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Fixed Point Theorems for Quadruple of Self Maps in Normed Boolean Vector Space

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Abstract: In this paper, we prove some common fixed point theorems for quadruple of weakly compatible self maps in normed Boolean vector space using property $(E.A)$ and its variants. Our results extend and unify various known results in literature.

Keywords: Normed Boolean vector space, Boolean metric, property $(E.A)$, common property $(E.A)$, $JCLR_{ST}$ property, common fixed point, coincidence point

1 Introduction

In 1964-65, Subrahmanyam [1,2] introduced the notion of Boolean vector spaces and studied the convergence of sequences in these spaces. For details on this aspect one may refer to [3,4,5] and references therein. Fixed point theory of Boolean functions has many applications in the field of Switching circuits, cryptography, the design of circuits and chips for digital computers, electrical engineering, reliability theory and many others. These applications have often provided motivation for the study of the problem in fixed point theory for Boolean valued functions. In 2011, Rao and Pant [6] utilized the concept of finite normed Boolean vector spaces and proved some common fixed point theorems for asymptotically regular maps. Recently, Mishra et al. [4] proved some common fixed point theorems using property $(E.A)$ (which was introduced by Aamri and Moutawakil [7]) in normed Boolean vector spaces.

In this paper, we prove some common fixed point theorems for four self maps in normed Boolean vector space by using property $(E.A)$ and its variants. Our results extend and unify various known results in literature such as Ghilzean [3], Rao and Pant [6], Mishra et al. [4] and Rudeanu [5].

2 Preliminaries

Definition 1.*[1] $V = (V,+)$ be an additive abelian group and $(\beta,+,.,')$ be a Boolean algebra. The set V is said to be a "Boolean vector space over β" (or simply, a "β -vector space") if for all $x,y \in V$ and $a,b \in \beta$,*
(2.1) $a(x+y) = ax+ay$;
(2.2) $(ab)x = a(bx) = b(ax)$;
(2.3) $1x = x$ *and*
(2.4) *if* $ab = 0$, *then* $(a+b)x = ax+bx$.

Remark.[1] The "zero element" of V and also the "null element" of B are both denoted by "0", while the "universal element" (= 0) of B is denoted by "1".

Example 1.[1] Let β be any Boolean algebra and V be the additive abelian group of the corresponding Boolean ring. Define for $a \in \beta$ and $x \in V$, ax = the Boolean product of a and x in β. Then V is a Boolean vector space over β.

Definition 2.*[1] A Boolean vector space V over a Boolean algebra β is said to be "β -normed" (or simply, "normed") if and only if there exists a map $||.|| : V \to \beta$ such that*
(i) $||x|| = 0$ *if and only if* $x = 0$, *and*
(ii) $||ax|| = a||x||$ *for all* $a \in \beta$ *and* $x \in V$.

Let V be a β -normed vector space and $V \times V \to \beta$ then $d(x,y) = ||x-y||$ defines a Boolean metric on V, i.e.,
(i) $d(x,y) = 0$ if and only if $x = y$;

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(ii) $d(x,y) = d(y,x)$ and
(iii) $d(x,z) < d(x,y) + d(y,z)$.

Definition 3.[8] *Two self maps A and S of a Boolean vector space V are weakly compatible if $ASx = SAx$ for all x at which $Ax = Sx$.*

Definition 4.[7] *Self maps A and S of a Boolean vector space V satisfies the property (E.A) if there exist a sequence $\{x_n\}$ in V such that*

$$lim_{n\to\infty}Ax_n = lim_{n\to\infty}Sx_n = z$$

for some $z \in V$.

Clearly, both compatible and noncompatible pairs enjoy property $(E.A)$.

Definition 5.[9] *Two pairs of self maps (A,S) and (B,T) on a Boolean vector space V satisfy common property (E.A) if there exists two sequences $\{x_n\}$ and $\{y_n\}$ in V such that*

$$lim_{n\to\infty}Ax_n = lim_{n\to\infty}Sx_n = lim_{n\to\infty}Ty_n = lim_{n\to\infty}By_n = p$$

for some $p \in V$.

