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Influence of Stark Shift and Kerr-like Medium on the Interaction of a Two-Level Atom with Two Quantized Field Modes: A Time-Dependent System

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Abstract: In this paper, some properties through a two-level atom interacting with a two-mode radiation field are presented. The model describes multi-photon process and includes a nonlinear Kerr-like medium and Stark shift. Also, the coupling parameter is taken in time-dependent. The results show that Stark shift, nonlinear Kerr-like medium and time-dependent coupling parameter play important roles in the evolution of the field entropy. We test this observation with accessible parameters and some new aspects are obtained.

Keywords: entropy, Kerr-like medium, Stark shift, time-dependent coupling parameter

1 Introduction

The Jaynes-Cummigs model (JCM) [1] is an exactly solvable model of a single-mode quantized field interacting with a single two level atom in a lossless cavity in the dipole and rotating-wave approximation (RWA). Many interesting physical features have been studied with this model such as atomic inversion [2] and entanglement [3,4,5]. Much attention has been focused on the properties of the entanglement between the field and the atom [6,7,8,9] and in particular on the entropy of the system. Knight and co-workers [10,11,12] employing the entropy theory regarding the interaction of the field with the atom. They have shown that the entropy is a very useful operational measure of the purity of the quantum state, which automatically includes all moments of the density operator. The effect of Stark shift on the evolution of field entropy and entanglement in two-photon process is presented in [13]. Quantum entropy and entanglement in the Jaynes-Cummings model without the rotating-wave approximation have been shown in [14]. Properties of quantum entropy evolution in the Jaynes-Cummings model with initial squeezed coherent states field have been studied in [15].

Abdel-Aty *et al*. [16] have studied the entropy evolution of the bimodal field interacting with an effective two-level atom via the Raman transition in Kerr medium. The results of this paper shown that the system are potentially interesting for their ability to process information in a novel way and might application in models of quantum logic gates. Liao *et al*. [17] considered a system of two two-level atoms interacting with a bimodal field in an ideal cavity and studied the time evolution of the single-atom entropy squeezing, atomic inversion and linear entropy of the system. In Ref. [18] the quantum treatement for two two-level atoms interacting with SU(1,1) quantum system has been investigated. The dynamics for the collective model of two atoms interacting with two-mode quantized radiation fields in a Raman type process has been investigated [19].

However, in recent years, the effect of the atomic motion and field mode structure on entanglement attracted much attention [20,21,22,23,24,25]. Abdalla *et al*. [26] considered the interaction of a two-level atom with a single-mode multi-photon field in a medium consisting of the Stark shift and the Kerr-medium effects, with the coupling term assumed to be a function of time but still linear with the intensity of light. The authors of Ref. [27] nonlinearized the atom-field system considered in Ref. [26]. More recently, the entanglement of two coupled atoms in the presence of a time-dependent external magnetic field has studied [28].

In this paper we extend these investigations to study the dynamics of a two-level atom interacting with a

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two-mode coherent field. Furthermore the field and the atom are assumed to be coupled with modulated coupling parameter which depends explicitly on time. An exact solution of a two-level atom in interaction with cavity field has been obtained. We investigate the effect of different parameters of the system on the field entropy and atomic inversion. The material of this paper is arranged as follows: In Sec. 2, we introduce the model and its solution under certain approximation similar to that of the rotating-wave approximation (RWA) at any time $t > 0$, In Sec. 3, We investigate the field entropy and the dynamical properties for different regemes. Numerical results for the field entropy and atomic inversion are discussed in Sec. 4. Finally, conclusions are presented in Sec. 5.

