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Modified Ratio-cum- Exponential Estimator for Appraising the Population Mean using Conventional and non-Conventional Parameters

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Abstract: In this present study a ratio type exponential estimator has been developed for estimating the populations mean using auxiliary information as Kurtosis, Skewness, Correlation, Trimmed Mean, Median, Mid-Range, Hodges-Lehman estimator etc. We obtain bias and mean square error of the proposed estimator up to first order of approximation and show that proposed estimator is efficient than existing estimators. In addition, we support theoretical results with the relieve of numerical examples

Keywords: Ratio estimator, Bias, Mean Square error, Study Variable, Auxiliary Variable, Efficiency, Simple Random Sampling.

1 Introduction

In survey sampling, we generally use the auxiliary information to raise precision of the estimators by using the advantages of correlation between the study variable y and the auxiliary variable x. When the relationship between y and x is positive, then the ratio estimator produces efficient results, and when the relationship is negative, then the product estimator performs better. However, several authors have researched the auxiliary qualities under different conditions. To improve precision, supplemental information might be quantified in the form of auxiliary proportions in specific cases. As a result, some researchers have employed one or more auxiliary proportions during the estimating step to improve the estimators' efficiency. In this context, the ratio, product, and regression estimation methods are appropriate examples. Under some conditions, these estimating approaches are more efficient than the traditional mean per unit estimator. Many authors have proposed or changed different estimators when employing mixed types estimators, such as regression and ratio type estimators or exponential type estimators. Individual estimators were outperformed by these mixed estimators. For example, the size of a tree can be utilized as a critical auxiliary variable when predicting the average height of trees in a forest, and the type of dairy animal is a significant auxiliary characteristic when determining usual milk yield (Ahmad et.al 2021). The ratio estimator suggested by Cochran (1977) plays an important role in the situation of positive correlation among study and auxiliary variable. The calculation of a finite population mean using various techniques is one of the hottest topics in sample survey theory. The main goal of sample survey theory is to make inferences about unknown population parameters such as the total population, population proportion, and population mean, and population variance, among others. The main aim of survey sampling is to increase efficiency in the ratio, product and regression estimators in presence of known auxiliary information are used to estimate the unknown population parameters of the study variable using various sampling techniques. When the correlation between the study variable and the auxiliary variable is positively strong, the ratio method of estimation proposed by Cochran (1940) performs better than the normal estimators of mean/total. In certain real-life cases, auxiliary variables such as a person's age and sleep period, income and expenditure, a person's age and blood pressure, and so on are strongly associated with the study variable. Robson (1957) proposed the product method of estimation, which Murthy rediscovered (1964). If the association between the research variable and the auxiliary variable is negative, it's best to use it. Watson (1937) suggested the regression method of estimation, which is the best alternative. When the regression

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line is linear and passes through a non-origin point. Many authors have proposed estimators for estimating the population mean in simple random sampling without substitution using auxiliary data (SRSWOR). Singh and Tailor (2003), Kadilar and Cingi (2004, 2006a, 2006b, 2006c), Koyuncu and Kadilar (2009), Singh et al. (2009), Yan and Tian (2010), Upadhyaya et al. (2011), Yadav and Kadilar (2013), Khan et al. (2013), and Khan et al (2015).(Sanaullah et.al 2018)Considered stratified two-phase sample design with non-respondent sub sampling in the context of non-response for population estimation entail taking into consideration the information from two auxiliary variables . (Singh et, al 2020 and C.E.R.A.N et.al 2021) also proposed estimators for estimating population mean in simple random sampling using auxiliary data. The aim of this paper is to propose an improved general family of exponential-type estimators for finite population mean estimation under SRSWOR. A brief overview of some traditional and exponential-type population mean estimators is given.

Notations used in this paper are:

N	Population size
n	Sample size
$f = \frac{n}{N}$	Sampling fractuion
Y	Study Variable
X	Auxialiary Variable
$\bar{Y}, \bar{X}$	Population means
$\bar{y}, \bar{x}$	Sample means
S_y, S_x	Population standard deviations
S_{yx}	Population covariance between variables
C_y, C_x	Population coefficient of variation
ρ	Population correlation coefficient
$B(.)$	Bias of estimator
$MSE(.)$	Mean square error of estimator
φ_{ZR}	Proposed estimator
M_d	Population Median of auxiliary variable
$\beta_{2(x)}$	Population kurtosisof auxiliary variable
$\beta_{1(x)}$	Population skewness of auxiliary variable

$\mathrm{HL} = \mathrm{median}\left[\frac{(X_i + X_k)}{2}, 1 \leq j \leq k \leq N\right]$ Hodges − Lehman estimator

$\mathrm{MR} = \frac{\left(X_{(1)} + X_{(N)}\right)}{2}$ Population mid range

$TM = \frac{Q_1 + 2Q_2 + Q_3}{4}$

2 Estimators in Literature

Consider a sample of size "n" is selected from a population of size "N" subject to the constraint n < N under SRSWOR. Let n pair of observations $(\boldsymbol{y_i x_i})\boldsymbol{i} = \mathbf{1, 2, 3, \dots,} \boldsymbol{n}$ for the study (y) and auxiliary variable (x) respectively. Let$\sum_{i=1}^{N} \boldsymbol{y_i} = \boldsymbol{N^{-1}\bar{Y}}$ and $\sum_{i=1}^{N}(\boldsymbol{y_i} - \boldsymbol{\bar{Y}})^2 = (\boldsymbol{N} - \mathbf{1})^{-1}\boldsymbol{S_y^2}$ denote the mean and the population variance of the study variable and $\sum_{i=1}^{N} \boldsymbol{x_i} = \boldsymbol{N^{-1}\bar{X}}$ and $\sum_{i=1}^{N}(\boldsymbol{x_i} - \boldsymbol{\bar{X}})^2 = (\boldsymbol{N} - \mathbf{1})^{-1}\boldsymbol{S_x^2}$ denote the mean and the population variance of the auxiliary variable.

1. The traditional unbiased estimator, which does not use auxiliary information, is characterized by

$$\boldsymbol{\eta_0} = \boldsymbol{\bar{y}}$$

Variance /MSE of $\boldsymbol{\eta_0}$ is given by

$$\boldsymbol{MSE}(\boldsymbol{\eta_0}) = \boldsymbol{Var}(\boldsymbol{\eta_0}) = \psi \boldsymbol{\bar{Y}^2 C_y^2} \quad (2.1)$$

2. Cochran (1940) proposed the traditional ratio estimator, which is given as

$$\hat{\bar{\boldsymbol{Y}}}_{\boldsymbol{R}} = \boldsymbol{\bar{y}}\left(\frac{\boldsymbol{\bar{X}}}{\boldsymbol{\bar{x}}}\right), \boldsymbol{\bar{x}} \neq \mathbf{0}$$

expressing for the bias and MSE of the $\hat{\bar{\boldsymbol{Y}}}_{\boldsymbol{R}}$ estimator, up to first degree of approximation, are respectively given by

