Journal of Analysis & Number Theory
*An International Journal*

http://dx.doi.org/10.18576/jant/030203

# Some Fixed Point Theorems Satisfying Contractive Conditions of Integral Type in Dislocated Quasi-Metric Space

*Mujeeb Ur Rahman* and Muhammad Sarwar**

Department of Mathematics, University of Malakand, Chakdara Dir(L), Pakistan.

Received: 2 Feb. 2015, Revised: 29 Mar. 2015, Accepted: 30 Mar. 2015
Published online: 1 Jul. 2015

**Abstract:** The aim of this note is to study some fixed point theorems of integral type in dislocated quasi-metric space. We have establish some fixed point theorems satisfying integral type contractive conditions which generalizes fixed point theorems proved by Aage and Salunke [1], Muraliraj and Hussain [6], kohli et al. [7] and Zeyada et al. [11].

**Keywords:** Complete dislocated quasi-metric space, self-mapping, Cauchy sequence, fixed point.

## 1 Introduction

The concept of dislocated metric space was introduced by Hitzler and Seda [5]. In such a space the self-distance of points need not to be zero necessarily. They also generalized famous Banach contraction principle in dislocated metric space. Dislocated metric space play a vital role in Topology, Logical programming and Electronic engineering etc. Zeyada et al. [11] developed the notion of complete dislocated quasi-metric space and generalized the result of Hitzler and Seda [5] in dislocated quasi-metric space. With the passage of time many papers have been published by various authors containing fixed point results in dislocated quasi-metric spaces for different type of contractive conditions (see [1], [2], [6], [7], [9], [10]).

In 2002, Branciari [3] obtained a fixed point theorem for a single self-mapping satisfying an analogous of Banach's contraction principle for integral type inequality in metric space. Recently, in 2014 Patel et al. [8] studied some fixed point theorems of integral type in dislocated quasi-metric space.

In this article, we have establish some fixed point results for integral type contractive conditions in dislocated quasi-metric space. Our obtain results generalizes some well-known results in the literature. Examples are constructed in the support of our establish theorems and corollaries.

## 2 Preliminaries

Througisout the paper $\mathbb{R}^+$ represent the set of non-negative real numbers.

**Definition 2.1.**[8]. Let $X$ be a non-empty set. Let $d : X \times X \to \mathbb{R}^+$ be a function satisfying the conditions for all $x, y, z \in X$,

$d_1$) $\int_0^{d(x,x)} \rho(t)dt = 0;$

$d_2$) $\int_0^{d(x,y)} \rho(t)dt = \int_0^{d(y,x)} \rho(t)dt = 0 \Rightarrow x = y;$

$d_3$) $\int_0^{d(x,y)} \rho(t)dt = \int_0^{d(y,x)} \rho(t)dt;$

$d_4$) $\int_0^{d(x,y)} \rho(t)dt \leq \int_0^{d(x,z)} \rho(t)dt + \int_0^{d(z,y)} \rho(t)dt.$

If $d$ satisfies all of the above conditions then $d$ is called a metric on $X$. If $d$ satisfies the conditions from $d_2 - d_4$ then $d$ is said to be dislocated metric (OR) shortly ($d$-metric) on $X$ and if $d$ satisfies only $d_2$ and $d_4$ then $d$ is called dislocated quasi-metric (OR) shortly ($dq$-metric) on $X$ and the pair $(X, d)$ is called dislocated quasi-metric space. Where $\rho : \mathbb{R}^+ \to \mathbb{R}^+$ is a Lebesque integrable mapping which is summable on each compact subset of $\mathbb{R}^+$, non-negative and such that for any $s > 0$ $\int_0^s \rho(t)dt > 0$.

* Corresponding author e-mail: mujeeb846@yahoo.com, sarwar@uom.edu.pk

**Note.** The above definition change to usual definition of metric space if $\rho(t) = I$.

It is clear that every metric space is dislocated metric and dislocated quasi metric space but the converse is not true. Also every dislocated metric space is dislocated quasi-metric space but the converse is not necessarily true.

The following definitions can be found in [8].

**Definition 2.2.** A sequence $\{x_n\}$ in $dq$-metric space is said to be $dq$-convergent to a point $x \in X$ if

$$\lim_{n\to\infty} \int_0^{d(x_n,x)} \rho(t)dt = \lim_{n\to\infty} \int_0^{d(x,x_n)} \rho(t)dt = 0.$$

In such a case $x$ is called $dq$-limit of the sequence $\{x_n\}$.