2 The model and its solution

The model considered here consisits of two-mode interacting with an effective two-level atom via multi-photon Raman transition. We consider the multiple photon case and the quantized radiation field in the rotating-wave approximation, taking into account both Kerr and Stark effects in an ideal cavity ($Q = \infty$) filled with a nonlinear Kerr-like medium. We also assume that the cavity mode interacts with both the atom and Kerr-like medium. Furthermore the field and the atom are assumed to be coupled with modulated coupling parameter which depends explicitly on time. The atomic levels $|e\rangle$ and $|g\rangle$ have identical parites. Each is dipole coupled with a different mode of the field to the set of the intermediate states $|i\rangle$. If we assume that there are no dipole transitions between the sates $|i\rangle$ and that the interacting field modes are far off resonance with these intermediate states, the atom can be treated as an effective two-level atom by means of the adibatic eliminations of the intermediate state [29]. We assume that the atom can be prepared in the excited state and the initial state of the field is given by

$$|\Psi(0)\rangle_F = \sum_{n_1,n_2=0}^{\infty} q_{n_1,n_2} |n_1,n_2\rangle , \tag{1}$$

where $q_{n_1,n_2} = q_{n_1} q_{n_2}$ and $q_{n_i} = b_{n_i} e^{i\theta_i}$ describes the amplitude of the state $|n_i\rangle$ of the ith mode of the cavity field and $b_{n_i} = \exp\{-\bar{n}_i\}\sqrt{\bar{n}_i^{\,n_i}/n_i!}$. The state function of the total atom-field system at $t = 0$ is take the form

$$|\Psi(0)\rangle = |\Psi(0)\rangle_F \otimes |\Psi(0)\rangle_A = \sum_{n_1,n_2=0}^{\infty} q_{n_1,n_2} |n_1,n_2;e\rangle . \tag{2}$$

To obtain the wave function of the system at any time $t > 0$ we will solve Schrödinger equation

$$\frac{id\,|\Psi(t)\rangle}{dt} = H_{in}\,|\Psi(t)\rangle , \tag{3}$$

where that the Hamiltonian is given by [30] ($\hbar = c = 1$),

$$\begin{aligned}\hat{H} =\ & \Omega_1 \hat{a}_1^\dagger \hat{a}_1 + \Omega_2 \hat{a}_2^\dagger \hat{a}_2 + \frac{\omega}{2}\sigma_z + \hat{a}_1^\dagger \hat{a}_1 \beta_1 |g\rangle\langle g| \\ & + \hat{a}_2^\dagger \hat{a}_2 \beta_2 |e\rangle\langle e| + \chi_1 a_1^{\dagger 2} \hat{a}_1^2 + \chi_2 a_2^{\dagger 2} \hat{a}_2^2 \\ & + \lambda(t)(a_2^{\dagger k_2} a_1^{\dagger k_1}\sigma_- + \hat{a}_2^{k_2}\hat{a}_1^{k_1}\sigma_+).\end{aligned} \tag{4}$$

Where Ω_1 and Ω_2 are the field frequencies and ω is the transition frequency between the excited and ground states of the atom, $\hat{a}_1$ and $\hat{a}_2^\dagger$ respectively, are the annihilation and creation operators for the ith mode of the cavity field, β_1 and β_2 are parameters describing the intensity-dependent Stark shifts of the two levels that are due two the virtual transitions to the intermediate relay level, χ_i ($i = 1,2$) are related to the third order nonlinear susceptibilities for the processes of self-phase-modulation of the two modes, $\lambda(t)$ is the effective coupling parameter and considered to be time-dependent, and σ_z and $\sigma_\pm$ are the atomic pseudo-spin operators. Now if we take the coupling parameter $\lambda(t) = \lambda\cos(\mu t)$ where λ and μ are an arbitrary constatnts, then the Hamiltonian (4) can be rewritten in the form

$$\begin{aligned}\hat{H} =\ & \Omega_1 \hat{a}_1^\dagger \hat{a}_1 + \Omega_2 \hat{a}_2^\dagger \hat{a}_2 + \frac{\omega}{2}\sigma_z + \hat{a}_1^\dagger \hat{a}_1 \beta_1 |g\rangle\langle g| \\ & + \hat{a}_2^\dagger \hat{a}_2 \beta_2 |e\rangle\langle e| + \chi_1 a_1^{\dagger 2} \hat{a}_1^2 + \chi_2 a_2^{\dagger 2} \hat{a}_2^2 \\ & + \lambda\cos\mu t(a_2^{\dagger k_2} a_1^{\dagger k_1}\sigma_- + \hat{a}_2^{k_2}\hat{a}_1^{k_1}\sigma_+).\end{aligned} \tag{5}$$

