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Assessing the Lifetime Performance Index of Extreme Value Model Based on Progressive Type-II Censored Samples

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Abstract: In practice, effective management and assessment of quality performance for products is important in modern enterprises and the process capability analysis is utilized to measure business performance. Hence, lifetime performance index C_L is used to measure the potential and performance of a process, where L is the lower specification limit. In the technology of data transformation, this study constructs a maximum likelihood estimator (MLE) of C_L under the extreme value distribution (EVD) with the progressive type-II censored sample. The MLE of C_L is then utilized to develop the new hypothesis testing procedure in the condition of known L. Also we assuming the conjugate prior distribution and squared error loss function, this study constructs a Bayes estimator of C_L. The Bayes estimator of C_L is then utilized to develop a credible interval in the condition of known L. Moreover, we propose a Bayesian test to assess the lifetime performance of products. Finally, we give example and the Monte Carlo simulation to assess the behavior of the lifetime performance index.

Keywords: Performance index; Extreme value distribution; Progressive type-II censoring; Maximum likelihood estimator; Bayes estimator.

1 Introduction

Lifetime performance assessment is important in service (or manufacturing) industries, process capability indices (*PCIs*) are used to measure process potential and performance, process capability indices are utilized to assess whether product quality meets the required level. For instance, Montgomery [1] and Kane [2] proposed the process capability index C_L (or *CPL*) for evaluating the lifetime performance of electronic components, where L is the lower specification limit, since the lifetime of electronic components exhibits the larger-the-better quality characteristic of time orientation. All of the above (*PCIs*) have been developed or investigated under normal lifetime model. Nevertheless, in many process which including manufacture process, service process and business operation process, the assumption of normality is common in process capability analysis, and is often not valid. Many researcher e.g. [3,4,5,6] noted that the lifetime of products frequently possesses an exponential, gamma or Weibull distribution, etc. Since the lifetime of products exhibits the larger-the-better quality characteristic of time orientation.

Recently, there have been many works on the statistical inference for lifetime performance index based on the usual type-II and progressive type-II censoring schemes with various lifetime distributions, see for example Hong et al. [7,8], Lee et al. [9,10], Lee et al [11]. Also, Lee et al. [12] have constructed a credible interval for C_L using a Bayesian approach and proposed a Bayesian test for evaluating the lifetime performance of the products, Ahmadi et al. [13] have constructed a confidence interval and the maximum likelihood estimator for C_L based on the progressive first-failure censored sample under Weibull distribution and Mahmoud et al. [14] have constructed ML-estimator and a Bayes estimator of C_L based on a progressively type-II censored sample under the assumption of Lomax distribution.

In this paper, we consider the case of the progressive type-II censoring. A progressive type-II censoring is a useful scheme in which a specific fraction of individuals

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at risk may be removed from the experiment at each of several ordered failure times, see Cohen [15,16], Sen [17], Balakrishnan and Cohen [18], Viveros and Balakrishnan [19], Balakrishnan and Sandhu [20], Balakrishnan and Aggarwala [21], Balakrishnan et al. [22], Balakrishnan and Lin [23], Fernandez [24], Asgharzadeh [25] and Wu et al. [26].

A schematic illustration of progressively type-II censored sample can be described as follows, suppose that n independent items are put on a life test with continuous identically distributed failure times $X_1, X_2, ..., X_n$. Suppose further that a censoring scheme $(R_1, R_2, ..., R_m)$ is previously fixed such that immediately following the first failure X_1, R_1 surviving items are removed from the experiment at random, and immediately following the second failure X_2, R_2 surviving items are removed from the experiment at random. This process continues until, at the time of the m th observed failure X_m, the remaining R_m surviving items are removed from the test. The m ordered observed failure times denoted by $X_{1:m:n}^{(R_1,...,R_m)}, X_{2:m:n}^{(R_1,...,R_m)}, ..., X_{m:m:n}^{(R_1,...,R_m)}$ are called progressively type-II right censored order statistics of size m from a sample of size n with progressive censoring scheme $(R_1, R_2, ..., R_m)$. It is clear that $n = m + R_1 + R_2 + ... + R_m$. The special case when $R_1 = R_2 = ... = R_{m-1} = 0$ so that $R = n - m$ is the case of conventional type-II right censored sampling. Also when $R_1 = R_2 = ... = R_{m-1} = 0$, so that $m = n$, the progressively type-II right censoring scheme reduces to the case of no censoring (ordinary order statistics).

