject\ to\ \ \textstyle\sum_{j=1}^{n} x_{ij}\{\leq / = / \geq\} a_i, i = 1,2,\ldots,m \\ &\qquad\qquad\ \ \textstyle\sum_{i=1}^{m} x_{ij}\{\leq / = / \geq\} b_j, j = 1,2,\ldots,n \\ &\qquad\qquad\ \ 0 \leq x_{ij} \leq r_{ij} \end{aligned}\right\} \tag{1}$$

where
c_{ij}^{k} denotes the transportation costs, delivery time and damage charges (loss of quality and quantity of transported items).
x_{ij} be the variable that represents the unknown quantity transported from i^{th} origin to j^{th} destination.
r_{ij} be the maximum amount of quantity transported from i^{th} source to j^{th} destination i.e. $x_{ij} \leq r_{ij}$ or the capacitated restriction on the route i to j.

3 Solution Procedure

3.1 Fuzzy multi objective programming method

For a multi objective programming, Zimmermann (1978) extends fuzzy programming by introducing fuzzy goals for all the objective functions. Let us assume that the DM has a fuzzy goal for each of the objective functions, then the corresponding membership functions are defined as

3.1.1 Linear membership function

For each objective function a linear membership function $\mu_k^L(Z^k)$ is defined as:

$$\mu_k^L\{Z^k\} = \begin{cases} 1 & if Z^k \leq Z_l^k \\ \frac{Z_u^k - Z^k}{Z_u^k - Z_l^k} & if Z_l^k \leq Z^k \leq Z_u^k \\ 0 & if Z^k > Z_u^k \end{cases}$$

where Z_l^k and Z_u^k are respectively the lower and upper tolerance limits of the objective functions such that the degrees of the membership function are 0 and 1, respectively, and it is depicted in Fig. 1. These tolerance limits are obtained from the following payoff matrix:

$$Payoff Matrix = \begin{bmatrix} & Z^1 & Z^2 & \cdots & Z^k \\ x_{ij}^{(1)} & Z^1(x_{ij}^{(1)}) & Z^2(x_{ij}^{(1)}) & \cdots & Z^k(x_{ij}^{(1)}) \\ x_{ij}^{(2)} & Z^1(x_{ij}^{(2)}) & Z^2(x_{ij}^{(2)}) & \cdots & Z^k(x_{ij}^{(2)}) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ x_{ij}^{(k)} & Z^1(x_{ij}^{(k)}) & Z^2(x_{ij}^{(k)}) & \cdots & Z^k(x_{ij}^{(k)}) \end{bmatrix}; i = 1,2,\ldots,m; j = 1,2,\ldots,n$$

where $x_{ij}^{k}; k = 1,2,\ldots,K$ is the individual optimum solution of the k^{th} objective function. The maximum value of each column gives the upper tolerance limit and the minimum value of each column gives lower tolerance limit for the objective functions respectively.

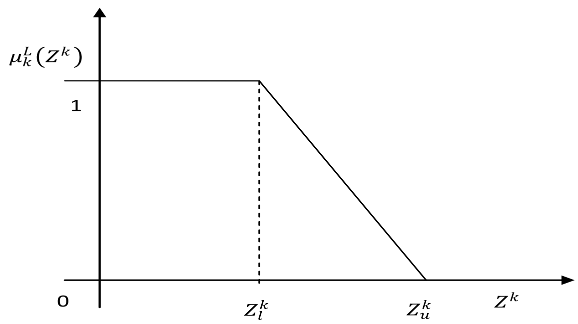

Fig. 1: *Linear membership function for k-th fuzzy goal*

The multi objective capacitated transportation problem with mixed constraints given in eq. (1) can be written as an equivalent linear model as follows:

$$\left.\begin{array}{l} Minimize\ \ \lambda \\ Subject\ to\ \ \frac{Z_u^k - Z^k}{Z_u^k - Z_l^k} \geq \lambda \\ \qquad \sum_{j=1}^{n} x_{ij}\{\leq / = / \geq\} a_i, i = 1,2,\ldots,m \\ \qquad \sum_{i=1}^{m} x_{ij}\{\leq / = / \geq\} b_j, j = 1,2,\ldots,n \\ \qquad \lambda \geq 0 \\ \qquad 0 \leq x_{ij} \leq r_{ij} \end{array}\right\} \tag{2}$$