The solution of Schrödinger equation in the interaction picture, i.e., the wave function of the system at any time $t > 0$ is given by

$$\begin{aligned}|\Psi(t)\rangle =\ & \sum_{n_1,n_2=0}^{\infty} [A_{n_1,n_2}(t)\,|n_1,n_2;e\rangle \\ & + B_{n_1,n_2}(t)\ |n_1+k_1,n_2+k_2;g\rangle].\end{aligned} \tag{6}$$

The coefficients $A_{n_1,n_2}(t)$ and $B_{n_1,n_2}(t)$ are given by

$$\begin{aligned}A_{n_1,n_2}(t) =\ & q_{n_1,n_2} e^{-i\lambda t F_{n_1,n_2}} \times \\ & \left[\cos\lambda t\,\nu_{n_1,n_2} - iW_{n_1,n_2}\frac{\sin\lambda t\,\nu_{n_1,n_2}}{2\nu_{n_1,n_2}}\right],\end{aligned} \tag{7}$$

$$B_{n_1,n_2}(t) = -i\,q_{n_1,n_2} V_{n_1,n_2}\frac{\sin\lambda t\,\nu_{n_1,n_2} e^{-i\lambda t G_{n_1,n_2}}}{2\nu_{n_1,n_2}}, \tag{8}$$

$$\begin{aligned}F_{n_1,n_2} =\ & \frac{1}{2r}[n_2 + r^2(n_1+k_1)] \\ & + \frac{\chi_1}{\lambda}[n_1(n_1-1) + k_1 n_1 + \frac{k_1(k_1-1)}{2}] \\ & + \frac{\chi_2}{\lambda}[n_2(n_2-1) + k_2 n_2 + \frac{k_2(k_2-1)}{2}] + \frac{\mu}{2\lambda},\end{aligned} \tag{9}$$

$$\begin{aligned}G_{n_1,n_2} =\ & \frac{1}{2r}[n_2 + r^2(n_1+k_1)] \\ & + \frac{\chi_1}{\lambda}[n_1(n_1-1) + k_1 n_1 + \frac{k_1(k_1-1)}{2}] \\ & + \frac{\chi_2}{\lambda}[n_2(n_2-1) + k_2 n_2 + \frac{k_2(k_2-1)}{2}] - \frac{\mu}{2\lambda},\end{aligned} \tag{10}$$

$$X_{n_1,n_2} = \Delta + \beta_2 n_2 - \beta_1(n_1+k_1) \\ -2\chi_1[k_1 n_1 + \frac{k_1(k_1-1)}{2}] \\ -2\chi_2[k_2 n_2 + \frac{k_2(k_2-1)}{2}], \quad (11)$$

$$\nu_{n_1,n_2} = \frac{\sqrt{W^2_{n_1,n_2}+V^2_{n_1,n_2}}}{2}, V_{n_1,n_2} = \sqrt{\frac{(n_1+k_1)!(n_2+k_2)!}{n_1!n_2!}}. \quad (12)$$

$$r = \sqrt{\beta_1/\beta_2},\ W_{n_1,n_2} = (X_{n_1,n_2}-\mu)/\lambda. \quad (13)$$