Definition 6.[10] *Two pairs of self maps (A,S) and (B,T) on a Boolean vector space V satisfy the $(JCLR_{ST})$ property (with respect to mappings S and T) if there exist two sequences $\{x_n\}$ and $\{y_n\}$ in V such that $lim_{n\to\infty}Ax_n = lim_{n\to\infty}Sx_n = lim_{n\to\infty}Ty_n = lim_{n\to\infty}By_n$ $= Sz = Tz$ where $z \in V$.*

Definition 7.[9] *Two finite families of self maps $\{A_i\}_{i=1}^m$ and $\{B_j\}_{j=1}^n$ on a set V are pairwise commuting if (i) $A_iA_j = A_jA_i$, $i,j \in \{1,2,3,...,m\}$,*
(ii) $B_iB_j = B_jB_i$, $i,j \in \{1,2,3,...,n\}$,
(iii) $A_iB_j = B_jA_i$, $i \in \{1,2,3,...,m\}$, $j \in \{1,2,3,...,n\}$.

3 Main Results

Let Φ be the set of all continuous functions $\Psi : \beta \to \beta$ satisfying $\Psi(a) < a'$ for all $a \in \beta$.

Theorem 1.*Let A,B,S and T be four self maps in normed Boolean vector space V satisfying:*
(3.1) $A(V) \subset T(V)$ and $B(V) \subset S(V)$;
(3.2) there exist $\Psi \in \Phi$ such that

$$d(Ax,By) = \Psi(M(x,y))$$

where

$$M(x,y) = max\{d(Sx,Ty), d(Sx,Ax), d(By,Ty)\}$$

for all $x,y \in V$;
(3.3) pair (A,S) or (B,T) satisfies the property (E.A).
(3.4) range of one of the maps A,B,S or T is a closed subspace of V.
Then pairs (A,S) and (B,T) have coincidence point. Further if (A,S) and (B,T) be weakly compatible pairs of self maps of V then A,B,S and T have a unique common fixed point in V.

*Proof.*If the pair (B,T) satisfies the property $(E.A)$, then there exist a sequence $\{x_n\}$ in V such that $Bx_n \to z$ and $Tx_n \to z$ for some $z \in V$ as $n \to \infty$.

Since, $B(V) \subset S(V)$, therefore, there exist a sequence $\{y_n\}$ in V such that $Bx_n = Sy_n$. Hence, $Sy_n \to z$ as $n \to \infty$. Also, since $A(V) \subset T(V)$, there exist a sequence $\{z_n\}$ in V such that $Tx_n = Az_n$. Hence, $Az_n \to z$ as $n \to \infty$. Suppose that $S(V)$ is a closed subspace of V. Then $z = Su$ for some $u \in V$. Therefore, $Az_n \to Su$, $Bx_n \to Su$, $Tx_n \to Su$, $Sy_n \to Su$ as $n \to \infty$.
First we claim that $Au = Su$. Suppose not, then by (3.2), take $x = u$, $y = x_n$, we get

$$d(Au,Bx_n) = \Psi(M(u,x_n)).$$

As $n \to \infty$,

$$d(Au,Su) = \Psi(lim_{n\to\infty}M(u,x_n))...(3.5)$$

where

$$M(u,x_n) = max\{d(Su,Tx_n), d(Su,Au), d(Bx_n,Tx_n)\}.$$

As $n \to \infty$
$lim_{n\to\infty}M(u,x_n)$
$= max\{d(Su,Su), d(Su,Au), d(Su,Su)\} = d(Su,Au)$.
(3.5) gives,

$$d(Au,Su) = \Psi(d(Au,Su)) < (d(Au,Su))'$$

a contradiction, hence, $Au = Su$. As A and S are weakly compatible. Therefore, $ASu = SAu$ and then $AAu = ASu = SAu = SSu$.

