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Entropy based Redundancy Allocation in Series-Parallel Systems with Choices of a Redundancy Strategy and Component Type: A Multi-Objective Model

Roya Soltani[1,*], *Seyed Jafar Sadjadi*[1] *and Reza Tavakkoli-Moghaddam*[2]

[1] Department of Industrial Engineering, Iran University of Science and Technology, Tehran, Iran
[2] School of Industrial Engineering and Engineering Optimization Research Group, College of Engineering, University of Tehran, Tehran, Iran

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Abstract: This paper presents a new multi-objective mathematical model for the redundancy allocation problem with the choices of a redundancy strategy and component type in series-parallel systems. The model considers entropy measure which is a measure of uncertainty in the information theory. For the first time, the model maximizes the reliability and entropy of the system and minimizes the nonlinear cost of the system simultaneously. In addition, this paper considers entropy in distribution of the weights of components within subsystems as another form of entropy, which is more realistic than considering entropy in distribution of the number of components. The subsystems can choose a redundancy strategy, which can be active or cold standby, or consider no redundancy. A mathematical compromise programming approach is employed to deal with this problem. As different weights of the objectives and norm of the L_p metric result in various solutions, appropriate criterion is employed to choose the best compromise solution. Finally, the results and conclusion are presented.

Keywords: Entropy measure, Multi-objective redundancy allocation, Cold-standby strategy, Active strategy, Compromise programming

1 Introduction

Redundancy allocation is a method of reliability optimization, in which optimal numbers of redundant components or redundancy levels are determined such that the system reliability is maximized. In the literature, the redundancy allocation problem (RAP) is considered with different active or standby redundancy strategies. [32] presented a comprehensive review on reliability optimization problems especially RAP. In the followings, we summarize the more relevant works whose focuses are mainly on RAP models. There are different studies on active redundancy. The first model for the RAP with an active strategy was proposed by [8], in which the system reliability is maximized subject to cost and weight constraints. Other studies focused on exact and meta-heuristic approaches for this problem (see [32]). [19] considered the problem that minimizes the cost subject to the requirement of meeting the minimum system reliability. [14] formulated the RAP using a max-min approach such that the reliability of the subsystem with the minimum reliability is maximized [33] studied RAP with discount consideration and presented heuristic and meta-heuristic approaches to deal with the problem. For more study on heuristic and meta-heuristic approaches for RAP with active strategy readers are referred to the works by [34,35]. Recently, [36] presented a robust possibilitic programming approach and developed robust models for RAP with active strategy. [40,41] also studied RAP with active strategy with respect to interval and budgeted uncertainties, respectively. It is obvious that adding redundancy increases the cost of the system, which is not desirable. Therefore, most designers wish to simultaneously optimize these objectives and deal with multi-objective models.

In the context of multi-objective models with active strategy, [6] considered a reliability optimization problem

* Corresponding author e-mail: royasoltani@iust.ac.ir

where the reliability of each subsystem is maximized and solved the resulted multi-objective model by multiple weighted objectives heuristic approach, [21] studied the problem of maximizing the system reliability and minimizing the system cost, which is nonlinear in terms of objective function, subject to weigh and volume constraints. They solved the problem by the NSGA-II. [25] studied the triple objectives of maximizing the system reliability and minimizing the system cost and weight. They used the non- dominated sorting genetic algorithm (NSGA). [26] proposed a multi-objective evolutionary algorithm to solve a multi-objective RAP, in which the objectives were maximization of the system reliability and minimizing the system cost and weight. [28] studied the problem of maximizing the system reliability and minimizing the system cost with respect to nonlinear constraints on cost, weigh and reliability. They also solved this problem using the NSGA-II.[12] considered the problem of maximizing the system reliability along with maximizing the system entropy subject to a nonlinear constraint on the system cost and coped with it via a global criterion method. [23] considered the problem of maximizing the minimum subsystem reliability along with minimizing the overall system cost. They found the Pareto solutions of this problem by the augmented epsilon-constraint approach for small and medium-sized instances, concurrently. Then, they applied a well-known sorting procedure, UTADIS, to categorize the solutions into preference ordered classes. [10] studied the same problem as [26] using epsilon-constraint, multi-start partial bound enumeration algorithm and data envelopment analysis (DEA).