**Definition 2.3.** A sequence $\{x_n\}$ in $dq$-metric space $(X,d)$ is said to be Cauchy sequence if for $\varepsilon > 0$ there exist $n_0 \in N$ such that $m,n \geq n_0$ implies

$$\int_0^{d(x_n,x_m)} \rho(t)dt = \int_0^{d(x_m,x_n)} \rho(t)dt < \varepsilon$$

(OR)

$$\lim_{n\to\infty} \int_0^{d(x_n,x_m)} \rho(t)dt = \lim_{n\to\infty} \int_0^{d(x_m,x_n)} \rho(t)dt = 0$$

**Definition 2.4.** A $dq$-metric space $(X,d)$ is said to be complete if every Cauchy sequence in $X$ converge to a point in $X$.

The following simple but important results can be seen in [11].

**Lemma 2.5.** Limit of a convergent sequence in $dq$-metric space is unique.

**Theorem 2.6.** Let $(X,d)$ be a complete $dq$-metric space $T : X \to X$ be a contraction. Then $T$ has a unique fixed point.

Branciari [3] proved the following theorem in metric spaces.

**Theorem 2.7.** Let $(X,d)$ be a complete metric space for $\alpha \in (0,1)$. Let $T : X \to X$ be a mapping such that for all $x,y \in X$ satisfying

$$\int_0^{d(Tx,Ty)} \rho(t)dt \leq \alpha \cdot \int_0^{d(x,y)} \rho(t)dt.$$

Where $\rho : \mathbb{R}^+ \to \mathbb{R}^+$ is a Lebesque integrable mapping which is summable on each compact subset of $\mathbb{R}^+$, non-negative and such that for any $s > 0$ $\int_0^s \rho(t)dt > 0$. Then $T$ has a unique fixed point in $X$.

## 3 Main Results

**Theorem 3.1.** Let $(X,d)$ be a complete dislocated quasi-metric space, for $a,b,c,e,f \geq 0$ with $a+b+c+e+f < 1$ and let $T : X \to X$ be a continuous self-mapping such that for all $x,y \in X$, satisfying the condition

$$\int_0^{d(Tx,Ty)} \rho(t)dt \leq a \cdot \int_0^{d(x,y)} \rho(t)dt + b \cdot \int_0^{d(x,Tx)} \rho(t)dt +$$

$$c \cdot \int_0^{d(y,Ty)} \rho(t)dt + e \cdot \int_0^{\frac{d(y,Ty)[1+d(x,Tx)]}{1+d(x,y)}} \rho(t)dt + f \cdot \int_0^{\frac{d(x,Ty)d(y,Ty)}{d(x,y)+d(y,Ty)}} \rho(t)$$

where $\rho : \mathbb{R}^+ \to \mathbb{R}^+$ is a Lebesque integrable mapping which is summable on each compact subset of $\mathbb{R}^+$, non-negative and such that for any $s > 0$ $\int_0^s \rho(t)dt > 0$.

Then $T$ has a unique fixed point.

**Proof.** Let $x_0$ be arbitrary in $X$ we define a sequence $\{x_n\}$ in $X$ defined as follows

$$x_0, x_1 = Tx_0, \ldots\ldots\ldots\ldots\ldots, x_{n+1} = Tx_n.$$

To show that $\{x_n\}$ is a Cauchy sequence in $X$ consider

$$\int_0^{d(x_n,x_{n+1})} \rho(t)dt = \int_0^{d(Tx_{n-1},Tx_n)} \rho(t)dt$$

By given condition in the theorem we have

$$\leq a \cdot \int_0^{d(x_{n-1},x_n)} \rho(t)dt + b \cdot \int_0^{d(x_{n-1},Tx_{n-1})} \rho(t)dt +$$

$$c \cdot \int_0^{d(x_n,Tx_n)} \rho(t)dt + e \cdot \int_0^{\frac{d(x_n,Tx_n)[1+d(x_{n-1},Tx_{n-1})]}{1+d(x_{n-1},x_n)}} \rho(t)dt +$$

$$f \cdot \int_0^{\frac{d(x_{n-1},Tx_n)d(x_n,Tx_n)}{d(x_{n-1},x_n)+d(x_n,Tx_n)}} \rho(t).$$

