Sohag Journal of Mathematics
An International Journal

http://dx.doi.org/10.18576/sjm/040205

Fractional Derivative Associated with Dirichlet Averages of Generalized Mainardi Function

*Jitendra Daiya** and *R. K. Saxena*

Department of Mathematics & Statistics, Jai Narain Vyas University, Jodhpur, India

Received: 3 May 2016, Revised: 2 Mar. 2017, Accepted: 5 Mar. 2017
Published online: 1 May 2017

Abstract: The object of this article is to investigate the Dirichlet averages of generalized Mainardi function defined by the authors. The results derived in this paper provide extension of the results given by Sharma et al [18]. Representations of such relations are obtained in terms of fractional derivative. Several others new results can also be obtained from our main theorems. The results obtained are useful in applied problems of science, engineering and technology.

Keywords: Generalized Mainardi function, Dirichlet averages, Fractional Derivative.

1 Introduction

The Dirichlet average of a function is certain kind of integral average with respect to Dirichlet measure. The concept of Dirichlet average was introduced by Carlson in [1], [2], [4]. It is studied, among others, by zu Castell [5] , Massopust and Forster [12], Neuman [14], Neuman and Van Fleet [15] and others. A detailed and comprehensive account of various types of Dirichlet averages has been given by Carlson in his monograph [3].

In this article we introduce generalized Mainardi hypergeometric function in the following form

$$ {}_pS_q\left[\begin{matrix}(b_1,B_1),\ldots,(b_q,B_q)\\(a_1,A_1),\ldots,(a_p,A_p);(\eta-\alpha,-\alpha);-Z\end{matrix}\right] $$

$$ =\sum_{n=0}^{\infty}\frac{[\Gamma(b_1+nB_1),\ldots,\Gamma(b_q+nB_q)]}{[\Gamma(a_1+nA_1),\ldots,\Gamma(a_p+nA_p)]} $$

$$ \frac{(-1)^n z^n}{\Gamma[-\alpha(n+1)+\eta]\,n!} \tag{1} $$

where $a_i, b_j \in C$ and $A_i, B_j \in R$ $(i=1,\ldots,p;\ j=1,\ldots,q)$ and the defining series (1) converges for

$$ \sum_{j=1}^{q}B_j-\sum_{i=1}^{p}A_i-\alpha>-1. $$

If we set $p=q=0$ *and* $\eta=1$,equation (1) it yields Mainardi function defined by Mainardi [11] as

$$ M(z,\alpha)=\frac{(-1)^n z^n}{\Gamma[-\alpha(n+1)+1]\,n!} \tag{2} $$

We will use some more notations in the further exposition. In the sequel, the symbol E_{n-1} will be denote by the Euclidean simplex, defined by

$$ E_{n-1}=\left\{\begin{matrix}(u_1,\ldots,u_{n-1}):u_j\geq 0,\\ j=1,\ldots,n,u_1+\ldots+u_{n-1}\leq 1\end{matrix}\right\}. \tag{3} $$

Next we need the concept of Dirichlet average. Following Carlson [3] [Definition 5.2-1]

let Ω be a convex set in C and $(z)=(z_1,\ldots,z_n)\in\Omega^n, n\geq 2$ and let f be a measurable function on Ω. Define

$$ F(b;z)=\int_{E_{n-1}}f(u\text{ o }z)\,d\mu_b(u), \tag{4} $$

in $d\mu_b$is the Dirichlet measure

$$ d\mu_b(u)=\frac{1}{B(b)}u_1^{b_1-1},\ldots,u_{n-1}^{b_{n-1}-1} $$

$$ (1-u_1-\ldots-u_{n-1})^{b_n-1}du_1,\ldots,du_{n-1} \tag{5} $$

* Corresponding author e-mail: jitendra.daiya25@gmail.com

with the multivariate Beta-function

$$B(b)=\frac{\Gamma(b_1),\ldots,\Gamma(b_k)}{\Gamma(b_1+\ldots+b_k)}$$

$$\Re(b_j)>0,\ (j=1,\ldots,k) \tag{6}$$

and

$$(\mathrm{u\ o\ z})\ =\sum_{j=1}^{\mathrm{n}-1}\mathrm{u}_j z_j+(1-u_1-\ \ldots\ -u_{n-1})\,z_n\,. \tag{7}$$

