Mathematical Sciences Letters
An International Journal

Common Fixed Point Theorems on Fuzzy Metric Spaces Using Implicit Relation

Sunny Chauhan[1] and Neeraj Dhiman[2]
[1]*Near Nehru Training Centre, H. No. 274, Nai Basti B-14, Bijnor-246701, Uttar Pradesh, India*
[2]*Department of Mathematics, Graphic Era University, Dehradun-248001, Uttarakhand, India*
Email: neeraj.dhiman1@gmail.com, sun.gkv@gmail.com

Received: 27 March 2012; Accepted: 25 July 2012.

Abstract: In this paper, we prove a common fixed point theorem for occasionally weakly compatible mappings on fuzzy metric space satisfying an implicit relation. Our result never requires the completeness (or closedness) of the whole space (or subspaces), containment of ranges amongst involved mappings and continuity of one or more mappings. Our result improves and extends the results of Altun and Turkoglu [*Commun. Korean Math. Soc.,* **23**, 111-124 (2008)].

Keywords: t-norm, fuzzy metric space, occasionally weakly compatible mappings, implicit relations, fixed point.

1 Introduction

In 1965, Zadeh [33] introduced the concepts of fuzzy sets. Since then, to use this concept in topology and analysis, many authors have expansively developed the theory of fuzzy sets and applications. For example, Kramosil and Michalek [22], Erceg [12], Deng [11], Kaleva and Seikkala [20], Grabiec [15], Fang [13], George and Veeramani [14], Subrahmanyam [32], Gregori and Sapena [16] and Singh and Jain [31] have introduced the concept of fuzzy metric spaces in different ways. In applications of fuzzy set theory, the field of engineering has undoubtedly been a leader. All engineering disciplines such as civil engineering, electrical engineering, mechanical engineering, robotics, industrial engineering, computer engineering, nuclear engineering etc. have already been affected to various degrees by the new methodological possibilities opened by fuzzy sets.

Jungck and Rhoades [18] introduced the notion of weakly compatible mappings in metric space. Mishra et al. [23] formulated the notion of weakly compatible mappings in fuzzy settings and proved some fixed point theorems on fuzzy metric space. In 2008, Al-Thagafi and Shahzad [2] introduced the notion of occasionally weakly compatible mappings in metric space, while Khan and Sumitra [21] extended the notion of occasionally weakly compatible mappings to fuzzy metric spaces. It is worth to mention that every pair of weak compatible self mappings is occasionally weak compatible but the reverse is not true. Many authors proved common fixed point theorems for occasionally weakly compatible mappings on various spaces (see [1]-[3], [6]-[10], [19], [21], [24]).

In 1998, Popa and Turkoglu [27] proved some fixed point theorems for hybrid mappings satisfying implicit relations. Popa [25] used the family of implicit real functions for the existence of fixed points (see [26]).

The aim of this paper is to prove a common fixed point theorem for even number of occasionally weakly compatible mappings on fuzzy metric space satisfying implicit relation. Our result improves and extends the results of Altun and Turkoglu [4].

2 Preliminaries

Definition 2.1 [33] Let X be any set. A fuzzy set A in X is a function with domain X and values in $[0,1]$.

Definition 2.2 [28, 29] A mapping $*:[0,1]\times[0,1]\to[0,1]$ is a t-norm if $*$ is satisfying the following conditions:

(1) $*$ is commutative and associative;
(2) $a*1=1$ for all $a\in[0,1]$;
(3) $a*b\le c*d$ whenever $a\le c$ and $b\le d$ and $a,b,c,d\in[0,1]$.

Examples of t-norms are: $a*b=\min\{a,b\}$, $a*b=ab$ and $a*b=\max\{a+b-1,0\}$.

Definition 2.3 [22] A 3-tuple $(X,M,*)$ is said to be a fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm and M is a fuzzy set on $X^2\times(0,\infty)$ satisfying the following conditions: for all $x,y,z\in X$ and $t,s>0$,

(1) $M(x,y,t)>0$;
(2) $M(x,y,t)=1$ if and only if $x=y$;
(3) $M(x,y,t)=M(y,x,t)$;
(4) $M(x,z,t+s)\ge M(x,y,t)*M(y,z,s)$;
(5) $M(x,y,\cdot):(0,\infty)\to(0,1]$ is continuous.