Using the definition of the defined sequence we have

$$\leq a \cdot \int_0^{d(x_{n-1},x_n)} \rho(t)dt + b \cdot \int_0^{d(x_{n-1},x_n)} \rho(t)dt +$$

$$c \cdot \int_0^{d(x_n,x_{n+1})} \rho(t)dt + e \cdot \int_0^{\frac{d(x_n,x_{n+1})[1+d(x_{n-1},x_n)]}{1+d(x_{n-1},x_n)}} \rho(t)dt +$$

$$f \cdot \int_0^{\frac{d(x_{n-1},x_{n+1})d(x_n,x_{n+1})}{d(x_{n-1},x_n)+d(x_n,x_{n+1})}} \rho(t).$$

Simplification yields

$$\leq a\cdot\int_0^{d(x_{n-1},x_n)}\rho(t)dt+b\cdot\int_0^{d(x_{n-1},x_n)}\rho(t)dt+$$

$$c\cdot\int_0^{d(x_n,x_{n+1})}\rho(t)dt+e\cdot\int_0^{d(x_n,x_{n+1})}\rho(t)dt+f\cdot\int_0^{d(x_n,x_{n+1})}\rho(t)$$

$$\int_0^{d(x_n,x_{n+1})}\rho(t)dt\leq\left(\frac{a+b}{1-(c+e+f)}\right)\cdot\int_0^{d(x_{n-1},x_n)}\rho(t)dt.$$

Let $h=\frac{a+b}{1-(c+e+f)}$, so the above inequality become

$$\int_0^{d(x_n,x_{n+1})}\rho(t)dt\leq h\cdot\int_0^{d(x_{n-1},x_n)}\rho(t)dt.$$

Also

$$\int_0^{d(x_{n-1},x_n)}\rho(t)dt\leq h\cdot\int_0^{d(x_{n-2},x_{n-1})}\rho(t)dt.$$

So

$$\int_0^{d(x_n,x_{n+1})}\rho(t)dt\leq h^2\cdot\int_0^{d(x_{n-2},x_{n-1})}\rho(t)dt.$$

Similarly proceeding we get

$$\int_0^{d(x_n,x_{n+1})}\rho(t)dt\leq h^n\cdot\int_0^{d(x_0,x_1)}\rho(t)dt.$$

Since $h<1$ and taking limit $n\to\infty$, we have $h^n\to 0$. Hence

$$\int_0^{d(x_n,x_{n+1})}\rho(t)dt\to 0.$$

Which implies that $d(x_n,x_{n+1})\to 0$ as $n\to\infty$. Hence $\{x_n\}$ is a Cauchy sequence in complete $dq$-metric space. So there must exists $u\in X$ such that

$$\lim_{n\to\infty}x_n=u.$$

Since $T$ is continuous so

$$Tu=\lim_{n\to\infty}Tx_n=\lim_{n\to\infty}x_{n+1}=u.$$

Thus $u$ is the fixed point of $T$.

**Uniqueness.** If $u\in X$ is a fixed point of $T$. Then by given condition in the theorem we have

$$\int_0^{d(u,u)}\rho(t)dt=\int_0^{d(Tu,Tu)}\rho(t)dt$$

$$\int_0^{d(u,u)}\rho(t)dt\leq(a+b+c+e+f)\int_0^{d(u,u)}\rho(t)dt.$$

Since $a+b+c+e+f<1$, so the above inequality is possible if $d(u,u)=0$ similarly if $v\in X$ is the fixed point of $T$. Then we can show that $d(v,v)=0$. Now consider that $u,v$ are two distinct fixed points of $T$ then again by given condition in the theorem We have

$$\int_0^{d(u,v)}\rho(t)dt=\int_0^{d(Tu,Tv)}\rho(t)dt$$

$$\leq a\cdot\int_0^{d(u,v)}\rho(t)dt+b\cdot\int_0^{d(u,Tu)}\rho(t)dt+c\cdot\int_0^{d(v,Tv)}\rho(t)dt+$$

$$e\cdot\int_0^{\frac{d(v,Tv)[1+d(u,Tu)]}{1+d(u,v)}}\rho(t)dt+f\cdot\int_0^{\frac{d(u,Tv)d(v,Tv)}{d(u,v)+d(v,Tv)}}\rho(t)dt.$$

Now using the fact that $u,v$ are fixed points of $T$ and then simplifying We get the following inequality

$$\int_0^{d(u,v)}\rho(t)dt\leq a.\int_0^{d(u,v)}\rho(t)dt.$$

Since $a<1$ so the a above inequality is possible if $d(u,v)=0$ similarly we can show that $d(v,u)=0$ which implies that $u=v$. Hence fixed point of $T$ is unique.