For n = 1, $F\,(b;z)\ =\ f\,(z)$. For n = 2 we have

$$\mathrm{d}\mu_{\beta\beta'}(\mathrm{u})=\frac{\Gamma\,(\beta+\beta')}{\Gamma\,(\beta)\,\Gamma\,(\beta')}\,u^{\beta-1}(1-u)^{\beta'-1}du. \tag{8}$$

The Dirichlet averages of the generalized Mainardi function (1) is given by

$$M_{p,q}\left[\begin{matrix}(b_1,B_1),\ldots,(b_q,B_q)\\(a_1,A_1),\ldots,(a_p,A_p);(\eta-\alpha,-\alpha);\ (\beta,\beta';x,y)\end{matrix}\right]$$
$$=\int\limits_{E_1}{}_pS_q\left[\begin{matrix}(b_1,B_1),\ldots,(b_q,B_q)\\(a_1,A_1),\ldots,(a_p,A_p);(\eta-\alpha,-\alpha);-(uoz)\end{matrix}\right]\mathrm{d}\mu_{\beta\beta'}(\mathrm{u}) \tag{9}$$

where for n = 1, $F\,(b;z)\ =\ f\,(z)$.

For n = 2 we have

$$\mathrm{d}\mu_{\beta\beta'}\,(\mathrm{u})=\frac{\Gamma\,(\beta+\beta')}{\Gamma\,(\beta)\,\Gamma\,(\beta')}\,u^{\beta-1}\,(1-u)^{\,\beta'-1}du. \tag{10}$$

The general Dirichlet average of a function has been defined by Carlson [2] as

$$F\,(b,z)=\int_{E_{n-1}}f(\mathrm{u\ o\ z})\ \mathrm{d}\mu_b\,(u) \tag{11}$$

Carlson [2] investigated the average (11) for $f(z)\ =\ z^k$, $k\in R$ in the form

$$R_k\,(b,z)=\int_{E_{n-1}}(\mathrm{u\ o\ z})^{\mathrm{k}}\ \mathrm{d}\mu_b\,(u) \tag{12}$$

If n = 2 . Carlson [2], [3] proved that

$$R_k\,(\beta,\beta';x,y)=\frac{\Gamma\left(\beta+\beta'\right)}{\Gamma(\beta)\Gamma\left(\beta'\right)}$$
$$\int_0^1\,[ux+(1-u)y]^k\,u^{\beta-1}\,(1-u)^{\,\beta'-1}du \tag{13}$$

where $\beta,\beta'\in C$, $R(\beta)>0$, $R\left(\beta'\right)>0$ and $x,y\in R$

In this connection, one can refer to the works of Erdélyi et al. [6],[8] Samko et al. [16] and Saxena et al. [17], Mathai et al [13] and Haubold et al [9].

2 Fractional Derivative

Fractional derivative with respect to an arbitrary function has bee used by Erdelyi [7] [p 181]. The general definition for the fractional derivative of order $\alpha\in C_\infty$, $Re\,(\alpha)>0$, the Riemann-Liouville integral is defined as

$$\left(D_x^\alpha F\right)(x)=\frac{1}{\Gamma\,(-\alpha)}\int_0^x\frac{F\,(t)}{(x-t)^{\,1-\alpha}}\,dt \tag{14}$$

and (see Lavoie et al.[10] eqn. 5.1, p. 245)

$$\left(D_{x-x_0}^\alpha F\right)(x)\ =\ \frac{1}{\Gamma(-\alpha)}\int_{x_0}^x\frac{F\,(t)}{(x-t)^{\,1-\alpha}}\,dt \tag{15}$$

where $Re(\alpha)\ <\ 0$, $F(t)$is of the form $x^p f(x)$ and $f(x)$is analytic at $x=0$.

