Applied Mathematics & Information Sciences
An International Journal

Solving Engineering Optimization Problems by a Deterministic Global Optimization Approach

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Received 22 Jun. 2011; Revised 21 Nov. 2011; Accepted 11 Mar. 2012

Abstract: Engineering optimization problems are normally formulated as nonlinear programming problems and adopted in a lot of research to show the effectiveness of new optimization algorithms. These problems are usually solved through deterministic or heuristic methods. Because non-convex functions exist in most engineering optimization problems that possess multiple local optima, the heuristic methods cannot guarantee the global optimality of the obtained solution. Although many deterministic approaches have been developed for treating non-convex engineering optimization problems, these methods use too many extra binary variables and constraints in reformulating the problems. Therefore, this study applies an efficient deterministic approach for solving the engineering optimization problem to find a global optimum. The deterministic global approach transforms a non-convex program into a convex program by convexification strategies and piecewise linearization techniques and is thus guaranteed to reach a global optimum. Some practical engineering design problems are presented and solved to demonstrate that this study is able to obtain a better solution than other methods.

Keywords: Engineering Optimization, Convex, Linearization, Global Optimization

1 Introduction

Optimization techniques have been applied in various engineering problems such as civil and material engineering design, production planning, and chemical engineering etc. These applications are extensively surveyed by Floudas and Gounaris [1] and Floudas et al. [2] The engineering problems normally have mixed continuous and discrete design variables, nonlinear objective functions, and nonlinear constraints. In general, the approaches for solving engineering optimization problems can be classified into two categories: heuristic and deterministic. The heuristic methods include genetic algorithm [3-5], simulated annealing [6], particle swarm optimization algorithm [7,8], harmony search algorithm [9-11], and evolutionary algorithm [12,13]. For example, Lee and Geem [14] developed a harmony search algorithm-based approach for various engineering problems, including mathematical function minimization and structural engineering optimization problems. Su and Hsieh [7] used cooperative particle swarm optimization to search the optimal solution of engineering design. Although these heuristic methods have the advantages of easy implementation for complex problems, they cause convergence difficulties and cannot guarantee the global optimality of the solution. Moreover, the probability of finding the global solution decreases when the problem size increases.

The deterministic methods developed to solve engineering optimization problems include branch and bound method [15,16], extended cutting plane method [7], sequential linearization algorithm [18], and generalized disjunctive programming [19]. These methods were discussed in Rao [20]. The branch and bound method can find the global solution only when each subproblem can be solved to global optimality. The extended cutting plane method cannot solve the problems with non-convex constraints or non-convex objective functions because the subproblems may not have a unique optimum in the solution process. Therefore, some transformation techniques have been developed for convexifying the non-convex functions. Pörn et al.

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[21] introduced different convexification strategies for non-convex programs with posynomial and negative binomial terms in the constraints. Maranas and Floudas [22] proposed exponential transformation methods to treat non-convex terms. Björk et al. [23] and Tsai et al. [24] proposed global optimization techniques based on convexifying signomial terms.

Most of the engineering optimization problems include non-convex functions that cannot be dealt with by the standard local optimization techniques to guarantee global optimality. Consequently, this study applies an efficient optimization approach for globally solving engineering optimization problems. The deterministic global approach transforms a non-convex program into a convex program by convexification strategies and piecewise linearization techniques and is thus guaranteed to reach a global optimum. Compared with other deterministic methods, the proposed method utilizes less additional binary variables and constraints to reformulate the problem, thereby decreasing the computational complexity.

Following this introduction, the convexification and piecewise linearization methods adopted in this study are described in Section 2. Three practical engineering design problems are solved in Section 3. Finally, conclusion remarks are made in Section 4.

2 Convexification and Linearization Techniques

Convexification is one of the important techniques for global optimization. Tsai et al. [24] and Li and Tsai [25] discussed non-convex minimization problems that can be transformed into convex ones, and are thus be globally solved by the local optimization techniques. To convexify the non-convex functions in engineering problems by the conventional convex techniques, we consider the following propositions [24-26]:

PROPOSITION 1 A twice-differentiable function $f(\mathbf{x}) = c\prod_{i=1}^{n} x_i^{\alpha_i}$, $\mathbf{x} = (x_1, x_2, \cdots, x_n)$, $c, x_i, \alpha_i \in \Re$, $\forall i$, is convex if $c \ge 0$, $x_i \ge 0$ and $\alpha_i \le 0$ for $i = 1,2,\ldots,n$.