In this paper, process capability analysis is utilized to assess the non-normal quality data under a specific non-normal distribution. Hence, the lifetime performance index (or larger-the-better process capability index) C_L is also utilized to measure product quality with the EVD. The two parameters extreme value distribution $EVD(\beta, \lambda)$, has the probability density function (PDF), and cumulative distribution function (CDF), given respectively, by

$$f(x) = \frac{1}{\beta} \exp\left\{\frac{x-\lambda}{\beta}\right\} \times \exp\left(-\exp\left\{\frac{x-\lambda}{\beta}\right\}\right), \quad (1)$$

$$F(x) = 1 - \exp\left(-\exp\left\{\frac{x-\lambda}{\beta}\right\}\right), \quad (2)$$

for $-\infty < x < \infty$, $-\infty < \lambda < \infty$ and $\beta > 0$. The aim of this paper apply data transformation technology to constructs MLE of C_L under the EVD with the progressively type-II censored sample. The MLE of C_L is then utilized to develop a new hypothesis testing procedure in the condition of known L. Also we propose a Bayesian test to assess the lifetime performance of products. The new testing procedure can be employed by managers to assess whether the lifetime of products adheres to the required level in the condition of known L.

The rest of this paper is organized as follows: Section 2 contains some properties of C_L for lifetime of product with the EVD based on the progressively type-II censored sample. Section 3 investigates the relationship between C_L and the conforming rate of products. We propose the MLE of C_L and its statistical properties in Section 4. Section 5 then presents the Bayes estimator under the conjugate prior distribution and squared error loss function of C_L and its statistical properties. Section 6 develops a $100(1-\alpha)\%$ one-sided credible interval, a Bayesian test and a $100(1-\alpha)\%$ one-sided confidence interval for C_L. Numerical example to illustrate the use of testing procedure based on the Bayes estimator and the MLE under the given significance level are given in Sections 7. A comparison between the MLE and Bayes estimator is made through a Monte Carlo simulation study in Sections 8. Finally, concluding remarks are given in Section 9.

2 The Lifetime Performance Index

Suppose that the lifetime X of products has the two-parameter EVD with the PDF and CDF are given as (1) and (2). Clearly, a longer lifetime implies a better product quality. Hence, the lifetime is a larger-the-better type quality characteristic. The lifetime is generally required to exceed L unit times to be both financially profitable and satisfy customers where L is the known lower specification limit. Montgomery [1] developed a capability index C_L to measure the larger-the-better type quality characteristics. Then C_L is defined as follows:

$$C_L = \frac{\mu - L}{\sigma} \quad (3)$$

where μ denotes the process mean, σ represents the process standard deviation, and L is the known lower specification limit. To assess the lifetime performance of products, C_L can be defines as the lifetime performance index. Under X has the EVD and the data transformation $Y = \frac{1}{\lambda} \exp\left(\frac{x-\lambda}{\beta}\right)$, for $-\infty < x < \infty$, $\lambda > 0$ and $\beta > 0$, the distribution of Y is a exponential distribution. Hence, the PDF of Y is

$$f(y) = \lambda \exp\{-\lambda y\}, \quad y > 0, \ \lambda > 0. \quad (4)$$

Moreover, there are several important properties, as follows:

– The lifetime performance index C_{Ly} can be rewritten as

$$C_{Ly} = \frac{\mu_y - L_y}{\sigma_y} = 1 - \lambda L_y, \ C_{Ly} < 1, \quad (5)$$

where the process mean $\mu_y = E(Y) = 1/\lambda$, the process standard deviation $\sigma_y = \sqrt{VARY} = 1/\lambda$ and L_y is known lower specification limit.

– The CDF of Y is given by

$$F(y) = 1 - \exp\{-\lambda y\}, \ y > 0, \ \lambda > 0, \quad (6)$$

The failure rate function $r(y)$ is defined by

$$r(y;) = \frac{f(y)}{1-F(y)} = \frac{\lambda \exp\{-\lambda y\}}{1-(1-\exp\{-\lambda y\})} = \lambda. \tag{7}$$

Hence, the data transformation $Y = \frac{1}{\lambda}\exp\left(\frac{x-\lambda}{\beta}\right)$, $\lambda > 0$ and $\beta > 0$ for $-\infty < x < \infty$ is one-to-one and strictly increasing, so data set of X and transformed data set of Y have the same effect in assessing the lifetime performance of products. Moreover, the data transformation $Y = \frac{1}{\lambda}\exp\left(\frac{x-\lambda}{\beta}\right)$, for $-\infty < x < \infty$, $\lambda > 0$ and $\beta > 0$, enables the calculation of important properties to be easy, when the mean $1/\lambda$ ($> L_y$) then the lifetime performance index $C_{Ly} > 0$. From (5) and (7), we can see that the larger the mean $1/\lambda$ the smaller the failure rate and the larger the lifetime performance index C_{Ly}. Therefore, the lifetime performance index C_{Ly} reasonably and accurately represents the lifetime performance of new product.