Bias $(\hat{\bar{\boldsymbol{Y}}}_{\boldsymbol{R}}) \cong \psi \boldsymbol{\bar{Y}}\left[\boldsymbol{C_x^2} - \boldsymbol{\rho_{yx} C_y C_x}\right]$

and

$$\text{MSE}(\hat{\bar{Y}}_R) \cong \psi \bar{Y}^2 [C_y^2 - C_x^2(1-2A)] \quad (2.2)$$

Where $A = \rho_{yx}\frac{C_y}{C_x}$

3. The population mean is estimated using the exponential ratio form estimators introduced by Bahl and Tuteja (1991).

$$\hat{\bar{Y}}_{BT} = \bar{y}exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right)$$

Bias and MSE of the $\hat{\bar{Y}}_{BT}$ estimator, up to first order of approximation, are respectively given as

$$\text{Bias}(\hat{\bar{Y}}_{BT}) \cong \frac{\psi}{2}\bar{Y}\left[\frac{3C_x^2}{4} - \rho_{yx}C_yC_x\right]$$

and

$$\text{MSE}\left(\hat{\bar{Y}}_{BT}\right) \cong \frac{\psi}{4}\bar{Y}^2[4C_y^2 + C_x^2 - 4\rho_{yx}C_yC_x] \quad (2.3)$$

4. Singh et.al (2008) suggested a ratio-product exponential type estimator for population mean given as

$$\hat{\bar{Y}}_{SI} = \bar{y}\left[\mu exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right) + (1-\mu)exp\left(\frac{\bar{x}-\bar{X}}{\bar{X}+\bar{x}}\right)\right]$$

Where μ is a real constant.

The optimal value of μ is as follows

$$\mu = \frac{1}{2} + \frac{\rho_{yxC_y}}{C_x}$$

Minimum MSE of $\hat{\bar{Y}}_{SI}$ estimator, up to first degree of approximation as follows

$$MSE_{min}\left(\hat{\bar{Y}}_{SI}\right) \cong \psi\bar{Y}^2C_y^2(1-\rho_{yx}^2) \cong MSE\left(\hat{\bar{Y}}_{Reg}\right) \quad (2.4)$$

5. Motivated by Khoshnevisan et.al (2007), Singh et.al (2009) proposed an exponential family of estimators for $\bar{Y}$ as

$$\eta = \bar{y}\exp\left[\frac{(\alpha\bar{X}+\beta)-(\alpha\bar{X}+\beta)}{(\alpha\bar{X}+\beta)+(\alpha\bar{X}+\beta)}\right]$$

where α and β are either real numbers or function of the known parameters of the auxiliary variables such as coefficient of skewness $\left(\beta_{1(x)}\right)$, coefficient of kurtosis $\left(\beta_{2(x)}\right)$ standard deviation (S_x), coefficient of variation (C_x) and coefficient of correlation $\left(\rho_{yx}\right)$ of the population

Following are the bias and MSE up to first degree of approximation

$$\text{Bias}(\eta) \cong \psi\bar{Y}\left[\frac{3}{4}\theta_i^2 C_x - \theta_i\rho_{yx}C_y\right]C_x$$

And

$$\text{MSE}(\eta) \cong \psi\bar{Y}^2[C_y^2 + \theta_i^2C_x^2 - 2\theta_i\rho_{yx}C_yC_x] \quad (2.5)$$

Where

$$\theta_i = \frac{\alpha\bar{X}}{2(\alpha\bar{X}+\beta)}, i = 1,2,3,\ldots,10$$

And

$$\theta_1 = 0.5;\ \theta_2 = \frac{\bar{X}}{2(\bar{X}+\beta_{2(X)})}; \theta_3 = \frac{\bar{X}}{2(\bar{X}+C_x)}; \theta_4 = \frac{\bar{X}}{2(\bar{X}+\rho_{yx})};$$

$$\theta_5 = \frac{\beta_{2(X)}\bar{X}}{2(\beta_{2(X)}\bar{X} + C_x)}; \theta_6 = \frac{C_x\bar{X}}{2(C_x\bar{X} + \beta_{2(X)})}; \theta_7 = \frac{C_x\bar{X}}{2(C_x\bar{X} + \rho_{yx})};$$

$$\theta_8 = \frac{\rho_{yx}\bar{X}}{2(\rho_{yx}\bar{X} + C_x)}; \theta_9 = \frac{\beta_{2(X)}\bar{X}}{2(\beta_{2(X)}\bar{X} + \rho_{yx})}; \theta_{10} = \frac{\rho_{yx}\bar{X}}{2(\rho_{yx}\bar{X} + \beta_{2(X)})}.$$

Table 1: Singh et.al (2009) family of estimators are:

Estimators	α	β
$\eta_1 = \bar{y}exp\left[\frac{[\bar{X}-\bar{x}]}{[\bar{X}+\bar{x}]}\right]$	1	0
$\eta_2 = \bar{y}exp\left[\frac{[\bar{X}-\bar{x}]}{[\bar{X}+\bar{x}]+2\beta_{2(x)}}\right]$	1	$\beta_{2(x)}$
$\eta_3 = \bar{y}exp\left[\frac{[\bar{X}-\bar{x}]}{[\bar{X}+\bar{x}]+2C_x}\right]$	1	C_x
$\eta_4 = \bar{y}exp\left[\frac{[\bar{X}-\bar{x}]}{[\bar{X}+\bar{x}]+2\rho_{yx}}\right]$	1	ρ_{yx}
$\eta_5 = \bar{y}exp\left[\frac{\beta_{2(x)}[\bar{X}-\bar{x}]}{\beta_{2(x)}[\bar{X}+\bar{x}]+2C_x}\right]$	$\beta_{2(x)}$	C_x
$\eta_6 = \bar{y}exp\left[\frac{C_x[\bar{X}-\bar{x}]}{C_x[\bar{X}+\bar{x}]+2\beta_{2(x)}}\right]$	C_x	$\beta_{2(x)}$
$\eta_7 = \bar{y}exp\left[\frac{C_x[\bar{X}-\bar{x}]}{C_x[\bar{X}+\bar{x}]+2\rho_{yx}}\right]$	C_x	ρ_{yx}
$\eta_8 = \bar{y}exp\left[\frac{\rho_{yx}[\bar{X}-\bar{x}]}{\rho_{yx}[\bar{X}+\bar{x}]+2C_x}\right]$	ρ_{yx}	C_x
$\eta_9 = \bar{y}exp\left[\frac{\beta_{2(x)}[\bar{X}-\bar{x}]}{\beta_{2(x)}[\bar{X}+\bar{x}]+2\rho_{yx}}\right]$	$\beta_{2(x)}$	ρ_{yx}
$\eta_{10} = \bar{y}exp\left[\frac{\rho_{yx}[\bar{X}-\bar{x}]}{\rho_{yx}[\bar{X}+\bar{x}]+2\beta_{2(x)}}\right]$	ρ_{yx}	$\beta_{2(x)}$