Then M is called a fuzzy metric on X. Here $M(x,y,t)$ denotes the degree of nearness between x and y with respect to t.

Example 2.1 [14] Let (X,d) be a metric space. Define $a*b=ab$ (or $a*b=\min\{a,b\}$) for all $a,b\in[0,1]$ and let M_d be fuzzy sets on $X^2\times(0,\infty)$ defined as follows:

$$M_d(x,y,t)=\frac{t}{t+d(x,y)}.$$

Then $(X,M_d,*)$ is a fuzzy metric space. We call this fuzzy metric induced by a metric.

Lemma 2.1 [5] For all $x,y\in X$, $(X,M,\cdot)$ is non-decreasing.

Lemma 2.2 [23] Let $(X,M,*)$ be a fuzzy metric space. If there exists a constant $k\in(0,1)$ such that

$$M(x,y,kt)\ge M(x,y,t),$$

for all $t>0$ with fixed $x,y\in X$ then $x=y$.

Definition 2.4 [18] Two self mappings A and S of a non-empty set X are said to be weakly compatible if they commute at their coincidence points, that is, if $Ax=Sx$ for some $x\in X$ then $ASx=SAx$.

Definition 2.5 [2] Two self mappings A and S of a non-empty set X are said to be occasionally weakly compatible if and only if there is a point $x\in X$ which is a coincidence point of A and S at which A and S commute.

From the following example it is clear that the notion of occasionally weakly compatible mappings is more general than weakly compatible mappings.

Example 2.2 Let $X=[0,\infty)$ with usual metric. Define $A,S:X\to X$ by $A(x)=3x$ and $S(x)=x^2$ for all $x\in X$. Then $A(x)=S(x)$ for $x=0,3$ but $AS(0)=SA(0)$ and $AS(3)\ne SA(3)$. Thus the mappings are occasionally weakly compatible but not weakly compatible.

Lemma 2.3 [19] Let X be a non-empty set and let A and S be occasionally weakly compatible self mappings of X if A and S have a unique point of coincidence, $w=Ax=Sx$, then w is the unique common fixed point of A and S.

3 Implicit relation

Many authors proved a number of common fixed point theorems by using implicit relations in different settings (see [25]-[27], [17]). In 2008, Altun and Turkoglu [4] proved two common fixed point theorems for continuous compatible mappings of type (α) and (β) on complete fuzzy metric spaces by using the following implicit relation.

Let $I=[0,1]$, $*$ be a continuous t-norm and $\varphi: I^6 \to R$ be a continuous function. Now we consider the following conditions:

(φ -1) φ is non-increasing in the fifth and sixth variables,

(φ -2) If, for some $k \in (0,1)$, we have

(φ_a) $\varphi\left(u(kt), v(t), v(t), u(t), 1, u\left(\frac{t}{2}\right) * v\left(\frac{t}{2}\right)\right) \geq 1$,

or

(φ_b) $\varphi\left(u(kt), v(t), u(t), v(t), u\left(\frac{t}{2}\right) * v\left(\frac{t}{2}\right), 1\right) \geq 1$,

for any fixed $t>0$ and any non-decreasing functions $u, v:(0,\infty) \to I$ with $0 < u(t), v(t) \leq 1$, then there exists $h \in (0,1)$ with $u(ht) \geq v(t) * u(t)$.

(φ -3) If, for some $k \in (0,1)$, we have

$$\varphi(u(kt), u(t), 1, 1, u(t), u(t)) \geq 1,$$

for any fixed $t>0$ and any non-decreasing function $u:(0,\infty) \to I$, then $u(kt) \geq u(t)$.

Now, let Φ be the set of all real continuous functions $\varphi: I^6 \to R$ satisfying the conditions (φ -1)~(φ -3).