In the area of cold standby strategy, [3] proposed a cold-standby redundancy optimization problem for non-repairable systems and extended a zero-one linear programming model to solve the problem. [4] studied the same redundancy allocation problems where there were redundancy strategy choices for subsystems. In application of meta-heuristics, [27] was the first that developed a genetic algorithm to solve the same problem proposed by [4]. There are also some studies in this area associated with bi-objective or multi-objective problems. [20] and [2] separately considered a bi-objective model to optimize the reliability and cost of the system with a choice of redundancy strategy and solved the resulted model through NSGA-II. [37] considered a multi-objective RAP with the choice of a redundancy strategy and reliability, cost and weight as objective functions. [38] presented an interval programming approach for RAP with the choice of a redundancy strategy. [39] considered cold standby RAP with interval uncertainty of components and formulated the model through Min-Max regret approach to deal with uncertainty.

There are also some studies on the applications of entropy for reliability problems. [13] presented the entropy-based reliability assessment technique in a case study of a robotic system. [17] introduced the entropy function in order to study the reliability and reparability of systems. [16] investigated the application and usability of the cross-entropy method for rare event simulation in Markovian reliability models. [18] discussed and calculated the reliability function during the system again through the stochastic entropy. [11]introduced a new approach based on cross-entropy method for optimization of network reliability. To the best of our knowledge, no model has been presented to consider maximizing the system reliability, in which the redundancy strategy and component choice are incorporated along with maximizing the entropy within subsystems and minimizing the nonlinear system cost, simultaneously. [12]considered the entropy regarding the number of components along with an active redundancy strategy. We consider a more realistic case, in which entropy is considered in relation to the distribution of weights since in most cases the goal is to make a balance in subsystems weights. As we present a new nonlinear mathematical model for this problem, we deal with it through a mathematical programming technique.

According to [9] the methods for solving multi objective mathematical problems are classified into three categories: The ”a priori” methods, the ”interactive” methods and the ”a posteriori” or ”generation” methods. The main difficulty with the ”a priori” methods such as utility function, lexicographic method, goal programming, etc., is to find the goals and/or preference information from the decision maker prior to any preliminary solution. In fact, a priori methods bear the risk of proposing a solution which would not have been selected if more information on the available trade-offs was available. The interactive methods such as interactive goal programming rely on the progressive definition of the decision maker’s preferences along with the exploration of the criterion space. The most widely used posteriori or generation methods are the weighted sum method and the ε -constraint method. The weighted sum method has this drawback that for a nonconvex set, some points in the nondominated set cannot be found for any set of weights. For the ε -constraint method, it is difficult to find interesting values of the parameter ε. In particular the problem may become infeasible due to the new constraints on objective functions. Global criterion or compromise programming method is a method that needs no articulation of given preference information. In fact, it does not need any interobjective or other subjective preference information from the decision makers once the problem constraints and objectives have been defined. This method is theoretically interesting because it can find any Pareto point, even for non-convex problems. Therefore we adopt this method in this paper which is also proposed in [12].

The rest of the paper is organized as follows. In Section 2, the given problem is described and the proposed mathematical model is presented. The compromise programming technique as a solution procedure is presented in Section 3. The experimental results are presented in Section 4. Finally, conclusion is presented in Section 5 along with some future research directions.

2 Problem definition and formulation

This paper presents a series-parallel system similar to Fig. 1, in which a number of subsystems work in series that means failing a subsystem cause failing the whole system. To prevent such damage, the reliabilities of subsystems need to be amplified. One way to enhance subsystem reliability is to allocate redundant components in parallel. Redundant components can be allocated according to an active or standby strategy depending on whether the replacement is allowable or not. In the active strategy, all redundant components are required to operate simultaneously whilst in the standby strategy, one of the redundant components begins to work only when the active component fails. The standby strategy can be in three forms (i.e., warm, hot and cold) depending on whether the system cessation is tolerable or not. In the warm standby redundancy, the component is more prone to failure before operation than the cold standby components. In the hot standby redundancy, the failure pattern of component does not depend on whether the component is idle or in operation. Finally, in the cold standby strategy, components do not fail before being put into operation. In this paper, both active and cold standby strategies are considered. In addition, it is possible that some subsystems choose no redundancy strategy and only one component is placed.