6. Yadav and Kadilar (2013) suggested an improved exponential family of estimators for population mean $\bar{Y}$ by utilizing Singh et.al (2009) as

$$t = k\bar{y}exp\left[\frac{(\alpha\bar{X}+\beta)-(\alpha\bar{X}+\beta)}{(\alpha\bar{X}+\beta)+(\alpha\bar{X}+\beta)}\right]$$

where k is a suitable constant and $\alpha$and β are defined earlier. some members of the t family of estimators are given in Table 2

Following are the expressions of bias and MSE of the **t** family of estimators

$$\text{Bias}(t) \cong \psi k\bar{Y}\left[\frac{3}{2}\theta_i^2 C_x - \theta_i\rho_{yx}C_y\right]C_x + \bar{Y}(k-1)$$

And

$$\text{MSE}(t) \cong \psi\bar{Y}^2\left[k^2C_y^2 + k\theta_i^2C_x^2(4k-3) - 2k\theta_i\rho_{yx}C_yC_x(2k-1)\right] + \bar{Y}^2(k-1)^2 \qquad (2.6)$$

Where θ_i is defined earlier.

Partially differentiating eq (2.6) w.r.t k and equating to zero the optimal value of k is given by

$$k = \frac{1+\psi\left[\frac{3}{2}\theta_i^2 C_x - \theta_i \rho_{yx} C_y C_x\right]}{1+\psi\left[C_y^2 + 4\theta_i^2 C_x^2 - 4\theta_i \rho_{yx} C_y C_x\right]} = \frac{C_{1i}}{C_{2i}} \qquad (2.7)$$

$$C_{1i} = 1+\psi\left[\frac{3}{2}\theta_i^2 C_x^2 \quad - \theta_i \rho_{yx} C_y C_x\right]$$

And

$$C_{2i} = 1+\psi\left[C_y^2 + 4\theta_i^2 C_x^2 - 4\theta_i \rho_{yx} C_y C_x\right]$$

Substituting eq(2.7) in eq(2.6) we get the minimum MSE of the t family of estimators as

$$\mathrm{MSE_{min}}(t) \cong \bar{Y}^2\left(1-\frac{C_{1i}^2}{C_{2i}^2}\right) \qquad (2.8)$$

Table 2: Yadav and Kadilar (2013) family of estimators are:

Estimators	α	β
$t_1 = k\bar{y}exp\left[\frac{[\bar{X}-\bar{x}]}{[\bar{X}+\bar{x}]}\right]$	1	0
$t_2 = k\bar{y}exp\left[\frac{[\bar{X}-\bar{x}]}{[\bar{X}+\bar{x}]+2\beta_{2(x)}}\right]$	1	$\beta_{2(x)}$
$t_3 = k\bar{y}exp\left[\frac{[\bar{X}-\bar{x}]}{[\bar{X}+\bar{x}]+2C_x}\right]$	1	C_x
$t_4 = k\bar{y}exp\left[\frac{[\bar{X}-\bar{x}]}{[\bar{X}+\bar{x}]+2\rho_{yx}}\right]$	1	ρ_{yx}
$t_5 = k\bar{y}exp\left[\frac{\beta_{2(x)}[\bar{X}-\bar{x}]}{\beta_{2(x)}[\bar{X}+\bar{x}]+2C_x}\right]$	$\beta_{2(x)}$	C_x
$t_6 = k\bar{y}exp\left[\frac{C_x[\bar{X}-\bar{x}]}{C_x[\bar{X}+\bar{x}]+2\beta_{2(x)}}\right]$	C_x	$\beta_{2(x)}$
$t_7 = k\bar{y}exp\left[\frac{C_x[\bar{X}-\bar{x}]}{C_x[\bar{X}+\bar{x}]+2\rho_{yx}}\right]$	C_x	ρ_{yx}
$t_8 = k\bar{y}exp\left[\frac{\rho_{yx}[\bar{X}-\bar{x}]}{\rho_{yx}[\bar{X}+\bar{x}]+2C_x}\right]$	ρ_{yx}	C_x
$t_9 = k\bar{y}exp\left[\frac{\beta_{2(x)}[\bar{X}-\bar{x}]}{\beta_{2(x)}[\bar{X}+\bar{x}]+2\rho_{yx}}\right]$	$\beta_{2(x)}$	ρ_{yx}
$t_{10} = k\bar{y}exp\left[\frac{\rho_{yx}[\bar{X}-\bar{x}]}{\rho_{yx}[\bar{X}+\bar{x}]+2\beta_{2(x)}}\right]$	ρ_{yx}	$\beta_{2(x)}$

7. Upadhyaya et.al (2011) proposed ratio exponential type estimator as

$$\hat{\bar{Y}}_{UP} = \bar{y}\exp\left(1-\frac{2\bar{X}}{\bar{X}+\bar{x}}\right)$$

$$\mathrm{MSEof}\hat{\bar{Y}}_{UP} \cong \psi\bar{Y}^2 C_y^2\left(1-\rho_{yx}^2\right) \cong MSE\left(\hat{\bar{Y}}_{Reg}\right) \qquad (2.9)$$

Kadilar (2016) suggested a modified exponential type estimator for the population mean as

$$\hat{\hat{Y}}_K = \bar{y}\left(\frac{\bar{x}}{\bar{X}}\right)^{\delta} exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right)$$

Where $\boldsymbol{\delta}$ is a real constant .

$$\textbf{The optimal value of } \delta = C_x - \frac{2\rho_{yx}C_y}{2C_x}$$

Minimum MSE of $\hat{\hat{Y}}_K$ estimator, up to first degree of approximation as follows

$$\text{MSE}_{\min}\ (\hat{\hat{Y}}_{UP})) \cong \psi\bar{Y}^2 C_y^2(1-\rho_{yx}^2) \cong MSE(\hat{\hat{Y}}_{Reg}) \qquad (3.1)$$

Minimum MSE of $\hat{\hat{Y}}_{SI}, \hat{\hat{Y}}_{UP}, \hat{\hat{Y}}_K$ estimators, up to the first order of approximation is exactly equal to the variance of the usual regression estimator $\hat{\hat{Y}}_{Reg}$ .Regression estimator suggested by Watson (1973) is given by

$$\hat{\hat{Y}}_{Reg} = \bar{y} + b_{yx}(\bar{X}-\bar{x})$$

Where $b_{yx} = \frac{S_{yx}}{S_x^2}$ is the sample regression coefficient.