PROPOSITION 2 A twice-differentiable function $f(\mathbf{x}) = c\prod_{i=1}^{n} x_i^{\alpha_i}$, $\mathbf{x} = (x_1, x_2, \cdots, x_n)$, $c, x_i, \alpha_i \in \Re$, $\forall i$, is convex if $c \le 0$, $x_i \ge 0$, $\alpha_i \ge 0$ (for $i = 1,2,\ldots,n$), and $1 - \sum_{i=1}^{n} \alpha_i \ge 0$.

Remark 1 A function $f(x) = x^{\alpha}$ for $x > 0$ is convex when $\alpha \le 0$ or $\alpha \ge 1$. $f(x)$ is concave when $0 \le \alpha \le 1$. For a given product term s, if s satisfies Propositions 1 or 2, then s is a convex term without any transformations. For instance, $s = x_1^{-1} x_2^{-2} x_3^{-1}$ with $x_1, x_2, x_3 > 0$ is a convex term requiring no transformation by Proposition 1, and $s = -x_1^{0.3} x_2^{0.5}$ with $x_1, x_2 > 0$ is also a convex term by Proposition 2.

This study utilizes different strategies for positive and negative non-convex terms, respectively. For the non-convex terms with positive coefficient, we apply the exponential transformation to convexify a positive monomial by the following remark [27-29]:

Remark 2 If $\alpha_j > 0$ for some j, $j \notin I$, $I = \{k \mid \alpha_k < 0, k = 1,2,\ldots,n\}$, then we convert $f(\mathbf{x}) = c\prod_{i=1}^{n} x_i^{\alpha_i}$, $c > 0$, $x_i > 0$, into another function $h(\mathbf{x}, \mathbf{y}) = c\left(\prod_{i \in I} x_i^{\alpha_i}\right) e^{\sum_{j \notin I} \alpha_j y_j}$ where $y_j = L(\ln x_j)$ and $L(\ln x_j)$ is a piecewise linear function of $\ln x_j$. Then $h(\mathbf{x}, \mathbf{y})$ where $x_i > 0$, $i \in I$, $y_j \in \Re$, $j \notin I$ is a convex function.

For the non-convex terms with negative coefficient, we apply the power transformation to convexify a negative monomial by the following remark [27-31]:

Remark 3 If $0 \le \alpha_1 \le \alpha_2 \le \ldots \le \alpha_p$, $0 \ge \alpha_{p+1} \ge \alpha_{p+2} \ge \ldots \ge \alpha_n$, and $\sum_{i=1}^{r} \alpha_i < 1$ for some largest integer *r*, such that $r \le p$, $I = \{k \mid k = 1,2,\ldots,r\}$, then we convert $f(\mathbf{x}) = c\prod_{i=1}^{n} x_i^{\alpha_i}$, $c < 0$, $x_i > 0$, into another function $h(\mathbf{x}, \mathbf{y}) = c\prod_{i \in I} x_i^{\alpha_i} \prod_{j \notin I} y_j^{\beta}$, $\beta = (1 - \sum_{i=1}^{r} \alpha_i)/(n - r)$, where $y_j = L(x_j^{\frac{\alpha_j}{\beta}})$ and $L(x_j^{\frac{\alpha_j}{\beta}})$ is a piecewise linear function of $x_j^{\frac{\alpha_j}{\beta}}$. Then $h(\mathbf{x}, \mathbf{y})$ where $x_i > 0$, $i \in I$, $y_j \in \Re$, $j \notin I$ is a convex function.

Herein we adopt the piecewise linearization techniques presented by Vielma and Nemhauser [32] to deal with a continuous univariate generated in the convexification process. Vielma and Nemhauser [32] formulated a piecewise linear function with a logarithmic number of binary variables and constraints and presented experimental results showing that it significantly outperforms other models. Tsai and Lin [33] used the techniques developed by Vielma and Nemhauser [32] in their algorithm for solving posynomial geometric programming problems with continuous variables. For linearizing a discrete univariate, Vielma and Nemhauser [32] also used a logarithmic number of binary variables and constraints. Compared with the Li and Lu [34] method for expressing a discrete variable or a univariate, both methods use the same number of binary variables but the Vielma and Nemhauser [32] method uses fewer constraints and continuous variables.