Allocating redundant components increases the finished cost of the system, which is not desirable in most of the times. Therefore, making a trade-off between the system cost and reliability is needed. Another objective considered in the presented model is to maximize the system's entropy, which represents the lack of information about the state of each subsystem. The positive entropy is a measure of disorder whilst the negative entropy measures the order of the system, which we wish to maximize it. The negative entropy is calculated by Equation (1).

$$En(X) = -\sum_{x} pr(x) \log pr(x), \tag{1}$$

where $pr(x)$ is the probability that X is in state x. Regarding the redundancy allocation problems, X represents the number of components allocated in each subsystem or weight of each subsystem. Therefore, the probability $pr(x)$ is calculated as the portion of the number of the components or weight of each subsystem to the total number of system components or weight as presented in Equation (2).

$$pr_i(x) = \frac{x_i}{\sum_i x_i}, \tag{2}$$

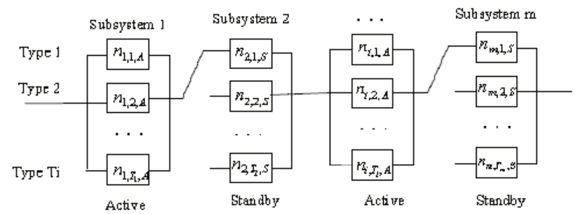

Fig. 1: General series-parallel system with a choice of redundancy strategy

where x_i is the number or weight of components allocated in subsystem i. The sum of probabilities is one and all probabilities are greater than zero (i.e., there is $pr_i(x) \geq 0$ for each i). In redundancy allocation problems, entropy measures the diversity in distribution of components or weights and tries to allocate the same number of redundant components or weights in all subsystems.

We formulate the above-mentioned problem as a multi-objective nonlinear model described below.

Assumptions:

– The components are in two states of functioning or non-functioning, i.e. binary state.

– Components' time to failures follow Erlang distribution.

– The standby strategy is of cold type and the standby units do not fail before they are put into operation.

– The switch reliability to cold standby component is assumed to be imperfect.

– There are different component types with different specifications.

– Just one component type can be allocated in each subsystem.

– There is no repair or preventive maintenance.

– The replacement time is negligible.

Decision variables:

$n_{i,j,h}$: Number of components of type j used in subsystem i under strategy h ($h \in A$:Active, S:Standby, N:No redundancy)

$Z_{i,j,h}$: A binary variable that is one if the component of type j is used in subsystem i under strategy h; and zero, otherwise

$X_{q,i,j}$: A binary variable that is one if q of the component of type j is used in subsystem i under standby strategy. This variable is used because the upper limit of the summation contains variable q.

Parameters:

$\lambda_{i,j}, k_{i,j}$: Scale and shape parameters of an Erlang distribution for component j in subsystem i

t : Mission time

$r_{i,j}(t)$: Reliability of component j available for subsystem i at time t

$\delta_{i,j}(t)$: Reliability of switch to component j in subsystem i at time t (imperfect switching)

$c_{i,j}, w_{i,j}$: Cost and weight associated with component j available for subsystem i

W : Total allowable weight for the system

T_i : Number of component types in subsystem i

m : Number of subsystems

n_{max} : Maximum number of components in each subsystem

$R(t)$: System reliability at time t

Presented mathematical model (1):

$$MaxR(t) = \prod_{i=1}^{m}(1-\sum_{j=1}^{T_i} Z_{i,j,A}\times\prod_{j=1}^{T_i}(1-r_{i,j}(t))^{n_{i,j,A}}) \times\prod_{i=1}^{m}(1-\sum_{j=1}^{T_i} Z_{i,j,S}) + \sum_{j=1}^{T_i} Z_{i,j,S}\times(r_{i,j}(t)+\delta_{i,j}(t) \times\exp(-\lambda_{i,j}t)\sum_{q=2}^{n_{max}} X_{q,i,j}\sum_{l=k_{i,j}}^{k_{i,j}\times q-1}\frac{(\lambda_{i,j}t)^l}{l!})\times\prod_{i=1}^{m}\prod_{j=1}^{T_i}(r_{i,j}(t))^{Z_{i,j,N}}. \quad (3)$$

$$MaxComponentEntropy = -\sum_{i=1}^{m}\left(\frac{\sum_{j=1}^{T_i}\sum_{h\in A,S,N} n_{i,j,h}}{\sum_{i=1}^{m}\sum_{j=1}^{T_i}\sum_{h\in A,S,N} n_{i,j,h}}\right)\log\left(\frac{\sum_{j=1}^{T_i}\sum_{h\in A,S,N} n_{i,j,h}}{\sum_{i=1}^{m}\sum_{j=1}^{T_i}\sum_{h\in A,S,N} n_{i,j,h}}\right). \quad (4)$$

$$MaxweightEntropy = -\sum_{i=1}^{m}\left(\frac{\sum_{j=1}^{T_i}\sum_{h\in A,S,N} w_{i,j}n_{i,j,h}}{\sum_{i=1}^{m}\sum_{j=1}^{T_i}\sum_{h\in A,S,N} w_{i,j}n_{i,j,h}}\right)\log\left(\frac{\sum_{j=1}^{T_i}\sum_{h\in A,S,N} w_{i,j}n_{i,j,h}}{\sum_{i=1}^{m}\sum_{j=1}^{T_i}\sum_{h\in A,S,N} w_{i,j}n_{i,j,h}}\right). \quad (5)$$

