

# Modeling and Optimization of Train Scheduling Network Based on Invulnerability Analysis

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Received: 17 Jan. 2011; Revised 11 Oct. 2011; Accepted 16 Oct. 2011

Published online: 1 Jan. 2013

**Abstract:** At present, the railway transport are always held up or even suspended because of the natural disasters occurred frequently. In order to guarantee the smooth operation of the railway transport and reduce the number of trains affected by the disasters as few as possible, proposal of train scheduling scheme with optimal invulnerability is proposed in this paper. Firstly, train scheduling network model is proposed based on present train scheduling scheme in China, in which the real railway station is described as a vertex, and the railway line is described as an edge, the number of trains on the railway lines is designed as the weight of edge. Secondly a new indicator of invulnerability measurement is proposed and regarded as the object function to be optimized by using improved particle swarm optimization algorithm. Finally comparison of invulnerability between the original scheme and optimal scheme with selective attack and random attack are analyzed and the simulation results show that the optimal train scheduling scheme have better invulnerability.

**Keywords:** Train scheduling network, invulnerability, particle swarm optimization algorithm, degree and weight effect.

## 1. Introduction

Railway is the primary transportation way in China. Chinese railway network has been formed due to long-term development. However, the spatial accessibility [1] of our railway network is still poor because of harsh geographical environment and expensive construction cost. Many previous examples proved that if some emergencies occurred such as natural disasters, the train operation will be influenced and railway transportation will be interrupted. Therefore, a reasonable train operation schedule with better invulnerability to ensure train operating normally as more as possible when some emergencies occur is an urgent problem needed to solve.

The invulnerability research on complex network becomes a new focus, its theoretical significance and application value are recognised increasingly. Invulnerability usually means the reliability of the network under artificial destruction, it assumes the destroyer own all the information of network structure and adopt a destructed strategy [2], so it also called resistance to attack. Generally, the average shortest path length is used to measure the damage extent of the network after attack. However, with the ex-

tent of damage increase, the average shortest path length increases at first, and then decreases, this particularity has brought much inconvenience in the research of invulnerability. Therefore, Holme et al [3] make use of the global efficiency  $E$  and the maximum connected subgraph size  $S$  to measure the network's performance after attack, the global efficiency is defined as formula (1).

$$E = \frac{1}{N \cdot (N - 1)} \sum_{i \neq j} \frac{1}{d_{ij}}. \quad (1)$$

Where  $N$  represents the total node number of the network;  $d_{ij}$  represents the average path length from node  $i$  to node  $j$ . Chinese railway geographic network is a tree network [4], which has approximate zero average clustering coefficient and very large average path length. If formula (1) is used to analysis the network's invulnerability which will result in very small analysis results, so be hard to tell which is better. Therefore, the global efficiency index proposed by Holme is not suitable for evaluating the railway network's invulnerability. Wu et al [5] proposed a new complex network connectivity measure  $C$  which consists of the global efficiency  $E$  and the maximum connected subgraph size  $S$ , as formula (2). It divides the complex network into

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multiple connected branches, reduce the average shortest path length, but is not suitable to analysis the weighted network's invulnerability.

$$C = \frac{1}{w \sum_{i=1}^w \frac{N_i N}{L_i}} \tag{2}$$

Where  $w$  represents the network's connected branch number,  $N_i$  represents the node number of connected branch  $i$ ,  $N$  represents the total node number of the network,  $L_i$  represents the average shortest path length of connected branch  $i$ . Zhang et al [6] proposed network coverage measure, which analysis the scale-free network's invulnerability by the object of information fusion, namely research the ratio value of coverage area of the information fusion system in the total coverage area system can cover, and also it can be expressed the proportional of the network biggest clusters after attack in the initial network size in network model. However, railway geographic network does not have scale-free network character [7] result in the network biggest clusters and the initial network's size almost is same, this way also is not suitable to analysis network's invulnerability.

