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New Modular Relations for the Ratio's of Ramanujan Function $\chi(-q)$ of degree 5

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Abstract: In this paper, we establish some new modular equations using ratio's of Ramanujan function $\chi(-q)$ of degree 5. Further, we obtain some explicit evaluations of class invariant g_n from them.

Keywords: Theta functions.

1 Introduction

In Chapter 16 of his second notebook [1,7], Ramanujan develops the theory of theta-function and is defined by

$$f(a,b) := \sum_{n=-\infty}^{\infty} a^{\frac{n(n+1)}{2}} b^{\frac{n(n-1)}{2}}, |\, ab\, | < 1, \tag{1}$$

$$= (-a;ab)_\infty(-b;ab)_\infty(ab;ab)_\infty$$

where $(a;q)_0 = 1$ and $(a;q)_\infty = (1-a)(1-aq)(1-aq^2)\cdots$.

As special cases, we have

$$\varphi(q) := f(q,q) = \sum_{n=-\infty}^{\infty} q^{n^2} = \frac{(-q;-q)_\infty}{(q;-q)_\infty}, \tag{2}$$

$$\psi(q) := f(q,q^3) = \sum_{n=0}^{\infty} q^{\frac{n(n+1)}{2}} = \frac{(q^2;q^2)_\infty}{(q;q^2)_\infty}, \tag{3}$$

$$f(-q) := f(-q,-q^2) = \sum_{n=-\infty}^{\infty} (-1)^n q^{\frac{n(3n-1)}{2}} = (q;q)_\infty \tag{4}$$

and

$$\chi(q) := (-q;q^2)_\infty. \tag{5}$$

The ordinary hypergeometric series ${}_2F_1(a,b;c;x)$ is defined by

$${}_2F_1(a,b;c;x) := \sum_{n=0}^{\infty} \frac{(a)_n(b)_n}{(c)_n n!} x^n,$$

where $(a)_0 = 1, (a)_n = a(a+1)(a+2)\cdots(a+n-1)$ for any positive integer n, and $|\, x\, | < 1$.
Let

$$z := z(x) := {}_2F_1\left(\frac{1}{2},\frac{1}{2};1;x\right) \tag{6}$$

and

$$q := q(x) := \exp\left(-\pi\, \frac{{}_2F_1(\frac{1}{2},\frac{1}{2};1;1-x)}{{}_2F_1(\frac{1}{2},\frac{1}{2};1;x)}\right), \tag{7}$$

where $0 < x < 1$.
Let r denote a fixed natural number and assume that the following relation holds:

$$r\frac{{}_2F_1\left(\frac{1}{2},\frac{1}{2};1;1-\alpha\right)}{{}_2F_1\left(\frac{1}{2},\frac{1}{2};1;\alpha\right)} = \frac{{}_2F_1\left(\frac{1}{2},\frac{1}{2};1;1-\beta\right)}{{}_2F_1\left(\frac{1}{2},\frac{1}{2};1;\beta\right)}. \tag{8}$$

Then a modular equation of degree r in the classical theory is a relation between α and β induced by (8). We often say that β is of degree r over α and $m := \dfrac{z(\alpha)}{z(\beta)}$ is called the multiplier. We also use the notations $z_1 := z(\alpha)$ and $z_r := z(\beta)$ to indicate that β has degree r over α.

The function $\chi(q)$ [1, Entry 12(v),(vi), p.56] is intimately connected to Ramanujan's class invariants G_n and g_n which are defined by

$$G_n = 2^{-1/4}q^{-1/24}\chi(q), \;\; g_n = 2^{-1/4}q^{-1/24}\chi(-q) \tag{9}$$

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where $q = e^{-\pi\sqrt{n}}$ and n is a positive rational number,

$$\chi(q) = 2^{1/6}\left\{\alpha(1-\alpha)q^{-1}\right\}^{-1/24}, \tag{10}$$

$$\chi(-q) = 2^{1/6}(1-\alpha)^{1/12}\alpha^{-1/24}q^{-1/24}. \tag{11}$$

2 Preliminary results

Lemma 1.[5] *If* $P := \dfrac{\psi(q)}{q^{1/2}\psi(q^5)}$ *and* $Q := \dfrac{\psi(q^2)}{q\psi(q^{10})}$, *then*

$$\left(\frac{P}{Q}\right)^2 + \left(\frac{Q}{P}\right)^2 + 4 = P^2 + \frac{5}{P^2}. \tag{12}$$

