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A Fuzzy Goal Programming Approach for Solving Chance Constrained Bi-Level Multi-Objective Quadratic Fractional Programming Problem

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Abstract: In this paper a chance constrained bi-level multi-objective quadratic fractional programming problem (BLMO-QFP) has been solved by using a fuzzy goal programming approach to get a compromise solution. Inside the model formation, individual best solutions of the quadratic fractional objective function to the equivalent deterministic constraints are used to formulate the quadratic fractional membership function. In this work, we convert the quadratic fractional programming model into equal non-linear quadratic problem with the help of Taylor series and the obtained quadratic problem is again converted into an equivalent linear membership function. Conclusively a numerical example is provided to expose the applicability and performance of the proposed method.

Keywords: Bi-level programming, Multi-objective programming, Fractional programming, Quadratic programming, Chance constraints, Fuzzy goal programming.

1 Introduction

Bi-level programming problem (BLPP) [5,6,10,23] is a hierarchal structure of two level decision makers. Here the decision executes sequentially from first to second level and a set of decision variables are controlled autonomously by each level decision maker. These systems were created to tackle the decentralized problem, with two decision makers in various leveled association. Here, every decision maker (DM) autonomously optimizes their own benefits, any way it is influenced by the activity or response of the other DM.

The formal formulation of BLPP offered through Candler and Townsley [10] as well as Fortuny-Amat and McCarl [14]. Anandalingam [1] has given Stackelberg arrangement idea to multi-level programming problem (MLPP) as well as bi-level decentralized programming problem (BLDPP). Lai [18] used tolerance membership function to use the concept of fuzzy set principle at the beginning to MLPP. This concept became extended by Shih et al. [32], Shih and Lee [34]. They delivered non-compensatory max-min aggregation operator and compensatory fuzzy operator for MLPP respectively. The interactive Fuzzy programming for MLPP was established by Sakawa et al. [33]. Sinha [35, 36] Presented alternative multi-level programming based on fuzzy mathematical programming. Arora and Gupta [2] offered interactive fuzzy goal programming (FGP) approach for linear BLPP with the characteristics of dynamic programming. Mishra, Verma and Dey [22] elongated their conceptions for solving bi-level multi-objective programming problem based on FGP.

Quadratic bi-level programming problem (QBLPP) is also a non-linear bi-level programming problem. In this text, the goal feature of each level DM is chosen as quadratic fractional feature with deterministic constraints. For greater details on QBLPP, one should go through, Calvete and Gale [11], Pal and Moitra [26], Saraj and Sadeghi [37] and Pramanik and Dey [28]. Lately, Pramanik and Dey [29] brought procedure totally based FGP approach to multi objective quadratic programming problem. They [30] additionally gave the idea to clear up QBLPP based on FGP. Mishra and Dey [21] have given answer of Linear Membership Function. Hosseini and Kamalabadi [16] offered a genetic algorithm to deal with linear-quadratic bi-level and linear-fractional bi-level programming problems. Singh and Haldar [38] added a new technique for solving bi-level quadratic linear partial programming problem.

According to [8], “In the area of optimization the fractional optimization problem is one of the most arduous problems

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that is the ratio of Optimization of the ratio of two functions". [15] Supplied a new technique for the answer of multi-objective linear fractional programming. FGP approach for multi-objective quadratic fractional programming (MOQFP) problem has been offered in [19]. An interactive FGP set of rules for decentralized bi-level multi-objective fractional programming problem was presented in [13]. Baky et al. [3] has given fuzzy goal programming procedure to solve the bi-level multi-objective fractional programming. FGP technique to resolve stochastic fuzzy multi-level multi-objective fractional programming problem change into prolonged in [24]. Adhami et.al [4] have given the concept of multi-level decision making models for advertising allocation problem under fuzzy environment. Parametric multi-level multi-objective partial programming problems with fuzziness in the constraints had been introduced in [25].

In the decision making process uncertainties may be occurred. They can also be described by fuzzily or stochastically. Dantzig [12] has given the concept of stochastic programming by using probability theory. There are two types of stochastic programming, one is chance constrained programming (CCP) and other one is two-stage programming. Chance constrained programming technique has been developed by Charnes and Cooper [9]. They have developed a new conceptual and analytic optimal method which contains temporary planning of optimal stochastic decision rules under uncertainty.

In the present study, we have used the fuzzy goal programming method to solve the chance constrained bi-level multi-objective quadratic fractional programming problem. In the model creation first we convert the chance constraints into equivalent deterministic constraints. By assigning the greatest degree of membership functions, the fuzzy goal of the problem has been transformed into flexible form. A linearization technique is used to convert the quadratic fractional goal into linear form and to get the most satisfactory solution in the decision making process.