After convexifying the non-convex terms and linearizing the univariates, the original problem is reformulated as a convex program that can be solved by conventional mixed-integer nonlinear programming methods to find a global solution.

3 Examples

*E*xample 1 This example is a stepped cantilever beam design problem shown in Figure 1. Chen and Chen [12] and Erbatur et al. [35] applied heuristic techniques to solve this problem. The objective of the design is to minimize the volume of the beam. The design variables are the widths and the depths of the rectangular cross-sections.

The mathematical model of the problem can be expressed as follows:

Minimize $100(x_5x_6 + x_7x_8 + x_9x_{10} + x_1x_2 + x_3x_4)$

subject to

$$10.7143 - \frac{x_5x_6^2}{10^3} \le 0,$$

$$8.5714 - \frac{x_7x_8^2}{10^3} \le 0,$$

$$6.4286 - \frac{x_9x_{10}^2}{10^3} \le 0,$$

$$4.2857 - \frac{x_1x_2^2}{10^3} \le 0,$$

$$2.1429 - \frac{x_3x_4^2}{10^3} \le 0,$$

$$10^4\left(\frac{244}{x_5x_6^3} + \frac{148}{x_7x_8^3} + \frac{76}{x_9x_{10}^3} + \frac{28}{x_1x_2^3} + \frac{4}{x_3x_4^3}\right) - 1086 \le 0,$$

$$x_6 - 20x_5 \le 0,$$

$$x_8 - 20x_7 \le 0,$$

$$x_{10} - 20x_9 \le 0,$$

$$x_2 - 20x_1 \le 0,$$

$$x_4 - 20x_3 \le 0,$$

$x_{1,3} \in [1,5]$, $x_{2,4} \in [30,65]$, $x_5 \in (1,2,\ldots,5)$, $x_6 \in (30,31,\ldots,65)$, $x_{7,9} \in (2.4,2.6,2.8,3.1)$, $x_{8,10} \in (45,50,55,60)$.

According to Proposition 1, $x_1^{-1}x_2^{-3}$, $x_3^{-1}x_4^{-3}$, $x_5^{-1}x_6^{-3}$, $x_7^{-1}x_8^{-3}$ and $x_9^{-1}x_{10}^{-3}$ are convex and do not require any transformations. By Remark 2, the objective function can be convexified as $100(e^{y_5+y_6} + e^{y_7+y_8} + e^{y_9+y_{10}} + e^{y_1+y_2} + e^{y_3+y_4})$, where $y_i = L(\ln x_i)$, $i = 1,2,\ldots,8$. By Remark 3, the negative monomials $-x_1x_2^2$, $-x_3x_4^2$, $-x_5x_6^2$, $-x_7x_8^2$ and $-x_9x_{10}^2$ can be convexified as $-z_1^{0.5}z_2^{0.5}$, $-z_3^{0.5}z_4^{0.5}$, $-z_5^{0.5}z_6^{0.5}$, $-z_7^{0.5}z_8^{0.5}$ and $-z_9^{0.5}z_{10}^{0.5}$, respectively, where $z_i = L(x_i^2)$, $i = 1,3,5,7,9$, and $z_i = L(x_i^4)$, $i = 2,4,6,8,10$. The piecewise linearization techniques presented by Vielma and Nemhauser [32] is utilized to generate the piecewise linear functions**.** The program therefore can be completely transformed to a convex mixed-integer program solvable to obtain a globally optimal solution. Solving this reformulated program by LINGO [36], the obtained global optimum is $(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10})$ = (2.204, 44.091, 1.749, 34.994, 3, 60, 3.1, 55, 2.6, 50) and the objective is 63893.24. The CPU time required for solving the transformed program is about 22 minutes.