Lemma 2.[2, *Ch. 25, Entry 66, p.233*] *If* $P := \dfrac{\psi(q)}{q^{1/2}\psi(q^5)}$ *and* $Q = \dfrac{\psi(q^3)}{q^{3/2}\psi(q^{15})}$, *then*

$$PQ + \frac{5}{PQ} = -\left(\frac{P}{Q}\right)^2 + \left(\frac{Q}{P}\right)^2 + 3\left(\frac{P}{Q} + \frac{Q}{P}\right). \tag{13}$$

Lemma 3.[4] *If* $P := \dfrac{\psi(q)}{q^{1/2}\psi(q^5)}$ *and* $Q = \dfrac{\psi(q^4)}{q^2\psi(q^{20})}$, *then*

$$\frac{P^4}{Q^4} + \frac{Q^4}{P^4} + 24\left(\frac{P^2}{Q^2} + \frac{Q^2}{P^2}\right) + 8\left(P^2Q^2 + \frac{5^2}{P^2Q^2}\right) - 20\left(Q^2 + \frac{5}{Q^2}\right) + 120 + 3\left(P^4 + \frac{5^2}{P^4}\right) - 32\left(P^4 + \frac{5}{P^4}\right) = P^4\left(Q^2 + \frac{3}{Q^2}\right) + \frac{5}{P^4}\left(3Q^2 + \frac{5^2}{Q^2}\right). \tag{14}$$

Lemma 4.[4] *If* $P := \dfrac{\psi(q)}{q^{1/2}\psi(q^5)}$ *and* $Q := \dfrac{\psi(q^5)}{q^{5/2}\psi(q^{25})}$ *then*

$$\frac{Q^3}{P^3} - \frac{5Q^2}{P^2} - \frac{15Q}{P} + 5\left(PQ + \frac{5}{PQ}\right) + 5\left(Q^2 + \frac{5}{P^2}\right) = P^2Q^2 + \frac{5^2}{P^2Q^2} + 15. \tag{15}$$

Lemma 5.[4] *If* $P := \dfrac{\psi(q)\psi(q^7)}{\psi(q^5)\psi(q^{35})}$ *and* $Q := \dfrac{\psi(q)\psi(q^{35})}{\psi(q^5)\psi(q^7)}$, *then*

$$Q^4 - \frac{1}{Q^4} - 14\left[\left(Q^3 + \frac{1}{Q^3}\right) - \left(Q^2 - \frac{1}{Q^2}\right) + 10\left(Q + \frac{1}{Q}\right)\right] + P^3 + \frac{5^3}{P^3} = 7\left\{\left(P^2 + \frac{5^2}{P^2}\right)\left(Q + \frac{1}{Q}\right) - \left(P + \frac{5}{P}\right) \times \left[2\left(Q^2 + \frac{1}{Q^2}\right) + 9\right]\right\}. \tag{16}$$