In our model, we should say that the solution given by Eqs. (7) and (8) is only valid for slowly oscillating term ($X_{n_1,n_2} \simeq \mu$) and the Rabi frequency in the present case is different from that of the JCM Rabi frequency. At any time $t > 0$ the state vector of the system is given by

$$|\Psi(t)\rangle = |C\rangle\,|e\rangle + |D\rangle\,|g\rangle\,, \quad (14)$$

where we written the bimodal field state as

$$|C\rangle = \sum_{n_1,n_2=0}^{\infty} q_{n_1,n_2} e^{-i\lambda t F_{n_1,n_2}} \times \\ \left[\cos\lambda t\nu_{n_1,n_2} - \frac{iW_{n_1,n_2}}{2\nu_{n_1,n_2}}\sin\lambda t\nu_{n_1,n_2}\right]|n_1,n_2\rangle\,, \quad (15)$$

$$|D\rangle = -i\sum_{n_1,n_2=0}^{\infty} q_{n_1,n_2} V_{n_1,n_2} e^{-i\lambda t G_{n_1,n_2}} \times \\ \frac{\sin\lambda t\nu_{n_1,n_2}}{2\nu_{n_1,n_2}}|n_1+k_1,n_2+k_2\rangle\,, \quad (16)$$

therefor, at any time $t > 0$ the density matrix for the system is given by

$$\rho(t) = |\Psi(t)\rangle\langle\Psi(t)| = \begin{pmatrix} |C\rangle\langle C| & |C\rangle\langle D| \\ |D\rangle\langle C| & |D\rangle\langle D| \end{pmatrix}, \quad (17)$$

and the reduced density matrix of the filed of system can be written as

$$\rho_f(t) = Tr_a[\rho(t)] = |C\rangle\langle C| + |D\rangle\langle D|\,. \quad (18)$$

Using the standard teqnique in Ref. [12], it can be shown that the eigenvalues of the reduced density operator $\rho_f(t)$ are given by

$$\pi_f^{\pm} = \langle C|C\rangle \pm \exp(\mp\delta)\,|\langle C|D\rangle| \\ = \langle D|D\rangle \pm \exp(\pm\delta)\,|\langle C|D\rangle|\,, \quad (19)$$

where

$$\delta = \sinh^{-1}\left[\frac{\langle C|C\rangle - \langle D|D\rangle}{2\,|\langle C|D\rangle|}\right],$$

$$\langle C|C\rangle = \sum_{n_1,n_2=0}^{\infty} |q_{n_1,n_2}|^2 \times \\ \left[\cos^2\lambda t\nu_{n_1,n_2} + \frac{W^2_{n_1,n_2}}{4\nu^2_{n_1,n_2}}\sin^2\lambda t\nu_{n_1,n_2}\right], \quad (20)$$

$$\langle D|D\rangle = \sum_{n_1,n_2=0}^{\infty} |q_{n_1,n_2}|^2 V^2_{n_1,n_2}\frac{\sin^2\lambda t\nu_{n_1,n_2}}{4\nu^2_{n_1,n_2}}, \quad (21)$$

$$\langle C|D\rangle = R(t) + iU(t) \quad (22)$$

where

$$R(t) = \sum_{n_1,n_2=0}^{\infty} q^*_{n_1+k_1,n_2+k_2} q_{n_1,n_2}\frac{V_{n_1,n_2}}{2\nu_{n_1,n_2}}[\frac{W_{n_1,n_2}}{2\nu_{n_1+k_1,n_2+k_2}} \times \\ \sin\lambda t\nu_{n_1+k_1,n_2+k_2}\sin\lambda t\nu_{n_1,n_2} \times \\ \cos\lambda t(F_{n_1+k_1,n_2+k_2} - G_{n_1,n_2}) \\ +\sin\lambda t\nu_{n_1,n_2}\cos\lambda t\nu_{n_1+k_1,n_2+k_2} \times \\ \sin\lambda t(F_{n_1+k_1,n_2+k_2} - G_{n_1,n_2})], \quad (23)$$

$$U(t) = \sum_{n_1,n_2=0}^{\infty} q^*_{n_1+k_1,n_2+k_2} q_{n_1,n_2}\frac{V_{n_1,n_2}}{2\nu_{n_1,n_2}}[\frac{W_{n_1,n_2}}{2\nu_{n_1+k_1,n_2+k_2}} \times \\ \sin\lambda t\nu_{n_1+k_1,n_2+k_2}\sin\lambda t\nu_{n_1,n_2} \times \\ \sin\lambda t(F_{n_1+k_1,n_2+k_2} - G_{n_1,n_2}) \\ -\sin\lambda t\nu_{n_1,n_2}\cos\lambda t\nu_{n_1+k_1,n_2+k_2} \times \\ \cos\lambda t(F_{n_1+k_1,n_2+k_2} - G_{n_1,n_2})]. \quad (24)$$