Example 3.1 [4] Let $\varphi(u_1, \cdots, u_6) = \dfrac{u_1}{\min\{u_2, \cdots, u_6\}}$ and $a * b = \min\{a, b\}$. Let $t>0$, $0 < u(t), v(t) \leq 1$, $k \in \left(\frac{0,1}{2}\right)$, where $u, v:[0,\infty) \to I$ are non-decreasing functions. Now, suppose that

$$\varphi\left(u(kt), v(t), v(t), u(t), 1, u\left(\frac{t}{2}\right) * v\left(\frac{t}{2}\right)\right) \geq 1,$$

that is,

$$\varphi\left(u(kt), v(t), v(t), u(t), 1, u\left(\frac{t}{2}\right) * v\left(\frac{t}{2}\right)\right) = \frac{u(kt)}{\min\left\{v(t), v(t), u(t), 1, u\left(\frac{t}{2}\right) * v\left(\frac{t}{2}\right)\right\}}$$

$$= \frac{u(kt)}{\min\left\{u\left(\frac{t}{2}\right), v\left(\frac{t}{2}\right)\right\}} \geq 1.$$

Thus $u(ht) \geq v(t) * u(t)$ if $h = 2k \in (0,1)$. A similar argument works if (φ_b) is assumed. Finally, suppose that $t>0$ is fixed, $u:(0,\infty) \to I$ is a non-decreasing function and

$$\varphi(u(kt), u(t), 1, 1, u(t), u(t)) = \frac{u(kt)}{u(t)} \geq 1,$$

for some $k \in (0,1)$. Then we have $u(kt) \geq u(t)$ and thus $\varphi \in \Phi$.

4 Results

Theorem 4.1 Let $P_1, P_2, \cdots, P_{2n}, A$ and B be self mappings on a fuzzy metric space $(X, M, *)$ with $a * b = \min\{a, b\}$ for all $a, b \in I$ satisfying the following conditions:

(4.1) there exist $k \in (0,1)$ and $\varphi \in \Phi$ such that

$$\varphi\begin{pmatrix} M(Ax, By, kt), M(P_1P_3 \cdots P_{2n-1}x, P_2P_4 \cdots P_{2n}y, t), M(Ax, P_1P_3 \cdots P_{2n-1}x, t), \\ M(By, P_2P_4 \cdots P_{2n}y, t), M(Ax, P_2P_4 \cdots P_{2n}y, t), M(By, P_1P_3 \cdots P_{2n-1}x, t) \end{pmatrix} \geq 1,$$

for all $x, y \in X$ and $t > 0$.

(4.2) Suppose that

$P_1(P_3 \cdots P_{2n-1}) = (P_3 \cdots P_{2n-1})P_1$,

$P_1P_3(P_5 \cdots P_{2n-1}) = (P_5 \cdots P_{2n-1})P_1P_3$,

$\vdots$

$P_1 \cdots P_{2n-3}(P_{2n-1}) = (P_{2n-1})P_1 \cdots P_{2n-3}$,

$A(P_3 \cdots P_{2n-1}) = (P_3 \cdots P_{2n-1})A$,

$A(P_5 \cdots P_{2n-1}) = (P_5 \cdots P_{2n-1})A$,

$\vdots$

$AP_{2n-1} = P_{2n-1}A$,

similarly,

$P_2(P_4 \cdots P_{2n}) = (P_4 \cdots P_{2n})P_2$,

$P_2P_4(P_6 \cdots P_{2n}) = (P_6 \cdots P_{2n})P_2P_4$,

$\vdots$

$P_2 \cdots P_{2n-2}(P_{2n}) = (P_{2n})P_2 \cdots P_{2n-2}$,

$B(P_4 \cdots P_{2n}) = (P_4 \cdots P_{2n})B$,

$B(P_4 \cdots P_{2n}) = (P_4 \cdots P_{2n})B$,

$\vdots$

$BP_{2n} = P_{2n}B$.

Then, if the pairs $(A, P_1P_3 \cdots P_{2n-1})$ and $(B, P_2P_4 \cdots P_{2n})$ are each occasionally weakly compatible, it follows that $P_1, P_2, \cdots, P_{2n}, A$ and B have a unique common fixed point in X.