See also Lavoie et al [10] and Samko et al [16].

Theorem 1 Let $x,y\in R$ be real number such that $x>y$, $\beta,\beta'\in C$, $R(\beta)>0$, $R\left(\beta'\right)>0\,a_i$, $b_j\ \in\ C$ and A_i, $B_j\ \in\ R\ (i=1,\ldots,p;\ j=1,\ldots,q)$ then the Dirichlet average of generalized Mainardi function (1) is given by

$$M_{p,q}\left[\begin{matrix}(b_1,B_1),\ldots,(b_q,B_q)\\(a_1,A_1),\ldots,(a_p,A_p);(\eta-\alpha,-\alpha);\ (\beta,\beta';x,y)\end{matrix}\right]$$

$$=\frac{\Gamma\,(\beta+\beta')}{\Gamma\,(\beta)}\,(x-y)^{1-\beta-\beta'}$$

$$\left\{D_{x-y}^{-\beta'}\,{}_pS_q\left[\begin{matrix}(b_1,B_1),\ldots,(b_q,B_q)\\(a_1,A_1),\ldots,(a_p,A_p);(\eta-\alpha,-\alpha);-x\end{matrix}\right]\right\}(x-y)^{\beta-1} \tag{16}$$

Proof:- By virtue equation (1) and (9), we have

$$M_{p,q}\left[\begin{matrix}(b_1,B_1),\ldots,(b_q,B_q)\\(a_1,A_1),\ldots,(a_p,A_p);(\eta-\alpha,-\alpha);\ (\beta,\beta';x,y)\end{matrix}\right]$$
$$=\int\limits_{E_1}{}_pS_q\left[\begin{matrix}(b_1,B_1),\ldots,(b_q,B_q)\\(a_1,A_1),\ldots,(a_p,A_p);(\eta-\alpha,-\alpha);-(uoz)\end{matrix}\right]\mathrm{d}\mu_{\beta\beta'}(\mathrm{u})$$

$$= \frac{\Gamma\left(\beta+\beta'\right)}{\Gamma(\beta)\Gamma(\beta')} \int_0^1 \sum_{n=0}^{\infty} \frac{\left[\Gamma(b_1+nB_1),\ldots,\Gamma\left(b_q+nB_q\right)\right]}{\left[\Gamma(a_1+nA_1),\ldots,\Gamma\left(a_p+nA_p\right)\right]}$$

$$\frac{(-1)^n[ux+y(1-u)]^n}{\Gamma(-\alpha(n+1)+\eta)\,n!} u^{\beta-1}(1-u)^{\beta'-1}du$$

$$= \frac{\Gamma\left(\beta+\beta'\right)}{\Gamma(\beta)\Gamma(\beta')} \int_0^1 \sum_{n=0}^{\infty} \frac{\left[\Gamma(b_1+nB_1),\ldots,\Gamma\left(b_q+nB_q\right)\right]}{\left[\Gamma(a_1+nA_1),\ldots,\Gamma\left(a_q+nA_q\right)\right]}$$

$$\frac{(-1)^n[y+u(x-y)]^n}{\Gamma(-\alpha(n+1)+\eta)\,n!} u^{\beta-1}(1-u)^{\beta'-1}du$$

$$= \frac{\Gamma\left(\beta+\beta'\right)}{\Gamma(\beta)\Gamma(\beta')} \sum_{n=0}^{\infty} \frac{\left[\Gamma(b_1+nB_1),\ldots,\Gamma\left(b_q+nB_q\right)\right]}{\left[\Gamma(a_1+nA_1),\ldots,\Gamma\left(a_q+nA_q\right)\right]}$$

$$\frac{(-1)^n}{\Gamma(-\alpha(n+1)+\eta)\,n!} \int_0^1 [y+u(x-y)]^n u^{\beta-1}(1-u)^{\beta'-1}du$$