3 Proposed Estimator

Several authors have considered exponential type estimator of the population mean after the initiate work carried out by Bahl and Tetuja. Motivated by Irfan et.al [5], a modified ratio type exponential estimator for estimating the population mean in Simple Random sampling.

$$\varphi_{ZR} = \mu_1\bar{y}\left(\frac{\bar{X}'}{\bar{x}'}\right) + \mu_2(\bar{X}-\bar{x})exp\left[\frac{\bar{X}'-\bar{x}'}{\bar{X}'+\bar{x}'}\right] \qquad (3.2)$$

Where $\bar{X}' = \alpha\bar{X}+\beta$ and $\bar{x}' = \alpha\bar{x}+\beta$

To obtain the bias and MSE of the modified estimator φ_{ZR} given by (3.2), we write

$\varepsilon_y = \frac{\bar{y}-\bar{Y}}{\bar{Y}}$ and $\varepsilon_x = \frac{\bar{x}-\bar{X}}{\bar{X}}$

Such that $E(\varepsilon_y) = E(\varepsilon_x) = 0$ and

$$E(\varepsilon_y^2) = \tau_1 = \psi C_y^2 E(\varepsilon_x^2) = \tau_2 = \psi C_x^2 E(\varepsilon_y\varepsilon_x) = \tau_3 = \psi C_{yx}$$

Expressing (3.2) in terms of ε_y and ε_x, neglecting the terms of $\varepsilon's$ having power greater than two, we have

$$\varphi_{ZR} = \mu_1\bar{Y}(1+\varepsilon_y)(1+\gamma_i)^{-1} + \mu_2\bar{X}\varepsilon_x exp\left[\frac{\gamma_i\varepsilon_x}{2}\left(1+\frac{\gamma_i\varepsilon_x}{2}\right)^{-1}\right]$$

Where $\gamma_i = \frac{\alpha\bar{X}}{\alpha\bar{X}+\beta}$

And

$$\gamma_1 = \frac{\bar{X}Md}{\bar{X}Md+TM}\gamma_2 = \frac{\bar{X}TM}{\bar{X}TM+Md}\gamma_3 = \frac{\bar{X}TM}{\bar{X}TM+C_x}$$

$$\gamma_4 = \frac{\bar{X}}{\bar{X}C_x+S_{yx}}\gamma_5 = \frac{\bar{X}Md}{\bar{X}Md+\rho_{yx}}\gamma_6 = \frac{\bar{X}MR}{\bar{X}MR+HL}$$

$$\gamma_7 = \frac{\bar{X}HL}{\bar{X}HL+MR}\gamma_8 = \frac{\bar{X}\beta_{1(x)}}{\bar{X}\beta_{2(x)}+\beta_{2(x)}}\gamma_9 = \frac{\bar{X}\beta_{1(x)}HL}{\bar{X}\beta_{1(x)}HL+MR}$$

$$\gamma_{10}=\frac{\bar{X}S_{yx}}{\bar{X}S_{yx}+\beta_{1(x)}\rho_{yx}}\gamma_{11}=\frac{\bar{X}TM}{\bar{X}TM+HL}\gamma_{12}=\frac{\bar{X}MR.HL}{\bar{X}MR.HL+TM}$$

$$\gamma_{13}=\frac{\bar{X}HL}{\bar{X}HL+\beta_{2(x)}TM}\gamma_{14}=\frac{\bar{X}MR}{\bar{X}MR+\beta_{1(x)}\beta_{2(x)}}\gamma_{15}=\frac{\bar{X}S_{yx}HL}{\bar{X}S_{yx}HL+TM}$$

$$\gamma_{16}=\frac{\bar{X}\beta_{2(x)}HL}{\bar{X}\beta_{2(x)}HL+\beta_{1(x)}}\gamma_{17}=\frac{\bar{X}\rho_{yx}HL}{\bar{X}\rho_{yx}HL+\beta_{2(x)}}\gamma_{18}=\frac{\bar{X}MR.TM}{\bar{X}MR.TM+HL}$$

$$\gamma_{19}=\frac{\bar{X}C_x}{\bar{X}C_x+\beta_{2(x)}}\gamma_{20}=\frac{\bar{X}\rho_{yx}MR}{\bar{X}\rho_{yx}MR+\beta_{1(x)}}$$

Table 3: The proposed families of estimators are:

Proposed Estimators	α	β
$\varphi_{ZR1}=\mu_1\bar{y}\left(\frac{Md\bar{X}+TM}{Md\bar{x}+TM}\right)+\mu_2(\bar{X}-\bar{x})exp\left[\frac{Md(\bar{X}-\bar{x})}{Md(\bar{X}+\bar{x})+2TM}\right]$	Md	TM
$\varphi_{ZR2}=\mu_1\bar{y}\left(\frac{TM\bar{X}+Md}{TM\bar{x}+Md}\right)+\mu_2(\bar{X}-\bar{x})exp\left[\frac{TM(\bar{X}-\bar{x})}{TM(\bar{X}+\bar{x})+2Md}\right]$	TM	Md
$\varphi_{ZR3}=\mu_1\bar{y}\left(\frac{TM\bar{X}+C_x}{TM\bar{x}+C_x}\right)+\mu_2(\bar{X}-\bar{x})exp\left[\frac{TM(\bar{X}-\bar{x})}{TM(\bar{X}+\bar{x})+2C_x}\right]$	TM	C_x
$\varphi_{ZR4}=\mu_1\bar{y}\left(\frac{S_{yx}\bar{X}+Md}{S_{yx}\bar{x}+Md}\right)+\mu_2(\bar{X}-\bar{x})exp\left[\frac{S_{yx}(\bar{X}-\bar{x})}{S_{yx}(\bar{X}+\bar{x})+2Md}\right]$	S_{yx}	Md
$\varphi_{ZR5}=\mu_1\bar{y}\left(\frac{Md\bar{X}+\rho_{yx}}{Md\bar{x}+\rho_{yx}}\right)+\mu_2(\bar{X}-\bar{x})exp\left[\frac{Md(\bar{X}-\bar{x})}{Md(\bar{X}+\bar{x})+2\rho_{yx}}\right]$	Md	ρ_{yx}
$\varphi_{ZR6}=\mu_1\bar{y}\left(\frac{MR\bar{X}+HL}{MR\bar{x}+HL}\right)+\mu_2(\bar{X}-\bar{x})exp\left[\frac{MR(\bar{X}-\bar{x})}{MR(\bar{X}+\bar{x})+2HL}\right]$	MR	HL
$\varphi_{ZR7}=\mu_1\bar{y}\left(\frac{HL\bar{X}+MR}{HL\bar{x}+MR}\right)+\mu_2(\bar{X}-\bar{x})exp\left[\frac{HL(\bar{X}-\bar{x})}{HL(\bar{X}+\bar{x})+2MR}\right]$	HL	MR
$\varphi_{ZR8}=\mu_1\bar{y}\left(\frac{\beta_{1(x)}\bar{X}+\beta_{2(x)}}{\beta_{1(x)}\bar{x}+\beta_{2(x)}}\right)+\mu_2(\bar{X}-\bar{x})exp\left[\frac{\beta_{1(x)}(\bar{X}-\bar{x})}{\beta_{1(x)}(\bar{X}+\bar{x})+2\beta_{2(x)}}\right]$	$\beta_{1(x)}$	$\beta_{2(x)}$
$\varphi_{ZR9}=\mu_1\bar{y}\left(\frac{\beta_{1(x)}HL\bar{X}+MR}{\beta_{1(x)}HL\bar{x}+MR}\right)+\mu_2(\bar{X}-\bar{x})exp\left[\frac{\beta_{1(x)}HL(\bar{X}-\bar{x})}{\beta_{1(x)}HL(\bar{X}+\bar{x})+2MR}\right]$	$\beta_{1(x)}HL$	MR
$\varphi_{ZR10}=\mu_1\bar{y}\left(\frac{S_{yx}\bar{X}+\beta_{1(x)}\rho_{yx}}{S_{yx}\bar{x}+\beta_{1(x)}\rho_{yx}}\right)+\mu_2(\bar{X}-\bar{x})exp\left[\frac{S_{yx}(\bar{X}-\bar{x})}{S_{yx}(\bar{X}+\bar{x})+2\beta_{1(x)}\rho_{yx}}\right]$	S_{yx}	$\beta_{1(x)}\rho_{yx}$
$\varphi_{ZR11}=\mu_1\bar{y}\left(\frac{TM\bar{X}+HL}{TM\bar{x}+HL}\right)+\mu_2(\bar{X}-\bar{x})exp\left[\frac{TM(\bar{X}-\bar{x})}{TM(\bar{X}+\bar{x})+2HL}\right]$	TM	HL
$\varphi_{ZR12}=\mu_1\bar{y}\left(\frac{MR.HL\bar{X}+TM}{MR.HL\bar{x}+TM}\right)+\mu_2(\bar{X}-\bar{x})exp\left[\frac{MR.HL(\bar{X}-\bar{x})}{MR.HL(\bar{X}+\bar{x})+2TM}\right]$	MR.HL	TM

