

Fractional Order Study of the Impact of a Photo Thermal Wave on a Semiconducting Medium under Magnetic Field and Thermoplastic Theories

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Abstract: The study aims to estimate the extent to which the reflection wave is affected by the initial stress, magnetic field, and fractional parameter within a semiconductor photothermal diffusion medium when taking into account the Classical (CT) theory and the dual-phase-lag (DPL) theory. Its problem is defined in examining the generalized framework concerning plasma, thermoelastic waves under the thermomechanical responding of reflecting the photothermal diffusion for the semiconductor constituent. It applies the Maxwell's equations while considering the absence of the medium of infinite conducting and displacement current. Moreover, it applies the boundary settings for the Maxwell's and mechanical stress, diffusion, chemical reaction, as well as temperature gradient on the interface next to the vacuum. It obtains analytically and displays graphically the ratios of the reflection coefficient as tasks of the angle of incidence, diffusion, initial stress, magnetic field, semiconducting, and photothermal. It compares the CT to the DPL theory. It also compares its findings to the results of the literature. The results of the paper include generalizing the photothermal semiconductor medium and deducing to a special case when neglecting the novel parameters. When neglecting the magnetic field, fractional parameter, and initial stress, the findings of the current study deduce to the findings of Lotfy et al. (2020) as a special case of study.

Keywords: Magnetic field, Photothermal, Initial stress, Diffusion, Thermoelasticity, Semiconductor, Reflection, CT, DPL, Conformable fractional derivative, differential equations.

1 Introduction

Several natural materials are significant in the industry because they are highly applicable in renewable energy. For instance, semiconductor materials that are significant economically in the field of solare cells are found abundantly in nature. Exposing a semiconductor medium to the focus of laser or sunlight beams excites the surface electrons at the free surface. Many kinds of fractional integral and differential operators exist. The first one is the Riemann-Liouville definition [1–5] that contains the fractional differential of a constant, which is not zero. Caputo puts definitions that provide the value of zero concerning fractional differential of a constant. However, these definition require that the function are differentiable and smooth. Jumarie introduced modified Riemann-Liouville [16–23] - another concept of the fractional derivative and integral- which fit continuous and non-differentiable functions having the fractional differential of a constant, which equals zero [16–24]. Lately, the authors of [25] provided a novel fractional derivative known as “the

conformable fractional derivative” according to the familiarity boundary definition of the function's derivative. Abdeljawad [26] developed this new model. Yet, the fractional derivative and the conformal fractional derivative are not similar. Rather, the later represents a basic derivative increased using a further simple factor. Consequently, the novel definition is a natural extension of the classical derivative. Though, it differs from other model. It integrates the typical characteristics of fractional derivatives. It fits several extensions of the traditional calculus theory, including the derivative of a product and combining two issues, the Rolle's and the mean value theory, conformable integration by parts, expansion of fractional power series, etc. [27–33]. Abd-alla [34] investigated the relaxation impacts on reflecting the generalized magneto-thermoelastic waves. The authors of [35] examined the generalized wave impact on a micropolar thermodiffusion elastic half-space based on the initial stress and electromagnetic field. Abo-Dahab et al. [36] investigated the thermal stress of generalized magneto-thermoelasticity on a non-homogeneous orthotropic continuum solid having a sphere-shaped cavity. Abo-Dahab et al. [37] discussed reflecting plane waves on a generalized

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thermoelastic medium affected by initial stresses and temperature-dependent characteristics with the three-phase-lag system. Lotfy et al. [38] explored the model of thermomechanical response of the Reflection Photothermal Diffusion Waves (RPTD) to semiconductor media.

The present paper explored how reflection wave is affected by the magnetic field, fractional parameter, and initial stress within a semiconductor photothermal diffusion medium. In this model technique, we depend on the interaction between plasma waves, magnetic field, fractional parameter and initial stress, thermal waves, elastic waves, and diffusion of mass. The study's problem is defined in examining the generalized model for plasma, thermoelastic waves affected by the thermomechanical responding of reflecting the photothermal diffusion for the semiconductor constituent. It applies the Maxwell's equations while considering the absence of the medium of infinite conducting and displacement current. Moreover, it applies the boundary settings for the Maxwell's and mechanical stress, diffusion, chemical reaction, as well as temperature gradient on the interface next to the vacuum. It obtains analytically and displays graphically the ratios of the reflection coefficient as tasks of the incidence angle, diffusion, initial stress, magnetic field, semiconducting, and photothermal. It compares the CT to the DPL theory. It also compares its findings to the results of the literature. It derives analytically and displays in graphs the ratios of the reflection coefficient as roles of the semiconducting, diffusion, initial stress, magnetic field, as well as angle of incidence.

2 Fractional Problem Formulations

The authors take into account a standardized isotropic generalized magneto-thermoelastic half-space with initial stress. Analyzing the transport process theoretically within a semiconductor involves taking into account elastic waves, thermal waves, and coupled plasma waves concurrently as the major variable quantities. The transport process shows in a semiconductor medium during the three types of waves.

Using the Cartesian coordinates (x, y, z) in two dimensions (x, z) ($\vec{r}$) denotes the position vector and the time variable t , the problem is studied. We must consider that the circular plate is very thin, linear homogeneous, and having isotropic features.

We take the governing equations in generalized cases while having the parameter of thermal activation coupling that is describable via transporting coupled plasma, thermal, magnetic field, initial stress, and elastic properties of the medium.