3.1.2 Exponential membership function

For each objective function an exponential membership function $\mu_k^E(Z^k)$ is defined as:

$$\mu_k^E\{Z^k\} = \begin{cases} 1 & if Z^k \leq Z_l^k \\ \frac{exp\left(\frac{-\alpha(Z^k - Z_l^k)}{Z_u^k - Z_l^k}\right) - exp(-\alpha)}{1 - exp(-\alpha)} & if Z_l^k \leq Z^k \leq Z_u^k \\ 0 & if Z^k > Z_u^k \ \ and \ \ \alpha \to \infty \end{cases}$$

where α is a non-zero parameter, prescribed by the decision maker and Z_l^k, Z_u^k have the usual meaning as described in section 3.1.1. This is graphically depicted in Fig 2. Now the multi objective capacitated transportation problem with mixed

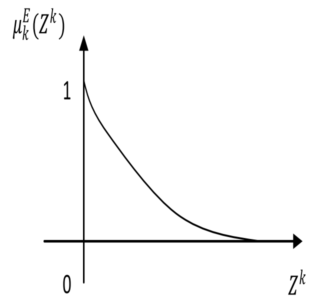

Fig. 2: *Exponential membership function for k-th fuzzy goal*

constraints given in eq. (1) can be written as an equivalent non linear model as follows:

$$\left.\begin{array}{l} Minimize \ \ \lambda \\ Subject\ to \ \ \frac{exp\left(\frac{-\alpha(Z^k-Z_l^k)}{Z_u^k-Z_l^k}\right)-exp(-\alpha)}{1-exp(-\alpha)} \geq \lambda \\ \qquad \sum_{j=1}^{n} x_{ij}\{\leq / = / \geq\} a_i, i=1,2,\ldots,m \\ \qquad \sum_{i=1}^{m} x_{ij}\{\leq / = / \geq\} b_j, j=1,2,\ldots,n \\ \qquad \lambda \geq 0 \\ \qquad 0 \leq x_{ij} \leq r_{ij} \end{array}\right\} \tag{3}$$

3.1.3 Hyperbolic membership function

For each objective function a hyperbolic membership function $\mu_k^H(Z^k)$ is defined as:

$$\mu_k^H\{Z^k\} = \begin{cases} 1 & if Z^k \leq Z_l^k \\ \frac{1}{2}tanh\left(\left(\frac{Z_u^k+Z_l^k}{2} - Z^k\right)\alpha_k\right) + \frac{1}{2} & if Z_l^k \leq Z^k \leq Z_u^k \\ 0 & if Z^k > Z_u^k \end{cases}$$

where $\alpha_k = \frac{6}{(Z_u^k - Z_l^k)}$ and Z_l^k, Z_u^k have the usual meaning as described in section 3.1.1.
This membership function has the following formal properties given by Zimmermann, 1985 which is graphically depicted in Fig 3:

– $\mu_k^H(Z^k)$ is strictly monotonously decreasing function with respect to Z^k.
– $\mu_k^H(Z^k) = \frac{1}{2} \Leftrightarrow Z^k = \frac{1}{2}(Z_u^k + Z_l^k)$.
– $\mu_k^H(Z^k)$ is strictly convex for $Z^k \geq \frac{1}{2}(Z_u^k + Z_l^k)$ and strictly concave for $Z^k \leq \frac{1}{2}(Z_u^k + Z_l^k)$.
– $\mu_k^H(Z^k)$ satisfies $0 < \mu_k^H(Z^k) < 1$ for $Z_l^k < \mu_k^H(Z^k) < Z_u^k$ and approaches asymptotically $\mu_k^H(Z^k) = 0$ and $\mu_k^H(Z^k) = 1$ as $Z^k \to \infty$ and $-\infty$ respectively.