2 Formulation of Chance Constrained Bi-Level Multi-Objective Quadratic Fractional Programming Problem

Here, we assume that there are two level in a hierarchal structure having first and second level decision maker. Accept that the vector of decision variables $X = (X_1, X_2) \in R^n$ be isolated into two planners. The first-level decision maker (FLDM) has control over the vector $X_1 \in R^{n_1}$ and the second-level decision maker has control (SLDM) over the vector $X_2 \in R^{n_2}$, where $n = n_1 + n_2$.we also assume that

$$Z_i(X_1, X_2): R^{n_1} \times R^{n_2} \to R^{k_i} \quad i = 1,2.$$

[FLDM]

$$\min_{X_1} Z_1(X_1, X_2) = \min_{X_1} \left(f_{11}(X_1, X_2), f_{12}(X_1, X_2), \dots, f_{1k_1}(X_1, X_2) \right).$$

Where X_2 solves

[SLDM]

$$\min_{X_2} Z_2(X_1, X_2) = \min_{X_2} \left(f_{21}(X_1, X_2), f_{22}(X_1, X_2), \dots, f_{2k_2}(X_1, X_2) \right).$$

Subject to

$$X \in S = \{X = (X_1, X_2) \in R^n : \Pr\left(A_1X_1 + A_2X_2 \overset{\leq}{\geq} \mathrm{d}\right) > I - m, X \geq 0, d \in R^m\} \qquad (2.1)$$

Where

$$f_{ij}(X_1, X_2) = \frac{P_{ij}(X_1, X_2)}{Q_{ij}X_1, X_2)} \qquad (2.2)$$

Here $P_{ij}(X_1, X_2) = C_{ij}X + \frac{1}{2}X^T D_{ij} X$ and $R_{ij}(X_1, X_2) = \bar{C}_{ij}X + \frac{1}{2}X^T \bar{D}_{ij} X$ are quadratic functions

$j = 1,2, \dots, m_1,\quad i = 1$ for FLDM objective functions,

$j = 1,2, \dots, m_2,\quad i = 2$ for SLDM objective functions,

Here the decision vector $X_1 = (x_1^1, x_1^2, x_1^3, \dots, x_1^{n_1})$ is

Controlled by FLDM and $X_2 = \left(x_2^1, x_2^2, x_2^3, \dots, x_2^{n_2}\right)$ is

Controlled by SLDM. $X_1 \cup X_2 = X \in R^n, n_1 + n_2 = n. I, m, d$ are vectors of order $p \times 1$. k_1 and k_2 denote the number of objective functions in first-level and second-level and also m denotes the number of constraints. $C_{ij}, \bar{C}_{ij}$ are constant vectors.$D_{ij}, \bar{D}_{ij}$ are constant symmetric matrices. $Q_{ij}(X_1, X_2) > 0$ for all $X \in S$. A_i is $m \times n_i$ matrix $i = 1,2$. S is considered as a non-empty polyhedron

3 Conversions of Chance Constraints into Deterministic Constraints [31]

Here we considered a chance constraints form as follows

$$\Pr\left(\sum_{j=1}^{n} c_{ij}x_j \leq d_i\right) \geq 1 - m_i,$$

$$i = 1,2, \dots, p_1 \qquad (3.1)$$

$$\Rightarrow \Pr\left(\frac{\sum_{j=1}^{n} c_{ij}x_j - E(d_i)}{\sqrt{var(d_i)}} \leq \frac{d_i E(d_i)}{\sqrt{var(d_i)}}\right) \geq 1 - m_i, i = 1,2 \dots, p_1$$

$$\Rightarrow m_i \geq 1 - \Pr\left(\frac{\sum_{j=1}^{n} c_{ij}x_j - E(d_i)}{\sqrt{var(d_i)}} \leq \frac{d_i - E(d_i)}{\sqrt{var(d_i)}}\right)$$

$$\Rightarrow m_i \geq \Pr\left(\frac{\sum_{j=1}^{n} c_{ij}x_j - E(d_i)}{\sqrt{var(d_i)}} > \frac{d_i - E(d_i)}{\sqrt{var(d_i)}}\right)$$

$$\Rightarrow \psi^{-1}(m_i) \geq \frac{\sum_{j=1}^{n} c_{ij}x_j - E(d_i)}{\sqrt{var(d_i)}}$$

$$\Rightarrow \psi^{-1}(m_i)\sqrt{var(d_i)} \geq \sum_{j=1}^{n} c_{ij}x_j - E(d_i)$$

$$\Rightarrow \sum_{j=1}^{n} c_{ij}x_j \leq E(d_i) + \psi^{-1}(m_i)\sqrt{var(d_i)}, \quad 1,2 \ldots, p_1 \tag{3.2}$$

Here the distribution function and inverse of distribution function of standard normal variable are denoted by $\psi(.)$ and $\psi^{-1}(.)$ respectively.

Considering the case when

$$\Pr\left(\sum_{j=1}^{n} c_{ij}x_j \geq d_i\right) \geq 1 - m_i, \quad i = p_1 + 1, p_1 + 2, \ldots, p \tag{3.3}$$