Put $u(x-y)=t$

$$= \frac{\Gamma(\beta+\beta')}{\Gamma(\beta)\Gamma(\beta')} \sum_{n=0}^{\infty} \frac{[\Gamma(b_1+nB_1),\ldots,\Gamma(b_q+nB_q)]}{[\Gamma(a_1+nA_1),\ldots,\Gamma(a_q+nA_q)]} \frac{(-1)^n}{\Gamma(-\alpha(n+1)+\eta)\,n!}$$

$$\int_0^{x-y} [y+t]^n \left(\frac{t}{x-y}\right)^{\beta-1}\left(1-\frac{t}{x-y}\right)^{\beta'-1}\frac{du}{x-y}$$

$$= \frac{\Gamma(\beta+\beta')}{\Gamma(\beta)\Gamma(\beta')} \sum_{n=0}^{\infty} \frac{[\Gamma(b_1+nB_1),\ldots,\Gamma(b_q+nB_q)]}{[\Gamma(a_1+nA_1),\ldots,\Gamma(a_q+nA_q)]} \frac{(-1)^n[y+t]^n}{\Gamma(-\alpha(n+1)+\eta)\,n!}$$

$$\int_0^{x-y} t^{\beta-1}(x-y)^{1-\beta-\beta'}(x-y-t)^{\beta'-1}dt$$

$$= \frac{\Gamma(\beta+\beta')}{\Gamma(\beta)\Gamma(\beta')}(x-y)^{1-\beta-\beta'} \int_0^{x-y} t^{\beta-1} \sum_{n=0}^{\infty} \frac{\left[\Gamma\left(b_1+nB_1\right),\ldots,\Gamma\left(b_q+nB_q\right)\right]}{\left[\Gamma\left(a_1+nA_1\right),\ldots,\Gamma\left(a_q+nA_q\right)\right]}$$

$$\frac{(-1)^n[y+t]^n}{\Gamma(-\alpha(n+1)+\eta)\,n!}(x-y-t)^{\beta'-1}dt$$

which yields (16) and thus the theorem 1 is proved.

3 Special Cases

For $\beta' = \gamma-\beta$ *and* $y=0$, Theorem 1 yields.

Corollary 1.1

$$M_{p,q}\left[\begin{matrix}(b_1,B_1),\ldots,(b_q,B_q)\\(a_1,A_1),\ldots,(a_p,A_p);\ (\eta-\alpha,-\alpha);\ (\beta,\gamma-\beta;x,0)\end{matrix}\right]$$

$$= \frac{\Gamma(\gamma)}{\Gamma(\beta)} x^{1-\gamma}\left\{D_x^{\beta-\gamma}\,{}_pS_q\left[\begin{matrix}(b_1,B_1),\ldots,(b_q,B_q)\\(a_1,A_1),\ldots,(a_p,A_p);(\eta-\alpha,-\alpha);-x\end{matrix}\right]\right\}(x-y)^{\beta-1}$$

Modification of

$$\gamma M_{p,q}\left[\begin{matrix}(b_1,B_1),\ldots,(b_q,B_q)\\(a_1,A_1),\ldots,(a_p,A_p);\ (\eta-\alpha,\ -\alpha);\ (\beta,\beta';x,y)\end{matrix}\right]$$

A modification of the (9) is taken in the form

$$\gamma M_{p,q}\left[\begin{matrix}(b_1,B_1),\ldots,(b_q,B_q)\\(a_1,A_1),\ldots,(a_p,A_p);\ (\eta-\alpha,\ -\alpha);\ (\beta,\beta';x,y)\end{matrix}\right]$$

$$= \int_{E_2} (uoz)^{\gamma-1}\,{}_pS_q\left[\begin{matrix}(b_1,B_1),\ldots,(b_q,B_q)\\(a_1,A_1),\ldots,(a_p,A_p);(\eta-\alpha,-\alpha);-(uoz)\end{matrix}\right] d\mu_{\beta\beta'}(u)$$