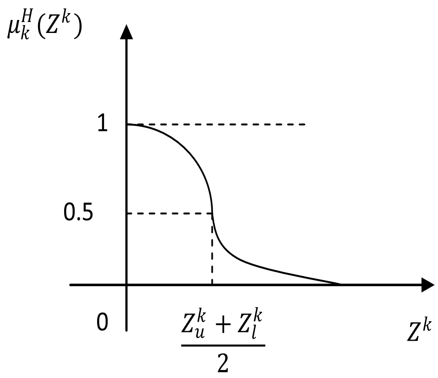

Fig. 3: *Hyperbolic membership function for k-th fuzzy goal*

Now the multi objective capacitated transportation problem with mixed constraints given in eq. (1) can be written as an equivalent non linear model as follows:

$$\left.\begin{array}{l} \textit{Minimize}\ \ \lambda \\ \textit{Subject to}\ \ \frac{1}{2}tanh\left(\left(\frac{Z_u^k+Z_l^k}{2}-Z^k\right)\alpha_k\right)+\frac{1}{2}\geq\lambda \\ \qquad\qquad \sum_{j=1}^{n}x_{ij}\{\leq/=/\geq\}a_i, i=1,2,\ldots,m \\ \qquad\qquad \sum_{i=1}^{m}x_{ij}\{\leq/=/\geq\}b_j, j=1,2,\ldots,n \\ \qquad\qquad \lambda\geq 0 \\ \qquad\qquad 0\leq x_{ij}\leq r_{ij} \end{array}\right\} \tag{4}$$

4 Numerical Illustration

To demonstrate the suggested approach, we consider the following example. Here, we consider three origins and three destinations. The TP cost, time and the damage charges (both quality and quantity damage) during the transportation are represented by the following matrices given below:

Table 1: Cost Matrix

	b_1	b_2	b_3	Supply
a_1	3	4	13	≤ 12
a_2	12	14	7	$= 15$
a_3	15	10	8	≥ 20
Demand	≥ 9	$= 13$	≤ 21	

Table 2: Time Matrix

	b_1	b_2	b_3	Supply
a_1	9	1	3	≤ 12
a_2	2	4	6	= 15
a_3	8	12	10	≥ 20
Demand	≥ 9	= 13	≤ 21	

Table 3: Damage charge Matrix

	b_1	b_2	b_3	Supply
a_1	8	9	11	≤ 12
a_2	3	4	7	= 15
a_3	2	1	8	≥ 20
Demand	≥ 9	= 13	≤ 21	

Using the data given in table 1,2 and 3, the multi objective capacitated transportation problem with mixed constraints can be given as:

$$Min\ \ Z_1 = (3x_{11}+4x_{12}+13x_{13})+(12x_{21}+14x_{22}+7x_{23})+(15x_{31}+10x_{32}+8x_{33})$$

$$Min\ \ Z_2 = (9x_{11}+x_{12}+3x_{13})+(2x_{21}+4x_{22}+6x_{23})+(8x_{31}+12x_{32}+10x_{33})$$

$$Min\ \ Z_3 = (8x_{11}+9x_{12}+11x_{13})+(3x_{21}+4x_{22}+7x_{23})+(2x_{31}+x_{32}+6x_{33})$$

$$Subject\ to$$

$$\sum_{j=1}^{3} x_{1j} \leq 12;\ \sum_{j=1}^{3} x_{2j} = 15;\ \sum_{j=1}^{3} x_{3j} \geq 20$$

$$\sum_{i=1}^{3} x_{i1} \geq 9;\ \sum_{j=1}^{3} x_{1j} = 13;\ \sum_{j=1}^{3} x_{1j} \leq 21$$

The capacitated constraints are given below:

$0 \leq x_{11} \leq 6, 0 \leq x_{12} \leq 7, 0 \leq x_{13} \leq 13, 0 \leq x_{21} \leq 6, 0 \leq x_{22} \leq 2, 0 \leq x_{23} \leq 13, 0 \leq x_{31} \leq 4,$

$0 \leq x_{32} \leq 7, 0 \leq x_{33} \leq 14.$

Individual optimum solutions are obtained by solving the above problem separately for each objective using the optimizing software LINGO as follows:

Table 4: Individual optimum solution

Objectives	Objective values	Allocations								
		x_{11}	x_{12}	x_{13}	x_{21}	x_{22}	x_{23}	x_{31}	x_{32}	x_{33}
Cost	345	3	5	0	6	1	8	0	7	13
Time	269	0	7	0	6	2	7	4	4	12
Damage Charges	180	0	4	0	6	2	7	4	7	9