The constraints can be modified as:

$$\Rightarrow \Pr\left(\frac{\sum_{j=1}^{n} c_{ij}x_j - E(d_i)}{\sqrt{var(d_i)}} \geq \frac{d_i - E(d_i)}{\sqrt{var(d_i)}}\right) \geq 1 - m_i, i = p_1 + 1, p_1 + 2, \ldots, p$$

$$\psi\left(\frac{\sum_{j=1}^{n} c_{ij}x_j - E(d_i)}{\sqrt{var(d_i)}}\right) \geq 1 - m_i$$

$$1 - \psi\left(-\frac{\sum_{j=1}^{n} c_{ij}x - E(d_i)}{\sqrt{var(d_i)}}\right) \geq 1 - m_i$$

$$\Rightarrow \psi^{-1}(m_i) \geq \left(-\frac{\sum_{j=1}^{n} c_{ij}x_j - E(d_i)}{\sqrt{var(d_i)}}\right)$$

$$\Rightarrow \sum_{j=1}^{n} c_{ij}x_j \geq E(d_i) - \psi^{-1}(m_i)\sqrt{var(d_i)}, \quad i = p_1 + 1, p_1 + 2, \ldots, p \tag{3.4}$$

Also,

$$X \geq 0 \tag{3.5}$$

Here the equivalent deterministic system constraints (3.2), (3.4) and (3.5) are denoted by X. Here, X and S are equivalent set of constraints.

4 Construction of Membership Function

Let the optimal solution of first level and second level objective functions are denoted by $(X_1^{1j}, X_2^{1j}; f_{1j}^{min}, j = 1,2,\ldots,m_1)$ and $(X_1^{2j}, X_2^{2j}; f_{2j}^{min}, j = 1,2,\ldots,m_2)$.

We have taken the maximum value of each objective functions individually that is corresponding aspiration level, for defining the membership functions of the fuzzy goals in [7, 27] as follows:

$$u_{ij} = max f_{ij}(X_1, X_2) \quad i = 1,2, \quad j = 1,2,\ldots,m_i$$

Denote the membership function of objective function for the upper tolerance limit or aspired level. It can be assumed that the values of $f_{ij}(X_1, X_2) \geq u_{ij}$ $i = 1,2,$ $j = 1,2,\ldots,m_i$ are acceptable and all values less than $l_{ij} = min f_{ij}(X_1, X_2)$ are absolutely not acceptable. Then, the minimum value of each objective functions are individually taken as the corresponding aspiration level and the membership functions $\mu_{ij}(f_{ij}(X_1, X_2))$ for the ij^{th} fuzzy goal can be formulated as follows:

$$\mu_{ij}\left(f_{ij}(X_1, X_2)\right) = \begin{cases} 1 & f_{ij} \leq l_{ij} \\ \dfrac{u_{ij} - f_{ij}(X_1, X_2)}{u_{ij} - l_{ij}} & l_{ij} \leq f_{ij} \leq u_{ij} \\ 0 & f_{ij} \geq u_{ij} \end{cases} \tag{4.1}$$

Let

$$X_1^L = min\left\{X_1^{2j} \middle| j = 1,2,\ldots,m_2\right\}, X_2^L = min\left\{X_2^{1j} \middle| j = 1,2,\ldots,m_1\right\}$$

$$X_1^U \leq max\left\{X_1^{1j} \middle| j = 1,2,\ldots,m_1\right\}, X_2^U \leq max\left\{X_2^{2j} \middle| j = 1,2,\ldots,m_2\right\}$$

The decision lower than X_1^L and X_2^L are absolutely acceptable to the respective DMs. But, for making a balance of decision powers, the leader and follower would have to give possible relaxations of their decisions X_1^L and X_2^L, respectively and that depends on the decision making.

Let X_1^U and X_2^U be the upper tolerance limits of respective decisions. Then

$$X_1^L \leq X_1 \leq X_1^U$$
$$X_2^L \leq X_2 \leq X_2^U \tag{4.2}$$

5 Fuzzy Goal Programming Model

While dealing with the multiple conflicting objectives at a time, it is not possible to have a single solution that optimizes each objective simultaneously but one may obtain a compromise solution. Among various techniques that deal with multi-objective optimization problems in an imprecise environment, one of the most commonly used method is fuzzy goal programming. In fuzzy goal programming technique, the positive and negative ideal deviation along with the fuzzy membership function is optimized in order to reach the prescribed goal assign by decision maker. So, as in Mohamed [20], for the described membership functions in equations (4.1), the flexible membership goals having the aspired level unity may also be written as follows:

$$\mu_{ij}\left(f_{ij}(X_1,X_2)\right)+d_{ij}^{-}-d_{ij}^{+}=1, \quad i=1,2. \quad j=1,2,\dots,m_i$$

Or equivalently as

$$\frac{u_{ij}-f_{ij}(X_1,X_2)}{u_{ij}-l_{ij}}+d_{ij}^{-}-d_{ij}^{+}=1, \quad , \quad i=1,2. \quad j=1,2,\dots,m_i \qquad (5.1)$$

Over and under deviation is represented by $d_{ij}^{-},d_{ij}^{+}\geq 0$ respectively with $d_{ij}^{-}\times d_{ij}^{+}=0$, from the aspired stage . In this paper FGP approach to fuzzy multi objective decision making problems have been used which is introduced by Mohamed [20] and is extended to solve BLMO-QFP problems. As a result considering the goals of each individual objective function, the equivalent fuzzy quadratic bi-level fractional multi-objective goal programming model of the problem may be written as

$$minZ=\sum_{j=1}^{m_1}w_{1j}^{-}d_{1j}^{-}+\sum_{j=1}^{m_2}w_{2j}^{-}d_{2j}^{-}$$