On the other hand, since $A(V) \subset T(V)$, there exist $v \in V$ such that $Au = Tv$. We now show that, $Tv = Bv$. Suppose not, then by (3.2), take $x = u, y = v$, we have,

$$d(Au,Bv) = \Psi(M(u,v))$$

or

$$d(Tv,Bv) = \Psi(M(u,v))...(3.6)$$

where

$$M(u,v) = max\{d(Su,Tv), d(Su,Au), d(Bv,Tv)\}$$

$$M(u,v) = max\{d(Tv,Tv), d(Au,Au), d(Bv,Tv)\}$$

$$M(u,v) = d(Bv,Tv).$$

Thus, (3.6) gives,

$$d(Tv,Bv) = \Psi(d(Bu,Tv)) < (d(Bv,Tv))'$$

a contradiction, hence, $Bv = Tv$.

As B and T are weakly compatible, therefore, $BTv = TBv$ and hence,

$$BTv = TBv = TTv = BBv.$$

Next we claim that $AAu = Au$.
Suppose not, then by (3.2), take $x = Au, y = v$, we get

$$d(AAu, Bv) = \Psi(M(Au, v))...(3.7)$$

where

$$M(Au, v) = max\{d(SAu, Tv), d(SAu, AAu), d(Bv, Tv)\}$$

$$M(Au, v) = max\{d(AAu, Bv), d(AAu, AAu), d(Tv, Tv)\}$$

$$M(Au, v) = d(AAu, Bv).$$

(3.7) gives,

$$d(AAu, Bv) = \Psi(d(AAu, Bv)) < (d(AAu, Bv))'$$

again a contradiction, hence $AAu = Au$. Therefore, $Au = AAu = SAu$ and Au is a common fixed point of A and S. Similarly, we can prove that Bv is a common fixed point of B and T. As $Au = Bv$, we conclude that Au is a common fixed point of A, B, S and T.

The proof is similar when $T(V)$ is assumed to be a closed subspace of V. The cases in which $A(V)$ or $B(V)$ is a closed subspace of V are similar to the cases in which $T(V)$ or $S(V)$ respectively, is closed since $A(V) \subset T(V)$ and $B(V) \subset S(V)$.

For uniqueness, let u and v are two common fixed points of A, B, S and T. Therefore, by definition, $Au = Bu = Tu = Su = u$ and $Av = Bv = Tv = Sv = v$. Then by (3.2), take $x = u$ and $y = v$, we get

$$d(Au, Bv) = \Psi(M(u, v))$$

or

$$d(u, v) = \Psi(M(u, v))...(3.8)$$

where

$$M(u, v) = max\{d(Su, Tv), d(Su, Au), d(Bv, Tv)\}$$

$$M(u, v) = max\{d(u, v), d(u, u), d(v, v)\}$$

$$M(u, v) = d(u, v).$$

Equation (3.8) gives,

$$d(u, v) = \Psi(d(u, v)) < (d(u, v))'$$

a contradiction, therefore, $u = v$. Hence A, B, S and T have a unique common fixed point in V.

Taking $B = A$ and $T = S$ in Theorem 1, we get following result:

Corollary 1. *Let A and S be two self maps in normed Boolean vector space V over Boolean algebra β such that*
(3.9) *there exist $\Psi \in \Phi$ such that*

$$d(Ax, By) = \Psi(M(x, y))$$

where

$$M(x, y) = max\{d(Sx, Sy), d(Sx, Ax), d(Ay, Sy)\}$$

for all $x, y \in V$;
(3.10) *pair (A, S) satisfies the property (E.A);*
(3.11) *the range of one of the maps A or S is a closed subspace of V.*
Then A and S have a coincidence point in V. Further if (A, S) be weakly compatible pair of self maps then A and S have a unique common fixed point in V.

Now we attempt to drop containment of subspaces by replacing property $(E.A)$ by a weaker condition common property $(E.A)$ in Theorem 1.

Theorem 2. *Let A, B, S and T be four self maps in normed Boolean vector space V satisfying condition (3.2) of Theorem 1 and*
(3.12) *pairs (A, S) and (B, T) satisfies the common property (E.A);*
(3.13) *$S(V)$ and $T(V)$ are closed subspace of V.*
Then pairs (A, S) and (B, T) have coincidence point. Further if (A, S) and (B, T) be weakly compatible pairs of self maps of V then A, B, S and T have a unique common fixed point in V.