The fractional-order governing equations for coupled plasma, thermal and elastic transport are represented as:

$$D_t^\alpha N(\vec{r}, t) = D_e \nabla_a^2 N(\vec{r}, t) - \frac{N(\vec{r}, t)}{\tau} + \kappa T(\vec{r}, t), \quad \nabla_a^2 = D_x^{\alpha\alpha} + D_y^{\alpha\alpha} + D_z^{\alpha\alpha}, \quad (1)$$

$$k(1 + \tau_\theta D_t^\alpha) \nabla_a^2 T(r, t) + \frac{E}{\tau} N(r, t) = (D_t^\alpha + \tau_q D_t^{\alpha\alpha}) (\rho C T(r, t) + \beta_1 T_0 D_t^\alpha u(r, t) + c T_0 C), \quad (2)$$

The fractional equation of motion with carrier density considering Lorenz's body forces and initial stress takes the form:

$$\rho D_t^{\alpha\alpha} u_i = \mu D_j^{\alpha\alpha} u_i + (\mu + \lambda) D_i^\alpha (D_j^\alpha u_j) - (1 + \tau_\theta D_t^\alpha) (\beta_1 D_i^\alpha T - \beta_2 D_i^\alpha C) - \delta_n D_i^\alpha N + D_i^\alpha F - p D_j^\alpha \omega_{ij}, \quad (3)$$

The diffusion of the mass equation is represented as:

$$D_c \beta_2 e_{ii} + D_c c D_t^{\alpha\alpha} T(r, t) + (D_t^\alpha + \bar{\tau} D_t^{\alpha\alpha}) C(r, t) = D_c b D_t^{\alpha\alpha} C(r, t). \quad (4)$$

where

$$F_i = \left(\vec{J} \times \vec{B} \right)_i. \quad (5)$$

The various values of the electric and magnetic fields provided by Maxwell's equation neglecting the displacement current take the following form:

$$\begin{aligned} \text{curl}_a \vec{h} = \vec{J}, \quad \nabla_a = D_x^\alpha \vec{i} + D_y^\alpha \vec{j} + D_z^\alpha \vec{k}, \quad \text{curl}_a \vec{h} = \nabla_a \times \vec{h} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ h_1 & h_2 & h_3 \end{vmatrix}, \\ -\mu_e D_t^\alpha \vec{h} = \text{curl}_a \vec{E}, \quad \text{curl}_a \vec{E} = \nabla_a \times \vec{E} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ E_1 & E_2 & E_3 \end{vmatrix}, \end{aligned} \quad (6)$$

$$\text{div}_a \vec{h} = 0, \quad \text{div}_a \vec{h} = \nabla_a \cdot \vec{h},$$

$$\text{div}_a \vec{E} = 0, \quad \text{div}_a \vec{E} = \nabla_a \cdot \vec{E},$$

$$\vec{E} = -\mu_e (D_t^\alpha \vec{u} \times \vec{H}_0), \quad \vec{h} = \text{curl}(\vec{u} \times \vec{H}_0), \quad \vec{F}_i = \mu_e (\vec{J} \times \vec{H}_0)_i,$$

where

$$\vec{H} = \vec{H}_0 + \vec{h}(x, y, t), \quad \vec{H}_0 = (0, H_0, 0).$$

Utilizing Eq. (6) gives

$$F_x = \mu_e H_0^2 D_x^{\alpha\alpha} e, \quad (7)$$

$$F_z = \mu_e H_0^2 D_z^{\alpha\alpha} e, \quad (8)$$

$$F_y = 0. \quad (9)$$

Once more, Maxwell's stress equation is

$$\tau_{ij} = \mu_e [H_i h_j + H_j h_i - H_k h_k \delta_{ij}]$$

This decreases to

$$\tau_{11} = \tau_{22} = \mu_e H^2 (D_x^{\alpha\alpha} u + D_y^{\alpha\alpha} v), \quad \tau_{12} = 0. \quad (10)$$

In equation (2), the second concept to the right denotes the impact of heat generated by the surface de-excitations and carrier volume in the sample, whereas the third one defines stress wave-generated heat. The third and fourth concepts in Eq. (3) in RHS define the source term and the impact of the thermal and plasma waves on the fractional elastic wave, correspondingly.

Because the analysis is limited to xz -plane, the

fractional displacement is identified using

$$\vec{u} = (u, 0, w), \quad u(x, z, t), \quad w(x, z, t).$$

The constitutive relationships are represented as:

$$\sigma_{xx} = (2\mu + \lambda)D_x^\alpha u + \lambda D_z^\alpha w - (1 + \tau_\theta D_t^\alpha)(\beta_1(T - T_0) - \beta_2 C) - (3\lambda + 2\mu)d_n N - p, \quad (11)$$

$$\sigma_{zz} = (2\mu + \lambda)D_z^\alpha w + \lambda D_x^\alpha u - (1 + \tau_\theta D_t^\alpha)(\beta_1(T - T_0) - \beta_2 C) - (3\lambda + 2\mu)d_n N - p, \quad (12)$$

$$\sigma_{xx} = (\mu - \frac{p}{2})D_x^\alpha w + (\mu + \frac{p}{2})D_z^\alpha u. \quad (13)$$

The chemical potential of the material equation takes the form of generalizing the Fourier law and standard thermal conductivity equation.

Thus, the equations of chemical potential are

$$P = -\beta_2 e_{nn} + bC - c(T - T_0), \quad (14)$$

By Helmholtz's theory, the vector of fractional displacement $\vec{u}$ takes the form of the functions of the fractional displacement scalar potential $\Pi(x^\alpha, z^\alpha, t^\alpha)$ and $\Psi(x^\alpha, z^\alpha, t^\alpha)$ identified using the relationships non-dimensionally:

$$\vec{u} = \text{grad}_\alpha \Pi + \text{curl}_\alpha \Psi, \quad (15)$$

$$\Psi = (0, \psi, 0)$$

That declines to

$$u = D_x^\alpha \Pi - D_z^\alpha \psi, \quad w = D_z^\alpha \Pi + D_x^\alpha \psi, \quad (16)$$

The equations of the fractional field and constitutive relationships at the plane surface of (equation (3)) in generalized linear elasticity influenced by gravity, without thermal sources and body forces take the form:

$$\rho D_t^\alpha u = \left(\mu - \frac{p}{2} \right) \nabla_x^\alpha u + (\mu + \lambda + \mu_e H_o^2 + p) D_x^\alpha e - (1 + \tau_\theta D_t^\alpha)(\beta_1 D_x^\alpha T - \beta_2 D_x^\alpha C) + (3\lambda + 2\mu)d_n D_x^\alpha N, \quad (17)$$

$$\rho D_t^\alpha w = \left(\mu - \frac{p}{2} \right) \nabla_z^\alpha w + (\mu + \lambda + \mu_e H_o^2 + p) D_z^\alpha e - (1 + \tau_\theta D_t^\alpha)(\beta_1 D_z^\alpha T - \beta_2 D_z^\alpha C) + (3\lambda + 2\mu)d_n D_z^\alpha N. \quad (18)$$