3 Quantum Entropy

Following the work by P-K in Ref. [12], we can express the reduced quantum entropy $S_f(t)$ of the two-mode coherent field in term of eigenvalues $\pi_f^{\pm}$ of the reduced density matrix $\rho_f(t)$, given by expression (19), that is

$$S_f(t) = -\left[\pi_f^+\ln(\pi_f^+) + \pi_f^-\ln(\pi_f^-)\right]. \quad (25)$$

If the minimum value of $S_f(t)$ is taken to zero, the two-mode coherent field and the two-level atom are disentangled, if the maximum value of $S_f(t)$ is taken to be one, the two-mode coherent field and the two-level atom are in maximal quantum entangled state, if the value of $S_f(t)$ is between zero and one, the two-mode coherent field and the two-level atom are in usual quantum entangled states.

The atomic population inversion of the atom is important quantity in the atomic dynamics. It is measure the difference in the populations of the two levels of the atom and plays a fundemental role in the laser theory [31]. After determining $A_{n_1,n_2}(t)$ and $B_{n_1,n_2}(t)$ for the initial field state in Eq. (1), we can investigate the quantity

$$W(t) = \sum_{n=0}^{\infty} |q_{n_1,n_2}|^2\left\{|A_{n_1,n_2}(t)|^2 - |B_{n_1,n_2}(t)|^2\right\}. \quad (26)$$

By inserting Eqs. (7) and (8) in equation (26), we obtain

$$W(t) = \sum_{n=0}^{\infty} |q_{n_1,n_2}|^2\{\cos^2\lambda t\nu_{n_1,n_2} + \\ [W^2_{n_1,n_2} - V^2_{n_1,n_2}]\frac{\sin^2\lambda t\nu_{n_1,n_2}}{4\nu^2_{n_1,n_2}}\}. \quad (27)$$

4 Discussion of results

In this section we examine the temporal evolution of the field quantum entropy and atomic inversion related to the present model as a function of the scaled time λt. We have invoked numerically sound truncation criteria. To ensure an excellent accuracy the behavior of both of the field entropy and atomic inversion functions have been determined with great precision. For all our plots the initial condition has been chosen, with coherence parameter α_j real. Its square is equal to the mean photon number. we have taken $k_1 = k_2 = 1,\ \chi_1 = \chi_2 = \chi$ and $\bar{n} = \bar{n}_1 = \bar{n}_2 = 5$.

As we see in Fig. 1(a), when all parameter equal zero, the field entropy evolves collapse and revival with decreases of the amplitude of oscillation in the time evolution process. Also, in this case the number of flactuations is less than the case that in one-mode. As soon as we increase the value of the parameter μ/λ, the amplitude of oscillation decreases more and more in the time evolution process (see Fig. 1(e) compared to Fig. 1(a)). The behavior of the atomic inversion has shown in Fig. 1((b), (d), (f)). It is clear that with increasing the value of parameter μ/λ, the mean value of oscillations shifts upward. To visualize the influence of the Kerr-like medium and the time-dependent coupling parameter in the field entropy and atomic inversion, we have plotted Figs. 2-4. In Fig. 2(a), we set $\chi/\lambda = 0.01$, and all the other parameters are the same as in Fig. 1(a). It is clearly the the number of oscillations is decrease and the field entropy reached its maximum value faster than the case that in Fig. 1(a). With the increase of the value of the parameter μ/λ, the amplitude of oscillation is decrease and the field entropy reaches its maximum value in the time evolution process (see Fig. 2((c), (e))). This means that, when the nonlinear interaction of the Kerr-like medium with the field mode is very weak the degree of entanglement between the field and the atom is unreduced. The behavior of the atomic inversion in Fig. 2((b), (d), (f)) as the same that in Fig. 1((b), (d), (f))). Since the increase of the nonlinear interaction of the Kerr-like medium with the field mode ($\chi/\lambda = 0.1$), the value of the maximum field entropy still approximately at 0.7 (see Fig. 3(a)). We note that the amplitude of the field entropy decreases as χ/λ increases. Also, with the increase of value of the parameter μ/λ, the value of maximum field entropy decreases (see Fig. 3(e)). In this case, the degree of entanglement between the field and the atom reduces. The mean value of the atomic inversion in Fig. 3(b) become positive compared to Fig. 2(b). When $\mu/\lambda = 2$, the mean value of the atomic inversion shifted upward greater than the case that in Fig. 2(f). When $\chi/\lambda = 0.5$, and all the other parameters equall zero, the value of the maximum field entropy decreased more than the previous case with decreases of the amplitude of oscillation in the time evolution process (see Fig. 4(a)).