Subject to

$$\frac{u_{1j}-f_{1j}(X_1,X_2)}{u_{1j}-l_{1j}}+d_{1j}^{-}-d_{1j}^{+}=1, \quad , \quad j=1,2,\dots,m_1$$

$$\frac{u_{2j}-f_{2j}(X_1,X_2)}{u_{2j}-l_{2j}}+d_{2j}^{-}-d_{2j}^{+}=1, \quad , \quad j=1,2,\dots,m_2$$

$$X_1^L\leq X_1\leq X_1^U$$

$$X_2^L\leq X_2\leq X_2^U$$

$$(X_1,X_2)\in S \qquad (5.2)$$

$d_{ij}^{-},d_{ij}^{+}\geq 0$ with $d_{ij}^{-}\times d_{ij}^{+}=0, \quad i=1,2. \quad j=1,2,\dots,m_i$

Where w_{ij}^{-} are the relative weights by considering the reciprocal of the difference of upper and lower bounds respective i^{th} level and j^{th} objective functions. Which can be determined as follows.

$$w_{ij}^{-}=\frac{1}{u_{ij}-l_{ij}}, \quad i=1,2. \quad j=1,2,\dots,m_i \qquad (5.3)$$

It is able to be without difficulty realized that the membership goals in (11) are in the form of quadratic fractional. So, the present FGP procedure cannot be used directly to solve the problem. To keep away from such problems, a linearization procedure is provided within the following phase

6 Linearization of Membership Goals

A linearization problem has been discussed in order to solve the existing FGP process. By using the under and over deviational variable has been done. Therefor by considering the ij^{th} membership goal can be represented as goal (5.2).

$h_{ij}u_{ij}=h_{ij}f_{ij}(X_1,X_2)+d_{ij}^{-}-d_{ij}^{+}=1$ Where $h_{ij}=\frac{1}{u_{ij}-l_{ij}}, \quad i=1,2. \quad j=1,2,\dots,m_i$

The expression $f_{ij}(X_1,X_2)$ are represented in equation (2.2). The above goal can be written as

$$h_{ij}u_{ij}-h_{ij}\frac{P_{ij}(X_1,X_2)}{Q_{ij}(X_1,X_2)}+d_{ij}^{-}-d_{ij}^{+}=1,$$

or equivalently as

$$\left(h_{ij}u_{ij}-1\right)Q_{ij}(X_1,X_2)-h_{ij}P_{ij}(X_1,X_2)+d_{ij}^{-}Q_{ij}(X_1,X_2)-d_{ij}^{+}Q_{ij}(X_1,X_2)=0$$

Here we have

$$G_{ij}+d_{ij}^{-}Q_{ij}(X_1,X_2)-d_{ij}^{+}Q_{ij}(X_1,X_2)=1 \qquad (6.1)$$

In which

$$G_{ij} = \left(h_{ij}u_{ij} - 1\right)Q_{ij}(X_1, X_2) - h_{ij}P_{ij}(X_1, X_2) + 1 \quad (6.2)$$

By Kornbluth [17] Pal et al. use the variable change method that represented in (6.1) and may be linearized as follows.

Let $D_{ij}^- = d_{ij}^- Q_{ij}(X_1, X_2)$ and $D_{ij}^+ = d_{ij}^+ Q_{ij}(X_1, X_2)$; the expression of quadratic form can be obtained as

$$G_{ij} + D_{ij}^- - D_{ij}^+ = 1 \quad (6.3)$$

With $D_{ij}^-, D_{ij}^+ \geq 0$ and $D_{ij}^- D_{ij}^+ = 0$ since $d_{ij}^-, d_{ij}^+ \geq 0$ and $Q_{ij}(x_1, x_2) > 0$.

Here, clearly the equation (6.3) contains only quadratic forms without any fractional part. Further, to obtain the linear membership function we transform the quadratic membership function (6.3) with the help of Taylor series.

By maximizing the each objectives in upper and lower level membership functions $\mu f_{ij}(X)$ that is associated with upper and lower level $f_{ij}(X_1, X_2)(i = 1,2, \quad j = 1,2, \dots, m_i)$, we have determined $X^0 = (X_1^0, X_2^0)$ with the help of Taylor polynomial series the linear approximation of ij^{th} membership goal in (6.3) can be obtained as

$$1 - D_{ij}^- + D_{ij}^+ = G_{ij}(X) \cong G_{ij}X^0 + \left((x_1 - x_1^0)\frac{\partial}{\partial x_1} + (x_2 - x_2^0)\frac{\partial}{\partial x_2} + \cdots + (x_n - x_n^0)\frac{\partial}{\partial x_n}\right)G_{ij}(X^0).$$

Or equivalently as

$$1 - D_{ij}^- + D_{ij}^+ = G_{ij}(X) \cong G_{ij}(X^0) + \sum_{k=1}^{n}(x_k - x_k^0)\frac{\partial G_{ij}(X^0)}{\partial x_k}, \quad (6.4)$$

Where n is the number of decision variables, x_0^k is the k^{th} component of $X^0 = (X_1^0, X_2^0)$ and x_k is the k^{th} component of the new solution $X = (X_1, X_2)$.