$$MinSystemCost = \sum_{i=1}^{m}\sum_{j=1}^{T_i}\sum_{h\in A,S,N} c_{i,j}Z_{i,j,h}(n_{i,j,h}+\exp(0.25n_{i,j,h}). \quad (6)$$

$$r_{i,j}(t) = \exp(-\lambda_{i,j}t)\sum_{l=0}^{k_{i,j}-1}\frac{(-\lambda_{i,j}t)^l}{l!}, i=1,...,m, j=1,...,T_i. \quad (7)$$

$$\sum_{i=1}^{m}\sum_{j=1}^{T_i}\sum_{h\in A,S,N} w_{i,j}\times n_{i,j,h}\le W \quad (8)$$

$$\sum_{j=1}^{T_i} Z_{i,j,A}+\sum_{j=1}^{T_i} Z_{i,j,S}+\sum_{j=1}^{T_i} Z_{i,j,N} = 1, i=1,...,m. \quad (9)$$

$$\sum_{q=2}^{n_{max}} X_{q,i,j} = 1, i=1,...,m, j=1,...,T_i. \quad (10)$$

$$n_{i,j,S} = Z_{i,j,S}\times\sum_{q=2}^{n_{max}} q\times X_{q,i,j}, i=1,...,m, j=1,...,T_i. \quad (11)$$

$$2\times Z_{i,j,A}\le n_{i,j,A}\le n_{max}\times Z_{i,j,A}, i=1,...,m, j=1,...,T_i. \quad (12)$$

$$n_{i,j,N} = 1\times Z_{i,j,N}, i=1,...,m, j=1,...,T_i. \quad (13)$$

$$1\le\sum_{j=1}^{T_i}\sum_{h\in A,S,N} n_{i,j,h}\le n_{max} \quad (14)$$

$$X_{i,j,q}\in 0,1, i=1,...,m, j=1,...,T_i, q=2,...,n_{max}. \quad (15)$$

$$Z_{i,j,h}\in 0,1, i=1,...,m, j=1,...,T_i, h\in A,S,N \quad (16)$$

The objective function presented in Equation (3) maximizes the system reliability, which consists of three terms. The first term multiplies the reliability of those subsystems, whose components are in active redundancy. In this case, when there is no active component in any subsystem (i.e., all $Z_{i,j,A}$ are zero), this term is one which is neutral in multiplication. The second term multiplies the reliability of those subsystems, whose components are in standby redundancy and it consists of three parts. The first part is to ensure that in a case that no component is selected for standby redundancy in a subsystem, the multiplication is not zero. In other words, the value of this part is zero if a component type is selected in standby and it is one otherwise which is neutral in calculating the system reliability. The second and third parts are considered only if $Z_{i,j,S}$ is one. The second part includes the reliability of the working component and the third part considers between one and $n_{i,j,S}-1$ failures regarding the reliability of the switch. The third term multiplies the reliability of those subsystems that choose no redundancy strategy. When all $Z_{i,j,N}$ are zero, this term is also one.

Objective functions presented in Equations (4) and (5) maximize the negative entropy within subsystems. These functions consider entropy in distribution of components and weights, respectively. Note that at a time either objective function (4) or (5) is considered in the model. Objective function (6) minimizes the cost of the system, which is in nonlinear form. Since a specific $n_{i,j,h}$ can be zero and $\exp(0)=1$, it is needed to multiply each summation's term by $Z_{i,j,h}$. Constraint (7) calculates the reliability of components considering Erlang's parameters. Constraint (8) poses a restriction on the total weight of the system. Constraint set (9) states that only one component type and strategy is selected for each subsystem. Constraint set (10) declares that the number of components can only be a value between 2 and n_{max} for the cold standby strategy. Constraint set (11) calculates the number of components in subsystems with the cold standby strategy. This constraint is equivalent to the constraint $2\times Z_{i,j,S}\leq n_{i,j,S}\leq n_{max}\times Z_{i,j,S}, i=1,\ldots,m, j=1,\ldots,T_i$. However, for the purpose of calculating the upper limit of the sigma in the standby term of the objective function, Constraint set (11) is considered in the model. Constraint set (12) indicates that for each subsystem, variable $n_{i,j,A}$ gets value only when type j and strategy A are selected. Its value is at least 2 and at most n_{max}. Constraint set (13) ensures that in a case of no redundancy the variable $n_{i,j,N}$ gets one only when $Z_{i,j,N}$ is one; and otherwise, it is zero. This equation can be transformed into two less equal and greater equal constraints. Constraint set (14) indicate that the number of components in each subsystem is at least one and at mostn_{max} . Constraint sets (15-16) defines the binary nature of the variables.