To make it simpler, we employ these non-dimensional variables

$$\sigma'_{ij} = \frac{\sigma_{ij}}{\mu}, \quad P' = \frac{P}{\beta_2}, \quad (x'^\alpha, z'^\alpha, u', w') = \frac{(x^\alpha, z^\alpha, u, w)}{C_T t^*}, \quad p' = \frac{p}{\mu},$$

$$(t'^\alpha, \tau'_q, \tau'_\theta) = \frac{(t^\alpha, \tau_q, \tau_\theta)}{t^*}, \quad H' = \frac{H}{\rho C_T}, \quad N' = \frac{\delta_n N}{2\mu + \lambda + \mu_e H_o^2},$$

$$(\Pi', \psi') = \frac{(\Pi, \psi)}{(C_T t^*)^2}, \quad T' = \frac{\beta_1(T - T_0)}{2\mu + \lambda + \mu_e H_o^2}, \quad C' = \frac{\beta_2 C}{2\mu + \lambda + \mu_e H_o^2},$$

Thus, utilizing the scalar function (9) and Eq. (10) in Eqs. (1), (2), (4), (11), and (12), results in (neglecting the dashed for convenience)

$$(\nabla_\alpha^2 - q_1 - q_2 D_t^\alpha)N + \varepsilon_3 T = 0, \quad (19)$$

$$(1 + \tau_\theta D_t^\alpha) \nabla_\alpha^2 T - (D_t^\alpha + \tau_q D_t^\alpha) \{T + \varepsilon_1 \nabla_\alpha^2 \Pi + \varepsilon_4 C\} - \varepsilon_2 N = 0, \quad (20)$$

$$(\nabla_t^2 - D_t^\alpha) \Pi - (1 + \tau_\theta D_t^\alpha) T - N = 0, \quad (21)$$

$$(\nabla_\alpha^2 - \beta^2 D_t^\alpha) \psi = 0, \quad (22)$$

$$\nabla_\alpha^2 \Pi + q_4 \nabla_\alpha^2 T + q_5 (D_t^\alpha + \tau D_t^\alpha) C - q_3 \nabla_\alpha^2 C = 0, \quad (23)$$

$$P = -\nabla_\alpha^2 \Pi + q_3 C - q_4 T, \quad (24)$$

where

$$q_1 = \frac{kt^*}{D_E \rho \tau C_e}, \quad q_2 = \frac{k}{D_E \rho C_e}, \quad \varepsilon_1 = \frac{\gamma^2 T_0 t^{*2}}{k \rho},$$

$$\varepsilon_2 = -\frac{\alpha_T E_g t^*}{d_n \rho \tau C_e}, \quad \varepsilon_3 = \frac{d_n k \kappa t^*}{\alpha_T \rho C_e D_E}, \quad \varepsilon_4 = \varepsilon_3 \frac{c C_T^2}{\beta_1 \beta_2},$$

$$C_T^2 = \frac{2\mu + \lambda + \mu_e H_o^2 + p}{\rho}, \quad C_L^2 = \frac{\mu - \frac{p}{2}}{\rho},$$

$$\beta^2 = \frac{C_T^2}{C_L^2}, \quad \delta_n = (2\mu + 3\lambda)d_n, \quad t^* = \frac{k}{\rho C_e C_T^2},$$

$$q_3 = \frac{b \rho C_T^2}{\beta_2^2}, \quad q_4 = \frac{c \rho C_T^2}{\beta_1 \beta_2},$$

$$q_5 = \frac{(2\mu + \lambda)t^{*2} C_T^2}{D \beta_2^2}$$

Equations (11)-(13)) in the non-dimensional form become:

$$\sigma_{xx} = 2D_x^\alpha \Pi + \frac{\lambda}{\mu} \nabla_x^\alpha \Pi + 2D_x^\alpha D_z^\alpha \psi - \frac{(2\mu + \lambda)}{\mu} ((1 + \tau_\theta D_t^\alpha)(T + C) + N) - p, \quad (25)$$

$$\sigma_{zz} = 2D_z^\alpha \Pi + \frac{\lambda}{\mu} \nabla_z^\alpha \Pi - 2D_x^\alpha D_z^\alpha \psi - \frac{(2\mu + \lambda)}{\mu} ((1 + \tau_\theta D_t^\alpha)(T + C) + N) - p, \quad (26)$$

$$\sigma_{xz} = (1 + \frac{p}{2}) D_z^\alpha \psi + 2D_x^\alpha D_z^\alpha \Pi - (1 - \frac{p}{2}) D_x^\alpha \psi, \quad (27)$$

3 Solving the problem

Concerning a fractional propagated harmonic wave where the normal exists in the xz-plane and creates an angle θ with the positive direction of the z-axis, the authors argue that the solutions of the system in equations (19)-(24) are represented as:

$$\{N, \Pi, \psi, C, T\} = \{\bar{N}, \bar{\Pi}, \bar{\psi}, \bar{C}, \bar{T}\} e^{i\xi(x^\alpha \sin \theta + z^\alpha \cos \theta) - i\omega t^\alpha}, \quad (28)$$

where ξ the wave is numbered, and ω denotes the complex circular frequency.

We substitute from equation (28) into equations (19)-(24) and reach a 5 homogeneous algebraic equation- system, as follows:

$$(\alpha^2 \xi^2 + \alpha_1) \bar{N} - \varepsilon_3 \bar{T} = 0, \quad (29)$$

$$(\alpha^2 (1 - i\alpha\omega\tau_\theta) \xi^2 - i\alpha\omega(1 - i\alpha\omega\tau_q)) \bar{T} + i\alpha\omega(1 - i\alpha\omega\tau_q)(\varepsilon_1 \alpha^2 \xi^2 \bar{\Pi} - \varepsilon_4 \bar{C}) + \varepsilon_2 \bar{N} = 0, \quad (30)$$

$$\alpha^2 (\xi^2 - \omega^2) \bar{\Pi} + (1 - i\alpha\omega\tau_\theta) \bar{T} + \bar{N} = 0, \quad (31)$$

$$\alpha^2 \xi^2 \bar{\Pi} + \alpha^2 q_4 \xi^2 \bar{T} + [i\alpha\omega q_5 (1 - i\alpha\omega\tau) - \alpha^2 q_3 \xi^2] \bar{C} = 0, \quad (32)$$

$$\alpha^2 (\xi^2 - \beta^2 \omega^2) \bar{\psi} = 0, \quad (33)$$

where, $\alpha_1 = q_1 - i\alpha\omega q_2$.