At present, the research in view of the weighted network's invulnerability does not see more. Jin Lei [8] has analyzed the invulnerability of Xinjiang region's highway by grade. The research of Li et al [9] demonstrated that the invulnerability of complex network drops rapidly with the increase of the flow of network, and described the process of complex network's cascading failures. Namely a certain node suffer damage, the flow of this node is distributed to the around nodes of it, which make other nodes' flow load beyond their capacity limit to lead to cascading collapse. However, when one site breaks down, the adjacent sites will process the affected vehicles until the original site's operation plan is in normal in the actual rail network. This circumstance is not the same as the cascading collapse of the power grid, so this method is also not suitable for studying railway network. Deng et al [10] use the overload function to carry through the weights' dynamic allocation of the double small-world network's cascading failures, but the nodes or edges' invalidation in the actual rail network will lead to cancel or postpone the schedule, namely the weights' cancel is different from the weights' dynamic allocation. Therefore, it is necessary to put forward a new invulnerability evaluation index for studying the weighted railway network.

Because of its high construction cost, railway network cannot extend in all directions as the ordinary highway, this paper make use of adjusting the train scheduling scheme, namely reasonable allocating railway network's traffic flow, to promote regional railway network invulnerability, but not by increasing railway lines. Under the consideration of the actual application in railway network, degree and weight effect invulnerability measure is proposed and make it as target function. Then improved particle swarm optimization algorithm is used to optimize the network's flow distribution for obtaining train operation schedule with better invulnerability. Subsequently, the invulnerability of the

original scheme and the optimal scheme using selective and random attack patterns respectively are analyzed in this paper. Simulation results show that the network after optimization is more stable than before.

## 2. Invulnerability analysis of train scheduling network

### 2.1. Model of train scheduling network

With Chinese geographic network as carrier, we use number of trains operated between two stations to describe the edge weight according to present train scheduling scheme, so the weighted railway network can be constructed as shown in Figure 1. In this diagram, the network's nodes are

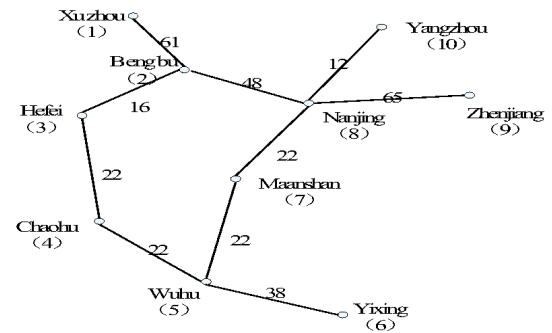


Figure 1 Train scheduling network model diagram.

actual railway stations and the network's edges are actual railway lines. Single-line or multi-line railway is considered as one edge in this model. And the weight of edge is defined as number of trains operated on the line. And then the topology of train scheduling network can be described in matrix form as  $A$ :

$$A = \begin{bmatrix} 0 & 61 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 61 & 0 & 16 & 0 & 0 & 0 & 0 & 48 & 0 & 0 \\ 0 & 16 & 0 & 22 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 22 & 0 & 22 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 22 & 0 & 38 & 22 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 38 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 22 & 0 & 0 & 22 & 0 & 0 \\ 0 & 48 & 0 & 0 & 0 & 0 & 22 & 0 & 65 & 12 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 65 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 12 & 0 & 0 \end{bmatrix}$$

where numerical vector  $e = [61, 16, 22, 22, 38, 22, 22, 65, 12, 48]$  represents the network's edge weight, which means

the actual train operation quantity between the adjacent sites. Sum vector of non-zero element in every row or column  $d = [1, 3, 2, 2, 3, 1, 2, 4, 1, 1]$  represents the degree of each node. This diagram shows that the degree of node is one for majority. The stations with higher degree are the important nodes in regional network, and their nearby edges often own higher weight. The damage of these stations will make quite a large number of running trains affected. Adequately considering this actual situation, we will propose an optimization algorithm to redistribute the edge weights and construct new train scheduling scheme which will have a less number of affected train when some stations are damaged to promote the invulnerability of the railway network.