Proof. In view of (3.12), there exist two sequences $\{x_n\}$ and $\{y_n\}$ in V such that

$$lim_{n\to\infty}Ax_n = lim_{n\to\infty}Sx_n = lim_{n\to\infty}Ty_n = lim_{n\to\infty}By_n = z$$

for some $z \in V$.
Since $S(V)$ is a closed subset of V, therefore, there exists a point $u \in V$ such that $z = Su$.
We claim that $Au = z$. Suppose not, then by (3.2), take $x = u, y = y_n$,

$$d(Au, By_n) = \Psi(M(u, y_n))$$

taking $n \to \infty$, we get

$$d(Au, z) = \Psi(lim_{n\to\infty}M(u, y_n))...(3.14)$$

where

$$M(u, y_n) = max\{d(Su, Sy_n), d(Su, Au), d(Ay_n, Sy_n)\}$$

$n \to \infty$, we get
$lim_{n\to\infty}M(u, y_n)$
$= max\{d(Su, Su), d(z, Au), d(z, z)\} = d(z, Au).$
(3.14) becomes,

$$d(Au, z) = \Psi(d(Au, z)) < (d(Au, z))'$$

a contradiction, This gives, $Au = z$.

Therefore, $Au = z = Su$ which shows that u is a coincidence point of the pair (A,S). Since $T(V)$ is also a closed subset of V, therefore $lim_{n\to\infty}Ty_n = z$ in $T(V)$ and hence there exists such that $Tv = z = Au = Su$. Now, by taking $x = u, y = v$ in (3.2) we can easily show that $Bv = z$. Therefore, $Bv = z = Tv$ which shows that v is a coincidence point of the pair (B,T).

Since the pairs (A,S) and (B,T) are weakly compatible and $Au = Su, Bv = Tv$, therefore,

$$Az = ASu = SAu = Sz,$$

$$Bz = BTv = TBv = Tz.$$

Next, we claim that $Az = z$. Suppose not, then by using inequality (3.2), take $x = z$ and $y = v$, we have

$$d(Az,Bv) = \Psi(M(z,v))$$

or

$$d(Az,z) = \Psi(M(z,v))...(3.15)$$

where

$$M(z,v) = max\{d(Sz,Tv),d(Sz,Az),d(Bv,Tv)\}$$

$$M(z,v) = max\{d(Az,z),d(Az,Az),d(Bv,Bv)\}$$

$$M(z,v) = d(Az,z).$$

which gives (3.15) as

$$d(Az,z) = \Psi(d(Az,z)) < (d(Az,z))'$$

a contradiction. Hence, $Az = z = Sz$.

Similarly, one can prove that $Bz = Tz = z$. Hence, $Az = Bz = Sz = Tz$, and z is common fixed point of A,B,S and T. The uniqueness of common fixed point is an easy consequence of inequality (3.2).

Now we attempt to drop closedness of subspaces by using weaker condition $JCLR_{ST}$ property in Theorem 2.

Theorem 3.*Let A,B,S and T be four self maps in normed Boolean vector space V satisfying condition (3.2) of Theorem 1 and*
(3.16) (A,S) and (B,T) satisfy $JCLR_{ST}$ property.
Then pairs (A,S) and (B,T) have coincidence point. Further if (A,S) and (B,T) be weakly compatible pairs of self maps of V then A,B,S and T have a unique common fixed point in V.

*Proof.*As the pairs (A,S) and (B,T) satisfy the $JCLR_{ST}$ property, that is, there exists two sequences $\{x_n\}$ and $\{y_n\}$ in X such that
$lim_{n\to\infty}Ax_n = lim_{n\to\infty}Sx_n$
$= lim_{n\to\infty}Ty_n = lim_{n\to\infty}By_n = Sz = Tz$
for some $z \in V$.
Firstly, we assert that $Az = Sz$. Suppose not, then by (3.2), take $x = z$ and $y = y_n$, we have

$$d(Az,By_n) = \Psi(M(z,y_n))$$

taking $n \to \infty$, we get

$$d(Az,Sz) = \Psi(lim_{n\to\infty}M(z,y_n))...(3.17)$$

where

$$M(z,y_n) = max\{d(Sz,Ty_n),d(Sz,Az),d(By_n,Ty_n)\}$$

$n \to \infty$,
$lim_{n\to\infty}M(z,y_n)$
$= max\{d(Sz,Sz),d(Sz,Az),d(Sz,Sz)\} = d(Sz,Az)$.
(3.17) becomes,

$$d(Az,Sz) = \Psi(d(Az,Sz)) < (d(Az,Sz))'$$

a contradiction, $Az = Sz$ which shows that z is a coincidence point of the pair (A,S).