3 Compromise programming

Compromise programming is a mathematical programming technique which was developed by [29,30, 31]. The compromise solution is a feasible solution, which is closest to the ideal solution, and a compromise means an agreement established by mutual concessions [15]. Through the compromise programming method a discrete set of solutions is ranked according to their distance from an ideal solution. This method uses L_p distance metrics as presented in Equation (17) in order to measure the distance between each solution and its ideal and tries to minimize it.

$$L_p=\left(\sum_{i=1}^{n_0} w_i^0\left[\frac{f_i^+-f_i}{f_i^+-f_i^-}\right]^p\right)^{\dfrac{1}{p}}, \tag{17}$$

where n_0 is the number of objectives, in this paper $n_0=3$, p is a parameter ($p\in 1,2,\infty$),w_i^0 is the weight of the objective i, f_i is the actual value of the objective function i,f_i^+ and f_i^- are ideal and nadir solutions of the objective function i, respectively. For maximization problems, the former is achieved through maximizing each objective function subject to constraints whilst the latter is determined by minimizing those objectives. This procedure is for minimization problems vice versa. Compromise programming solves a single-objective model that minimizes the L_p distance presented in Equation (17).

For different values of p in L_p metrics and different values of weights different compromise solutions can be obtained. For $p=1$, the L_p metric (i.e.,L_1) is called the Manhattan metric. L_2 and L_∞ are called the Euclidean and Chebycheff metrics, respectively. In all cases, the corresponding metric needs to be minimized according to models presented in (18), (19) and (20) for L_1, L_2 and L_∞ , respectively.

$$\begin{aligned} &Min\, w_1^0\left|\frac{f_1^+-f_1}{f_1^+-f_1^-}\right|+w_2^0\left|\frac{f_2^+-f_2}{f_2^+-f_2^-}\right|+w_3^0\left|\frac{f_3^+-f_3}{f_3^+-f_3^-}\right| \\ &s.t. \\ &Constraints\ given\ in\ Model\ (1). \end{aligned} \tag{18}$$

$$\begin{aligned} &Min\sqrt{w_1^0\left[\frac{f_1^+-f_1}{f_1^+-f_1^-}\right]^2+w_2^0\left[\frac{f_2^+-f_2}{f_2^+-f_2^-}\right]^2+w_3^0\left[\frac{f_3^+-f_3}{f_3^+-f_3^-}\right]^2} \\ &s.t. \\ &Constraints\ given\ in\ Model\ (1). \end{aligned} \tag{19}$$

$$
\begin{aligned}
&MinD_\infty \\
&s.t. \\
&w_1^0 \frac{f_1^+ - f_1}{f_1^+ - f_1^-} \leq D_\infty \\
&w_2^0 \frac{f_2^+ - f_2}{f_2^+ - f_2^-} \leq D_\infty \\
&w_3^0 \frac{f_3^+ - f_3}{f_3^+ - f_3^-} \leq D_\infty \\
&Constraints\ given\ in\ Model\ (1). \quad (20)
\end{aligned}
$$

4 Computational results

To solve the proposed model using compromise programming, the data taken from [24] are considered representing a series-parallel system composed of three subsystems and four or five component choices. In that example, the reliability of the components is reported as a specific value between 0 and 1. To make the example compatible with our proposed model whose components, lifetimes follow an Erlang distribution, the scale and shape parameters are determined such that those reliabilities are obtained. The scale and shape parameters are shown in Table 1 along with components, costs and weights. The maximum number of allowable components is 8, the reliability of switch for all components equals 0.99 and the mission time is 100 unit of time.

To start with compromise programming, ideal and nadir solutions need to be calculated. From an ideal solution, we mean that the maximum and minimum values are achieved for the maximization and minimization problems, respectively. These values are obtained through maximizing the reliability and entropy objectives and minimizing the cost objective. On the other side, nadir solutions can be obtained by minimizing the problem which is of a maximization type and maximizing the problem which has a minimization nature. The nadir solutions are obtained by minimizing the reliability and entropy objectives and maximizing the cost objective. We use the branch-and-cut method to deal with the presented model. The model is solved using the Baron solver of the GAMS (General Algebraic Modelling System) version 23.8.2 and the nadir and ideal results are presented in Table 2. Solving the presented model using the compromise programming technique results in different Pareto solutions, which depend on the norm of the L_p metric and the weights of the objectives. The results are presented in Table 3 and also depicted in Fig. 2.