### 2.2. Invulnerability evaluating index

As already stated, the existing invulnerability evaluation index of complex network is not suitable for the weighted railway network, a new invulnerability evaluation index is proposed to solve this problem. Wang et al[11] puts forward a flow distribution method of network in analysing all sorts of transportation systems. Making use of the lessons from this idea, this paper regards the operation train quantity distribution of railway network as a network flow distribution problem. Network flow's reasonable distribution can make the nodes' importance of the railway network keep balance and so reduce the affected train quantity when some nodes are damaged. In fact, it is not possible to keep this balance in railway network, because some railway stations such as which locate in capital or some big cities. Considering degree of node and weight of edge synthetically, a new evaluation index called degree and weight effect is proposed, which can be used to evaluate importance of node, as shown in formula (3):

$$f_i = \frac{s_i}{S} \cdot \frac{d_i}{D} \tag{3}$$

Where  $s_i, S$  represent the total number of the train that pass by the railway station  $i$  and the total operating train number of the network respectively;  $d_i, D$  represent the node degree of the railway station  $i$  and the sum of the all nodes' degree in the railway network respectively. This index considers the degree and the flow distribution of the railway network comprehensively, if the nodes own higher degree or bigger network flow that shows these nodes are more important in the railway network. Furthermore, the standard deviation of nodes' degree and weight effect distribution is used as fitness function to evaluate invulnerability of the train scheduling network, as shown in formula (4):

$$F = \sqrt{\frac{\sum_{i=1}^N (f_i - \bar{f})^2}{N - 1}} \tag{4}$$

Where  $N$  represent the station number of the railway network;  $f_i$  represents the degree and weight effect value of

node  $i$ ;  $\bar{f}$  represents average value of all the nodes' degree and weight effect value. If the fitness  $F$  is smaller, the balance of the network flow is better and the invulnerability of this railway network will be better.

### 2.3. Particle swarm optimization

The optimal solution of the flow distribution will be solved by using an improved particle swarm optimization (PSO) algorithm. PSO [12] algorithm is an important branch of the group of intelligent optimization, which is inspired by the predatory behavior of birds proposed by American scholar Eberhart and Kennedy in 1995. This algorithm is of these features: less setting parameters, fast convergence speed, and it has been widely used in the engineering field after ten years development. The particles in PSO algorithm without quality and size fly across the  $n$  dimension space by a certain speed, and dynamically adjust the group to the optimal state gradually by their own flying experience and companions flying experience, the math expression shown in formula (5) and formula (6):

$$v_{\bar{i},d} = wv_{\bar{i},d} + c_1r_1(p_{\bar{i},d}^b - x_{\bar{i},d}) + c_2r_2(p_d^g - x_{\bar{i},d}) \tag{5}$$

$$x_{\bar{i},d} = x_{\bar{i},d} + v_{\bar{i},d} \tag{6}$$

Where  $x_{\bar{i}} = (x_{\bar{i},1}, x_{\bar{i},2}, \dots, x_{\bar{i},d})$  and  $v_{\bar{i}} = (v_{\bar{i},1}, v_{\bar{i},2}, \dots, v_{\bar{i},d})$  represent the position vector and velocity vector of particle  $\bar{i}$  respectively.  $p_{\bar{i}}^b = (p_{\bar{i},1}^b, p_{\bar{i},2}^b, \dots, p_{\bar{i},d}^b)$  and  $p_{\bar{i}}^g = (p_{\bar{i},1}^g, p_{\bar{i},2}^g, \dots, p_{\bar{i},d}^g)$  represents the position of particle  $\bar{i}$  searched by itself and the best position of all particles searched. Nonnegative constant  $c_1$  and  $c_2$  represent the learning factor,  $r_1$  and  $r_2$  represent random number of uniformly distributed in the interval  $(0, 1)$ . In formula (5), the three items on the right of the equation is corresponding to the inertia of itself, the best history position of itself and the best position of the particle population. Adjust particles to their own best position and population best position learning weights according to the study factor.