3 Conforming Rate

If the lifetime of a product X which $Y = \frac{1}{\lambda}\exp\left(\frac{x-\lambda}{\beta}\right)$, for $-\infty < x < \infty$, $\lambda > 0$ and $\beta > 0$ exceeds the lower specification limit L_y, then the product is defined as a conforming product. The ratio of conforming products is known as the conforming rate P_r, and can be defined as

$$P_r = P(Y \geq L_y) = \int_{L_y}^{\infty} \lambda \exp\{-\lambda y\}\, dy = \exp\{-\lambda L_y\} = \exp\{C_{Ly} - 1\}, \tag{8}$$

where $-\infty < C_{Ly} < 1$. Obviously, a strictly positive relationship exists between P_r and C_{Ly}. Thus, the larger the index value C_{Ly}, the larger conforming rate P_r. Table 1 lists various C_{Ly} values and the corresponding P_r. For the C_{Ly} values which are not listed in **Table 1**, the conforming rate P_r can be easily calculated. The conforming rate can be calculated by dividing the number of conforming products by the total number of products sampled. To accurately estimate P_r, Montgomery [1] suggests to use a large sample size. However, a large sample size is usually not practical from the perspective of cost, since collecting the lifetime data of new products involves damaging the products which may prove to be cost prohibitive if not practically in feasibility. In addition, a complete sample is, as mentioned earlier, not practical. Since a one-to-one mathematical relationship exists between P_r and C_{Ly}, utilizing the one-to-one relationship between P_r and C_{Ly}, lifetime performance index can be a flexible and effective tool, not only for evaluating product quality, but also for estimating P_r.

Table 1: The lifetime performance index v.s. the conforming rate.

C_{Ly}	P_r	C_{Ly}	P_r
$-\infty$	0.00000	0.15	0.42741
-9.00	0.00004	0.20	0.44933
-8.00	0.00012	0.25	0.47237
-7.00	0.00033	0.30	0.49659
-6.00	0.00091	0.35	0.52205
-5.00	0.00248	0.40	0.54881
-4.50	0.00409	0.45	0.57695
-4.00	0.00673	0.50	0.60653
-3.50	0.01111	0.55	0.63763
-3.00	0.01832	0.60	0.67032
-2.50	0.03019	0.65	0.70469
-2.00	0.04979	0.70	0.74082
-1.50	0.08208	0.75	0.77880
-1.00	0.13534	0.80	0.81873
-0.50	0.22313	0.85	0.86071
0.00	0.36788	0.90	0.90484
0.05	0.38674	0.95	0.95123
0.10	0.40657	1.00	1.00000

4 MLE of C_{Ly}

Let Y denote the lifetime of such a product and Y has the one-parameter exponential distribution with the PDF as (4). With progressive type-II censoring, n products (or items) are placed on test. Consider that $Y_{1:m:n}, Y_{2:m:n}, \ldots, Y_{m:m:n}$ is the corresponding progressive type-II censored sample, with censoring scheme $R = (R_1, R_2, \ldots, R_m)$. The joint PDF of all m progressive type-II censored order statistic is given by (see Balakrishnan and Aggarwala [21])

$$L(\underline{y}|\lambda) = C \prod_{i=1}^{m} f(y_{i,m,n}|\lambda)\,[1 - F(y_{i,m,n}|\lambda)]^{R_i}, \tag{9}$$

where $C = n(n-1-R_1)(n-2-R1-R_2)\cdots(n-\sum_{i=1}^{m-1}(R_i+1))$, $f(y_{i,m,n}|\lambda)$ is the PDF of Y Eq. (4) and $F(y_{i,m,n}|\lambda)$ is the CDF of Y Eq. (6). So, the likelihood function is given by

$$L(\underline{y}|\lambda) = C\lambda^m \exp\left\{-\lambda \sum_{i=1}^{m}(R_i+1)y_i\right\}. \tag{10}$$

It is easy to obtain that the MLE of λ is given by

$$\hat{\lambda}_{ML} = \frac{m}{\sum_{i=1}^{m}(R_i+1)y_i}. \tag{11}$$

By using the invariance of MLE see Zehna [27], the MLE of C_{Ly} can be written as

$$\hat{C}_{Ly_{ML}} = 1 - \hat{\lambda}_{ML} L_y = 1 - \frac{m}{T} L_y, \tag{12}$$

where

$$T = \sum_{i=1}^{m}(R_i+1)y_i. \tag{13}$$

5 Bayes Estimator of C_{Ly}

Bayesian approach provides the methodology for incorporation of previous information with the current data, and λ is considered a random variable having some specified distribution. In this paper, we consider that the conjugate prior distribution is gamma distribution with PDF

$$\pi(\lambda|a,b)=\frac{b^a}{\Gamma(a)}\lambda^{a-1}\exp\{-b\lambda\},\lambda>0. \tag{14}$$