Theorem 3.1 yields the following corollaries.

**Corollary 3.2.**. Let $(X,d)$ be a complete dislocated quasi-metric space, for $a\geq 0$, with $a\in(0,1)$ and let $T:X\to X$ be a continuous self-mapping such that for all $x,y\in X$ satisfying the condition

$$\int_0^{d(Tx,Ty)}\rho(t)dt\leq a\cdot\int_0^{d(x,y)}\rho(t)dt.$$

Where $\rho:\mathbb{R}^+\to\mathbb{R}^+$ is a Lebesque integrable mapping which is summable on each compact subset of $\mathbb{R}^+$, non-negative and such that for any $s>0$ $\int_0^s\rho(t)dt>0$. Then $T$ has a unique fixed point.

**Corollary 3.3.** Let $(X,d)$ be a complete dislocated quasi-metric space, for $a,b,c\geq 0$, with $a+b+c<1$ and let $T:X\to X$ be a continuous self-mapping such that for all $x,y\in X$ satisfying the condition

$$\int_0^{d(Tx,Ty)}\rho(t)dt\leq a\cdot\int_0^{d(x,y)}\rho(t)dt+b\cdot\int_0^{d(x,Tx)}\rho(t)dt+$$

$$c\cdot\int_0^{d(y,Ty)}\rho(t)dt.$$

Where $\rho : \mathbb{R}^+ \to \mathbb{R}^+$ is a Lebesque integrable mapping which is summable on each compact subset of $\mathbb{R}^+$, non-negative and such that for any $s > 0$ $\int_0^s \rho(t)dt > 0$. Then $T$ has a unique fixed point.

**Corollary 3.4.** Let $(X,d)$ be a complete dislocated quasi metric space, for $a,b,c \geq 0$, with $a+b+c < 1$ and let $T : X \to X$ be a continuous self-mapping such that for all $x,y \in X$ satisfying the condition

$$\int_0^{d(Tx,Ty)} \rho(t)dt \leq a \cdot \int_0^{d(x,y)} \rho(t)dt + b \cdot \int_0^{d(y,Ty)} \rho(t)dt + c \cdot \int_0^{\frac{d(y,Ty)[1+d(x,Tx)]}{1+d(x,y)}} \rho(t)dt$$

Where $\rho : \mathbb{R}^+ \to \mathbb{R}^+$ is a Lebesque integrable mapping which is summable on each compact subset of $\mathbb{R}^+$, non-negative and such that for any $s > 0$ $\int_0^s \rho(t)dt > 0$. Then $T$ has a unique fixed point.

**Corollary 3.5.** Let $(X,d)$ be a complete dislocated quasi-metric space, for $a,b \geq 0$, with $a+b < 1$ and let $T : X \to X$ be a continuous self-mapping such that for all $x,y \in X$ satisfying the condition

$$\int_0^{d(Tx,Ty)} \rho(t)dt \leq a \cdot \int_0^{d(x,y)} \rho(t)dt + b \cdot \int_0^{\frac{d(x,Ty)d(y,Ty)}{d(x,y)+d(y,Ty)}} \rho(t)dt.$$

Where $\rho : \mathbb{R}^+ \to \mathbb{R}^+$ is a Lebesque integrable mapping which is summable on each compact subset of $\mathbb{R}^+$, non-negative and such that for any $s > 0$ $\int_0^s \rho(t)dt > 0$. Then $T$ has a unique fixed point.

We have the following remarks from the above corollaries.

**Remarks.**

– In Corollary 3.2 if $\rho(t) = I$. Then we get the result of Zeyada et al. [11].
– In Corollary 3.3 if $\rho(t) = I$. Then we get the result of Aage and Salunke [1].
– In Corollary 3.4 if $\rho(t) = I$. Then we get the result of Kohli et al. [7].
– In Corollary 3.5 if $\rho(t) = I$. Then we get the result of Muraliraj and Hussain [6].