Proof. Since the pairs $(A, P_1P_3 \cdots P_{2n-1})$ and $(B, P_2P_4 \cdots P_{2n})$ are each occasionally weakly compatible, there exist points $u, v \in X$ such that $Au = P_1P_3 \cdots P_{2n-1}u$, $A(P_1P_3 \cdots P_{2n-1})u = (P_1P_3 \cdots P_{2n-1})Au$ and $Bv = P_2P_4 \cdots P_{2n}v$, $B(P_2P_4 \cdots P_{2n})v = (P_2P_4 \cdots P_{2n})Bv$. Now we assert that $Au = Bv$. On using inequality (4.1) with $x = u$, $y = v$, we get

$$\varphi\begin{pmatrix} M(Au, Bv, kt), M(P_1P_3 \cdots P_{2n-1}u, P_2P_4 \cdots P_{2n}v, t), M(Au, P_1P_3 \cdots P_{2n-1}u, t), \\ M(Bv, P_2P_4 \cdots P_{2n}v, t), M(Au, P_2P_4 \cdots P_{2n}v, t), M(Bv, P_1P_3 \cdots P_{2n-1}u, t) \end{pmatrix} \geq 1$$

$$\varphi(M(Au, Bv, kt), M(Au, Bv, t), M(Au, Au, t), M(Bv, Bv, t), M(Au, Bv, t), M(Bv, Au, t)) \geq 1$$

$$\varphi(M(Au, Bv, kt), M(Au, Bv, t), 1, 1, M(Au, Bv, t), M(Bv, Au, t)) \geq 1.$$

Thus, from (φ -3), we have

$$M(Au, Bv, kt) \geq M(Au, Bv, t).$$

From Lemma 2.2, we have $Au = Bv$. Moreover, if there is another point z such that $Az = P_1P_3 \cdots P_{2n-1}z$. Then using inequality (4.1), it follows that $Az = P_1P_3 \cdots P_{2n-1}z = Bv = P_2P_4 \cdots P_{2n}v$, or $Au = Az$. Hence $w = Au = P_1P_3 \cdots P_{2n-1}u$ is the unique point of coincidence of A and $P_1P_3 \cdots P_{2n-1}$. By Lemma 2.3, it follows that w is the unique common fixed point of A and $P_1P_3 \cdots P_{2n-1}$. By symmetry, $q = Bv = P_2P_4 \cdots P_{2n}v$ is the unique common fixed point of B and $P_2P_4 \cdots P_{2n}$. Since $w = q$, we obtain that w is the unique common fixed point of B and $P_2P_4 \cdots P_{2n}$. Now we show that w is the fixed point of all the component mappings. On using inequality (4.1) with $x = P_3 \cdots P_{2n-1}w$, $y = w$, $P_1' = P_1P_3 \cdots P_{2n-1}$, $P_2' = P_2P_4 \cdots P_{2n}$, we have

$$\varphi\begin{pmatrix} M(AP_3 \cdots P_{2n-1}w, Bw, kt), M(P_1'P_3 \cdots P_{2n-1}w, P_2'w, t), \\ M(AP_3 \cdots P_{2n-1}w, P_1'P_3 \cdots P_{2n-1}w, t), M(Bw, P_2'w, t), \\ M(AP_3 \cdots P_{2n-1}w, P_2'w, t), M(Bw, P_1'P_3 \cdots P_{2n-1}w, t) \end{pmatrix} \geq 1$$

$$\varphi\begin{pmatrix} M(P_3 \cdots P_{2n-1}w, w, kt), M(P_3 \cdots P_{2n-1}w, w, t), M(P_3 \cdots P_{2n-1}w, P_3 \cdots P_{2n-1}w, t), \\ M(w, w, t), M(P_3 \cdots P_{2n-1}w, w, t), M(w, P_3 \cdots P_{2n-1}w, t) \end{pmatrix} \geq 1$$

$$\varphi\begin{pmatrix} M(P_3 \cdots P_{2n-1}w, Bw, kt), M(P_3 \cdots P_{2n-1}w, w, t), 1, \\ 1, M(P_3 \cdots P_{2n-1}w, w, t), M(w, P_3 \cdots P_{2n-1}w, t) \end{pmatrix} \geq 1.$$

Thus, from (φ -3), we have

$$M(P_3 \cdots P_{2n-1}w, w, kt) \geq M(P_3 \cdots P_{2n-1}w, w, t).$$

From Lemma 2.2, we have $P_3 \cdots P_{2n-1}w = w$. Hence, $P_1w = w$. Continuing this procedure, we have $Aw = P_1w = P_3w = \cdots = P_{2n-1}w = w$. So, $Bw = P_2w = P_4w = \cdots = P_{2n}w = w$. That is, w is the unique common fixed point of $P_1, P_2, \cdots, P_{2n}, A$ and B. ■

The following result is a slight generalization of Theorem 4.1.