To decide about the best compromise solution amongst Pareto solutions, the objective functions are normalized through Equation (21). $f_i^{min}(x)$ and $f_i^{max}(x)$ are the minimum and maximum values for $f_i(x)$ in the Pareto optimal set on condition that all objectives are in minimization form. In other words, the reliability and entropy functions are multiplied by -1 to be comparable with the cost objective. The results for p=2 are shown in Table 4. The results show that solution 20 is the best compromise solution with the lowest L_2 norm.

$$\frac{f_i(x) - f_i^{min}(x)}{f_i^{max}(x) - f_i^{min}(x)}, i = 1, \ldots, n_0 \quad (21)$$

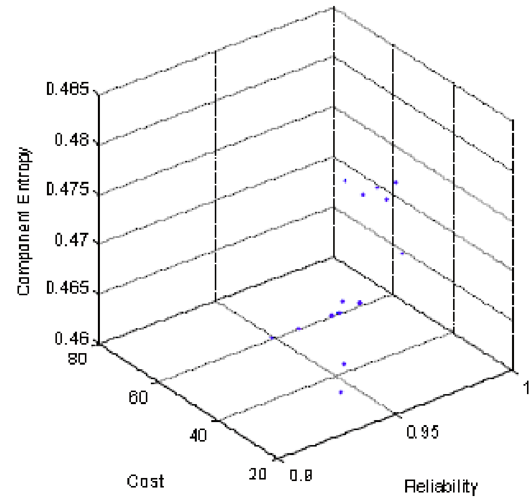

Fig. 2: Solutions with respect to component entropy

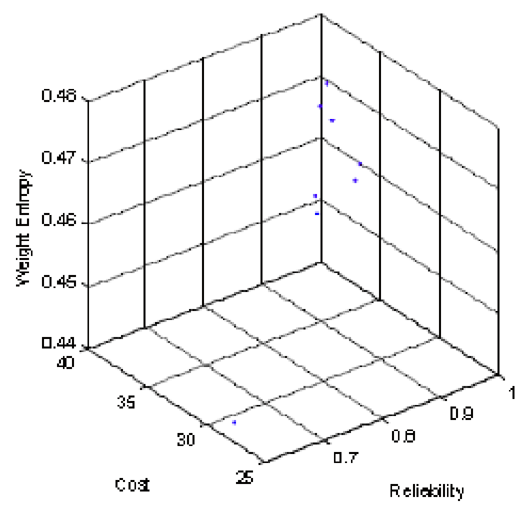

Fig. 3: Solutions with respect to the weight entropy

Table 1: Experimental data

	Subsystem 1				Subsystem 2				Subsystem 3			
Component	k	λ	c	w	k	λ	c	w	k	λ	c	w
1	1	0.000619	9	9	3	0.00665	12	5	1	0.000408	10	6
2	2	0.00499	6	6	3	0.01288	3	7	2	0.00564	6	8
3	2	0.00564	6	4	1	0.00356	2	3	1	0.00328	4	2
4	1	0.00287	3	7	1	0.00415	2	4	1	0.00342	3	4
5	1	0.00328	2	8	-	-	-	-	1	0.004	2	4

Table 2: Ideal and Nadir solutions

	Ideal solution	Nadir solution
Objective1(Reliability)	0.99935125	0.33790179
Objective2(Component Entropy)	0.477	0.330
Objective2(Weight Entropy)	0.478	0.208
Objective3(Cost)	13.704	162.325

Table 3: Experimental results with different L_p metrics and weights with respect to the component entropy