PSO algorithm has numerous improvement methods, predatory search strategy combined with PSO algorithm is one of the important methods. Predatory search strategy is a kind of new bionic computing thought put forward by Linhares[13]. This thought simulating animal feeding strategy realize local search and global search by controlling the limits size of the search space and the conversion between them, which have good local concentrated search and jump out of local optimal ability. Predatory strategy can balance the local search ability and the global search ability of the PSO algorithm, and this just makes up the defect of the algorithm easily falling in local optimum.

Predatory search strategy combined with PSO algorithm still has its limitations. Because this strategy affect the algorithm search process only by adjusting the search range size, and the search range limitation is randomly generated, when particles search in a small range but the search speed keep fast which is extremely easy to cause the

algorithm to skip the optimal solution. In complex problems, this combined strategy will have no obvious effect. Aiming at this shortcomings, this paper proposes a speed adaptive control strategy, which will increase the particle search speed with the search range increasing, and vice versa. The math expression shown in formula (7):

$$x_{\bar{i},d} = x_{i,d} + \frac{re(\bar{j})}{\delta} \left(1 - \frac{t}{it_{max}}\right) \gamma v_{\bar{i},d} \tag{7}$$

Where  $re_{max}$  represents the range restriction quantity,  $\delta$  and  $\gamma$  represent two adjustable parameters,  $t$  represents the current iteration times, and  $it_{max}$  represents the maximum iterating times of every range limitations,  $\bar{j} = 1, 2, \dots, max$ .

When particles search in a wide range limitation, particle keeping faster speed can quickly find a relatively optimal solution. Then, the algorithm search in a small scope centered on this relatively optimal solution, and at the same time the particle speed will reduce in order to get a more optimal solution around the relatively optimal solution. Because particle search speed and the current iteration related, make particle velocity get a better adjustment. What's more, to prevent algorithm to entrap into local solution, the algorithm will re-generate range limitations when the particles can't find a more optimal solution.

As shown in Figure 1, the train scheduling scheme can be described as a  $N * N$  dimension adjacency matrix. In this adjacency matrix, the nonzero elements indicate that these two stations are adjacency and the value represents the weight of these two stations, the zero elements indicate that these two stations aren't adjacency. In PSO algorithm, a particle code for the train operation number of the adjacent stations, namely consists of the nonzero elements of the triu or the tril in adjacency matrix, and make formula (4) as the objective function. In this problem, the fitness value is smaller and the result is better under the constraints. This paper make the consistency of the train operation number between the original train scheduling network and the optimal train scheduling network as the first constraint. Another, passenger flow volume of every stations are different, just like the description of the evaluation index degree and weight effect, so we define the other constraint condition, as shown in formula(8):

$$\|s_i - s_i^{opt}\| < 0.2 * s_i \tag{8}$$

where  $s_i$  represents the total number of the train that pass by the railway station  $i$  in the original train scheduling network, as the same  $s_i^{opt}$  represents the total train number of the optimal train scheduling network.

Algorithm procedure is as follows:

1) Set the range limitation of all levels and the search times  $it_{max}$  of every level. Determine the total station number  $N$  and the degree number  $d_i$  of every station, the particle number  $P$  of the PSO algorithm. Calculate the total train number of the original train scheduling scheme, and make it as the constraint of search results.

2) Randomly initialize all particles speed  $v_{\bar{i},d}$  in range limitation  $re(1)$ . Using formula (5) formula (7) update par-

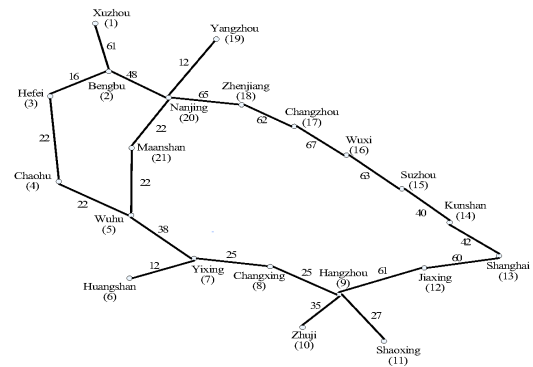


Figure 2 The original train scheduling network diagram of the Yangtze River Delta regional diagram.

ticles position information and speed information, and then turn all the values into integers. The particles searching ranges are defined as formula(9):

$$0.5 * e_d < x_{\bar{i},d} < 1.5 * e_d \tag{9}$$

where  $e_d$  represents the original train scheduling network's edge weight, namely the operation train number of the between the adjacent stations.