Theorem 2 Let $x,y \in R$ be real number such that $x>y$, $\beta,\beta' \in C$, $R(\beta)>0$, $R\left(\beta'\right)>0\, a_i,\, b_j \in C$, and $A_i,\, B_j \in R$ $(i=1,\ldots,p;\ j=1,\ldots,q)$ then the modification of Dirichlet average generalized Mainardi function (1) is given by

$$M_{p,q}\left[\begin{matrix}(b_1,B_1),\ldots,(b_q,B_q)\\(a_1,A_1),\ldots,(a_p,A_p);(\eta-\alpha,-\alpha);\ (\beta,\beta';x,y)\end{matrix}\right]$$

$$= \frac{\Gamma(\beta+\beta')}{\Gamma(\beta)}(x-y)^{1-\beta-\beta'}$$

$$\left\{D_{x-y}^{-\beta'}(y+t)^{\gamma-1}\,{}_pS_q\left[\begin{matrix}(b_1,B_1),\ldots,(b_q,B_q)\\(a_1,A_1),\ldots,(a_p,A_p);(\eta-\alpha,-\alpha);-x\end{matrix}\right]\right\}(x-y)^{\beta-1} \tag{17}$$

Proof :- Using equation (1) and (10), we have

$$\gamma M_{p,q}\left[\begin{matrix}(b_1,B_1),\ldots,(b_q,B_q)\\(a_1,A_1),\ldots,(a_p,A_p);(\eta-\alpha,-\alpha);\ (\beta,\beta';x,y)\end{matrix}\right]$$

$$= \frac{\Gamma\left(\beta+\beta'\right)}{\Gamma(\beta)\Gamma(\beta')} \int_0^1 [ux+y(1-u)]^{\gamma-1}$$

$$\sum_{n=0}^{\infty} \frac{\left[\Gamma(b_1+nB_1),\ldots,\Gamma\left(b_q+nB_q\right)\right]}{\left[\Gamma(a_1+nA_1),\ldots,\Gamma\left(a_p+nA_p\right)\right]}$$

$$\frac{(-1)^n[ux+y(1-u)]^n}{\Gamma(-\alpha(n+1)+\eta)\,n!} u^{\beta-1}(1-u)^{\beta'-1}du$$

$$= \frac{\Gamma\left(\beta+\beta'\right)}{\Gamma(\beta)\Gamma(\beta')} \sum_{n=0}^{\infty} \frac{\left[\Gamma(b_1+nB_1),\ldots,\Gamma\left(b_q+nB_q\right)\right]}{\left[\Gamma(a_1+nA_1),\ldots,\Gamma\left(a_p+nA_p\right)\right]}$$

$$\frac{-1^n}{\Gamma(-\alpha(n+1)+\eta)\,n!} \int_0^1 [y+(x-y)u]^{\gamma+n-1}$$

$$u^{\beta-1}(1-u)^{\beta'-1}du$$

Put $(x-y)\,u,=t$, we get

$$=\frac{\Gamma\left(\beta+\beta'\right)}{\Gamma(\beta)\Gamma(\beta')}\sum_{n=0}^{\infty}\frac{\left[\Gamma(b_1+nB_1),\ldots,\Gamma\left(b_q+nB_q\right)\right]}{\left[\Gamma(a_1+nA_1),\ldots,\Gamma\left(a_p+nA_p\right)\right]}$$

$$\frac{(-1)^n}{\Gamma(-\alpha(n+1)+\eta)\,n!}\int_y^x (y+t)^{\gamma+n-1}\left(\frac{t}{x-y}\right)^{\beta-1}$$

$$\left(1-\frac{t}{x-y}\right)^{\beta'-1}\frac{dt}{(x-y)}$$

$$=\frac{\Gamma\left(\beta+\beta'\right)}{\Gamma(\beta)\Gamma(\beta')}(x-y)^{1-\beta-\beta'}$$