Similarly, we can easily prove that $Bz = Tz$ by taking $x = y = z$ in (3.2) which shows that z is a coincidence point of the pair (B,T). Thus, we have $Tz = Bz = Az = Sz$.

Now, we assume that $u = Tz = Bz = Az = Sz$. Since the pairs (A,S) and (B,T) are weakly compatible, this gives,

$$Au = ASz = SAz = AAz = SSz = Su,$$

$$Bu = BTz = TBz = TTz = BBz = Tu.$$

Finally, we assert that $Au = u$. Suppose not, again by (3.2), taking $x = u$ and $y = z$, we have

$$d(Au,Bz) = \Psi(M(u,z))$$

or

$$d(Au,u) = \Psi(M(u,z))...(3.18)$$

where

$$M(u,z) = max\{d(Su,Tz),d(Su,Au),d(Bz,Tz)\}$$

$$M(u,z) = max\{d(Au,u),d(Au,Au),d(Bz,Bz)\}$$

$$M(u,z) = d(Au,u).$$

(3.18) gives,

$$d(Au,u) = \Psi(d(Au,u)) < (d(Au,u))'$$

a contradiction, again. This gives, $Au = u = Su$ which gives, u is common fixed point of A and S. Similarly, by taking $x = z$ and $y = u$ in (3.2), one can easily prove that $Bu = u = Tu$, that is u is common fixed point of B and T. Therefore u is common fixed point of A,S,B and T. The uniqueness of common fixed point is an easy consequence of inequality (3.2).

*Remark.*Theorem 1 and 2 remains true if we replace condition (3.2) by any one of the following conditions:
(3.19) there exist $\Psi \in \Phi$ such that

$$d(Ax,By) = \Psi(M(x,y))$$

where

$$M(x,y) = max\{d(Sx,Ty),d(Sx,By),d(By,Ty)\}$$

for all $x,y \in V$.
(3.20) there exist $\Psi \in \Phi$ such that

$$d(Ax,By) = \Psi(M(x,y))$$

where
$M(x,y) = max\{d(Sx,Ty), d(Ax,Sx), d(Ty,Ax),$
$d(Sx,By), d(By,Ty)\}$ for all $x,y \in V$.

Now we give an example to illustrate Theorem 2.

Example 2(6). Let S be a non-empty set and β the class of all subsets of S. Then the class $\beta(+,.,') = \beta(\cup,\cap,^c)$ defines a Boolean algebra. Also, (β,Δ) defines a Boolean ring where Δ represents symmetric difference between two sets. Let $V =$ be the additive abelian group as defined in Example 1. Then $V = (V,\Delta)$ is a Boolean vector space over β. Let A,B,S,T be four self-maps on V defined by $Ax = Bx = \xi$ and $Sx = Tx = x$ (identity map) for all $x \in V$ and ξ is some element in V. Let $\Psi : \beta \to \beta$ defined by $\Psi(a) = a-1$ for all $a \in \beta$, where '1' is the universal element of β. Then, clearly, $\Psi \in \Phi$. Also, $S(V)$ and $T(V)$ are closed subspace of V.

Now there exist sequences $\{x_n\}$ and $\{y_n\}$ in V defined by $x_n = y_n = \xi$ for all $n = 1,2,...,$ such that

$$lim_{n\to\infty}Ax_n = lim_{n\to\infty}Sx_n = lim_{n\to\infty}Ty_n = lim_{n\to\infty}By_n = \xi.$$

Further, let $d(x,y) = ||x-y||$ for all $x,y \in V$.
Clearly, A,B,S and T satisfies equation (3.2). Thus all the hypothesis of Theorem 2 are satisfied and ξ is a common fixed point of A,B,S and T.

As an application of Theorem 1, we prove a common fixed point theorem for four finite families of maps. While proving our result, we utilize Definition 7 which is a natural extension of commutativity condition to two finite families.