Group	w_1^0	w_2^0	w_3^0	p Norm	Reliability	Cost	Comp. Entropy	Distance	Solution
1	0.5	0.3	0.2	p=1	0.98614605	54.373	0.473	0.073	$n_{1,3,s}=3, n_{2,3,s}=3, n_{3,5,s}=4$
				p=2	0.94966849	32.126	0.469	0.083	$n_{1,4,s}=2, n_{2,2,s}=2, n_{3,5,s}=3$
				$p=\infty$	0.95268936	39.734	0.461	0.035	$n_{1,4,s}=2, n_{2,3,s}=4, n_{3,4,s}=3$
2	0.3	0.5	0.2	p=1	0.93226081	27.765	0.470	0.073	$n_{1,5,s}=2, n_{2,4,s}=3, n_{3,5,s}=3$
				p=2	0.94966849	32.126	0.469	0.080	$n_{1,4,s}=2, n_{2,2,s}=2, n_{3,5,s}=3$
				$p=\infty$	0.93542663	27.765	0.470	0.029	$n_{1,5,s}=2, n_{2,3,s}=3, n_{3,5,s}=3$
3	0.3	0.2	0.5	p=1	0.93542660	27.765	0.470	0.086	$n_{1,5,s}=2, n_{2,3,s}=3, n_{3,5,s}=3$
				p=2	0.93542660	27.765	0.470	0.088	$n_{1,5,s}=2, n_{2,3,s}=3, n_{3,5,s}=3$
				$p=\infty$	0.90666716	27.765	0.470	0.047	$n_{1,5,s}=2, n_{2,4,A}=3, n_{3,5,s}=3$
4	0.4	0.3	0.3	p=1	0.93226081	27.765	0.470	0.083	$n_{1,5,s}=2, n_{2,4,s}=3, n_{3,5,s}=3$
				p=2	0.93542660	27.765	0.470	0.084	$n_{1,5,s}=2, n_{2,3,s}=3, n_{3,5,s}=3$
				$p=\infty$	0.94034738	30.968	0.461	0.036	$n_{1,5,s}=2, n_{2,3,s}=4, n_{3,5,s}=3$
5	0.3	0.3	0.4	p=1	0.98009951	60.778	0.473	0.144	$n_{1,3,A}=4, n_{2,3,s}=3, n_{3,5,s}=3$
				p=2	0.94151750	31.414	0.470	0.093	$n_{1,4,s}=2, n_{2,4,s}=3, n_{3,5,s}=3$
				$p=\infty$	0.91828675	27.765	0.470	0.038	$n_{1,5,s}=2, n_{2,3,A}=3, n_{3,5,s}=3$
6	0.6	0.2	0.2	p=1	0.91828675	27.765	0.470	0.102	$n_{1,5,s}=2, n_{2,3,A}=3, n_{3,5,s}=3$
				p=2	0.94966849	32.126	0.469	0.084	$n_{1,4,s}=2, n_{2,2,s}=2, n_{3,5,s}=3$
				$p=\infty$	0.96247973	40.936	0.477	0.037	$n_{1,5,A}=3, n_{2,3,s}=3, n_{3,3,s}=3$
7	0.7	0.2	0.1	p=1	0.98358890	49.256	0.473	0.046	$n_{1,4,s}=3, n_{2,3,s}=4, n_{3,3,s}=3$
				p=2	0.97675585	40.936	0.477	0.065	$n_{1,5,s}=3, n_{2,3,s}=3, n_{3,3,s}=3$
				$p=\infty$	0.98166742	43.072	0.469	0.020	$n_{1,3,s}=2, n_{2,2,s}=2, n_{3,5,s}=3$

Once again, the proposed model with respect to weight entropy is considered and solved using branch and cut technique of GAMS. The results with different norms of L_p and weights are recorded in Table 5 and illustrated in Fig. 3. Amongst them, solutions 4 and 10 are dominated solutions. After eliminating the dominated solutions, the best compromise solution is chosen through the L_2 norm which is calculated and recorded in Table 6. As illustrated, the minimum one belongs to the first solution that means the first solution is the best compromise solution.

5 Conclusion

Reliability is an important requirement of systems (e.g., military, electronic devices, telecommunication systems, and internet protocol networks). Adding redundancy increases the reliability of the systems whilst other requirements should be met as far as possible. In many systems, low cost and balance in weights or number of components in subsystems are desired. In this paper, a multi-objective model has been presented to meet these requirements. For the first time, this model maximizes the system reliability with a redundancy strategy and component choices along with maximizing the system entropy and minimizing the system cost. In addition of considering entropy in distribution of components, the

Table 4: Choosing the best compromise solution using L_2 norm with respect to the component entropy

Alternative	1	2	3	4	5	6	7	8	9	10	11
L_2	0.843868	0.691443	1.14396	0.806886	0.691443	0.773719	0.773719	0.773719	1.091516	0.806886	0.773719
Alternative	12	13	14	15	16	17	18	19	20	21	
L_2	1.158215	1.03358	0.720363	0.959367	0.959367	0.691443	0.497834	0.698082	0.41609	0.684223	

Table 5: Experimental results with different L_p metrics and weights with respect to the weight entropy