Corollary 4.1 Let $\{T_\alpha\}_{\alpha \in J}$ and $\{P_i\}_{i=1}^{2n}$ be two families of self mappings on a fuzzy metric space $(X, M, *)$ with $a * b = \min\{a, b\}$ for all $a, b \in I$ satisfying the following conditions:

(4.1) there exist a fixed $\beta \in J$, $k \in (0,1)$ and $\varphi \in \Phi$ such that

$$\varphi\begin{pmatrix} M(T_\alpha x, T_\beta y, kt), M(P_1P_3 \cdots P_{2n-1}x, P_2P_4 \cdots P_{2n}y, t), M(T_\alpha x, P_1P_3 \cdots P_{2n-1}x, t), \\ M(T_\beta y, P_2P_4 \cdots P_{2n}y, t), M(T_\alpha x, P_2P_4 \cdots P_{2n}y, t), M(T_\beta y, P_1P_3 \cdots P_{2n-1}x, t) \end{pmatrix} \geq 1,$$

for all $x, y \in X$ and $t > 0$.

(4.2) Suppose that

$P_1(P_3 \cdots P_{2n-1}) = (P_3 \cdots P_{2n-1})P_1$,

$P_1P_3(P_5 \cdots P_{2n-1}) = (P_5 \cdots P_{2n-1})P_1P_3$,

⋮

$P_1 \cdots P_{2n-3}(P_{2n-1}) = (P_{2n-1})P_1 \cdots P_{2n-3}$,

$T_\alpha(P_3 \cdots P_{2n-1}) = (P_3 \cdots P_{2n-1})T_\alpha$,

$T_\alpha(P_5 \cdots P_{2n-1}) = (P_5 \cdots P_{2n-1})T_\alpha$,

⋮

$T_\alpha P_{2n-1} = P_{2n-1} T_\alpha$,

similarly,

$P_2(P_4 \cdots P_{2n}) = (P_4 \cdots P_{2n})P_2$,

$P_2P_4(P_6 \cdots P_{2n}) = (P_6 \cdots P_{2n})P_2P_4$,

$\vdots$

$P_2 \cdots P_{2n-2}(P_{2n}) = (P_{2n})P_2 \cdots P_{2n-2}$,

$T_\beta(P_4 \cdots P_{2n}) = (P_4 \cdots P_{2n})T_\beta$,

$T_\beta(P_4 \cdots P_{2n}) = (P_4 \cdots P_{2n})T_\beta$,

$\vdots$

$T_\beta P_{2n} = P_{2n} T_\beta$.

Then, if the pairs $(T_\alpha, P_1P_3 \cdots P_{2n-1})$ and $(T_\beta, P_2P_4 \cdots P_{2n})$ are each occasionally weakly compatible, it follows that all $\{P_i\}$ and $\{T_\alpha\}$ have a unique common fixed point in X.

Proof. Since the proof is straightforward, we omit it. ■

Corollary 4.2 Let A, B, S and T be self mappings on a fuzzy metric space $(X, M, *)$ with $a * b = \min\{a, b\}$ for all $a, b \in I$. Suppose that

(4.1) there exist $k \in (0,1)$ and $\varphi \in \Phi$ such that

$$\varphi\begin{pmatrix} M(Ax, By, kt), M(Sx, Ty, t), M(Ax, Sx, t), \\ M(By, Ty, t), M(Ax, Ty, t), M(By, Sx, t) \end{pmatrix} \geq 1,$$

holds for all $x, y \in X$ and $t > 0$.

Then, there exists a unique point $w \in X$ such that $Aw = Sw = w$ and a unique point $z \in X$ such that $Bz = Tz = z$. Moreover, $z = w$, so that there is a unique common fixed point of A, B, S and T in X.

Now we give an example which illustrates Corollary 4.2.