The linear approximation of the ij^{th} membership goal may be rewritten as

$$G_{ij}(X^0) + \left[\nabla G_{ij}(X^0)\right]^T (X - X^0) + D_{ij}^- - D_{ij}^+ = 1 \quad (6.5)$$

Where $\nabla G_{ij}(X^0)$ is the gradient of $G_{ij}(X^0)$ and the superscript T denotes transpose of $\nabla G_{ij}(X^0)$.

In the following the linearization manner, the proposed FGP model of the BLMO-QFP problem can be represented as

$$MinF = \sum_{j=1}^{m_1} w_{1j}^- d_{1j}^- + \sum_{j=1}^{m_2} w_{2j}^- d_{2j}^-$$

Subject to

$$G_{1j}(X^0) + \left[\nabla G_{1j}(X^0)\right]^T (X - X^0) + D_{1j}^- - D_{1j}^+ = 1 \quad j = 1,2, \dots, m_1$$

$$G_{2j}(X^0) + \left[\nabla G_{2j}(X^0)\right]^T (X - X^0) + D_{2j}^- - D_{2j}^+ = 1 \quad j = 1,2, \dots, m_2$$

$$X_1^L \leq X_1 \leq X_1^U$$

$$X_2^L \leq X_2 \leq X_2^U$$

$$(X_1, X_2) \in S \quad (6.6)$$

$D_{ij}^-, D_{ij}^+ \geq 0$ with $D_{ij}^- \times D_{ij}^+ = 0 \quad i = 1,2 \quad j = 1,2, \dots, m_i$

In which F represents the fuzzy achievement function including the weighted under deviational variables. The FGP model (18) provides the maximum satisfactory decision for both the FLDM and the SLDM by way of accomplishing the aspired levels of the membership goals to the extent possible in the decision environment.

7 The FGP Algorithm for Chance Constrained BLMO-QFP

The stepwise solution algorithm for solving BLMO-QFP has been summarized as follows:

Step1. For each level, transform the chance constraints into equivalent deterministic constraints.

Step2. Calculate the individual best solution for each objective function for each level subject to the respective constraints.

Step3. With the help of individual best solution, we find the value of each objective function in both levels after that we set the goals and upper tolerance limits.

Step4. Construct the quadratic fractional membership functions for each of the objective function in both levels.

Step5. Obtain the quadratic programming problem from quadratic fractional membership functions after applying linearization techniques by using (6.3) and (6.5)

Step6. Decide the desire bounds on the decision variables provide by the DMs in (4.2)

Step7. Formulate the Model (6.6) for the BLMO-QFP problem.

Step8. Solve Model (6.6) to get an existing solution to the BLMO-QFP problem.

Step9. If the DM is satisfied with the existing solution in Step 8, then go to Step 10, else go to Step11.

Step10. Stop with a satisfactory solution to the BLMO-QFP problem.

Step11. Tune\adjust the upper tolerance limits of all the decision variables in order to get compromise solutions i.e. go to Step 6.

8 Numerical Examples

To illustrate the proposed FGP method, the following chance constraint bi-level multi-objective quadratic fractional programming problem is taken into consideration.

[FLDM]

$$\min_{X_1}\left(f_{11} = \frac{x_2^2 - x_1^2}{x_1^2 + x_2^2 + 2}, \qquad f_{12} = \frac{(x_1-2)^2 - x_2}{(x_2-2)^2 + 5} \right)$$

Where X_2 solves

[SLDM]

$$\min_{X_2}\left(f_{21} = \frac{(x_1-1)^2 + (x_2+3)^2}{x_1^2 + x_2 + 10}, f_{22} = \frac{8x_1^2 - 9x_2^2 - 4}{x_1^2 + x_2^2 + 8} \right)$$

Subject to

$$\Pr(x_1 + x_2 \le d_1) \ge 1 - m_1 \qquad (8.1)$$

$$\Pr(-2x_1 + 5x_2 \le d_2) \ge 1 - m_2 \qquad (8.2)$$

$$\Pr(3x_1 - 4x_2 \le d_3) \ge 1 - m_3 \qquad (8.3)$$

$$x_1 \ge 0, x_2 \ge 0.$$

The mean, variance and the confidence levels are given as follows:

$$E(d_1) = 3, var(d_1) = 2, m_1 = 0.03$$

$$E(d_2) = 12, var(d_2) = 8, m_2 = 0.01$$

$$E(d_3) = 10, var(d_3) = 18, m_3 = 0.05$$

Using (2.2) and (2.4), the chance constraints defined in (8.1),(8.2) and (8.3) can be converted into equivalent deterministic constraints as:

$$x_1 + x_2 \le 5.666$$

$$-2x_1 + 5x_2 \le 18.576$$

$$3x_1 - 4x_2 \le 3.021$$

The individual optimal solution of the leader and followers are $(x_1^{11}, x_2^{11}) = (0.100, 3.775)$ with $f_{11}^{min} = -0.874$, $(x_1^{12}, x_2^{12}) = (1.393, 4.272)$ with $f_{12}^{min} = -0.757$, $(x_1^{21}, x_2^{21}) = (1.007, 0)$ with $f_{21}^{min} = 0.817$, $(x_1^{22}, x_2^{22}) = (0.612, 3.96)$ with $f_{22}^{min} = -11.523$,respectively.