Here all the hyperparameters a and b are assumed to be known (a and b are obtained from the past history) and non-negative. From (10) and (14), we can obtain the posterior distribution of λ is given by

$$\begin{aligned}\pi^*(\lambda|Y_{1:m:n},...,Y_{m:m:n}) &= \frac{(b+T)^{a+m}}{\Gamma(a+m)}\lambda^{a+m-1}\\ &\quad\times\exp\{-\lambda(b+T)\},\end{aligned} \tag{15}$$

for $\lambda>0$, zero elsewhere. Under a squared error loss function and using (15), then the Bayes estimator of λ is

$$\hat{\lambda}_{BS}=E[\lambda|Y_{1:m:n},...,Y_{m:m:n}]=\frac{a+m}{b+T}. \tag{16}$$

Hence, the Bayes estimator $\hat{C}_{LyBS}$ of C_L can be written as

$$\begin{aligned}\hat{C}_{LyBS} &= 1-\hat{\lambda}_{BS}L_y\\ &= 1-\frac{(a+m)L_y}{T^*},\end{aligned} \tag{17}$$

where

$$T^*=b+\sum_{i=1}^{m}(R_i+1)y_i=b+T. \tag{18}$$

Next, given the observed values $(Y_{1:m:n},...,Y_{m:m:n})$ and Eq. (15), then we shall show that $2\lambda T^*\sim\chi^2_{2(m+a)}$. The derivation of processes is as follows:

Suppose that $Z=2\lambda T^*$, by using the change of variables see Casella and Berger [28], pp. 184–185, then we obtain that the PDF of Z is given by

$$\begin{aligned}f_Z(z) &= \pi^*\left(\frac{Z}{2T^*}|Y_{1:m:n},...,Y_{m:m:n}\right)\|J_z\|,\\ &= \frac{(2T^*)^{-1}Z^{m+a}}{\Gamma(a+m)2^{m+a}}\left(\frac{Z}{2T^*}\right)^{-1}\exp\left\{-\frac{Z}{2}\right\},\\ &= \frac{Z^{\left(\frac{2(m+a)}{2}\right)-1}}{2^{\left(\frac{2(m+a)}{2}\right)}\Gamma\left(\frac{2(m+a)}{2}\right)}\exp\left\{-\frac{Z}{2}\right\}.\end{aligned} \tag{19}$$

Hence, $Z=2\lambda T^*\sim\chi^2_{2(m+a)}$.

6 Testing procedure for C_{Ly}

To determine whether the lifetime of products meets the requirements, a credible (or confidence) interval is needed to objectively assess whether the lifetime performance index adheres to the required level. Assuming that the required index value of lifetime performance C_{Ly} is larger than c^*, where c^* denotes the lower bound of C_{Ly}, the null hypothesis $H_0: C_{Ly}\le c^*$ (the product is unreliable) and the alternative hypothesis $H_1: C_{Ly}>c^*$ (the product is reliable) are constructed.

In Bayesian approach, given the specified significance level α, a $100(1-\alpha)\%$ one-sided credible interval for C_{Ly} can be derived as follows:
Based on the pivotal quantity $2\lambda T^*\sim\chi^2_{2(m+a)}$ and the lower $(1-\alpha)$ percentile denoted by $\chi^2_{2(m+a)}(1-\alpha)$, we obtain:

$$\left.\begin{aligned}&P\left(2\lambda T^*\le\chi^2_{2(m+a)}(1-\alpha)|\underline{y}\right)=1-\alpha\\ &\Rightarrow P\left(\lambda\le\left(\frac{\chi^2_{2(m+a)}(1-\alpha)}{2T^*}\right)|\underline{y}\right)=1-\alpha,\\ &\Rightarrow P\left(1-\lambda L_y\ge 1-L_y\left(\frac{\chi^2_{2(m+a)}(1-\alpha)}{2T^*}\right)|\underline{y}\right)=1-\alpha,\\ &\Rightarrow P\left(C_{Ly}\ge 1-L_y\left(\frac{\chi^2_{2(m+a)}(1-\alpha)}{2T^*}\right)|\underline{y}\right)=1-\alpha,\\ &\Rightarrow P\left(C_{Ly}\ge 1+\left(-1+1-\frac{(a+m)L_y}{T^*}\right)\right.\\ &\qquad\left.\times\left(\frac{\chi^2_{2(m+a)}(1-\alpha)}{2(a+m)}\right)\right)=1-\alpha,\\ &\Rightarrow P\left(C_{Ly}\ge 1-\left(1-\hat{C}_{LyBS}\right)\left(\frac{\chi^2_{2(m+a)}(1-\alpha)}{2(a+m)}\right)\right)\\ &\qquad=1-\alpha,\end{aligned}\right\} \tag{20}$$

where $\hat{C}_{LyBS}$ and T^* are defined in (17) and (18). From (20), we obtain that a $100(1-\alpha)\%$ one-sided credible interval for C_{Ly} is

$$C_{Ly}\ge 1-\left(1-\hat{C}_{LyBS}\right)\left(\frac{\chi^2_{2(m+a)}(1-\alpha)}{2(a+m)}\right). \tag{21}$$