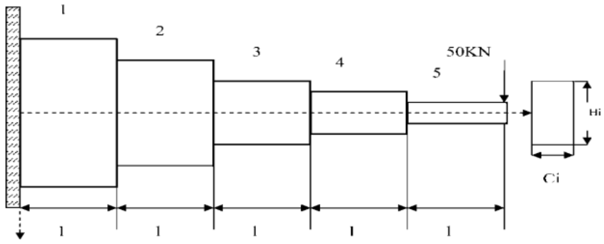

Figure 1: A stepped cantilever beam [35]

Table 1 lists the solutions obtained by the proposed method and the previous studies [12,35]. Chen and Chen [12] found the optimal value 64322 by using the genetic algorithm. Erbatur et al. [35] obtained the optimal value 64447 by using the evolutionary algorithm. The solution obtained by this study is better than the other two solutions.

Table 1 Comparison of results for stepped cantilever beam design

decision variables	Chen and Chen [12]	Erbatur et. al. [35]	proposed method
x_1	2.491	2.27	2.204
x_2	45.553	45.25	44.091
x_3	1.75	1.75	1.749
x_4	35.004	35	34.994
x_5	3	3	3
x_6	60	60	60
x_7	3.1	3.1	3.1
x_8	55	55	55
x_9	2.6	2.6	2.6
x_{10}	50	50	50
objective value	64322	64447	63893.24

Example 2 The second example is a heat exchanger design. This example has been solved previously by Jaberipour and Khorram [9] and Lee and Geem [14] using the harmony search algorithms. The mathematical model of the problem is given below:

Minimize $x_1 + x_2 + x_3$

subject to

$0.0025(x_4 + x_6) - 1 \le 0$,

$0.0025(x_5 + x_7 - x_4) - 1 \le 0$,

$0.01(x_8 - x_5) - 1 \le 0$,

$-x_1 x_6 + 833.33252 x_4 + 100 x_1 + 83333.333 \le 0$,

$-x_2 x_7 + 1250 x_5 + x_2 x_4 - 1250 x_4 \le 0$,

$-x_3 x_8 + x_3 x_5 + 2500 x_5 + 1250000 \le 0$,

$100 \le x_1 \le 10000$,

$1000 \le x_2, x_3 \le 10000$,

$10 \le x_i \le 1000,\ \ i = 4,5,\ldots,8.$

In this example, the objective function is a linear function without any transformations. The product terms $x_2 x_4$ and $x_3 x_5$ can be converted to convex functions as $e^{y_2+y_4}$ and $e^{y_3+y_5}$, respectively, where $y_i = L(\ln x_i)$, $i = 2,3,4,5$, by Remark 2. Furthermore, the product terms $-x_1 x_6$, $-x_2 x_7$, and $-x_3 x_8$ can be converted to convex terms $-z_1^{0.5} z_6^{0.5}$, $-z_2^{0.5} z_7^{0.5}$, and $-z_3^{0.5} z_8^{0.5}$, respectively, where $z_i = L(x_i^2)$, $i = 1,2,3,6,7,8$. The technique developed by Vielma and Nemhauser [32] is used to piecewisely linearizing each continuous univariate generated in the transformation process. The program therefore can be completely transformed to a convex mixed-integer program solvable to obtain a globally optimal solution within four hours. Table 2 lists the solutions obtained by the proposed method and the previous studies [9,14]. This study obtains a better solution than the other two solutions.

Table 2 Comparison of results for heat exchanger design

decision variables	Jaberipour and Khorram [9]	Lee and Geem [14]	proposed method
x_1	500.003	579.316	573.106
x_2	1359.311	1359.943	1355.386
x_3	5197.959	5110.071	5106.968
x_4	174.726	182.017	182.017
x_5	292.081	295.598	295.601
x_6	224.705	217.979	217.982
x_7	282.644	286.416	286.416
x_8	392.081	395.597	395.601
objective value	7057.274	7049.331	7035.461

Example 3 The third example is a more complicated example of a speed reducer design problem indicated in Figure 2. The design of the speed reducer is considered with the face width (b), module of teeth (m), number of teeth on pinion (z), length of shaft 1 between bearings (l_1), length of shaft 2 between bearings (l_2), diameter of shaft 1 (d_1), and diameter of shaft 2 (d_2) as design variables $x_1, x_2, \ldots, x_7$, respectively [9,20]. The objective is to minimize the total weight of the speed reducer. A detailed description of this problem with eleven behavioral constraints is outlined in Rao [20]. The mathematical model of the problem is given below:

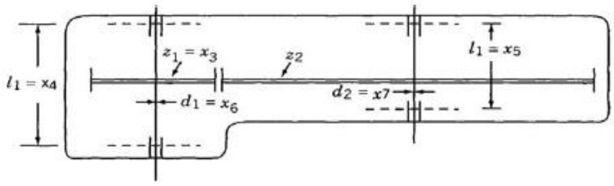

Figure 2: Speed reducer design [9]

Minimize

$$0.7854 x_1 x_2^2 (3.3333 x_3^2 + 14.9334 x_3 - 43.0934) - 1.508 x_1 (x_6^2 + x_7^2) + 7.4777(x_6^3 + x_7^3) + 0.7854(x_4 x_6^2 + x_5 x_7^2)$$

subject to

$$\frac{27}{x_1 x_2^2 x_3} \le 1,$$

$$\frac{397.5}{x_1 x_2^2 x_3^2} \le 1,$$

$\frac{1.93x_4^3}{x_2 x_3 x_6^4} \le 1$,

$\frac{1.93x_5^3}{x_2 x_3 x_7^4} \le 1$,

$x_2 x_3 \le 40$,

$\frac{x_1}{x_2} \ge 5$,

$\frac{x_1}{x_2} \le 12$,

$\frac{1.5x_6 + 1.9}{x_4} \le 1$,

$\frac{1.1x_7 + 1.9}{x_5} \le 1$,

$$\frac{\sqrt{\left(\frac{745x_4}{x_2 x_3}\right)^2 + 16.9(10^6)}}{110x_6^3} \le 1,$$

$$\frac{\sqrt{\left(\frac{745x_5}{x_2 x_3}\right)^2 + 157.5(10^6)}}{85x_7^3} \le 1,$$

$2.6 \le x_1 \le 3.6$, $0.7 \le x_2 \le 0.8$, $17 \le x_3 \le 28$, $7.3 \le x_4 \le 8.3$, $7.3 \le x_5 \le 8.3$, $2.9 \le x_6 \le 3.9$, $5 \le x_7 \le 5.5$.

Many heuristic algorithms have been proposed to solve this example. Such as particle swam optimization [7,8], genetic algorithm [5] and harmony search algorithm [9]. This example has been solved previously by Rao and Xiong [5] using a hybrid genetic algorithm. Li and Papalambros [37] solved the same problem using the global optimization knowledge.

Applying the linearization and convexification techniques described in Section 2, all non-convex terms can be transformed into convex terms. Then the original problem can be transformed into a convex program which is solvable to find a global optimum. The CPU time required for solving the transformed model by LINGO [36] is about two minutes. The smallest weight obtained by Li and Papalambros [37] is 2994.4, while the result obtained by this study is 2994.341. Compared with past literature as shown in Table 3, the solution obtained by this study is superior to those in past literature.

Table 3 Comparison of Results for speed reducer design

decision variables	Rao and Xiong [5]	Li and Papalambros [37]	proposed method
x_1	3.5	3.5	3.5
x_2	0.7	0.7	0.7
x_3	17	17	17
x_4	7.3	7.3	7.3
x_5	7.8	7.71	7.715
x_6	3.36	3.35	3.349
x_7	5.29	5.29	5.286
objective value	3000.83	2994.4	2994.341

4 Conclusions

This study utilizes an efficient deterministic approach for solving engineering optimization problems. The deterministic approach is capable of transforming a non-convex engineering optimization problem into a convex program by the linearization and convexification techniques and is thus guaranteed to reach a global optimum. Three illustrative examples in real applications are presented to demonstrate that the proposed method can effectively solve the engineering optimization problems for finding a global solution.

Acknowledgements

The authors would like to thank the anonymous referees for contributing their valuable comments regarding this paper, and thus significantly improving the quality of this paper. The research is supported by Taiwan NSC Grants NSC 99-2410-H-158-010-MY2 and NSC 99-2410-H-027-008-MY3.

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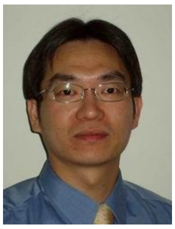

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