Now using the individual optimal solution, we formulate as follows:

X^{ij}	f_{11}	f_{12}	f_{21}	f_{22}
(0.10, 3.77)	(-0.87)	(-0.38)	(3.38)	(-5.93)
(1.39, 4.27)	(-0.74)	(0.25)	(3.27)	(-5.42)
(1.01, 0)	(0.34)	(0.16)	(0.82)	(0.46)
(0.61, 3.96)	(-0.85)	(0.15)	(3.39)	(5.91)

The following table summarizes the aspiration levels and upper tolerance limits, of all objective functions for the two levels of the BLMO-QFP problem.

	f_{11}	f_{12}	f_{21}	f_{22}
u_{ij}	0.34	0.67	3.39	5.91
l_{ij}	-0.87	-0.38	0.82	-5.93

Now, by using the above tolerance ranges the quadratic fractional membership functions of FLDM are:

$$\mu_{f_{11}}(f_{11}(x_1, x_2)) = \frac{0.34 - \frac{x_1^2 - x_2^2}{x_1^2 + x_2^2 + 2}}{0.34 + 0.87} + d_{11}^- - d_{11}^+ = 1$$

$$\mu_{f_{12}}(f_{12}(x_1, x_2)) = \frac{0.67 - \frac{(x_1-2)^2 - x_2}{(x_2-1)^2 + 5}}{0.67 + 0.38} + d_{12}^- - d_{12}^+ = 1$$

$d_{1j}^-, d_{1j}^+ > 0$ With $d_{1j}^- \times d_{1j}^+ = 0 \quad j = 1,2$

The quadratic fractional membership functions of SLDM are:

$$\mu_{f_{21}}(f_{21}(x_1, x_2)) = \frac{3.39 - \frac{(x_1-1)^2 + (x_2+3)^2}{x_1^2 + x_2 + 10}}{3.39 - 0.82} + d_{21}^- - d_{21}^+ = 1$$

$$\mu_{f_{22}}\big(f_{22}(x_1,x_2)\big) = \frac{5.91 - \frac{8x_1^2-9x_2^2-4}{x_1^2+x_2+8}}{5.91+5.93} + d_{22}^- - d_{22}^+ = 1$$

$d_{2j}^-, d_{2j}^+ > 0$ with $d_{2j}^- \times d_2^+ = 0 \quad j = 1,2.$

After transforming the quadratic fractional membership functions of FLDM into quadratic forms, in (15) the model takes the form as

$$-1.55x_1^2 + 0.11x_2^2 - 0.44 + D_{11}^- - D_{11}^+ = 1,$$

$$0.64(x_1-1)^2 - 0.95(x_2-2)^2 + 0.64x_2 - 0.81 + D_{12}^- - D_{12}^+ = 1,$$

Where

$D_{11}^- = d_{11}^-(x_1^2 + x_2^2 + 2), D_{11}^+ = d_{11}^+(x_1^2 + x_2^2 + 2)$ and $D_{12}^- = d_{12}^-((x_2-1)^2 + 5)$, $D_{12}^+ = d_{12}^+((x_2-1)^2+5), D_{1j}^-, D_{1j}^+ \geq 0$ with $D_{1j}^- \times D_{1j}^+ = 0 \quad j = 1,2.$

The transformed quadratic fractional membership functions of SLDM are:

$$-0.39(x_1-1)^2 + 0.32x_1^2 - 0.39(x_2+3)^2 + 0.32x_2^2 + 4.19 + D_{21}^- - D_{21}^+ = 1,$$

$$-0.13x_1^2 + 1.23x_2^2 - 2.60 + D_{22}^- - D_{22}^+ = 1,$$

Where

$D_{21}^- = d_{21}^-(x_1^2 + x_2^2 + 10), D_{21}^+ = d_{21}^+(x_1^2 + x_1^2 + 10), D_{22}^- = d_{22}^-(x_1^2 + x_2^2 + 8), D_{22}^+ = d_{22}^+(x_1^2 + x_2^2 + 8), D_{2j}^-, D_{2j}^+ \geq 0$ with $D_{2j}^- \times D_{2j}^+ = 0 \quad j = 1,2$

The membership functions for FLDM and SLDM are maximal at the points $\mu_{11}^*(0.10,3.77)$, $\mu_{12}^*(1.39,4.27), \mu_{21}^*(1.01,0), \mu_{22}^*(0.61,3.96)$ Respectively. Now the membership functions are converted using first-order Taylor series in (17) of FLDM and SLDM as

$$1 - D_{11}^- + D_{11}^+ \cong G_{11}(0.10,3.77) + \left((x_1-0.10)\frac{\partial}{\partial x_1} + (x_2-3.77)\frac{\partial}{\partial y_2}\right)G_{11}(0.10,3.77)$$

$$= -0.31x_1 + 0.83y_2 - 1.99$$

$$1 - D_{12}^- + D_{12}^+ \cong G_{12}(1.39,4.27) + \left((x_1-1.39)\frac{\partial}{\partial x_1} + (y_2-4.27)\frac{\partial}{\partial x_2}\right)G_{12}(1.39,4.27)$$