The system of equations (29)–(33) demonstrate significant solutions if and only if determining the factor matrix disappears. Thus,

$$\begin{vmatrix} \alpha^2 \xi^2 + \alpha_1 & -\varepsilon_3 & 0 & 0 & 0 \\ \varepsilon_2 & \alpha^2(1-i\alpha\omega\tau_\theta)\xi^2 - i\alpha\omega(1-i\alpha\omega\tau_q) & i\alpha^3\omega\varepsilon_3\xi^2(1-i\alpha\omega\tau_q) & -i\alpha\varepsilon_4\omega(1-i\alpha\omega\tau_q) & 0 \\ 1 & (1-i\alpha\omega\tau_\theta) & \alpha^2(\xi^2 - \omega^2) & 0 & 0 \\ 0 & 0 & 0 & 0 & \alpha^2(\xi^2 - \beta^2\omega^2) \end{vmatrix} = 0, \quad (34)$$

This denotes an algebraic equation $v = \frac{\omega}{\xi}$, in which v

represents the coupled wave's velocity:

$$\left(v^2 - \frac{1}{\beta^2}\right)(v^8 + A_1v^6 + A_2v^4 + A_3v^2 + A_4) = 0, \quad (35)$$

where

$$A_1 = \alpha\omega \frac{a_4}{a_6}, \quad (36)$$

$$A_2 = \alpha\omega \frac{a_3}{a_6}, \quad (37)$$

$$A_3 = \alpha^3\omega^3 \frac{a_2}{a_6}, \quad (38)$$

$$A_4 = \alpha^5\omega^5 \frac{a_1}{a_6}, \quad (39)$$

$$a_1 = -q_5(1-i\alpha\omega\tau_\theta), \quad (40)$$

$$a_2 = q_5(1-i\alpha\omega\tau_\theta)(i\alpha\omega(1-i\alpha\omega\tau) - \alpha^2\omega^2 - \alpha_1) + i\alpha\omega(1-i\alpha\omega\tau_q)(q_5 + q_4\varepsilon_4), \quad (41)$$

$$\begin{aligned} a_3 = & (1-i\alpha\omega\tau_q)(1-i\alpha\omega\tau_\theta)[- \alpha^2\omega^2\varepsilon_3q_5(1-i\alpha\omega\tau) + \alpha^2\omega^2\varepsilon_4] + (1-i\alpha\omega\tau_\theta)[i\alpha\omega q_5(1-i\alpha\omega\tau) \\ & - \alpha_1\alpha^2\omega^2q_5 + \alpha^2\omega^2q_5(1-i\alpha\omega\tau)] + (1-i\alpha\omega\tau_q)[\alpha^2\omega^2 + i\alpha^3\omega^3q_5 + i\alpha_1\alpha\omega q_5 + i\alpha^3\omega^3\varepsilon_4q_4 \\ & + i\alpha\omega\varepsilon_1\varepsilon_3q_5] + i\alpha\omega q_5(1-i\alpha\omega\tau) + \varepsilon_3q_5, \end{aligned} \quad (42)$$

$$\begin{aligned} a_4 = & (1-i\alpha\omega\tau)(1-i\alpha\omega\tau_q)[- \alpha_1\alpha^2\omega^2\varepsilon_3q_5(1-i\alpha\omega\tau_\theta) + \alpha^4\omega^4q_5 + \alpha_1\alpha^2\omega^2q_5 + \alpha^2\omega^2\varepsilon_1\varepsilon_3q_5] \\ & + (1-i\alpha\omega\tau_q)[i\alpha_1\alpha\omega\varepsilon_4(1-i\alpha\omega\tau_\theta) - i\alpha_1\alpha^3\omega^3q_5 + i\alpha_1\alpha^3\omega^3\varepsilon_4q_4 + i\alpha\omega\varepsilon_3\varepsilon_4] \\ & + i\alpha_1\alpha^3\omega^3q_5(1-i\alpha\omega\tau)(1-i\alpha\omega\tau_\theta) + i\alpha\omega\varepsilon_3\varepsilon_3q_5(1-i\alpha\omega\tau) + \alpha^2\omega^2\varepsilon_3q_5, \end{aligned} \quad (43)$$

$$a_5 = \alpha^3\omega^3(1-i\alpha\omega\tau)[\alpha_1\alpha\omega(1-i\alpha\omega\tau_q) - i\varepsilon_2\varepsilon_3q_5], \quad (44)$$

$$a_6 = (1+i\alpha\omega\tau)[\alpha_1\alpha\omega(1+i\alpha\omega\tau_q) + i\varepsilon_3^2q_5],$$

Then, we can obtain the velocities of five waves from the equation (35). They are denoted to thermal, elastic, plasma, as well as diffusion waves and have the velocities

v_i , $i = 1, 2, 3, 4$ and rotational wave, respectively, v_5 denoted to rotational wave are expressed analytically as $v_5 = 1/\beta$.

4 Solving incident p-wave:

Because equation (31) is fifth order in v^2 , four coupled waves have three diverse velocities and a rotational wave with the velocity $v = 1/\beta$. Neglect the radiation into the vacuum, when a coupled wave falls on the boundary ($z = 0$) from the inside of the elastic semiconducting means, which creates an angle θ with the z-axis's negative direction, and five reflected waves that create angle θ and θ_i ($i = 1, 2, \dots, 5$) in that direction (see Fig. 1).