$$\sum_{n=0}^{\infty}\frac{\left[\Gamma(b_1+nB_1),\ldots,\Gamma\left(b_q+nB_q\right)\right]}{\left[\Gamma(a_1+nA_1),\ldots,\Gamma\left(a_p+nA_p\right)\right]}\frac{(-1)^n(y+t)^n}{\Gamma(-\alpha(n+1)+\eta)\,n!}$$

$$\int_y^x (y+t)^{\gamma-1}(t)^{\beta-1}(x-y-t)^{\beta'-1}dt$$

$$=\frac{\Gamma\left(\beta+\beta'\right)}{\Gamma(\beta)\Gamma(\beta')}(x-y)^{1-\beta-\beta'}\int_y^x (y+t)^{\gamma-1}(t)^{\beta-1}$$

$${}_pS_q\left[\begin{matrix}(b_1,B_1),\ldots,(b_q,B_q)\\(a_1,A_1),\ldots,(a_p,A_p);(\eta-\alpha,-\alpha);-x\end{matrix}\right](x-y-t)^{\beta'-1}dt \tag{18}$$

This gives equation (1) and thus the proof of theorem 2 is completed.

For $\beta'=\gamma-\beta$ *and* $y=0$, Theorem 2 yields.

Corollary 2.1

$${}_\gamma M_{p,q}\left[\begin{matrix}(b_1,B_1),\ldots,(b_q,B_q)\\(a_1,A_1),\ldots,(a_p,A_p);\ (\eta-\alpha,-\alpha)\ ;\ (\beta,\gamma-\beta;x,0)\end{matrix}\right]$$

$$=\frac{\Gamma(\gamma)}{\Gamma(\beta)}x^{1-\gamma}$$

$$\left\{D_x^{\beta-\gamma}(y+t)^{\gamma-1}{}_pS_q\left[\begin{matrix}(b_1,B_1),\ldots,(b_q,B_q)\\(a_1,A_1),\ldots,(a_p,A_p);(\eta-\alpha,-\alpha);-x\end{matrix}\right]\right\}(x-y)^{\beta-1} \tag{19}$$

4 Concluding

In this present we have investigated the Dirichlet averages of generalized Mainardi function, the given results are in compact from. The modification of generalized Mainardi function is also obtained and special cases of our main finding are also sated. The results are new and the readers can obtained the generalized fractional calculus for the generalized Mainardi function given by authors.

References

[1] B.C. Carlson, Lauricella's hypergeometric function F_D. J. Math. Anal. Appl. 7 (1963), 452-470.

[2] B.C. Carlson, A connection between elementary and higher transcendental functions. SIAM J. Appl. Math. 17, No 1 (1969), 116-148.

[3] B.C. Carlson, Special Functions of Applied Mathematics. AcademicPress, New York (1977).

[4] B.C. Carlson, B-splines, hypergeoemtric functions and Dirichlet average, J. Approx, Theory 67, (1991), 311-325.

[5] W. zu Castell, Dirichlet splines as fractional integrals of B-splines, Rocky Mountain J. Math. 32(2002), No. 2, 545-559.

[6] A. Erdélyi, W. Magnus, F. Oberhettinger, F.G. Tricomi, Higher Tran-scendental Functions, Vol. I. McGraw-Hill, New York - Toronto - London (1953); Reprinted: Krieger, Melbourne, Florida (1981).

[7] A. Erdélyi, W. Magnus, F. Oberhettinger, F.G. Tricomi, Tables of Integral Transforms, Vol. 2, McGraw-Hill, New York (1954).

[8] A. Erdélyi, W. Magnus, F. Oberhettinger, F.G. Tricomi, Higher Transcendental Functions, Vol. III. McGraw-Hill, New York - Toronto London (1954); Reprinted: Krieger, Melbourne, Florida (1981).