Group	w_1^0	w_2^0	w_3^0	p Norm	Reliability	Cost	Weight Entropy	Distance	Solution
1	0.4	0.3	0.3	p=1	0.90533833	32.126	0.469	0.104	$n_{1,4,s}=2, n_{2,4,s}=3, n_{3,4,s}=2$
				p=2	0.95267342	37.243	0.474	0.098	$n_{1,4,s}=2, n_{2,2,s}=2, n_{3,4,s}=3$
				$p=\infty$	0.92793924	36.531	0.472	0.046	$n_{1,4,A}=3, n_{2,3,s}=3, n_{3,4,s}=2$
2	0.3	0.4	0.3	p=1	0.84776277	32.838	0.462	0.131	$n_{1,4,A}=2, n_{2,3,s}=2, n_{3,3,s}=2$
				p=2	0.95267342	37.243	0.474	0.095	$n_{1,4,s}=2, n_{2,2,s}=2, n_{3,4,s}=3$
				$p=\infty$	0.91317610	32.838	0.465	0.039	$n_{1,4,s}=2, n_{2,2,s}=2, n_{3,4,s}=2$
3	0.3	0.5	0.2	p=1	0.60229410	27.854	0.443	0.264	$n_{1,2,N}=1, n_{2,4,s}=2, n_{3,4,N}=1$
				p=2	0.95267342	37.243	0.474	0.081	$n_{1,4,s}=2, n_{2,2,s}=2, n_{3,4,s}=3$
				$p=\infty$	0.94770399	36.531	0.469	0.031	$n_{1,4,s}=2, n_{2,3,s}=3, n_{3,4,s}=3$
4	0.4	0.4	0.2	p=1	0.84220966	32.838	0.465	0.141	$n_{1,4,A}=2, n_{2,2,A}=2, n_{3,4,A}=2$
				p=2	0.94770399	36.531	0.469	0.087	$n_{1,4,s}=2, n_{2,3,s}=3, n_{3,4,s}=3$
				$p=\infty$	0.94770400	36.531	0.469	0.031	$n_{1,4,s}=2, n_{2,3,s}=3, n_{3,4,s}=3$

Table 6: Choosing the best compromise solution using L_2 norm with respect to the weight entropy

Alternative	1	2	3	4	5	6	7	8	9	10
L_2	0.50129	1	0.929101	1	0.615451	1.414214	1	0.938243	0.938243	0.938243

entropy in distribution of weights within subsystems has been considered in the presented model. In subsystems, one can either decide to allocate one component and choose no redundancy or choose a redundancy strategy from active or cold standby. The model has been dealt using a compromise programming technique with different L_p norms. For future research, other mathematical programming techniques can be implemented to deal with the presented model. Furthermore, heuristic and meta-heuristic algorithms can be employed to solve large-sized problems. Moreover, the model can be extended to allow component mixing.

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Roya Soltani received her PhD. degree in Industrial Engineering from Iran University of Science and Technology, Tehran, Iran. She obtained her B.S. and M.S. degrees in the field of Industrial Engineering in 2006 and 2008, respectively. Her research interests mainly include Operations research, Optimization under uncertainty, Reliability Engineering, Sequencing and Scheduling, Transportation Planning, Reverse logistic and supply chain management, Heuristic and Meta-Heuristic applications for combinatorial optimization problems.

Seyed Jafar Sadjadi finished his PhD. degree from University of Waterloo, Ontario, Canada. His interest is on nonlinear optimization problems, combinatorial optimization, optimal pricing, reliability engineering and multi-criteria decision making problems. Professor Sadjadi has been working at Iran University of Science and Technology since 2000. He is also responsible as editor-in-chief of International Journal of Industrial Engineering Computations.

Reza Tavakkoli-Moghaddam is a professor of Industrial Engineering at College of Engineering, University of Tehran in Iran. He obtained his Ph.D. in Industrial Engineering from the Swinburne University of Technology in Melbourne (1998), his M.Sc. in Industrial Engineering from the University of Melbourne in Melbourne (1994) and his B.Sc. in Industrial Engineering from the Iran University of Science and Technology in Tehran (1989). His research interests include facility layouts and location design, cellular manufacturing systems, sequencing and scheduling, and using meta-heuristics for combinatorial optimization problems. He serves as Editorial Boards of the International Journal of Engineering and Iranian Journal of Operations Research as well as three other Iranian journals. He is the recipient of the 2009 and 2011 Distinguished Researcher Award and the 2010 Distinguished Applied Research Award at the University of Tehran, Iran. He has been selected as National Iranian Distinguished Researcher for 2008 and 2010. Professor Tavakkoli-Moghaddam has published 4 books, 15 book chapters and more than 500 papers in reputable academic journals and conferences.