The potentials of the fractional displacement Π, Ψ and the quantities of the fractional field T, N , and C are expressed as

$$\Pi = E_1 e^{i\xi_1(x^\alpha \sin \theta_1 + z^\alpha \cos \theta_1) - i\omega t^\alpha} + \sum_{i=1}^4 F_i e^{i\xi_i(x^\alpha \sin \theta_i + z^\alpha \cos \theta_i) - i\omega t^\alpha},$$

$$\Psi = B_1 e^{i\xi_5(x^\alpha \sin \theta_5 + z^\alpha \cos \theta_5) - i\omega t^\alpha},$$

$$T = \eta_1 E_1 e^{i\xi_1(x^\alpha \sin \theta_1 + z^\alpha \cos \theta_1) - i\omega t^\alpha} + \sum_{i=1}^4 \eta_i F_i e^{i\xi_i(x^\alpha \sin \theta_i + z^\alpha \cos \theta_i) - i\omega t^\alpha},$$

$$N = \zeta_1 E_1 e^{i\xi_1(x^\alpha \sin \theta_1 + z^\alpha \cos \theta_1) - i\omega t^\alpha} + \sum_{i=1}^4 \zeta_i F_i e^{i\xi_i(x^\alpha \sin \theta_i + z^\alpha \cos \theta_i) - i\omega t^\alpha},$$

$$C = \varsigma_1 E_1 e^{i\xi_1(x^\alpha \sin \theta_1 + z^\alpha \cos \theta_1) - i\omega t^\alpha} + \sum_{i=1}^4 \varsigma_i F_i e^{i\xi_i(x^\alpha \sin \theta_i + z^\alpha \cos \theta_i) - i\omega t^\alpha},$$

where

$$\eta_i = \frac{(\alpha^2 \xi_i^2 + \alpha_1)(\alpha^2 \xi_i^2 - \alpha^2 \omega^2)}{(1-i\alpha\omega\tau_\theta)(\alpha^2 \xi_i^2 + \alpha_1) + \varepsilon_3}, \quad \zeta_i = \frac{-(\alpha^2 \xi_i^2 + \alpha_1)}{\varepsilon_3 \eta_i},$$

$$\begin{aligned} \varsigma_i = & [-\alpha^4 \xi_i^4 (1-i\tau_\theta\omega) + i\alpha^3 \omega \xi_i^2 (\alpha^2 \xi_i^2 + \alpha_1)(1-i\alpha\omega\tau_q) - \varepsilon_2 \varepsilon_3 \alpha^2 \xi_i^2 \\ & + i\alpha^5 \omega q_4 \varepsilon_1 \xi_i^4 (1-i\alpha\omega\tau_q)(\alpha^2 \xi_i^2 + \alpha_1)] \div [\alpha^2 \xi_i^2 (1-i\tau_\theta\omega)(\alpha^2 \xi_i^2 + \alpha_1) \\ & - i\alpha\omega(1-i\alpha\omega\tau_q)(\alpha^2 \xi_i^2 + \alpha_1) + \varepsilon_2 \varepsilon_3 [q_3 \alpha^2 \xi_i^2 - i\alpha\omega q_5(1-i\alpha\omega\tau)] \\ & - i\alpha^3 \omega \varepsilon_4 q_4 \xi_i^2 (\alpha^2 \xi_i^2 + \alpha_1)(1-i\alpha\omega\tau_q)], \end{aligned} \quad i = 1, 2, 3, 4.$$

The amplitude ratios of the fractional reflected waves and the fractional incident wave,

$$\frac{F_1}{E_1}, \frac{F_2}{E_1}, \frac{F_3}{E_1}, \frac{F_4}{E_1}, \frac{B_1}{E_1},$$

provide the conforming ratio of the reflection coefficients.

Moreover, the angle θ_i ($i = 1, 2, \dots, 5$) and the conforming

wave numbers ξ_i ($i = 1, 2, \dots, 5$) should be linked by the

following relationships based on Snell's law:

$$\xi_1 \sin \theta_1 = \xi_2 \sin \theta_2 = \xi_3 \sin \theta_3 = \xi_4 \sin \theta_4 = \xi_5 \sin \theta_5$$

On the interface $z = 0$ of the medium, the relationship (45) can take the form

$$\frac{\sin \theta_1}{v_1} = \frac{\sin \theta_2}{v_2} = \frac{\sin \theta_3}{v_3} = \frac{\sin \theta_4}{v_4} = \frac{\sin \theta_5}{v_5} \quad (46)$$

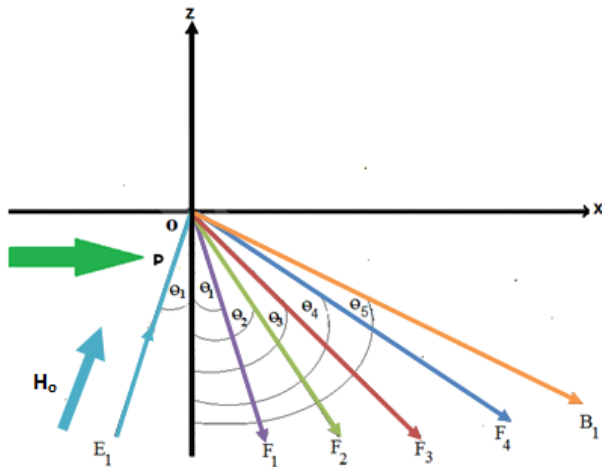


Fig.1: Outline of the problem.