**Example 3.6.** Let $X = [0,1]$ and the complete $dq$-metric defined on $X$ is given by $d(x,y) = |x|$ with self-mapping defined on $X$ is $Tx = \frac{x}{2}$ and $\rho(t) = \frac{t}{2}$. Then

$$\int_0^{d(Tx,Ty)} \rho(t)dt = \int_0^{|\frac{x}{2}|} \frac{t}{2}dt = \frac{1}{16}x^2 \leq \frac{1}{4}\left(\frac{1}{4}x^2\right) \leq a. \int_0^{d(x,y)} \rho(t)dt.$$

Satisfy all the conditions of the Corollary 3 for $a \in [\frac{1}{4}, 1)$ having $x = 0$ is its unique fixed point.

**Example 3.7.** Let $X = [0,1]$ and the complete $dq$-metric defined on $X$ is given by $d(x,y) = |x|$ with self-mapping defined on $X$ is $Tx = \frac{x}{2}$ and $\rho(t) = \frac{t}{2}$ for $a = \frac{1}{3}, b = \frac{1}{4}, c = \frac{1}{6}, e = \frac{1}{8}, f = \frac{1}{12}$. Satisfy all the conditions of Theorem **??** having $x = 0$ is the unique fixed point of $T$.

**Theorem 3.8.** Let $(X,d)$ be a complete dislocated quasi-metric space, for $\alpha \geq 0$, with $\alpha \in [0,1)$ and let $S,T : X \to X$ are continuous self-mappings such that for all $x,y \in X$ satisfying the condition

$$\int_0^{d(Sx,Ty)} \rho(t)dt \leq \alpha. \int_0^{M(x,y)} \rho(t)dt$$

with $M(x,y) = \alpha \cdot \max\{d(x,y), d(x,Sx), d(y,Ty)\}$ where $\rho : \mathbb{R}^+ \to \mathbb{R}^+$ is a Lebesque integrable mapping which is summable on each compact subset of $\mathbb{R}^+$, non-negative and such that for any $s > 0$ $\int_0^s \rho(t)dt > 0$. Then $S$ and $T$ have a unique common fixed point.

**Proof.** Let $x_0$ be arbitrary in $X$ we define a sequence $\{x_n\}$ for $n = 0,1,2,...$ by the rule

$$x_0, x_1 = Sx_0, x_3 = Sx_2, ........, x_{2n+1} = Sx_{2n}$$

and

$$x_2 = Tx_1, x_4 = Tx_3, .........., x_{2n} = Tx_{2n-1}.$$

Now we have to show that $\{x_n\}$ is a Cauchy sequence in $X$ for this consider

$$\int_0^{d(x_{2n+1},x_{2n+2})} \rho(t)dt = \int_0^{d(Sx_{2n},Tx_{2n+1})} \rho(t)dt$$

By given condition in the theorem and using the construction of the sequence defined above we have

$$\leq \alpha \cdot \int_0^{\max\{d(x_{2n},x_{2n+1}),d(x_{2n},Sx_{2n}),d(x_{2n+1},Tx_{2n+1})\}} \rho(t)dt$$

$$\leq \alpha \cdot \int_0^{\max\{d(x_{2n},x_{2n+1}),d(x_{2n},x_{2n+1}),d(x_{2n+1},x_{2n+2})\}} \rho(t)dt$$

$$\leq \alpha \cdot \int_0^{d(x_{2n},x_{2n+1})} \rho(t)dt.$$

Similarly

$$\int_0^{d(x_{2n},x_{2n+1})} \rho(t)dt \leq \alpha \cdot \int_0^{d(x_{2n-1},x_{2n})} \rho(t)dt.$$

So

$$\int_0^{d(x_{2n+1},x_{2n+2})} \rho(t)dt \le \alpha^2 \cdot \int_0^{d(x_{2n},x_{2n+1})} \rho(t)dt.$$

Proceeding in such a way we have

$$\int_0^{d(x_{2n+1},x_{2n+2})} \rho(t)dt \le \alpha^{2n} \cdot \int_0^{d(x_0,x_1)} \rho(t)dt.$$

Since $h < 1$ and taking limit $n \to \infty$, We have $h^{2n} \to 0$. Hence

$$\int_0^{d(x_{2n+1},x_{2n+2})} \rho(t)dt \to 0.$$

Which implies that $d(x_{2n+1},x_{2n+2}) \to 0$ as $n \to \infty$. Hence $\{x_n\}$ is a Cauchy sequence in complete $dq$-metric space. So there must exists $u \in X$ such that

$$\lim_{n\to\infty} x_n = u.$$

Also the sub-sequences $\{x_{2n}\}$ and $\{x_{2n+1}\}$ converges to $u$. Since $S$ and $T$ are continuous so

$$T \lim_{n\to\infty} x_{2n+1} = Tu \quad \Rightarrow \quad Tu = u.$$

Similarly we can show that $Su = u$. Therefore $u$ is the common fixed point of $S$ and $T$.