[9] H. J. Haubold , A. M. Mathai and R. K. Saxena, Mittag-Leffler function and Their Applications, Journal of Applie Mathematics, Article ID 292628, (2011), 1-5.

[10] J. L. Lavoie, T. J. Osler, R. Tremblay, Fractional derivatives and special functions, SLAM Review, Vol. 18, (1976), 240-268.

[11] F. Mainardi, Fractional calculus: some Basic problems in continuum and statistical mechanics, CISM Lecture Notes, international centre for mechanical sciences, palazzo del Troco piazza Garibaldi Udine, Italy (1996), 291-348.

[12] P. Massopust, B. Forster, Multivariate complex B-splines and Dirichlet averages, J.Approx. Theory Vol. 162, (2010), 252-269.

[13] A. M. Mathai, R. K. Saxena and H. J. Haubold, The H-function: Theory and Applications, Springer, (2010).

[14] E. Neuman, Stlarsky means of several variables, J. Inequal. Pure Appl. Math. 6, Art. 30, (2005), 1-10.

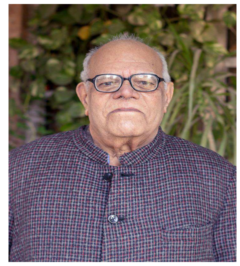
[15] E. Neuman, P. J. Van Fleet, Moments of Dirichlet splines and their applications to hypergeometric functions, J. Comput. Appl. Math. Vol. 53, (1994), 225-241.

[16] S.G. Samko, A.A. Kilbas, O.I. Marichev, Fractional integrals and derivatives. theory and applications. Gordon and Breach, New York at al. (1993).

[17] R.K. Saxena, T. K. Pogány, J. Ram and J. Daiya, Dirichlet Averages of Generalized Multi-index Mittag-Leffler Functions, Armenian Journal of Mathematics, Vol. 3 (2010), 174-187.

[18] M. Sharma, Mohd. Ferman Ali and Renu Jain, Dirichlet averages of Mainardi function and fractional derivative, international jour. of Innov. Res. in advanced engineering. Vol. 1, No. 2, 1-3.

Jitendra Daiya obtained M.Sc. (2007) and Ph.D. (2012) degrees from Jai Narain Vyas University, Jodhpur, India. He has published more than 30 research papers in National and International peer reviewed journals so far. Currently, he is an Post Doctoral Fellow (PDF) given by National Board for Higher Mathematics, Department of Atomic Energy, Government of INDIA. His research interest includes Fractional Calculus, Special Functions, Integral Transforms, Inequalities, q-Fractional Calculus, Mathematical Physics, Differential Equation, Basic Hypergeometric functions, and Operation Research. He is an active reviewer for Kuwait Journal of Science, Mathematical Reviews (American Mathematical Society) and reputed Indian and foreign journals. He is also the member of the editorial board for International and National journals.



R. K. Saxena obtained M.Sc. (1956), Ph.D. (1962) and D.Sc. (1971) from Jai Narain Vyas University, Jodhpur, India. He is Professor (Emeritus) of Mathematics at Jai Narain Vyas University, Jodhpur, India. Before retirement he has been Professor and Head, Department of Mathematics and Statistics and Dean, Faculty of Science at Jai Narain Vyas University, Jodphur, India. He has published more than 600 research papers in international esteemed journals and he has also published 5 monographs. He is Reviewer and for Referee for Mathematical Reviews, USA (American Mathematical Society), and Zentralblatt fuer Mathematik, Germany, since 1966. He is acting as a referee for a number of reputed Indian and foreign journals. He is also the member of the editorial board for number of International and National journals. He is also a member of many learned societies. Already twenty three candidates have received Ph.D. degree under his supervision and guidance and three post doctoral fellows have completed their researches under his guidance. He has been invited by number of international universities. His research interest includes Fractional Calculus, Special Functions, Integral Transforms, Statistical distributions, Generalized functions, Reaction-diffusion models, Differential Equation, Basic Hypergeometric functions, and Fractional differential equations.