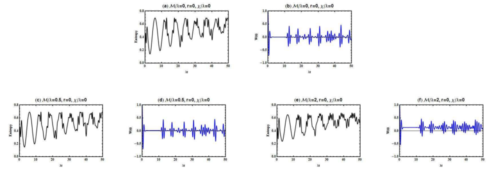

Fig. 1: Evolution of field entropy $S_f(t)$ (left curves) and atomic inversion $W(t)$ (right curves) of a two level a tom interacting with a two-mode coherent field for the parameters $\overline{n} = 5, \Delta/\lambda = 0,\ r = 0,\ \chi/\lambda = 0$.

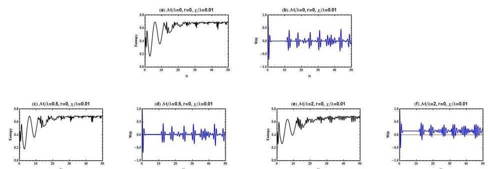

Fig. 2: The same as in Fig. 1 but for $\chi/\lambda = 0.01$.

With the increase of the value of the parameter μ/λ, the value of the maximum field entropy decreases more and more. This means that the degree of entanglement between the field and the atom decreases more and more (see Fig. 4((c), (e))). The effect of Kerr-like medium on the atomic occupation number results in inhibiting energy in the atomic system. The more χ/λ increases, the higher the mean values for $W(t)$ (see Fig. 4((b),(d),(f))). The effect of Stark shift parameter on the evolution of the field entropy and atomic inversion in the absence of Kerr-like medium has shown in Figs. 5-7. In our computations, we have taken $\bar{n}_1 = \bar{n}_2 = 5$. In Fig. 5(a), we show the case in which the two levels have unequal Stark shifts ($r < 1$). We see that the Stark shifts leads to decreasing of the values of the maximum field entropy but, with increasing the value of paramter μ/λ, the value of the maximum field entropy increases again (see Fig. 5((c), (e))) and the mean value of the atomic inversion decreases (see Fig. 5((d), (f))). It is remarkable that when $r = 1$ (i.e, $\beta_1 = \beta_2$), which corresponds to the case in which the two levels of the atom are equally strongly coupled with the intermediate relay level. We see that the evolution of the field entropy is almost similar to the case in Fig. 1. In Fig. 6(a), when $r = 2$, the maximum field entropy increased compared to the case in which $r < 1$. With the increase of the value of the parameter μ/λ, the value of the maximum field entropy decreases (see Fig. 6((c), (e))), and the mean value of the atomic inversion increases (see Fig. 6((d), (f))), While when $r \gg 1$, the values of the maximum field entropy decrease which indicates that the quantum entanglement between the field and the atom turns worse and worse (see Fig. 7((a), (c), (e))). This may be

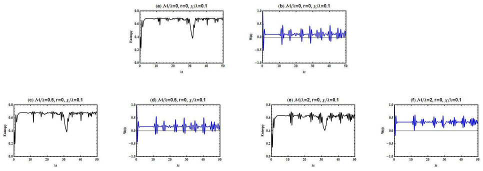

Fig. 3: The same as in Fig. 1 but for $\chi/\lambda = 0.1$.