5 Boundary Conditions

(i) Assume that the boundary $z = 0$ is next to vacuum. It is traction-free. Thus, the boundary setting takes the form:

$$\sigma_{zz} + \tau_{zz} = -p, \quad \sigma_{xz} + \tau_{xz} = -\frac{p}{2}(u_{z,x} - u_{x,z}),$$

(ii) Let the boundary $z = 0$ be insulated thermally, we get

$$\frac{\partial T}{\partial z} = 0$$

(iii) The carriers are able to arrive at the sample surface, with a finite probability of refusion. Therefore, the boundary setting of the density of the carrier takes the following form:

$$D \frac{\partial N}{\partial z} = -\lambda N$$

(iv) The chemical potential $P(x, z)$ at the boundary $z = 0$ is given by

$$P(x, 0) = 0$$

then,

$$\sum_{i=1}^4 \left[\left[(\lambda + 2\mu) \cos^2 \theta_i + \lambda \sin^2 \theta_i + \mu_e H_0^2 \right] \alpha^2 \xi_i^2 + \tau_\theta (\lambda + 2\mu) (\eta_i + \zeta_i) - \zeta_i \right] \frac{F_i}{E_1} \quad (47)$$

$$= - \left[\left[(\lambda + 2\mu) \cos^2 \theta_1 - \lambda \sin^2 \theta_1 + \mu_e H_0^2 \right] \alpha^2 \xi_1^2 - \tau_\theta (\eta_1 + \zeta_1) - \zeta_1 \right],$$

$$\sum_{i=1}^4 \xi_i^2 \sin 2\theta_i \frac{F_i}{E_1} + \xi_5^2 \cos 2\theta_5 \frac{B}{E_1} = \xi_1^2 \sin 2\theta_1 \quad (48)$$

$$\sum_{i=1}^4 \eta_i \cos \theta_i \frac{F_i}{E_1} = \eta_1 \cos \theta_1, \quad (49)$$

$$\sum_{i=1}^4 \zeta_i (D_e \cos \theta_i + S) \frac{F_i}{E_1} = \zeta_1 (D_e \cos \theta_1 - S), \quad (50)$$

$$\sum_{i=1}^4 (-1 + a_3 \zeta_i - a_4 \eta_i) \frac{F_i}{E_1} = 1 - a_3 \zeta_1 + a_4 \eta_1, \quad (51)$$

in which $\frac{F_i}{E_1}$ denote the coefficients of the amplitude

ratios of the reflected longitudinal waves, whereas $\frac{B}{E_1}$

denotes the coefficient of the amplitude ratio of the reflected shear waves from the following algebraic equation:

$$\sum_{j=1}^3 a_j Z_j = b_i, \quad Z_j = \frac{F_j}{E_1}, \quad j = 1, 2, 3, 4, \quad Z_5 = \frac{B}{E_1}, \quad \theta_1 = \theta_0 \quad (52)$$

where

$$a_{1j} = \left\{ \left[(\lambda + 2\mu) \cos^2 \theta_j - \lambda \sin^2 \theta_j + \mu_e H_0^2 \right] \alpha^2 \xi_j^2 + \tau_\theta (\lambda + 2\mu) (\eta_j + \zeta_j) - \zeta_j \right\}, \quad a_{15} = 0,$$

$$a_{2j} = \alpha^2 \xi_j^2 \sin 2\theta_j, \quad a_{25} = \alpha^2 \xi_5^2 \sin 2\theta_5,$$

$$a_{3j} = \eta_j \cos \theta_j, \quad a_{35} = 0,$$

$$a_{4i} = \zeta_i (D_e \cos \theta_i + S), \quad a_{45} = 0,$$

$$a_{5i} = (-1 + a_3 \zeta_i - a_4 \eta_i), \quad a_{55} = 0,$$

$$b_1 = - \left\{ \left[(\lambda + 2\mu) \cos^2 \theta_1 - \lambda \sin^2 \theta_1 + \mu_e H_0^2 \right] \alpha^2 \xi_1^2 - \tau_\theta (\eta_1 + \zeta_1) - \zeta_1 \right\},$$

$$b_2 = \alpha^2 \xi_1^2 \sin 2\theta_1,$$

$$b_3 = \eta_1 \cos \theta_1,$$

$$b_4 = \zeta_1 (D_e \cos \theta_1 - S),$$

$$b_5 = 1 - a_3 \zeta_1 + a_4 \eta_1.$$

6 Numerical Results

Si is selected as the constituent concerning numerical simulation. Its parameters are selected, see Table 1:

The thermal relaxation times are cited in the previous studies (Chandrasekharaiah 1986) for different types of materials to 10-14 sec. for metals. Concerning the semiconductor constituents, this parameter is behind metals and gases. We selected the relaxation times for silicon $\tau_0 = 1.85 \times 10^{-12} \text{ s}$, $\nu_0 = 1.334 \times 10^{-12} \text{ s}$

Table 1: Physical constants of Si (Song et al. 2014).

Parameter	Value	Parameter	Value	Parameter	Value
λ	$3.64 \times 10^{10} \text{ Nm}^{-2}$	k	$150 \text{ W m}^{-1} \text{ K}^{-1}$	T_0	800K
μ	$5.46 \times 10^{10} \text{ Nm}^{-2}$	C_e	$695 \text{ J Kg}^{-1} \text{ K}^{-1}$	b	0.9
ρ	$2.33 \times 10^3 \text{ Kg m}^{-3}$	c	1.2×10^4	D_c	0.85×10^{-8}
α_t	1.85×10^{-4}	D_e	$2.5 \times 10^{-3} \text{ m}^2 \text{ s}^{-1}$	dn	$-9 \times 10^{-31} \text{ m}^{-3}$
α_s	4.14×10^{-5}	s	2 m s^{-1}	E_g	1.11eV

To explore the behavior of solutions, we carried out numerical calculation of several values of parameters. We have to predict the behaviors of the temperature coefficients of reflection $|Z_i|$, $i = 1, 2, \dots, 5$. For this object, Figures 2-4 are displayed.

Figure 2 presents the variations of the coefficients of reflection $|Z_i|$, $i = 1, 2, \dots, 5$ over an incidence angle $\theta \in [0, 90]$. The reflection coefficients $Z_1 - Z_4$ decrease with increasing the incidence angle θ , while the reflection coefficients Z_5 increase with increasing over on $\theta \in [0, 15]$ but decrease over $\theta \in [15, 90]$.

Figure 3 shows the variations of the coefficients of reflection $|Z_i|$, $i = 1, 2, \dots, 5$ over an incidence angle $\theta \in [0, 90]$ concerning varieties of fractional parameter α . The coefficients of reflection Z_1 and Z_4 decline with rising α and θ , while the reflection coefficient Z_2 rises with higher α but declines over on $\theta \in [0, 90]$. One can observe that the reflection coefficients Z_3 and Z_5 are in oscillatory behavior. It is closes that an increment in α results in a decrease in Z_3 for $\theta \in [0, 15]$ an increase for $\theta \in [15, 90]$, and a decreases for $\theta \in [0, 30]$ and increase for $\theta \in [30, 90]$ with increasing in α , while they decrease with increasing an incidence angle.