Lemma 6.[4] *If* $P := \dfrac{\psi(q)\psi(q^{11})}{q^6\psi(q^5)\psi(q^{55})}$ *and* $Q := \dfrac{\psi(q)\psi(q^{55})}{q^{-5}\psi(q^5)\psi(q^{11})}$, *then*

$$\begin{aligned}
&33\left(Q^5 + \frac{1}{Q^5}\right) - 99\left(Q^4 + \frac{1}{Q^4}\right) + 1529\left(Q^3 + \frac{1}{Q^3}\right) + Q^6 + \frac{1}{Q^6} - 1683\left(Q^2 + \frac{1}{Q^2}\right) + 8800\left(Q + \frac{1}{Q}\right) \\
&= 6534 + \left(P^5 + \frac{5^5}{P^5}\right) - 11\left\{\left(P^4 + \frac{5^4}{P^4}\right)\left(Q + \frac{1}{Q}\right) - \left(P^3 + \frac{5^3}{P^3}\right)\left[11 + 4\left(Q^2 + \frac{1}{Q^2}\right)\right] - \left(P^2 + \frac{5^2}{P^2}\right)\right. \\
&\times\left[18 - 56\left(Q + \frac{1}{Q}\right) + 3\left(Q^2 + \frac{1}{Q^2}\right) - 8\left(Q^3 + \frac{1}{Q^3}\right)\right] - \left(P + \frac{5}{P}\right)\left[324 - 126\left(Q + \frac{1}{Q}\right) + 160\left(Q^2 + \frac{1}{Q^2}\right)\right. \\
&\left.-18\left(Q^3 + \frac{1}{Q^3}\right) + 9\left(Q^4 + \frac{1}{Q^4}\right)\right] - \left(P^3 + \frac{5^3}{P^3}\right) \left.\times\left[11 + 4\left(Q^2 + \frac{1}{Q^2}\right)\right]\right\}.
\end{aligned} \tag{17}$$

Lemma 7.[8, *p.56*],[6]

$$\frac{f^6(-q^2)}{f^6(-q^{10})} = \frac{\psi^4(q)}{\psi^4(q^5)}\left\{\frac{\psi^2(q) - 5q\psi^2(q^5)}{\psi^2(q) - q\psi^2(q^5)}\right\}. \tag{18}$$

Lemma 8.[1, *Ch. 16, Entry 24(iii), p.39*]

$$\chi(q) = \frac{f(-q^2)}{\psi(-q)}. \tag{19}$$

Lemma 9.[3]

$$2\left\{g_n^2 g_{25n}^2 + \frac{1}{g_n^2 g_{25n}^2}\right\} = \frac{g_{25n}^3}{g_n^3} - \frac{g_n^3}{g_{25n}^3}. \tag{20}$$

3 Modular Relation between $q^{1/6}\dfrac{\chi(-q)}{\chi(-q^5)}$ and $q^{n/6}\dfrac{\chi(-q^n)}{\chi(-q^{5n})}$

Theorem 1. *If* $P := q^{1/6}\dfrac{\chi(-q)}{\chi(-q^5)}$ *and* $Q := q^{1/3}\dfrac{\chi(-q^2)}{\chi(-q^{10})}$ *then*

$$\left[\left(\frac{P}{Q}\right)^3 + \left(\frac{Q}{P}\right)^3\right]\left[PQ + \frac{1}{PQ}\right] + 2 = (PQ)^2 + \frac{1}{(PQ)^2}. \tag{21}$$

*Proof.*Replace q by $-q$ and also replace q by $-q^5$ in the lemma (8), we obtain

$$\chi(-q) = \frac{f(-q^2)}{\psi(q)}, \tag{22}$$

$$\chi(-q^5) = \frac{f(-q^{10})}{\psi(q^5)}. \tag{23}$$

Dividing the equations (22) by (23), we get

$$\frac{\chi(-q)}{\chi(-q^5)} = \frac{\psi(q^5)}{\psi(q)}\frac{f(-q^2)}{f(-q^{10})}. \tag{24}$$

Raising the power six and also multiplying q on both side of the above equation (24), we get

$$q\frac{\chi^6(-q)}{\chi^6(-q^5)} = \frac{\psi^6(q^5)}{\psi^6(q)}\left\{q\frac{f^6(-q^2)}{f^6(-q^{10})}\right\}. \tag{25}$$