$\varphi_{ZR13}=\mu_1\bar{y}\left(\frac{HL\bar{X}+TM\beta_{2(x)}}{HL\bar{x}+TM\beta_{2(x)}}\right)+\mu_2(\bar{X}-\bar{x})exp\left[\frac{HL(\bar{X}-\bar{x})}{HL(\bar{X}+\bar{x})+2TM\beta_{2(x)}}\right]$	HL	$TM\beta_{2(x)}$
$\varphi_{ZR14}=\mu_1\bar{y}\left(\frac{MR\bar{X}+\beta_{1(x)}\beta_{2(x)}}{MR\bar{x}+\beta_{1(x)}\beta_{2(x)}}\right)+\mu_2(\bar{X}-\bar{x})exp\left[\frac{MR(\bar{X}-\bar{x})}{MR(\bar{X}+\bar{x})+2\beta_{1(x)}\beta_{2(x)}}\right]$	MR	$\beta_{1(x)}\beta_{2(x)}$
$\varphi_{ZR15}=\mu_1\bar{y}\left(\frac{S_{yx}HL\bar{X}+TM}{S_{yx}HL\bar{x}+TM}\right)+\mu_2(\bar{X}-\bar{x})exp\left[\frac{S_{yx}HL(\bar{X}-\bar{x})}{S_{yx}HL(\bar{X}+\bar{x})+2TM}\right]$	$S_{yx}HL$	TM
$\varphi_{ZR16}=\mu_1\bar{y}\left(\frac{\beta_{2(x)}HL\bar{X}+\beta_{1(x)}}{\beta_{2(x)}HL\bar{x}+\beta_{1(x)}}\right)+\mu_2(\bar{X}-\bar{x})exp\left[\frac{\beta_{2(x)}HL(\bar{X}-\bar{x})}{\beta_{2(x)}HL(\bar{X}+\bar{x})+2\beta_{1(x)}}\right]$	$\beta_{2(x)}HL$	$\beta_{1(x)}$
$\varphi_{ZR17}=\mu_1\bar{y}\left(\frac{\rho_{yx}HL\bar{X}+\beta_{2(x)}}{\rho_{yx}HL\bar{x}+\beta_{2(x)}}\right)+\mu_2(\bar{X}-\bar{x})exp\left[\frac{\rho_{yx}HL(\bar{X}-\bar{x})}{\rho_{yx}HL(\bar{X}+\bar{x})+2\beta_{2(x)}}\right]$	$\rho_{yx}HL$	$\beta_{2(x)}$
$\varphi_{ZR18}=\mu_1\bar{y}\left(\frac{MR.TM\bar{X}+HL}{MR.TM\bar{x}+HL}\right)+\mu_2(\bar{X}-\bar{x})exp\left[\frac{MR.TM(\bar{X}-\bar{x})}{MR.TM(\bar{X}+\bar{x})+2HL}\right]$	$MR.TM$	HL
$\varphi_{ZR19}=\mu_1\bar{y}\left(\frac{C_x\bar{X}+\beta_{2(x)}}{C_x\bar{x}+\beta_{2(x)}}\right)+\mu_2(\bar{X}-\bar{x})exp\left[\frac{C_x(\bar{X}-\bar{x})}{C_x(\bar{X}+\bar{x})+2\beta_{2(x)}}\right]$	C_x	$\beta_{2(x)}$
$\varphi_{ZR20}=\mu_1\bar{y}\left(\frac{\rho_{yx}MR\bar{X}+\beta_{1(x)}}{\rho_{yx}MR\bar{x}+\beta_{1(x)}}\right)+\mu_2(\bar{X}-\bar{x})exp\left[\frac{\rho_{yx}MR(\bar{X}-\bar{x})}{\rho_{yx}MR(\bar{X}+\bar{x})+2\beta_{1(x)}}\right]$	$\rho_{yx}MR$	$\beta_{1(x)}$

Expressing (2.1) in terms of ε_y and ε_x, neglecting the terms of $e's$ having power greater than two, we have

$$\varphi_{ZR}-\bar{Y}=(\mu_1-1)\bar{Y}+\mu_1\bar{Y}\varepsilon_y-\mu_1\bar{Y}\gamma_i\varepsilon_x-\mu_2\bar{X}\varepsilon_x+\mu_1\bar{Y}\gamma_i^2\varepsilon_x^2+\frac{\mu_2\bar{X}\gamma_i^2\varepsilon_x^2}{2}-\mu_1\bar{Y}\gamma_i\varepsilon_y\varepsilon_x$$

Taking expectation of both sides of, we have the bias of φ_{ZR} up to the first degree of approximation as:

$$Bias(\varphi_{ZR})=\bar{Y}\left[(\mu_1-1)+\gamma_i\{\mu_1(\gamma_i\tau_2-\tau_3\}+\frac{\mu_2R\tau_2}{2}\right] \quad (3.3)$$

Squaring both sides of (2.5) and neglecting the terms of $e's$ having power greater than two and then taking expectation of both sides, we get the MSE of the estimator φ_{ZR} to the first degree of approximation as

$$MSE(\varphi_{ZR})=\bar{Y}^2\left[1+\mu_1^2(1+\tau_1+3\gamma_i^2\tau_2-4\gamma_i\tau_3)+\mu_2^2R^2\tau_2-2\mu_1\mu_2R\left(\tau_3-\frac{3\gamma_i\tau_2}{2}\right)-2\mu_1(1+\gamma_i^2\tau_2-\gamma_i\tau_3)\right.$$
$$\left.-\mu_2R\gamma_i\tau_2\right]$$

$$MSE(\varphi_{ZR})=\bar{Y}^2[1+\mu_1^2S_1+\mu_2^2R^2\tau_2-2\mu_1\mu_2RS_2-2\mu_1S_3-\mu_2R\gamma_i\tau_2] \quad (3.4)$$