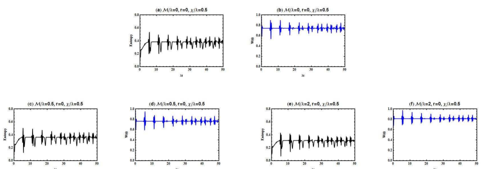

Fig. 4: The same as in Fig. 1 but for $\chi/\lambda = 0.5$.

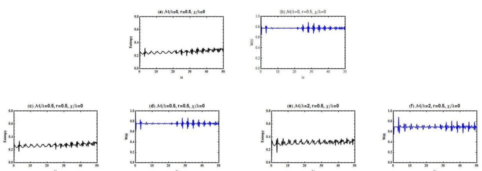

Fig. 5: The same as in Fig. 1 but for $\Delta/\lambda = 5,\ r = 0.5$.

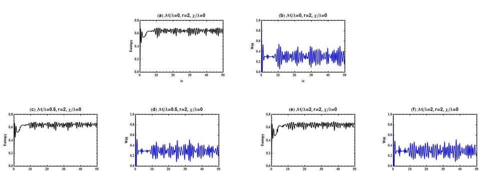

Fig. 6: The same as in Fig. 1 but for $\Delta/\lambda = 10,\ r = 2$.

interpreted the detuning due to the Stark shift results in disentanglement of the system. The influence of Stark shift in the presence of Kerr-like medium has plotted in Fig. 8. It is to be noted that Stark interaction behaves like the limiting case of the Kerr-interaction. This may be understood in the following way: the Kerr interaction produces two separate effects, a Kerr effect, which splits the field in phase space, producing a Schrödinger cat [32, 33], and a Stark interaction with the field in a cat state. The atom field interaction when the field is initially in a cat state has been shown to be less pure than for the field in a coherent state. It has been shown [34] taking into acount Stark shifts in the atom field interaction allows agreement with experimental results of micromasers [35], such as a shifted transition lineshapes and those asymmetrically distorted.

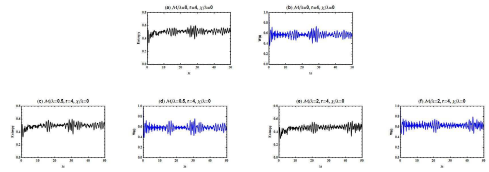

Fig. 7: The same as in Fig. 1 but for $\Delta/\lambda = 15,\ r = 4$.

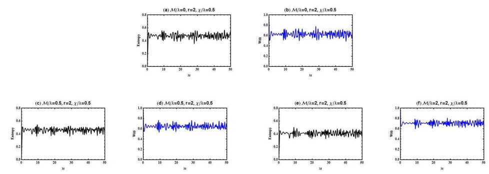

Fig. 8: The same as in Fig. 6 but for $\chi/\lambda = 0.5$.

5 Conclusion

In summary, we have investigated the time evolution of the field quantum entropy for the bimodal field interacting with an effective two-level atom via Raman transition. Also, The system has been chosen to include the effect of both Kerr-like medium as well as Stark shift. The coupling parameter between the atom and the bimodal field is modulated to be time-dependent. The exact expression of atom-field wave function is obtained, which provides the ability to detect the details of the field quantum entropy adjacent to the time-dependent or independent atom-field interactions. Under certain approximation similar to that of the rotating-wave approximation with $X_n \simeq \mu$ The field quantum entropy calculation of a two-level atomic system has been introduced and its time evolution has been discussed in a coparison with the atomic inversion behavior. The results show that the strong effect of Stark shift and Kerr-like medium on the field quantum entropy in the time-dependent system comparison with the time-independent system.

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