Thus, the level $100(1-\alpha)\%$ lower credible bound for C_{Ly} can be written as

$$LB=1-\left(1-\hat{C}_{LyBS}\right)\left(\frac{\chi^2_{2(m+a)}(1-\alpha)}{2(a+m)}\right). \tag{22}$$

Similarly, in the non-Bayesian approach, by using the pivotal quantity $2\lambda T\sim\chi^2_{(2m)}$, where $T=\sum_{i=1}^{m}(R_i+1)y_i$, we obtain that a $100(1-\alpha)\%$ one-sided confidence interval for C_{Ly} is

$$C_{Ly} \geq 1-\left(1-\hat{C}_{LyML}\right)\left(\frac{\chi^2_{2m}(1-\alpha)}{2m}\right), \tag{23}$$

where $\hat{C}_{LyML}$ is as in the above definition of Eq. (12), hence the level $100(1-\alpha)\%$ lower confidence bound for C_{Ly} can be written as

$$LB_{ML} = 1-\left(1-\hat{C}_{LyML}\right)\left(\frac{\chi^2_{2m}(1-\alpha)}{2m}\right). \tag{24}$$

The proposed testing procedure about C_{Ly} in Bayesian approach can be organized as follows:

Step 1.Let the transformation of $Y = \frac{1}{\lambda}\exp\left(\frac{x-\lambda}{\beta}\right)$, for $-\infty < x < \infty$, $\lambda > 0$, $\beta > 0$, $i = 1,2,\ldots,m$ and the progressive type-II censored sample $X_{1:m:n}, X_{2:m:n}, \ldots, X_{m:m:n}$.
Step 2.Determine the lower lifetime limit L_y for products and performance index value c^*, then the testing null hypothesis $H_0 : C_{Ly} \leq c^*$ and the alternative hypothesis $H_1 : C_{Ly} > c^*$ is constructed.
Step 3.Specify a significance level α.
Step 4.Calculate the level $100(1-\alpha)\%$ one-sided credible interval $[LB,\infty)$ for C_{Ly}, where LB is given in the above Eq. (22).
Step 5.The decision rule of statistical test is provided as follows:

If the performance index value $c^* \notin [LB,\infty)$, it is concluded that the lifetime performance index of the product meets the required level.
In the non-Bayesian approach, the managers can also employ the level $100(1-\alpha)\%$ one-sided confidence interval $[LB_{ML},\infty)$ to determine whether the product performance adheres to the required level, with LB_{ML} as in the definition of Eq. (24). Therefore, the decision rule of the statistical test is provided as follows:
If the performance index value $c^* \notin [LB_{ML},\infty)$, it is concluded that the lifetime performance index of the product meets the required level.

Based on the proposed testing procedure, the lifetime performance of products is easy to assess. Numerical example of the proposed testing procedure given in Section 7, and these numerical example illustrate the use of the testing procedure.

7 Numerical Example

A new hypothesis testing procedure is proposed to allow the application of the above testing procedure to a practical data set. Example 1 considered is the failure data of $n = 19$, $m = 9$ electrical insulating fluids from Nelson [29]. Also used by Al-Aboud [30].
Example 1 (*Real Life Data*)**.** Nelson [[29], p. 105] presents the results of a life-test experiment in which specimens of a type of electrical insulating fluid were subject to a constant voltage stress (34 KV/minutes). In analyzing the complete data, Nelson assumed a Weibull distribution for the times to breakdown. The 19 log-times to breakdown are

−1.66073,	−0.248461,	−0.040822,	0.270027,
1.02245,	1.15057,	1.42311,	1.54116,
1.57898,	1.8718,	1.9947,	2.08069,
2.11263,	2.48989,	3.45789,	3.48186,
3.52371,	3.60305,	4.28895	

In the numerical example, a progressively type-II censored sample of size $m = 9$ is generated randomly from the $n = 19$ observations recorded at 34 k.v. The observed failure times and the progressive censoring scheme are given in **Table 2**.

By using the graphical method introduced by Balakrishnan and Kateri [31], we obtained the estimation of the parameters as $\tilde{\lambda} = 2.2866$ and $\tilde{\beta} = 1.021$. Then the transformed progressive type-II censored sample with transformation $Y_i = \frac{1}{\lambda}\exp\left(\frac{X_i-\lambda}{\beta}\right)$, for $\lambda = 2.2866$, $\beta = 1.021$, $i = 1,2,\ldots,9$ and $X_{1:9:19}, X_{2:9:19}, \ldots, X_{9:9:19}$ is given in **Table 3**.