Where, $R=\frac{\bar{X}}{\bar{Y}}$

For minimum MSE, differentiate (3.4) with respect to μ_1 and μ_2 respectively and equating to zero, we get $\mu_{1(opt)}$ and $\mu_{2(opt)}$

$\frac{\partial MSE(\varphi_{ZR})}{\partial\mu_1}=0$ and $\frac{\partial MSE(\varphi_{ZR})}{\partial\mu_2}=0$

$\mu_{1(opt)}=\frac{\tau_2(2S_3+S_2\gamma_i)}{2(S_1\tau_2-S_2{}^2)}$ and $\mu_{2(opt)}=\frac{2S_2S_3+S_1\gamma_i\tau_2}{2R(S_1\tau_2-S_2{}^2)}$

Minimum MSE of φ_{ZR} up to first degree of approximation is obtained by substituting the optimal values of μ_1 and μ_2 in equation (3.4) and simplifying as

$$MSE_{min}(\varphi_{ZR})=\frac{\bar{Y}^2\tau_2}{4S_4{}^2}\left[\frac{4S_4{}^2}{\tau_2}+S_5{}^2(S_4+S_2{}^2)+S_6{}^2-2S_2S_5S_6-4S_3S_5S_4-2\gamma_iS_4S_6\right] \quad (3.5)$$

Where,

$S_1 = 1 + \tau_1 + 3\gamma_i^2\tau_2 - 4\gamma_i\tau_3$, $S_2 = \tau_3 - \frac{3\gamma_i\tau_2}{2}$, $S_3 = 1 + \gamma_i^2\tau_2 - \gamma_i\tau_3$,

$S_4 = S_1\tau_2 - {S_2}^2$, $S_5 = 2S_3 + S_2$. $S_6 = 2S_2S_3 + S_1\tau_2$

Interesting note:

Many more ratio-type estimators can be formulated by taking different measures of γ_i

4 Efficiency Comparisons

Comparison with existing estimators: This section deals with the derivation of algebraic situation, under which the proposed estimators will have minimum MSE as compared to estimators in literature,

1. φ_{ZR} Perform better than $\hat{\hat{Y}}_R$ estimator if,

$$MSE(\hat{\hat{Y}}_R) > MSE_{min}(\varphi_{ZR})$$

$$\psi\bar{Y}^2\left[C_y^2 - C_x^2(1 - 2\rho_{yx}\frac{C_y}{C_x})\right] > \frac{\bar{Y}^2\psi C_x^2}{4{S_4}^2}\left[\frac{4{S_4}^2}{\psi C_x^2} + B\right]$$

Where, $B = {S_5}^2(S_4 + {S_2}^2) + {S_6}^2 - 2S_2S_5S_6 - 4S_3S_5S_4 - 2\gamma_iS_4S_6$

$$\psi\left[C_y^2 - C_x^2(1 - 2\rho_{yx}\frac{C_y}{C_x} + \frac{B}{4{S_4}^2})\right] > 1$$

2. φ_{ZR} Perform better than $\hat{\hat{Y}}_{BT}$ estimator if,

$$MSE(\hat{\hat{Y}}_{BT}) > MSE_{min}(\varphi_{ZR})$$

$$\frac{\psi}{4}\bar{Y}^2\left[4C_y^2 + C_x^2 - 4\rho_{yx}C_yC_x\right] > \frac{\bar{Y}^2\psi C_x^2}{4{S_4}^2}\left[\frac{4{S_4}^2}{\psi C_x^2} + B\right]$$

$$4\left[C_y^2 - \rho_{yx}C_yC_x - \frac{1}{\psi}\right] + C_x^2\left(1 - \frac{B}{{S_4}^2}\right) > 0$$

3. φ_{ZR} Perform better than $\hat{\hat{Y}}_{SI}$ estimator if,

$$MSE_{min}(\hat{\hat{Y}}_{SI}) > MSE_{min}(\varphi_{ZR})$$

$$\psi\bar{Y}^2C_y^2(1 - \rho_{yx}^2) > \frac{\bar{Y}^2\psi C_x^2}{4{S_4}^2}\left[\frac{4{S_4}^2}{\psi C_x^2} + B\right]$$

$$\psi\left[C_y^2(1 - \rho_{yx}^2) - \frac{BC_x^2}{4{S_4}^2}\right] > 1$$

4. φ_{ZR} Perform better than η family of estimators if,

$$\text{MSE}\ (\eta) > MSE_{min}(\varphi_{ZR})$$

$$\psi\bar{Y}^2\left[C_y^2 + \theta_i^2C_x^2 - 2\theta_i\rho_{yx}C_yC_x\right] > \left[\frac{4{S_4}^2}{\psi C_x^2} + B\right]$$

$$\psi\left[C_y^2 + C_x^2\left(\theta_i^2 - \frac{B}{4{S_4}^2}\right) - 2\theta_i\rho_{yx}C_yC_x\right] > 1$$

5. φ_{ZR} Perform better than $\hat{\hat{Y}}_{UP}$ estimator if,

$$\text{MSE}_{\text{min}}\ (\hat{\hat{Y}}_{UP}) > MSE_{min}(\varphi_{ZR})$$

$$MSE_{min}(\hat{\hat{Y}}_{SI}) > MSE_{min}(\varphi_{ZR})$$

$$\boldsymbol{\psi\bar{Y}^2C_y^2(1-\rho_{yx}^2) > \frac{\bar{Y}^2\psi C_x^2}{4S_4{}^2}\left[\frac{4S_4{}^2}{\psi C_x^2}+B\right]}$$

$$\boldsymbol{\psi\left[C_y^2(1-\rho_{yx}^2)-\frac{BC_x^2}{4S_4{}^2}\right]} > 1$$

6. φ_{ZR} Perform better than t family of estimators if,

$\text{MSE}_{\min}(\text{t})) > MSE_{min}(\varphi_{ZR})$

$$\bar{Y}^2\left(1-\frac{C_{1i}^2}{C_{2i}^2}\right) > \frac{\bar{Y}^2\psi C_x^2}{4S_4{}^2}\left[\frac{4S_4{}^2}{\psi C_x^2}+B\right]$$

$$\frac{B\psi C_x^2}{4S_4{}^2}+\frac{C_{1i}^2}{C_{2i}^2} < 0$$

5 Empirical Studies

We use the data of Singh and Chaudhary (1986) page 177

N =34 , n=20, $\bar{Y}$=856.4117, $\bar{X}$=208.8823, ρ=0.4491, S_Y=733.1407 ,C_Y=0.8561, S_X=150.5059,C_X=0.7205, β_2=0.0978, TM =162.25, Median=150 β_1=0.9782

Table 4: Mean Square Error (MSE) of Existing estimators:

Estimators	MSE
$MSE(\bar{y})$	11073.41
MSE ($\hat{\bar{Y}}_R$)	10546
MSE ($\hat{\bar{Y}}_{BT}$)	8848.872
$MSE\left(\hat{\bar{Y}}_{Reg}\right)=MSE\left(\hat{\bar{Y}}_i\right), i=SI, UP, K$	8840.005

Table 5: Mean Square Error (MSE) of t and η family of Existing estimators.