**Uniqueness.** Let $u,v$ be two distinct common fixed points of $S$ and $T$. Then by using the given condition in the theorem we can easily show that

$$d(u,u) = d(v,v) = 0.$$

Now consider

$$\int_0^{d(u,v)} \rho(t)dt = \int_0^{d(Su,Tv)} \rho(t)dt$$

$$\le \alpha \cdot \int_0^{\max\{d(u,v),d(u,u),d(v,v)\}} \rho(t)dt$$

$$\le \alpha \cdot \int_0^{d(u,v)} \rho(t)dt.$$

Since $\alpha < 1$ so the above inequality is possible if $d(u,v) = 0$. Similarly we can show that $d(v,u) = 0$ implies that $u = v$. Hence $S$ and $T$ have a unique common fixed point.

**Remark.**

– In Theorem 3.8 if $S = T$ and $\rho(t) = I$. Then we get the result established by Aage and Salunke [2].

## Acknowledgement

The authors are grateful to the anonymous referees and editor for there careful review and valuable comments that improved this paper.

## References

[1] C.T. Aage and J.N. Salunke, Some results of fixed point theorem in dislocated quasi-metric space, *Bulletin of Marathadawa Mathematical Society*, **9**(2008), 1-5.
[2] C.T. Aage and J.N. Salunke, The results of fixed points in dislocated and dislocated quasi-metric space, *Applied Mathematical Sciences*, **2**(2008), 2941-2948.
[3] A. Branciari, A fixed point theorem for mappings satisfying general contractive condition of integral type, *Int. Journal of Mathematics and Mathematical Sciences*, **29**(2002), 531-536.
[4] Lj.B. Ciric, A generalization of Banach contraction principle, *Proc. Amer. Math. Soc.*, **45**(1974), 267-273.
[5] P. Hitzler, Generalized metrics and topology in logic programming semantics, Ph.D Thesis, National Univeristy of Ireland, University College Cork, (2001).
[6] A. Muraliraj and R.J. Hussain, New results On fixed point theorems in dislocated quasi-metric spaces, *Int. Journal of Mathematical Sciences and Applications*, **3**(2013), 321-326.
[7] M. Kohli, R. Shrivastava and A.D. Singh, Some results on fixed point theorems in dislocated quasi-metric space, *Int. Journal of Theoritical and Applied Sciences* , **2**(2010), 27-28.
[8] S.T. Patel, V.C. Makvana and C.H. Patel, Some results of fixed point theorem in dislocated quasi-metric space of integral type, *Scholars Journal of Engineering and Technology*, **2**(2014), 91-96.
[9] M.U. Rahman and M. Sarwar, Fixed point results in dislocated quasi-metric spaces, *International Mathematical Forum*, **9**(2014), 677-682.
[10] M. Sarwar, M.U. Rahman and G. Ali, Some fixed point results in dislocated quasi-metric ($dq$-metric) spaces, *Journal of Inequalities and Applications*, **278**(2014), 1-11.
[11] F.M. Zeyada, G.H. Hassan and M.A. Ahmad, A generalization of Fixed point theorem due to Hitzler and Seda in dislocated quasi-metric space, *Arabian J. Sci. Engg.*, **31**(2005), 111-114.

**Mujeeb Ur Rahman** Completed his M-Phil degree from the Department of Mathematics, University of Malakand, Dir(L) Khyber Pakhtunkhwa, Pakistan. His area of interest is Functional Analysis. He has eight research papers published in reputed international journals of Mathematics. He is referee of various mathematical journals.

**Muhammad Sarwar** Assistant Professor in the Department of Mathematics, University of Malakand, Dir(L) Khyber Pakhtunkhwa, Pakistan. He obtained has Ph.D degree from Abdul Salam school of Mathematical Sciences, Lahore, Pakistan in 2011. His areas of Interest are Fixed point theory and Weight measure in function spaces etc. He has Published various research papers in his fields of interest in international reputed journals of Mathematics. He is referee of various mathematical journals.