Theorem 4.*Let* $\{A_1,A_2,...,A_m\}$, $\{B_1,B_2,...,B_n\}$, $\{S_1,S_2,...,S_p\}$ *and* $T_1,T_2,...,T_q\}$ *be four finite families of self maps of a normed Boolean vector space* V *such that* $A = A_1.A_2.....A_m$, $B = B_1.B_2.....B_n$, $S = S_1.S_2.....S_p$ *and* $T = T_1.T_2.....T_q$ *satisfy the condition (3.2) and*
(3.21) $A(X) \subset T(X)$ (*or* $B(X) \subset S(X)$);
(3.22) *the pair* (A,S) (*or* (B,T)) *satisfy property (E.A).*
Then the pairs and have a point of coincidence each. Moreover finite families of self maps A_i,S_k,B_r *and* T_t *have a unique common fixed point provided that the pairs of families* $(\{A_i\},\{S_k\})$ *and* $(\{B_r\},\{T_t\})$ *commute pairwise for all* $i = 1,2,...,m$, $k = 1,2,...,p$, $r = 1,2,...,n$, $t = 1,2,...,q$.

*Proof.*Since self maps A,B,S,T satisfy all the conditions of Theorem 1, the pairs and have a point of coincidence. Also the pairs of families $(\{A_i\},\{S_k\})$ and $(\{B_r\},\{T_t\})$ commute pairwise, we first show that $AS = SA$ as
$AS = (A_1A_2A_m)(S_1S_2...S_p)$
$= (A_1A_2...A_{m-1})(A_mS_1S_2...S_p)$
$= (A_1A_2...A_{m-1})(S_1S_2...S_pA_m)$
$= (A_1A_2...A_{m-2})(A_{m-1}S_1S_2...S_pA_m)$
$= (A_1A_2...Am-2)(S_1S_2...S_pA_{m-1}A_m)$
$= ... = A_1(S_1S_2...S_pA_2...A_m)$
$= (S_1S_2...S_p)(A_1A_2...A_m) = SA.$
Similarly one can prove that $BT = TB$. And hence, obviously the pair (A,S) and (B,T) are weakly compatible.

Using Theorem 1, we conclude that A,S,B and T have a unique common fixed point in V, say z. Now, one needs to prove that z remains the fixed point of all the component maps. For this consider $A(A_iz) = ((A_1A_2...A_m)A_i)z$
$= (A_1A_2...A_{m-1})(A_mA_i)z$
$= (A_1A_2...A_{m-1})(A_iA_m)z$
$= (A_1A_2...A_{m-2})(Am-1A_iA_m)z$
$= (A_1A_2...A_{m-2})(A_iA_{m-1}A_m)z$
$= ... = A_1(A_iA_2...A_m)z$
$= (A_1A_i)(A_2...A_m)z$
$= (A_iA_1)(A_2...A_m)z$
$= A_i(A_1A_2...A_m)z$
$= A_iAz = A_iz.$
Similarly, one can prove that

$$A(S_kz) = S_k(Az) = S_kz,$$

$$S(S_kz) = S_k(Sz) = S_kz,$$

$$S(A_iz) = A_i(Sz) = A_iz,$$

$$B(B_rz) = B_r(Bz) = B_rz,$$

$$B(T_tz) = T_t(Bz) = T_tz,$$

$$T(T_tz) = T_t(Tz) = T_tz$$

and

$$T(B_rz) = B_r(Tz) = B_rz,$$

which shows that (for all i,r,k and t) A_iz and S_kz are other fixed point of the pair (A,S) whereas B_rz and T_tz are other fixed points of the pair (B,T). As A,B,S and T have a unique common fixed point, so, we get

$$z = A_iz = S_kz = B_rz = T_tz,$$

for all $i = 1,2,...,m$, $k = 1,2,...,p$, $r = 1,2,...,n$, $t = 1,2,...,q$

which shows that z is a unique common fixed point of $\{A_i\}_{i=1}^m, \{S_k\}_{k=1}^p$, $\{B_r\}_{r=1}^n$ and $\{T_t\}_{t=1}^q$.

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