Estimators (η family)	MSE	Estimators (t family)	MSE
η_1	8848.872	t_1	8722.471
η_2	8848.977	t_2	8722.689
η_3	8849.791	t_3	8723.772
η_4	8849.461	t_4	8723.335
η_5	8860.104	t_5	8736.695
η_6	8849.031	t_6	8722.760
η_7	8849.680	t_7	8723.626
η_8	8850.989	t_8	8725.344
η_9	8855.483	t_9	8731.047
η_{10}	8849.137	t_{10}	8722.903

Table 6: Mean Square Error (MSE) of proposed estimators

Estimators (φ family)	MSE	Estimators (φ family)	MSE
φ_{ZR1}	8690.811	φ_{ZR11}	8680.484
φ_{ZR2}	8679.624	φ_{ZR12}	8680.157
φ_{ZR3}	8680.263	φ_{ZR13}	8685.986
φ_{ZR4}	8680.157	φ_{ZR14}	8679.975
φ_{ZR5}	8680.157	φ_{ZR15}	8679.945
φ_{ZR6}	8694.000	φ_{ZR16}	8683.752
φ_{ZR7}	8686.885	φ_{ZR17}	8680.021
φ_{ZR8}	8687.164	φ_{ZR18}	8680.157
φ_{ZR9}	8689.245	φ_{ZR19}	8689.745
φ_{ZR10}	8679.945	φ_{ZR20}	8680.504

Table 7: Percent Relative Efficiencies of proposed estimators with respect to existing estimators.

Estimators	$\bar{y}$	$\hat{\bar{Y}}_R$	$\hat{\bar{Y}}_{BT}$	$\hat{\bar{Y}}_{Reg} = \hat{\bar{Y}}_i$, $i = SI, UP, K$
φ_{ZR1}	127.4152	121.3466	101.8187	101.7167
φ_{ZR2}	127.5794	121.5030	101.9499	101.8478
φ_{ZR3}	127.5700	121.4940	101.9424	101.8403
φ_{ZR4}	127.5715	121.4955	101.9437	101.8415
φ_{ZR5}	127.5715	121.4955	101.9437	101.8415
φ_{ZR6}	127.3684	121.3020	101.7814	101.6794
φ_{ZR7}	127.4727	121.4014	101.8647	101.7627
φ_{ZR8}	127.4686	121.3975	101.8615	101.7594
φ_{ZR9}	127.4381	121.3684	101.8371	101.7350
φ_{ZR10}	127.5747	121.4985	101.9462	101.8440
φ_{ZR11}	127.5667	121.4909	101.9398	101.8377
φ_{ZR12}	127.5715	121.4955	101.9437	101.8415
φ_{ZR13}	127.4859	121.4140	101.8753	101.7732
φ_{ZR14}	127.5742	121.4980	101.9458	101.8437
φ_{ZR15}	127.5747	121.4985	101.9462	101.8440
φ_{ZR16}	127.5187	121.4452	101.9015	101.7994
φ_{ZR17}	127.5735	121.4974	101.9453	101.8431
φ_{ZR18}	127.5715	121.4955	101.9437	101.8415
φ_{ZR19}	127.4308	121.3614	101.8312	101.7292
φ_{ZR20}	127.5664	121.4906	101.9396	101.8375

Table 8: Percent Relative Efficiencies of proposed estimators with respect to η family of estimators

Estimators	η1	η2	η3	η4	η5	η6	η7	η8	η9	η10
φ_{ZR1}	101.8187	101.8199	101.8293	101.8255	101.948	101.8205	101.828	101.8431	101.8948	101.8218
φ_{ZR2}	101.9499	101.9512	101.9605	101.9567	102.0794	101.9518	101.9593	101.9743	102.0261	101.953
φ_{ZR3}	101.9424	101.9437	101.953	101.9492	102.0718	101.9443	101.9518	101.9668	102.0186	101.9455
φ_{ZR4}	101.9437	101.9449	101.9543	101.9505	102.0731	101.9455	101.953	101.9681	102.0198	101.9467
φ_{ZR5}	101.9437	101.9449	101.9543	101.9505	102.0731	101.9455	101.953	101.9681	102.0198	101.9467

φ_{ZR6}	101.7814	101.7826	101.7919	101.7881	101.9106	101.7832	101.7907	101.8057	101.8574	101.7844
φ_{ZR7}	101.8647	101.8659	101.8753	101.8715	101.994	101.8666	101.874	101.8891	101.9408	101.8678
φ_{ZR8}	101.8615	101.8627	101.872	101.8682	101.9908	101.8633	101.8708	101.8858	101.9376	101.8645
φ_{ZR9}	101.8371	101.8383	101.8476	101.8438	101.9663	101.8389	101.8464	101.8614	101.9131	101.8401
φ_{ZR10}	101.9462	101.9474	101.9568	101.953	102.0756	101.948	101.9555	101.9706	102.0223	101.9492
φ_{ZR11}	101.9398	101.9411	101.9504	101.9466	102.0692	101.9417	101.9492	101.9642	102.016	101.9429
φ_{ZR12}	101.9437	101.9449	101.9543	101.9505	102.0731	101.9455	101.953	101.9681	102.0198	101.9467
φ_{ZR13}	101.8753	101.8765	101.8859	101.8821	102.0046	101.8771	101.8846	101.8996	101.9514	101.8783
φ_{ZR14}	101.9458	101.947	101.9564	101.9526	102.0752	101.9477	101.9551	101.9702	102.022	101.9489
φ_{ZR15}	101.9462	101.9474	101.9568	101.953	102.0756	101.948	101.9555	101.9706	102.0223	101.9492
φ_{ZR16}	101.9015	101.9027	101.9121	101.9083	102.0308	101.9033	101.9108	101.9259	101.9776	101.9045
φ_{ZR17}	101.9453	101.9465	101.9559	101.9521	102.0747	101.9471	101.9546	101.9697	102.0214	101.9483
φ_{ZR18}	101.9437	101.9449	101.9543	101.9505	102.0731	101.9455	101.953	101.9681	102.0198	101.9467
φ_{ZR19}	101.8312	101.8324	101.8418	101.838	101.9605	101.833	101.8405	101.8556	101.9073	101.8343
φ_{ZR20}	101.9396	101.9408	101.9502	101.9464	102.069	101.9414	101.9489	101.964	102.0158	101.9427