Figure 4 shows the variations of the coefficients of reflection $|Z_i|$, $i = 1, 2, \dots, 5$ over an angle of incidence $\theta \in [0, 90]$ concerning the variations of a magnetic field H_0 . The coefficients of reflection Z_1 and Z_4 decline with rising H_0 and θ , while the reflection coefficients Z_2 and Z_3 rise with increasing H_0 over on $\theta \in [0, 30]$ but decrease over on $\theta \in [30, 90]$. In sum, an increment in H_0 results in an increase in Z_5 for $\theta \in [0, 30]$ and a decrease for $\theta \in [30, 90]$.

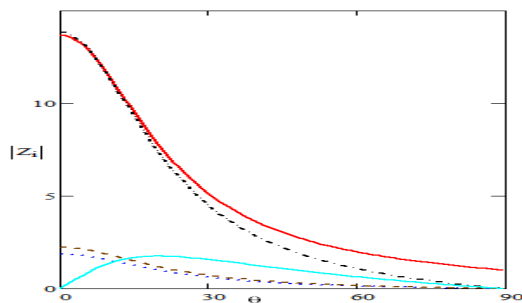
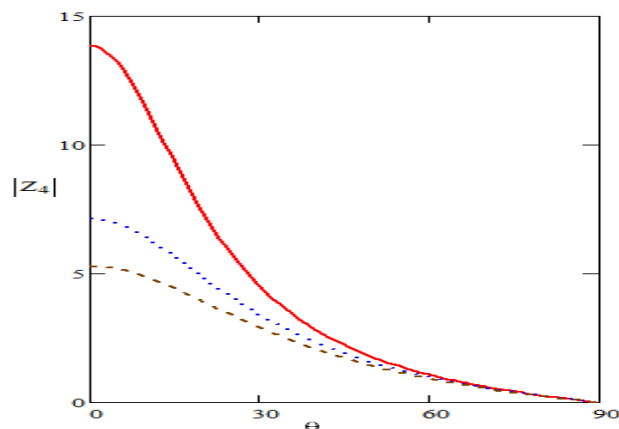
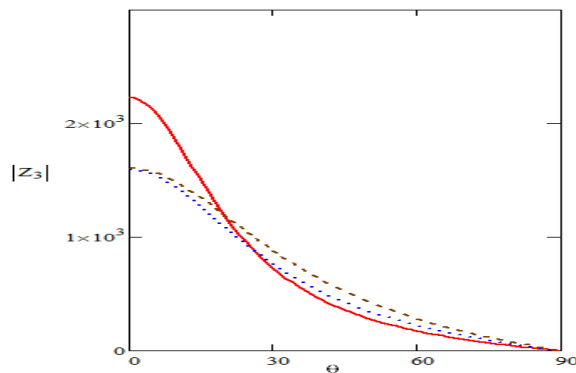
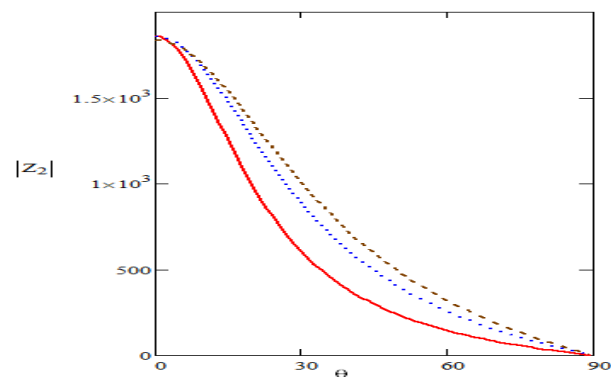
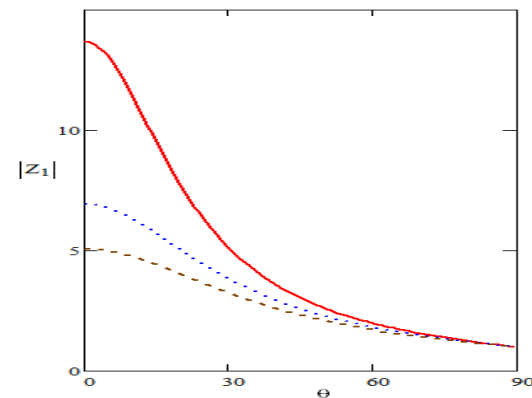


Fig.2: Various values of the coefficients of reflection

$|Z_i|$, $i = 1, 2, \dots, 5$ regarding the incidence angle θ :