$$= 0.49x_1 - 4.95x_2 + 17.58$$

$$1 - D_{21}^- + D_{21}^+ \cong G_{21}(1.01,0) + \left((x_1-1.01)\frac{\partial}{\partial x_1} + (x_2-0)\frac{\partial}{\partial x_2}\right)G_{21}(1.01,0)$$

$$= 0.64x_1 - 2.34x_2 - 3.82$$

$$1 - D_{22}^- + D_{22}^+ \cong G_{22}(0.61,3.96) + \left((x_1-0.61)\frac{\partial}{\partial x_1} + (x_2-3.96)\frac{\partial}{\partial x_2}\right)G_{22}(0.61,3.96)$$

$$= -0.16x_1 + 9.74x_2 - 21.83$$

The upper tolerance limits for x_1 and x_2 their tolerance ranges are obtained as

$$0.10 \leq x_1 \leq 1.39$$

$$0 \leq x_2 \leq 4.27$$

Then the proposed FGP model for solving BLMO-QFP is formulated as follows:

$$MinZ = 0.83D_{11}^- + 0.95D_{12}^- + 0.39D_{21}^- + 0.08D_{22}^-$$

Subject to

$$-0.31x_1 + 0.83x_2 + D_{11}^- - D_{11}^+ = 2.99$$

$$0.49x_1 - 4.95x_2 + D_{12}^- - D_{12}^+ = -16.58$$

$$0.64x_1 - 2.34x_2 + D_{21}^- - D_{21}^+ = 4.82$$

$$-0.16x_1 + 9.74x_2 + D_{22}^- - D_{22}^+ = 22.83$$

$$x_1 + x_2 \leq 5.67$$

$$-2x_1 + 5x_2 \leq 18.58$$

$$3x_1 - 4x_2 \leq 3.02$$

$$0.10 \leq x_1 \leq 1.39$$

$$0 \leq x_2 \leq 4.27$$

$$x_1, x_2 \geq 0$$

$D_{ij}^-, D_{ij}^+ \geq 0$ with $D_{ij}^- \times D_{ij}^+ = 0 \qquad i = 1,2 \qquad j = 1,2.$

The software LINGO is used to solve the problem. Optimal compromise solution of the problem is given by $x_1^* = 0.1$ and $x_2^* = 0$ with objective functions values $f_{11} = 0.01, f_{12} = 0.40, f_{21} = 0.98, f_{22} = -0.49$ with membership function values $\mu_{11} = 0.41, \mu_{12} = 0.07, \mu_{21} = 1.00$ and $\mu_{22} = 0.54$ respectively.

9 Conclusions

A fuzzy goal programming method for solving chance constrained BLMO-QFP based on first order Taylor series approximation is presented in this paper. Within the proposed method, chance constraints are converted into equal deterministic constraints and the quadratic fractional membership functions corresponding to the objective functions are transformed into the linear membership functions by first order Taylor series. Thereafter, a numerical problem to validate the proposed method is given.

References

[1] Anandalingam, G. (1988). A mathematical programming model of decentralized multi-level systems. Journal of the Operational Research Society., 1021-1033(1988).

[2] Arora, S. R., and Gupta, R. (2009).Interactive fuzzy goal programming approach for bilevel programming problem. European Journal of Operational Research., **194(2)**, 368-376(2009).

[3] Abo-Sinna, M. A., and Baky, I. A. (2010).Fuzzy goal programming procedure to bilevel multiobjective linear fractional programming problems. International Journal of Mathematics and Mathematical Sciences, 2010

[4] Adhami, A. Y., Muneeb, S. M., and Nomani, M. A. (2017). A multi-level decision making model for the supplier selection problem in a fuzzy situation. Operations Research and Decisions., **27(4)**, 5-26(2017).

[5] Bard, J. F. (1984). Optimality conditions for the bilevel programming problem. Naval Research Logistics Quaeterly., **31** , 13-26(1984).

[6] Bialas, W. F., and Karwan, M. H. (1984).Two-level linear programming. Management science.,**30(8)**,1004-1020(1984).

[7] Baky, I. A. (2010). Solving multi-level multi-objective linear programming problems through fuzzy goal programming approach. Applied Mathematical Modelling, **34(9)**, 2377-2387(2010).

[8] Bhati, D., and Singh, P. (2017). Branch and bound computational method for multi-objective linear fractional optimization problem. Neural Computing and Applications., **28(11)**, 3341-3351(2017)

[9] Charnes, A., and Cooper, W. W. (1959).Chance-constrained programming. Management science., **6(1)**, 73-79(1959).

[10] Candler, W. and Townsley, R., (1982) “A linear bilevel programming problem”, Computers and Operations Research., **9(1)**, 59-76(1982)

[11] Calvete, H. I., and Galé, C. (2004). Optimality conditions for the linear fractional/quadratic bilevel problem. MonografiasdelSeminarioMatematico Garcia de Galdeano, **31**, 285-294(2004).

[12] Dantzig, G.B.(1955). Linear programming under uncertainty, Management Science., **1**, 197-206(1955).