Example 4.1 Let $X = [0,20]$ with the metric d defined by $d(x, y) = |x - y|$ and for each $t \in [0,1]$, define

$$M(x, y, t) = \begin{cases} \dfrac{t}{t + d(x,y)}, & \text{if } t > 0; \\ 0, & \text{if } t = 0, \end{cases}$$

for all $x, y \in X$. Clearly $(X, M, *)$ be a fuzzy metric space, where $a * b = \min\{a, b\}$ for all $a, b \in [0,1]$. Let $\varphi: I^6 \to R$ be defined as in Example 3.1. Define the self mappings A, B, S and T by

$$A(x) = \begin{cases} x, \text{if } 0 \leq x \leq 10; \\ 20, \text{if } 10 < x \leq 20. \end{cases} \quad B(x) = \begin{cases} 10, \text{if } 0 \leq x \leq 10; \\ 20, \text{if } 10 < x \leq 20. \end{cases}$$

$$S(x) = \begin{cases} 10, \text{if } 0 \leq x \leq 10; \\ 0, \text{if } 10 < x \leq 20. \end{cases} \quad T(x) = \begin{cases} 10, \text{if } 0 \leq x \leq 10; \\ \dfrac{x}{20}, \text{if } 10 < x \leq 20. \end{cases}$$

Then A, B, S and T satisfy all the conditions of Corollary 4.2 for some fixed $k \in (0,1)$. First we have $A(10) = 10 = S(10)$, $AS(10) = 10 = SA(10)$ and $B(10) = 10 = T(10)$, $BT(10) = 10 = TB(10)$, that is, A and S as well as B and T are occasionally weakly compatible mappings and have a unique common fixed point 10. Also all the involved mappings are discontinuous at 10.

On taking $A = B$ and $S = T$ in Corollary 4.2, we get the following result for two self mappings:

Corollary 4.3 Let A and S be self mappings on a fuzzy metric space $(X, M, *)$ with $a * b = \min\{a, b\}$ for all $a, b \in I$. Suppose that

(4.1) there exist $k \in (0,1)$ and $\varphi \in \Phi$ such that

$$\varphi\begin{pmatrix} M(Ax, Ay, kt), M(Sx, Sy, t), M(Ax, Sx, t), \\ M(Ay, Sy, t), M(Ax, Sy, t), M(Ay, Sx, t) \end{pmatrix} \geq 1,$$

holds for all $x, y \in X$ and $t > 0$.

Then, if the pair (A, S) is occasionally weakly compatible, it follows that A and S have a unique common fixed point in X.

3 Conclusion

Theorem 4.1 is proved for even numbers of occasionally weakly compatible mappings on fuzzy metric space. Theorem 4.1 improves and extends the main results of Altun and Turkoglu [4, Theorem 1, Theorem 2]. Corollary 4.1 is derived as a slight generalization of Theorem 4.1. Corollary 4.2 and Corollary 4.3 are defined as natural results due to Theorem 4.1. Example 4.1 is furnished in support of Corollary 4.2.

Acknowledgements

The authors are thankful to anonymous referee for his valuable remarks to improve the paper. The first author is also grateful to Professor Lj. B. Ćirić for a reprint of the paper [10].

References

[1] C.T. Aage and J.N. Salunke, *On fixed point theorems in fuzzy metric spacess,* Int. J. Open Prob. Compt. Math., **3(2)**, 123-131 (2010).

[2] M.A. Al-Thagafi and N. Shahzad, *Generalized I-nonexpansive selfmaps and invariant approximations,* Acta Math. Sinica, **24(5)**, 867-876 (2008).

[3] M.A. Al-Thagafi and N. Shahzad, *A note on occasionally weakly compatible maps,* Int. J. Math. Anal. (Ruse), **3(2)**, 55-58 (2009).

[4] I. Altun and D. Turkoglu, *Some fixed point theorems on fuzzy metric spaces with implicit relations*, Commun. Korean Math. Soc., **23(1)**, 111-124 (2008).

[5] D. Butnariu, *Fixed points for fuzzy mappings*, Fuzzy Sets and Systems, **7**, 191-207 (1982).

[6] H. Chandra and A. Bhatt, *Fixed point theorems for occasionally weakly compatible maps in probabilistic semi-metric space*, Int. J. Math. Anal. (Ruse), **3(12)**, 563-570 (2009).

[7] S. Chauhan, M.A. Khan and N. Dhiman, *Coincidences and fixed point theorems in fuzzy metric spaces with integral type contractive conditions*, J. Adv. Res. Pure Math., (2012/13), Article in press.

[8] S. Chauhan and B.D. Pant, *Common fixed point theorems for occasionally weakly compatible mappings using implicit relation*, J. Indian Math. Soc. (N.S.), **77(1-4)**, 13-21 (2010).