— Z_1 , - - - Z_2 , . . . Z_3 , - . - Z_4 , — Z_5



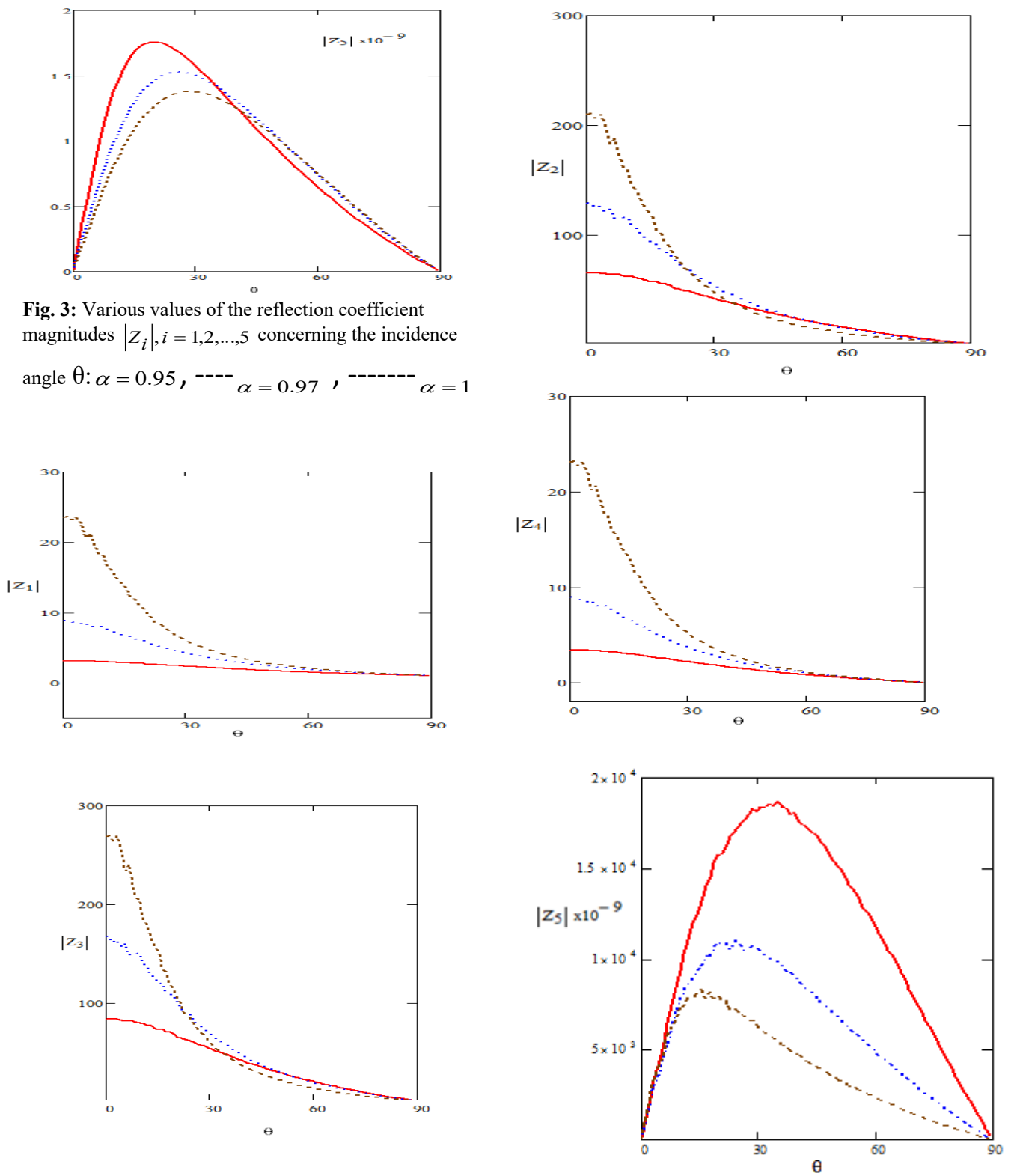


Fig. 3: Various values of the reflection coefficient magnitudes $|Z_i|, i = 1, 2, \dots, 5$ concerning the incidence angle θ : $\alpha = 0.95$, $----- \alpha = 0.97$, $----- \alpha = 1$

Fig. 4: Various values of the reflection coefficients magnitudes $|Z_i|, i = 1, 2, \dots, 5$ concerning the incidence

angle θ : $H_0 = 3 \times 10^6$, $----- H_0 = 2 \times 10^6$, $----- H_0 = 1 \times 10^6$

7 Conclusions

For studying waves' reflection within a semiconductor medium, we utilized the models of the fractional orderly study of the reflection of a photothermal wave within a semiconducting medium. We employed the harmonic wave technique to give the ratio of the reflection coefficient analytically and illustrate it in graphs.

We discussed the impacts of the fractional parameter and the magnetic field.

The following conclusions can be made:

1. The incidence angle is the base of the ratio of reflection coefficient, and the nature of dependence differs concerning various reflected waves.
2. The fractional parameter affects strongly the ratio of reflection coefficient.
3. The impact of the magnetic field is evident under the thermoelastic theories.
4. The findings motivate the investigation of vibration frequencies of an elastic medium, which represents a novel category of the materials of the application. They can benefit physicists, designers of new materials, authors of material science, and researchers of developing magneto-elasticity, as well as designing and optimal uses of nameplates and microplates. The paper employed methods that apply to various issues in elasticity and thermodynamics.

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Nomenclature

C	Measurement of the thermoelastic diffusion impact
C_e	Specific heat per heat mass
e	Cubical dilatation
E_g	The semiconductor's energy gap
ϵ_{ij}	Strain components tensor
F_i	Lorenz's body force tensor
$\kappa = \frac{\partial N_0}{\partial T} \frac{T}{\tau}$	Heat conduction of the sample
L_E	Coefficient of the carrier diffusion
$N(\vec{r}, t)$	The density of the carrier
N_0	Equilibrium carrier concentration at temperature T
p	Initial stress
$P(x, z)$	The chemical potential
T	Absolute temperature
T_0	Under the natural state, the temperature of the medium is $\left \frac{T - T_0}{T_0} \right < 1$
$T(\vec{r}, t)$	Distribution of temperature
$u(\vec{r}, t)$	Vector of displacement
α_c	Coefficient of linear diffusion expansion
α_T	Coefficient of linear thermal expansion
β_1 and β_2	Material constants given by $\beta_1 = \alpha_T(3\lambda + 2\mu)$ and $\beta_2 = \alpha_c(3\lambda + 2\mu)$
δ_n	The difference of deformation potential of conduction and valence
ϵ_1	Thermoelastic coupling parameter (according to the volume thermal expansion β_1)
ϵ_2	Thermoenergy coupling parameter (according to the semiconductor's energy gap)
ϵ_3	Parameter of thermo-electric coupling (debonded on the electronic deformation coefficient).
ϵ_4	The parameter of thermochemical coupling (The parameter of thermoelastic diffusion)
$\gamma = \alpha_T(3\lambda + 2\mu)$	The volume thermal expansion
λ, μ	Counterparts of Lamé's parameters
σ_{ij}	Stress tensor components
τ_{ij}	Maxwell's stress tensor components

- τ The photogenerated carrier lifetime
- τ_0 Time of thermal relaxation
- τ_θ, τ_q ($0 \leq \tau_\theta < \tau_q$) The phase-lags of temperature gradient and heat flux, respectively.