[13] Dalman, H. (2016). An interactive fuzzy goal programming algorithm to solve decentralized bi-level multi-objective fractional programming problem. Cornell University Library, 1(1), 1-12. arXiv:1606.00927

[14] Fortuny-Amat, J., and McCarl, B. (1981).A representation and economic interpretation of a two-level programming problem. Journal of the operational Research Society., 783-792(1981).

[15] Güzel, N. (2013). A proposal to the solution of multiobjective linear fractional programming problem, Abstract and Applied Analysis, Hindawi, Vol.(13), 1-4. http://dx.doi.org/10.1155/2013/435030

[16] Hosseini, E., and Kamalabadi, I. N. (2013).Solving linear-quadratic bi-level programming and linear-fractional bi-level programming problems using genetic algorithm. Applied Mathematics and Computational Intelligence., **2(2)**, 169-182(2013).

[17] Kornbluth, J. S., and Steuer, R. E. (1981).Goal programming with linear fractional criteria. European Journal of Operational Research., **8(1)**, 58-65(1981).

[18] Lai, Y. J. (1996). Hierarchical optimization: a satisfactory solution. Fuzzy Sets and Systems, **77(3)**, 321-335(1996)..

[19] Lachhwani, K. (2013).FGP approach to multi objective quadratic fractional programming problem. International Journal of Industrial Mathematics., **6(1)**, 49-57(2013).

[20] Mohamed, R. H. (1997). The relationship between goal programming and fuzzy programming. Fuzzy sets and systems., **89(2)**, 215-222(1997).

[21] Mishra, S., and Dey, I.(2016). Solution of Bi-level Quadratic Fractional Programming Problem using Fuzzy Linear Membership Function, American International Journal of Research in Science, Technology, Engineering & Mathematics, 1-6 (2016).

[22] Mishra, S., Verma, A. B., and Dey, I.(2017). Fuzzy Goal Programming Procedure to Bi-level Multi-objective Programming Problems, International Journal of Research in Engineering, Technology and Science, 7(I), 1-9(2017).

[23] Muneeb, S. M., Adhami, A. Y., Asim, Z., and Jalil, S. A.(2018) Bi-level decision making models for advertising allocation problem under fuzzy environment. International Journal of System Assurance Engineering and Management., 1-13(2018).

[24] Osman, M. S., Emam, O. E., and El Sayed, M. A. (2016). On parametric multi-level multi-objective fractional programming problems with fuzziness in the constraints. Br. J. Math. Comput.Sci, **18(5)**, 1-19(2016).

[25] Osman, M. S., Emam, O. E., and El Sayed, M. A. (2017). Stochastic fuzzy multi-level multi-objective fractional programming problem: A FGP approach. OPSEARCH, **54(4)**, 816-840(2017).

[26] Pal, B. B., and Moitra, B. N. (2003).A fuzzy goal programming procedure for solving quadratic bilevel programming problems. International Journal of Intelligent Systems., **18(5)**, 529-540(2003).

[27] Pramanik, S., and Roy, T. K.(2007) Fuzzy goal programming approach to multi-level programming problems, European Journal of Operational Research., **176 (2)** ,1151-1166(2007).

[28] Pramanik, S., and Dey, P. P. (2011). Multi-objective quadratic programming problem based on fuzzy goal programming. International Journal of pure and applied science and technology, **6(1)**, 45-53(2011).

[29] Pramanik, S., and Dey, P. P. (2011). Multi-objective quadratic programming problem: a priority based fuzzy goal programming. International Journal of Computer Applications., **26(10)**, 30-35(2011).

[30] Pramanik, S., and Dey, P. P. (2011). Multi-objective quadratic bi-level programming problem based on fuzzy goal programming approach, International Journal of Software Engineering Applications., **2(1)**, 30-35(2011).

[31] Pramanik, S., and Banerjee, D. (2012). Chance constrained quadratic bi-level programming problem. International Journal of Modern Engineering Research., **2(4)**, 2417-2424(2012).

[32] Shih, H. S., Lai, Y. J., and Lee, E. S. (1996).Fuzzy approach for multi-level programming problems. Computers and Operations Research., **23(1)**, 73-91(1996).

[33] Sakawa, M., Nishizaki, I., and Uemura, Y. (1998). Interactive fuzzy programming for multi-level linear problems", .Computers and Mathematics with applications,., **36(2)**, 71-86(1998).

[34] Shih, H. S., and Lee, E. S. (2000). Compensatory fuzzy multiple level decision making. Fuzzy Sets and Systems., **114(1)**, 71-87(2000).

[35] Sinha, S. (2003). Fuzzy programming approach to multi-level programming problems. Fuzzy Sets and Systems., **136(2)**, 189-202(2003).

[36] Sinha, S. (2003). Fuzzy mathematical programming applied to multi-level programming problems. Computers and Operations Research., **30(9)**, 1259-1268(2003).

[37] Saraj,M and Sadeghi,,S. (2014).Quadratic bi-level programming problems: a fuzzy goal programming approach. International Journal of Applied Operational Research., **4(2)**, 83-88(2014).

[38] Singh, S., and Haldar, N. (2015).A New method to solve bi-Level quadratic linear fractional programming problems. International Game Theory Review., **17(02)**, 1540017.