Using the equation (18) and the above equation (25), we obtain

$$a^4b - a^2(b+1) + 5 = 0, \tag{26}$$

where $a := \frac{\psi(q)}{\sqrt{q}\psi(q^5)}$, $b := q\frac{\chi^6(-q)}{\chi^6(-q^5)}$, the above equation (26) can be written as

$$c^2b - c(b+1) + 5 = 0, \tag{27}$$

where $c = a^2$. On solving the above equation (27). we get,

$$c = \frac{b+1+\sqrt{b^2-18b+1}}{2b}. \tag{28}$$

Substituting the above equation (28) in equation (12), we obtain

$$\begin{aligned}&(2Q^4P^4 + Q^2P^8 - Q^2P^2 - Q^6P^6 + P^6 + Q^6 + Q^8P^2)\\ &(Q^4P^{16} + Q^8P^{14} - Q^2P^{14} + P^{12}Q^{12} - 4P^{12}Q^6 + P^{12}\\ &+ 4Q^{10}P^{10} - 4Q^4P^{10} + Q^{14}P^8 + Q^8P^8 + Q^2P^8 + Q^{12}\\ &+ 4Q^6P^6 + Q^{16}P^4 - 4Q^{10}P^4 + Q^4P^4 - Q^{14}P^2 + Q^8P^2\\ &- 4Q^{12}P^6) = 0\end{aligned} \tag{29}$$

by examining the behavior of the above factors near $q = 0$, we can find a neighborhood about the origin, where the first factor is zero; whereas other factor are not zero in this neighborhood. By the Identity Theorem first factor vanishes identically. This completes the proof.

*Remark.*Using the definition of class invariant (9), the above theorem can be written as $P := \frac{g_n}{g_{25n}}$ and $Q := \frac{g_{4n}}{g_{100n}}$.

Theorem 2.*If* $P := q^{1/6}\frac{\chi(-q)}{\chi(-q^5)}$ *and* $Q := q^{1/2}\frac{\chi(-q^3)}{\chi(-q^{15})}$ *then*

$$\begin{aligned}&\left[\left(\frac{P}{Q}\right)^6 + \left(\frac{Q}{P}\right)^6\right] + 9\left[\left(\frac{P}{Q}\right)^3 + \left(\frac{Q}{P}\right)^3\right]\\ &+ 18 = (PQ)^3 + \frac{1}{(PQ)^3}.\end{aligned} \tag{30}$$

*Proof.*Employing the equation (28) and equation (13), we obtain

$$\begin{aligned}&(Q^{12} + 9P^9Q^3 + 9P^3Q^9 - P^9Q^9 + 18P^6Q^6 + P^{12}\\ &- P^3Q^3)(Q^{12} - 9P^9Q^3 - 9P^3Q^9 + P^9Q^9 + P^{12}\\ &+ 18P^6Q^6 + P^3Q^3) = 0\end{aligned} \tag{31}$$

By examining the behavior of the above factors near $q = 0$, we can find a neighborhood about the origin, where the first factor is zero; whereas other factor are not zero in this neighborhood. By the Identity Theorem first factor vanishes identically. This completes the proof.

*Remark.*Using the definition of class invariant (9), the above theorem can be written as $P := \frac{g_n}{g_{25n}}$ and $Q := \frac{g_{9n}}{g_{225n}}$.

Theorem 3.*If* $P := q^{1/6}\frac{\chi(-q)}{\chi(-q^5)}$ *and* $Q := q^{2/3}\frac{\chi(-q^4)}{\chi(-q^{20})}$ *then*

$$\begin{aligned}&\left[\left(\frac{P}{Q}\right)^6 + \left(\frac{Q}{P}\right)^6\right] + 13\left[\left(\frac{P}{Q}\right)^4 + \left(\frac{Q}{P}\right)^4\right]\\ &+ 52\left[\left(\frac{P}{Q}\right)^2 + \left(\frac{Q}{P}\right)^2\right] + \left[(PQ)^3 + \frac{1}{(PQ)^3}\right]\\ &\times\left[\left(\frac{P}{Q}\right)^5 + \left(\frac{Q}{P}\right)^5 + 19\left(\frac{P}{Q} + \frac{Q}{P}\right)\right] + 82\\ &+ 8\left[P^6 + \frac{1}{P^6}\right] + 8\left[Q^6 + \frac{1}{Q^6}\right] = (PQ)^6 + \frac{1}{(PQ)^6}.\end{aligned} \tag{32}$$