[9] S. Chauhan and B.D. Pant, *Common fixed point theorems in fuzzy metric spaces*, Bull. Allahabad Math. Soc., (2012), Article in press.

[10] Lj. Ćirić, B. Samet and C. Vetro, Common fixed point theorems for families of occasionally weakly compatible mappings, Math. Comput. Modelling, **53(5-6)**, 631-636 (2011).

[11] Z.K. Deng, *Fuzzy pseudo-metric spaces*, J. Math. Anal. Appl., **86**, 74-95 (1982).

[12] M.A. Erceg, *Metric spaces in fuzzy set theory*, J. Math. Anal. Appl., **69(1)**, 205-230 (1979).

[13] J.X. Fang, *On fixed point theorems in fuzzy metric spaces*, Fuzzy Sets and Systems, 46, 107-113 (1992).

[14] A. George and P. Veeramani, *On some result in fuzzy metric space*, Fuzzy Sets and Systems, 64, 395-399 (1994).

[15] M. Grabiec, *Fixed points in fuzzy metric spaces*, Fuzzy Sets and System, **27**, 385-389 (1988).

[16] V. Gregori and A. Sapena, *On fixed-point theorem in fuzzy metric spaces*, Fuzzy Sets and Systems, **125**, 245-252 (2002).

[17] M. Imdad, S. Kumar and M.S. Khan, *Remarks on some fixed point theorems satisfying implicit relations*, Rad. Math., **11**, 135-143 (2002).

[18] G. Jungck and B.E. Rhoades, *Fixed points for set valued functions without continuity*, Indian J. Pure Appl. Math., **29(3)**,

227-238 (1998).

[19] G. Jungck and B.E. Rhoades, *Fixed point theorems for occasionally weakly compatible mappings*, Fixed Point Theory, **7**, 286-296 (2006).

[20] O. Kaleva and S. Seikkala, *On fuzzy metric spaces*, Fuzzy Sets and Systems, **12**, 215-229 (1984).

[21] M.A. Khan and Sumitra, *Common fixed point theorems for occasionally weakly compatible maps in fuzzy metric spaces*, Far East J. Math. Sci., **41(2)**, 285-293 (2010).

[22] I. Kramosil and J. Michalek, *Fuzzy metric and statistical metric spaces*, Kybernetika, **11**, 336-344 (1975).

[23] S.N. Mishra, N. Sharma and S.L. Singh, *Common fixed points of maps in fuzzy metric spaces*, Int. J. Math. Math. Sci., **17**, 253-258 (1994).

[24] B.D. Pant and S. Chauhan, *Common fixed point theorem for occasionally weakly compatible mappings in Menger space*, Surv. Math. Appl., **6**, 1-7 (2011).

[25] V. Popa, *Some fixed point theorems for compatible mappings satisfying an implicit relation*, Demonsratio Math., **32**, 157-163 (1999).

[26] V. Popa, *A general coincidence theorem for compatible multivalued mappings satisfying an implicit relation*, Demonsratio Math., **33**, 159-164 (2000).

[27] V. Popa and D. Turkoglu, *Some fixed point theorems for hybrid contractions satisfying an implicit relation*, Stud. Cercet. Ştinţ. Ser. Math. Univ. Bacau, **8**, 75-86 (1998).

[28] B. Schweizer and A. Sklar, *Statistical metric spaces*, Pacific J. Math., **10**, 313-334 (1960).

[29] B. Schweizer and A. Sklar, *Probabilistic metric spaces*, Elsevier, North Holland, New York, (1983).

[30] S. Sharma and B. Deshpande, *On compatible mappings satisfying an implicit relation in common fixed point consideration*, Tamkang J. Math., **33**, 245-252 (2002).

[31] B. Singh, S. Jain, *Semicompatibility and fixed point theorems in fuzzy metric space using implicit relation*, Int. J. Math. Math. Sci., **16**, 2617-2629 (2005).

[32] P.V. Subrahmanyam, A *common fixed point theorem in fuzzy metric spaces*, Inf. Sci., **83**, 109-112 (1995).

[33] L. A. Zadeh, *Fuzzy sets*, Inf. Cont. **8**, 338-353 (1965).