In the Bayesian approach, we assumed that $a = 1$ and $b = 1$. Under the transformed progressive type-II censored sample and removed numbers are also reported in **Table 3**, the proposed testing procedure for C_{Ly} can be stated as the following algorithm

Step 1.The lower lifetime limit L_y is assumed to be 0.021. To deal with the product purchasers' concerns regarding operational performance, the conforming rate P_r of products is required to exceed 80%. Referring to Table 1, the C_{Ly} value is required to exceed 0.80. Thus, the performance index value is set at $c^* = 0.80$. The testing hypothesis $H_0 : C_{Ly} \leq 0.80$ vs. $H_1 : C_{Ly} > 0.80$ is constructed.
Step 2.Specify a significance level $\alpha = 0.05$.
Step 3.Calculate the 95% one-sided credible interval $[LB,\infty)$ for C_{Ly} by (22), where

$$LB = 1-(1-0.9575)\left(\frac{\chi^2_{2(9+1)}(1-0.05)}{2(9+1)}\right) = 0.9332.$$

Step 4.Because of the performance index value $c^* = 0.80 \notin [0.9332,\infty)$, we reject the null hypothesis $H_0 : C_{Ly} \leq 0.80$.

In the non-Bayesian approach, we also obtain that the 95% one-sided confidence interval $[LB_{ML},\infty)$ for C_{Ly} by (24)

$$LB_{ML} = 1-(1-0.9520)\left(\frac{\chi^2_{2(9)}(1-0.05)}{2\times 9}\right) = 0.9230.$$

Because of the performance index value $c^* = 0.80 \notin [0.9230, \infty)$, we reject the null hypothesis $H_0 : C_{Ly} \leq 0.80$. Hence, we can conclude that the lifetime performance index of insulating fluids does meet the required level for Bayesian and non-Bayesian approach.

Table 2: The observed failure times and progressive censoring scheme at 34 k.v.

i	1	2	3
x_i	-1.66073	-0.248461	-0.040822
R_i	0	0	0
i	4	5	6
x_i	0.270027	1.02245	1.15057
R_i	0	1	1
i	7	8	9
x_i	1.54116	1.57898	1.8718
R_i	1	1	6

Table 3: The transformed progressive type-II censored sample.

i	1	2	3	4	5
y_i	0.01	0.04	0.0491	0.0665	0.139
R_i	0	0	0	0	1
i	6	7	8	9	
y_i	0.1576	0.231	0.2397	0.3194	
R_i	1	1	1	6	

8 Simulation Study

In order to compare the MLEs and Bayes estimates of the lifetime performance index C_{Ly}, Monte Carlo simulations were performed utilizing 1000 progressively type-II censored samples for each simulations. The mean square error (MSE) is used to compare the estimators. The samples were generated by using the algorithm described in Balakrishnan and Sandhu [20] using $(\lambda, \beta) = (2.511, 1.3045)$, with different sample of sizes (n), different effective sample of sizes (m), different hyperparameters (a, b), and different of sampling schemes (i.e., different R_i values) we consider the following scheme (CS):

CS I:$R_1 = n - m$, $R_i = 0$ for $i \neq 1$.
CS II:$R_{(m+1)/2} = n - m$, $R_i = 0$ for $i \neq (m+1)/2$ if m odd;
$R_{m/2} = n - m$, $R_i = 0$ for $i \neq m/2$ if m even.
CS III:$R_m = n - m$, $R_i = 0$ for $i \neq m$.

Under X has EVD and the data transformation $Y = \frac{1}{\lambda}\exp\left(\frac{x-\lambda}{\beta}\right)$, for $-\infty < x < \infty$, $\lambda > 0$, $\beta > 0$ and $i = 1, 2, \ldots, m$, such that distribution of Y is a exponential distribution. Based on the lower lifetime limit $L_y = 0.0195$, the results of MSEs of the MLEs and Bayes estimates for C_{Ly}, and coverage probabilities (CPs) of the 95% credible interval (CRI) and confidence interval (CI) for C_{Ly} are reported in **Table 4, 5**, **6** and **7**.

Table 4: MSEs of the MLEs and Bayes estimates with $a = b = 1$ for C_{Ly}

n	m	CS	MLE	Bayes
				$a = b = 1$
20	15	I	0.000219	0.000118
		II	0.000227	0.000119
		III	0.000231	0.000123
30	20	I	0.000155	0.000099
		II	0.000159	0.000103
		III	0.000162	0.000112
30	25	I	0.000109	0.000087
		II	0.000112	0.000096
		III	0.000118	0.000104
50	30	I	0.000097	0.000070
		II	0.000103	0.000085
		III	0.000107	0.000092
50	40	I	0.000073	0.000051
		II	0.000092	0.000063
		III	0.000101	0.000087
70	50	I	0.000050	0.000041
		II	0.000055	0.000045
		III	0.000060	0.000052
90	60	I	0.000039	0.000034
		II	0.000047	0.000040
		III	0.000056	0.000049
90	70	I	0.000036	0.000032
		II	0.000038	0.000035
		III	0.000043	0.000039