*Proof.*Employing the equation (28) and equation (14), we obtain (32).

Theorem 4.*If* $P := q^{1/6}\frac{\chi(-q)}{\chi(-q^5)}$ *and* $Q := q^{5/6}\frac{\chi(-q^5)}{\chi(-q^{25})}$ *then*

$$\begin{aligned}&\left[\left(\frac{P}{Q}\right)^3 + \left(\frac{Q}{P}\right)^3\right]\left[(PQ)^2 + \frac{1}{(PQ)^2} - PQ - \frac{1}{PQ} + 1\right]\\ &+ 6\left[(PQ)^3 + \frac{1}{(PQ)^3}\right] + 16\left[PQ + \frac{1}{PQ}\right] = 22\\ &+ \left[(PQ)^4 + \frac{1}{(PQ)^4}\right] + 11\left[(PQ)^2 + \frac{1}{(PQ)^2}\right].\end{aligned} \tag{33}$$

*Proof.*Employing the equation (28) and equation (15), we obtain (33).

Theorem 5.*If* $P := q^{1/6}\dfrac{\chi(-q)}{\chi(-q^5)}$ *and* $Q := q^{7/6}\dfrac{\chi(-q^7)}{\chi(-q^{35})}$ *then*

$$\left[\left(\frac{P}{Q}\right)^4+\left(\frac{Q}{P}\right)^4\right]+7\left[\left(\frac{P}{Q}\right)^3+\left(\frac{Q}{P}\right)^3\right] +21\left[\left(\frac{P}{Q}\right)^2+\left(\frac{Q}{P}\right)^2\right]+42\left[\frac{P}{Q}+\frac{Q}{P}\right] +56=(PQ)^3+\frac{1}{(PQ)^3}. \quad (34)$$

*Proof.*Employing the equation (28) and equation (16), we obtain (34).

Theorem 6.*If* $P := q^{1/6}\dfrac{\chi(-q)}{\chi(-q^5)}$ *and* $Q := q^{11/6}\dfrac{\chi(-q^{11})}{\chi(-q^{55})}$ *then*

$$\left[\left(\frac{P}{Q}\right)^6+\left(\frac{Q}{P}\right)^6\right]+11\left[\left(\frac{P}{Q}\right)^3+\left(\frac{Q}{P}\right)^3\right] \times\left[(PQ)^2+\frac{1}{(PQ)^2}+PQ+\frac{1}{PQ}+3\right]+165 +11\left[(PQ)^4+\frac{1}{(PQ)^4}\right]=\left[(PQ)^5+\frac{1}{(PQ)^5}\right] +11\left[(PQ)^3+\frac{1}{(PQ)^3}\right]+88\left[(PQ)^2+\frac{1}{(PQ)^2}\right] +33\left[PQ+\frac{1}{PQ}\right]. \quad (35)$$

*Proof.*Employing the equation (28) and equation (17), we obtain (35).