3) Calculate the fitness value of every particle by using formula (4), get the global optimal fitness value and global optimal solution by comparing the fitness value of every particle. If the value is better than current global optimal solution, update the current global optimal solution. Then make  $x_{\bar{i}} = p^g (i = 1, 2, \dots, P)$ , and turn to step 2).

4) Otherwise repeat the search process until the search times exceeding the maximum search times  $it_{max}$ , and then randomly initialize all particles center on the current optimal particle in a larger range limitation and turn to step 3).

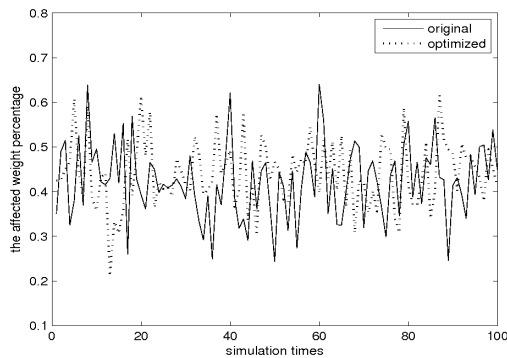
5) Output the optimal solution until all range limitations have been searched, and regress to a new train operation plan.

### 3. Simulation results

#### 3.1. Invulnerability optimization

The simulation analysis is based on the railway network of Yangtze River Delta region, as shown in Figure 2 and the topology matrix of the original train scheduling network is as  $A_i$ . The data of Figure 2 comes from Chinese Railway Passenger Transport Network in 2008, and choose twenty-one stations as the nodes of the train scheduling network. The weight of the station is the sum of all the edges connected to this station in the model. The bigger station weight shows more trains go through this station, and it is more important in the railway network system.





**Figure 4** Weight percentage affected by random attack.

conduct random attack 100 times to original train scheduling network and the optimized train scheduling network, shown in Figure 4.

From Figure 4, we can find that the number of trains affected by the random attack is approximately consistent in the original train scheduling network and the optimized train scheduling network. Because the total train number does not change, the optimized train scheduling network only adjust the flow of the stations to make it balance based on the original train scheduling network. This also explains that the strategy balancing the flow does not change the basic network feature of the original train scheduling network. Furthermore, the maximal percent value of the affected train in the optimized train scheduling network is lower than in original train scheduling network, and the minimal percent value is lower also. Overall, the invulnerability of the optimized weighted railway network under random attack is also improved.

#### 4. Conclusion

In this paper, a new invulnerability measure named degree and weight effect is put forward based on the model of train scheduling network. This model does not change the existing rail network structure and the total operation train number, and optimize the weight value of current train dispatching network to get an optimal train scheduling scheme based on improved particle swarm optimization algorithm. The weight values at some important stations (such as the stations in the capital city) have been reduced, which means the passenger flow volume was reduced at these stations and the passengers will be transferred to their neighbor cities in the optimal train scheduling scheme. As a result, the distribution of passenger flow on train scheduling network will be balanced and the invulnerability of the train scheduling network will be improved. What's more, the second-tier cities near the capital city are rising abruptly at present which certainly will result in floating population increasing, and the passenger

flow volume of these second-tier cities also will increase accordingly, so the strategy proposed in this paper is in accordance with the development of passenger flow in China.

#### Acknowledgement

The authors acknowledge the financial support National Natural Science Foundation of China, project No60804066. The author is grateful to the anonymous referee for a careful checking of the details and for helpful comments that improved this paper.

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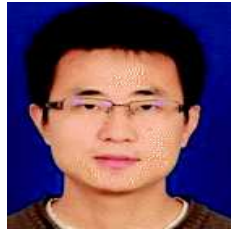
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