4.1 Compromise solution by Fuzzy multi objective programming method

To formulate the problem (2), (3) and (4), upper and lower tolerance limits are required which we obtained from the following payoff matrix as:

$$PayoffMatrix = \begin{bmatrix} & Z^1 & Z^2 & Z^3 \\ x_{ij}^{(1)} & 345 & 310 & 232 \\ x_{ij}^{(2)} & 373 & 269 & 222 \\ x_{ij}^{(3)} & 367 & 272 & 180 \end{bmatrix}$$

$$Z_u^1 = 373, Z_l^1 = 345, Z_u^2 = 310, Z_l^2 = 269, Z_u^3 = 232 \ \ and \ \ Z_l^3 = 180$$

Now if we are using linear membership function, an equivalent crisp problem (2) can be formulated as:

$$\left.\begin{array}{l} Minimize \quad \lambda \\ Subject\ to \\ (373-((3x_{11}+4x_{12}+13x_{13})+(12x_{21}+14x_{22}+7x_{23})+(15x_{31}+10x_{32}+8x_{33}))) \geq 28\lambda \\ (310-((9x_{11}+x_{12}+3x_{13})+(2x_{21}+4x_{22}+6x_{23})+(8x_{31}+12x_{32}+10x_{33}))) \geq 41\lambda \\ (232-((8x_{11}+9x_{12}+11x_{13})+(3x_{21}+4x_{22}+7x_{23})+(2x_{31}+x_{32}+6x_{33}))) \geq 52\lambda \\ \sum_{j=1}^3 x_{1j} \leq 12;\ \sum_{j=1}^3 x_{2j} = 15;\ \sum_{j=1}^3 x_{3j} \geq 20 \\ \sum_{i=1}^3 x_{i1} \geq 9;\ \sum_{i=1}^3 x_{2i} = 13;\ \sum_{i=1}^3 x_{i3} \leq 21 \\ 0 \leq x_{11} \leq 6, 0 \leq x_{12} \leq 7, 0 \leq x_{13} \leq 13, 0 \leq x_{21} \leq 6, 0 \leq x_{22} \leq 2, 0 \leq x_{23} \leq 13, \\ 0 \leq x_{31} \leq 4, 0 \leq x_{32} \leq 7, 0 \leq x_{33} \leq 14 \end{array}\right\}$$

By optimizing software LINGO, the optimum compromise allocation is obtained as:

$$x_{11}^* = 0, x_{12}^* = 6, x_{13}^* = 0, x_{21}^* = 5, x_{22}^* = 0, x_{23}^* = 10, x_{31}^* = 4, x_{32}^* = 7, x_{33}^* = 9$$

If we are using exponential membership function with parameter $\alpha = 1$, an equivalent crisp problem (3) can be formulated as:

$$\left.\begin{array}{l} Minimize \quad \lambda \\ Subject\ to \\ \frac{e^{-(Z_1-345)}-e^{-1}}{1-e^{-1}} \geq \lambda \\ \frac{e^{-(Z_2-269)}-e^{-1}}{1-e^{-1}} \geq \lambda \\ \frac{e^{-(Z_3-180)}-e^{-1}}{1-e^{-1}} \geq \lambda \\ \sum_{j=1}^3 x_{1j} \leq 12;\ \sum_{j=1}^3 x_{2j} = 15;\ \sum_{j=1}^3 x_{3j} \geq 20 \\ \sum_{i=1}^3 x_{i1} \geq 9;\ \sum_{i=1}^3 x_{2i} = 13;\ \sum_{i=1}^3 x_{i3} \leq 21 \\ 0 \leq x_{11} \leq 6, 0 \leq x_{12} \leq 7, 0 \leq x_{13} \leq 13, 0 \leq x_{21} \leq 6, 0 \leq x_{22} \leq 2, 0 \leq x_{23} \leq 13, \\ 0 \leq x_{31} \leq 4, 0 \leq x_{32} \leq 7, 0 \leq x_{33} \leq 14 \end{array}\right\}$$