4 Explicit Evaluations of Class Invariant g_n

Theorem 7.

$$g_{\frac{8}{5}}=\frac{(6+2\sqrt{10})^{3/8}\sqrt{\sqrt{2}+\sqrt{10}-4}}{2}, \quad (36)$$

$$g_{40}=\frac{(6+2\sqrt{10})^{3/8}\sqrt{\sqrt{2}-\sqrt{10}+4}}{2}. \quad (37)$$

*Proof.*Setting $n=\frac{2}{5}$ in Theorem (3) and using the fact that $g_{2n}g_{\frac{2}{n}}=1$, we obtain

$$2b^4a^8+b^2-b^2a^{12}-b^6a^4+a^4+b^6a^{16}+b^8a^{12}=0, \quad (38)$$

where $a=g_{10}=\left(\frac{1+\sqrt{5}}{2}\right)^{1/2}$ and $b=\dfrac{g_{\frac{8}{5}}}{g_{40}}$.

Solving the above equation (38) in terms of b, we get

$$\frac{g_{\frac{8}{5}}}{g_{40}}=\frac{\sqrt{(2\sqrt{2}-2)(\sqrt{5}-1)}}{2}. \quad (39)$$

Employing the equation (20) with $n=\frac{8}{5}$, we obtain

$$2\left\{g_{\frac{8}{5}}^2g_{40}^2+\frac{1}{g_{\frac{8}{5}}^2g_{40}^2}\right\}=\frac{g_{40}^3}{g_{\frac{8}{5}}^3}-\frac{g_{\frac{8}{5}}^3}{g_{40}^3}. \quad (40)$$

Substituting the value $\dfrac{g_{\frac{8}{5}}}{g_{40}}$ in the above equation, we get

$$g_{\frac{8}{5}}g_{40}=\left(6+2\sqrt{10}\right)^{\frac{1}{4}}. \quad (41)$$

Using the equations (41) and (39), we obtain (36) and (37).

Theorem 8.

$$g_{\frac{18}{5}}=\frac{r(\sqrt{30}+4\sqrt{5}+3\sqrt{6}+7)(11\sqrt{30}+26\sqrt{5}+s-24\sqrt{6}-59)}{10}, \quad (42)$$

$$g_{90}=\frac{r^2(26\sqrt{30}+64\sqrt{5}+59\sqrt{6}+141)(11\sqrt{30}+26\sqrt{5}+s-24\sqrt{6}-59)}{10}, \quad (43)$$

where $r:=(\sqrt{30}+2\sqrt{5}-2\sqrt{6}-5)^{1/3}$ *and* $s:=(9255-4140\sqrt{5}+3780\sqrt{6}-1690\sqrt{30})^{1/2}$.

*Proof.*Setting $n=\frac{2}{5}$ in Theorem (3) and using the fact that $g_{2n}g_{\frac{2}{n}}=1$, we obtain

$$b^{12}a^{24}+9a^6b^3+9a^{18}b^9-a^6b^9+18b^6a^{12}+1-a^{18}b^3=0, \quad (44)$$

where $a=g_{10}=\left(\frac{1+\sqrt{5}}{2}\right)^{1/2}$ and $b=\dfrac{g_{\frac{18}{5}}}{g_{90}}$.

Solving the above equation (44) in terms of b, we get

$$\frac{g_{\frac{18}{5}}}{g_{90}}=(2\sqrt{5}+\sqrt{30}-2\sqrt{6}-5)^{\frac{1}{3}}. \quad (45)$$

Employing the equation (20) with $n=\frac{18}{5}$, we obtain

$$2\left\{g_{\frac{18}{5}}^2g_{90}^2+\frac{1}{g_{\frac{18}{5}}^2g_{90}^2}\right\}=\frac{g_{90}^3}{g_{\frac{18}{5}}^3}-\frac{g_{\frac{18}{5}}^3}{g_{90}^3}. \quad (46)$$

Substituting the value $\dfrac{g_{\frac{18}{5}}}{g_{90}}$ in the above equation, we get

$$g_{\frac{18}{5}}g_{90}=\sqrt{\frac{(\sqrt{30}+2\sqrt{6}+5+2\sqrt{5})(9\sqrt{30}+22\sqrt{5}+s-20\sqrt{6}-49)}{2}}. \quad (47)$$

Using the equations (47) and (45), we obtain (42) and (43).

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