Table 5: MSEs of the MLEs and Bayes estimates with $a = b = 2$ for C_{Ly}

n	m	CS	MLE	Bayes
				$a = b = 2$
20	15	I	0.000221	0.000108
		II	0.000226	0.000115
		III	0.000230	0.000120
30	20	I	0.000157	0.000088
		II	0.000159	0.000091
		III	0.000161	0.000107
30	25	I	0.000110	0.000075
		II	0.000113	0.000083
		III	0.000116	0.000102
50	30	I	0.000094	0.000062
		II	0.000104	0.000081
		III	0.000109	0.000091
50	40	I	0.000067	0.000050
		II	0.000095	0.000058
		III	0.000100	0.000079
70	50	I	0.000053	0.000040
		II	0.000056	0.000043
		III	0.000061	0.000049
90	60	I	0.000038	0.000034
		II	0.000044	0.000038
		III	0.000057	0.000045
90	70	I	0.000035	0.000031
		II	0.000037	0.000033
		III	0.000044	0.000036

Table 6: CPs of 95% CI and CRI with $a=b=1$ for C_{Ly}.

n	m	SC	MLE	Bayes
20	15	I	0.957	0.958
		II	0.958	0.961
		III	0.951	0.953
30	20	I	0.941	0.963
		II	0.952	0.967
		III	0.960	0.964
30	25	I	0.947	0.952
		II	0.961	0.963
		III	0.948	0.957
50	30	I	0.955	0.959
		II	0.944	0.962
		III	0.948	0.957
50	40	I	0.957	0.947
		II	0.953	0.955
		III	0.958	0.948
70	50	I	0.955	0.940
		II	0.966	0.967
		III	0.948	0.951
90	60	I	0.952	0.949
		II	0.944	0.966
		III	0.965	0.967
90	70	I	0.956	0.958
		II	0.946	0.956
		III	0.947	0.953

Table 6: CPs of 95% CI and CRI with $a=b=2$ for C_{Ly}.

n	m	SC	MLE	Bayes
20	15	I	0.958	0.959
		II	0.946	0.966
		III	0.959	0.960
30	20	I	0.955	0.962
		II	0.968	0.971
		III	0.949	0.959
30	25	I	0.959	0.969
		II	0.950	0.952
		III	0.951	0.954
50	30	I	0.946	0.966
		II	0.947	0.954
		III	0.950	0.959
50	40	I	0.954	0.957
		II	0.947	0.961
		III	0.948	0.953
70	50	I	0.943	0.958
		II	0.956	0.971
		III	0.951	0.957
90	60	I	0.952	0.963
		II	0.954	0.975
		III	0.958	0.959
90	70	I	0.940	0.967
		II	0.954	0.956
		III	0.951	0.948

9 Conclusions

This paper purposes to utilize the lifetime performance index C_{Ly} in assessing the lifetime performance of businesses and products more generally and accurately. Under the assumption of the extreme value distribution, this paper constructs a MLEs and Bayes estimator of C_{Ly} with the progressively type-II censored sample. The MLEs and Bayes estimator of C_{Ly} is then utilized to develop the new hypothesis testing procedure in the condition of known L_y. The proposed testing procedure is easily applied and can effectively evaluate whether the lifetime of products meets requirements. A simulation study was conducted to examine the performance of the different estimators. From the results, we observe the following:

(i)From Table 4, when the effective sample proportion m/n increases, the MSE of different MLEs and Bayes estimators are reduced, also the censoring scheme $R=(n-m,0,...,0)$ is most efficient for all choices, it usualy provides the smallest MSE for all estimators.

(ii)From Table 4, the Bayes estimates are generally smaller than their corresponding MLEs, for the considered different n, m $(n>m)$, censoring scheme $R=(R_1,R_2,...,R_m)$ and hyperparameters a, b. So, the Bayes estimates are better than their corresponding MLEs for the considered cases. Hence, these results from simulation studies illustrate that the performance of our proposed Bayesian method is acceptable.

(iii)From Table 5, it is observed that the coverage probabilities of one-sided credible interval and one-sided confidence interval for lifetime performance index C_{Ly} close to the desired level of 0.95.

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References

[1] Montgomery, D. C. (1985): Introduction to Statistical Quality Control, John Wiley & Sons Inc., New York.
[2] Kane, V. E. (1986): Process capability indices, Journal of Quality Technology, 18, pp. 41–52.
[3] Epstein, B. and Sobel, M. (1953): Life-testing, Journal of the American Statistical Association, 48, pp. 486–502.
[4] Meyer, P. L. (1965): Introductory Probability and Statistical Applications, Addison-Wesley, Reading, MA.
[5] Anderson, D. R., Sweeney, D. J. and Williams, T. A. (1990): Statistics for Business and Economics, West Publishing Company, Saint Paul, MN.
[6] Keller, G., Warrack, B. and Bartel, H. (1994): Statistics for Management and Economics, Duxbury Press, Belmont, CA.