By optimizing software LINGO, the optimum compromise allocation is obtained as:

$$x_{11}^* = 0, x_{12}^* = 4, x_{13}^* = 1, x_{21}^* = 5, x_{22}^* = 2, x_{23}^* = 8, x_{31}^* = 4, x_{32}^* = 7, x_{33}^* = 9$$

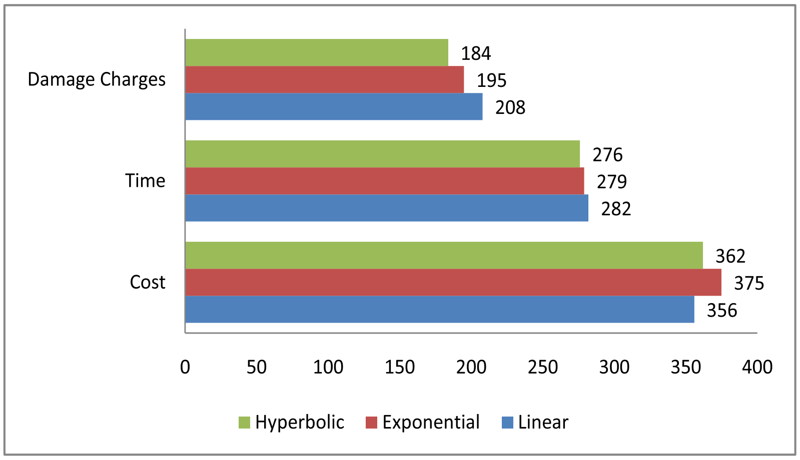

Fig. 4: *Graphical representation of results*

If we are using hyperbolic membership function, an equivalent crisp problem (4) can be formulated as:

$$\left.\begin{array}{l}
Minimize \quad \lambda \\
Subject\ to \\
\frac{1}{2}tanh((359-((3x_{11}+4x_{12}+13x_{13})+(12x_{21}+14x_{22}+7x_{23})+(15x_{31}+10x_{32}+8x_{33})))0.2143)+\frac{1}{2}\geq\lambda \\
\frac{1}{2}tanh((289.5-((9x_{11}+x_{12}+3x_{13})+(2x_{21}+4x_{22}+6x_{23})+(8x_{31}+12x_{32}+10x_{33})))0.1463)+\frac{1}{2}\geq\lambda \\
\frac{1}{2}tanh((206-((8x_{11}+9x_{12}+11x_{13})+(3x_{21}+4x_{22}+7x_{23})+(2x_{31}+x_{32}+6x_{33})))0.1154)+\frac{1}{2}\geq\lambda \\
\sum_{j=1}^{3}x_{1j}\leq 12;\ \sum_{j=1}^{3}x_{2j}=15;\ \sum_{j=1}^{3}x_{3j}\geq 20 \\
\sum_{i=1}^{3}x_{i1}\geq 9;\ \sum_{i=1}^{3}x_{2i}=13;\ \sum_{i=1}^{3}x_{i3}\leq 21 \\
0\leq x_{11}\leq 6, 0\leq x_{12}\leq 7, 0\leq x_{13}\leq 13, 0\leq x_{21}\leq 6, 0\leq x_{22}\leq 2, 0\leq x_{23}\leq 13, \\
0\leq x_{31}\leq 4, 0\leq x_{32}\leq 7, 0\leq x_{33}\leq 14
\end{array}\right\}$$

By optimizing software LINGO, the optimum compromise allocation is obtained as:

$$x_{11}^{*}=0, x_{12}^{*}=4, x_{13}^{*}=0, x_{21}^{*}=5, x_{22}^{*}=2, x_{23}^{*}=8, x_{31}^{*}=4, x_{32}^{*}=7, x_{33}^{*}=9$$

Table 5: Compromise optimum solution

Methods	Objective values		
	Cost	Time	Damage charges
Linear membership function	356	282	208
Exponential membership function	375	279	195
Hyperbolic membership function	362	276	184

5 Conclusion and Summary

Present article presents a solution procedure i.e Fuzzy multi objective programming with three different forms of membership functions viz. linear, exponential and hyperbolic to determine the optimum compromise solution of the MOCTP with mixed constraints. The solutions obtained has been summarized in table 5 and graphically shown in Fig 4.

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