[7] Hong, C. W., Wu, J. W. and Cheng, C. H. (2008): Computational procedure of performance assessment of lifetime index of Pareto lifetime businesses based on confidence interval, Applied Soft Computing, 8, pp. 698–705.

[8] Hong, C. W., Wu, J. W. and Cheng, C. H. (2009) Implementing lifetime performance index for the pareto lifetime businesses of the service industries, Quality and Quantity, 43, pp. 291–304.

[9] Lee, W. C., Wu, J. W. and Hong, C. W. (2009): Assessing the lifetime performance index of products with the exponential distribution under progressively type-II right censored samples, Journal of Computational and Applied Mathematics, 231, pp. 648–656.

[10] Lee, W. C., Wu, J. W. and Hong, C. W. (2009): Assessing the lifetime performance index of products from progressively type-II right censored data using Burr XII model, Mathematics and Computers in Simulation, 79, pp. 2167–2179.

[11] Lee, W. C., Wu, J. W. and Lei, C. L. (2010): Evaluating the lifetime performance index for the exponential lifetime products, Applied Mathematical Modelling, 34, pp. 1217–1224.

[12] Lee, W. C., Wu, J. W., Hong, M. L., Lin, L. S. and Chan, R. L. (2011): Assessing the lifetime performance index of Rayleigh products based on the Bayesian estimation under progressive type-II right censored samples, Journal of Computational and Applied Mathematics, 235, pp. 1676–1688.

[13] Ahmadi, M. V., Doostparast, M. and Ahmadi, J. (2013): Estimating the lifetime performance index with Weibull distribution based on progressive first-failure censoring scheme, Journal of Computational and Applied Mathematics, 239, pp. 93–102.

[14] Mahmoud, M. A. W., EL-Sagheer, R. M., Soliman, A. A. and Abd Ellah, A. H. (2014): Inferences of the lifetime performance index with Lomax distribution based on progressive type-II censored data, Economic Quality Control, 29(1), pp. 39–51

[15] Cohen, A. C. (1966): Life testing and early failure, Technometrics, 8, pp. 539-549.

[16] Cohen, A. C. (1991): Truncated and Censored Samples: Theory and Applications, Marcel Dekker, New York.

[17] Sen, P. K. (1986): Progressive censoring schemes, in: Encyclopedia of Statistical Sciences, John Wiley and Sons Inc., New York, pp. 296 299.

[18] Balakrishnan, N. and Cohen, A. C. (1991): Order Statistics and Inference: Estimation Methods, San Diego, Academic Press.

[19] Viveros, R. and Balakrishnan, N. (1994): Interval estimation of parameters of life from progressively censored data, Technometrics, 36, pp. 84-91.

[20] Balakrishnan, N. and Sandhu, R. A. (1996): Best linear unbiased and maximum likelihood estimation for exponential distributions under general progressive Type-II censored samples, Sankhya B, 58, pp. 1-9.

[21] Balakrishnan, N. and Aggarwala, R. (2000): Progressive Censoring: Theory, Methods and Applications, Birkhauser, Boston.

[22] Balakrishnan, N., Ng, H. K. T. and Kannan, N. (2002): A test of exponentiality based on spacings for progressively type-II censored data, Birkhauser, Boston, pp. 89-111.

[23] Balakrishnan, N. and Lin, C. T. (2003): On the distribution of a test for exponentiality based on progressively type-II right censored spacings, Journal of Statistical Computation and Simulation, 73, pp. 277-283.

[24] Fernandez, A. J. (2004): On estimating exponential parameters with general type II progressive censoring, Journal of statistical planning and inference, 121, pp. 135-147.

[25] Asgharzadeh, A. (2006): Point and interval estimation for a generalized logistic distribution under progressive type II censoring, Communications in Statistics-Theory and Methods, 35, pp. 1685 1702.

[26] Wu, S. J., Chen, D. H. and Chen, S. T. (2006): Bayesian inference for Rayleigh distribution under progressive censored sample, Applied Stochastic Models in Business and Industry, 22, pp. 269-279.

[27] Zehna, P. W. (1966): Invariance of maximum likelihood estimation, Annals of Mathematical Statistics, 37, pp. 744–755.

[28] Casella, G. and Berger, R. L. (2002): Statistical Inference, second ed., Duxbury, Pacific Grove, CA.

[29] Nelson, W. (2009): Applied Life Data Analysis, Wiley, New York.

[30] Al-Aboud, F. M. (2009): Bayesin estimations for the extreme value distribution using progressive censored data and asymmetric loss, International Mathematical Forum, 4, pp. 1603-1622.

[31] Balakrishnan, N. and Kateri, M. (2008): On the maximum likelihood estimation of parameters of Weibull distribution based on complete and censored data, Statistics and Probability Letters, 78, pp. 2971–2975.

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