Table 9: Percent Relative Efficiencies of proposed estimators with to t-family

Estimators	t1	t2	t3	t4	t5	t6	t7	t8	t9	t10
φ_{ZR1}	100.3643	100.3668	100.3793	100.3742	100.528	100.3676	100.3776	100.3974	100.463	100.3693
φ_{ZR2}	100.4937	100.4962	100.5086	100.5036	100.6575	100.497	100.507	100.5268	100.5925	100.4986
φ_{ZR3}	100.4863	100.4888	100.5012	100.4962	100.6501	100.4896	100.4996	100.5194	100.5851	100.4912
φ_{ZR4}	100.4875	100.49	100.5025	100.4974	100.6513	100.4908	100.5008	100.5206	100.5863	100.4925
φ_{ZR5}	100.4875	100.49	100.5025	100.4974	100.6513	100.4908	100.5008	100.5206	100.5863	100.4925
φ_{ZR6}	100.3275	100.33	100.3424	100.3374	100.4911	100.3308	100.3408	100.3605	100.4261	100.3324
φ_{ZR7}	100.4097	100.4122	100.4246	100.4196	100.5734	100.4130	100.4229	100.4427	100.5084	100.4146
φ_{ZR8}	100.4064	100.4089	100.4214	100.4164	100.5702	100.4098	100.4197	100.4395	100.5052	100.4114
φ_{ZR9}	100.3824	100.3849	100.3974	100.3923	100.5461	100.3857	100.3957	100.4154	100.4811	100.3874
φ_{ZR10}	100.4899	100.4924	100.5049	100.4999	100.6538	100.4933	100.5032	100.5230	100.5887	100.4949
φ_{ZR11}	100.4837	100.4862	100.4987	100.4936	100.6476	100.4870	100.4970	100.5168	100.5825	100.4887
φ_{ZR12}	100.4875	100.4900	100.5025	100.4974	100.6513	100.4908	100.5008	100.5206	100.5863	100.4925
φ_{ZR13}	100.4200	100.4226	100.4350	100.4300	100.5838	100.4234	100.4333	100.4531	100.5188	100.4250
φ_{ZR14}	100.4896	100.4921	100.5046	100.4995	100.6535	100.4929	100.5029	100.5227	100.5884	100.4946
φ_{ZR15}	100.4899	100.4924	100.5049	100.4999	100.6538	100.4933	100.5032	100.5230	100.5887	100.4949
φ_{ZR16}	100.4459	100.4484	100.4609	100.4558	100.6097	100.4492	100.4592	100.4790	100.5446	100.4509
φ_{ZR17}	100.4891	100.4916	100.5040	100.499	100.6529	100.4924	100.5024	100.5222	100.5879	100.4940
φ_{ZR18}	100.4875	100.49	100.5025	100.4974	100.6513	100.4908	100.5008	100.5206	100.5863	100.4925
φ_{ZR19}	100.3766	100.3791	100.3916	100.3865	100.5403	100.3799	100.3899	100.4097	100.4753	100.3816
φ_{ZR20}	100.4835	100.486	100.4985	100.4934	100.6473	100.4868	100.4968	100.5166	100.5823	100.4884

The percent relative efficiencies (PRE) of the proposed estimators, with respective to the existing estimators is computed by

$$PRE = \frac{MSE\ of\ Existing\ Estimator}{MSE\ of\ proposed\ estimator} \times 100$$

From these tables it is clear that the proposed estimators are quite efficient with respect to the estimators present in the literature.

6 Conclusions

Hence by using the auxiliary information of Kurtosis, Skewness, Correlation, Trimmed Mean, Median, Mid-Range, Hodges-Lehman estimator etc., our proposed estimators perform better than the classical and the existing the estimators as their mean square error is lower than the classical and the existing estimators in the literature. Thus, we strongly recommend that our proposed estimators proffered over the estimators in the literature and also over classical estimators, thus should be used in future for practical applications.

References

[1] Bahl, Shashi, and RK Tuteja. "Ratio and product type exponential estimators." Journal of information and optimization sciences., **12(1)**,159-164 (1991).

[2] Cochran, W. G. "The estimation of the yields of cereal experiments by sampling for the ratio of grain to total produce." The journal of agricultural science., **30(2)**, 262-275 (1940).

[3] Cochran, W.G. Sampling Techniques, 3rd edn. New York: John Wiley and Sons (1977).

[4] Irfan, Muhammad, Maria Javed, and Zhengyan Lin. "Enhanced estimation of population mean in the presence of auxiliary information." Journal of King Saud University-Science., **31(4),** 1373-1378 (2019)

[5] Kadilar, G.O., A new exponential type estimator for the population mean in simple random sampling. J. Mod. Appl. Stat. Methods.,**15 (2)**, 207–214 (2016).

[6] Kadilar, C., Cingi, H., Ratio estimators in stratified random sampling. Biometrical Journal.,**45 (2)**, 218–225 (2003).

[7] Kadilar, C., Cingi, H. Ratio estimators in simple random sampling. Appl. Math. Comput.. **151**, 893–902 (2004).

[8] Kadilar, C., Cingi, H.,. An improvement in estimating the population mean byusing the correlation coefficient. Hacettepe Journal. Math. Stat.,**35 (1)**, 103–109 (2006a).

[9] Kadilar, C., Cingi, H.New ratio estimators using correlation coefficient. Interstat., 1 11(2006b).

[10] Kadilar, C., Cingi, H., Improvement in estimating the population mean in simple random sampling. Appl. Math. Lett. 19, 75–79. (2006c).

[11] Khan, S.A., Ali, H., Manzoor, S., Alamgir,. A class of transformed efficient ratio

estimators of finite population mean. Pakistan J. Statist.,**31 (4)**, 353–362 (2015)

[12] Khoshnevisan, M., Singh, R., Chauhan, P., Sawan, N., Smarandache, F. A general

family of estimators for estimating population mean using known value of some population parameter(s). Far East J. Theor. Stat.,**22 (2)**, 181–191 (2007)

[13] Koyuncu, N., Kadilar, C., Efficient estimators for the population mean. Hacettepe J. Math. Statist.,**38 (2)**, 217–225 (2009).

[14] Singh, R., Chauhan, P., Sawan, N., On linear combination of ratio and product

type exponential estimator for estimating the finite population mean. Statist.Transition., **9(1)**, 105–115 (2008).

[15] Singh, R., Chauhan, P., Sawan, N., Smarandache, F. Improvement in estimating the population mean using exponential estimator in simple random sampling. Bull. Statist. Econ., **3**, 13–18 (2009).

[16] Singh, H.P., Tailor, R. Use of known correlation coefficient in estimating the finite population means. Statist. Transition.,**6 (4)**, 555–560. (2003)

[17] Srisodaphol, W., Kingphai, K., Tanjai, N. New ratio estimators of a population.

mean using one auxiliary variable in simple random sampling. Chiang Mai J. Sci.,**42 (2)**, 523–527 (2015).

[18] Upadhyaya, L.N., Singh, H.P. Chatterjee, S., Yadav, R. Improved ratio and product exponential type estimators. J. Statist. Theory Pract.,**5 (2)**, 285–302 (2011).

[19] Yan, Z., Tian, B., Ratio method to the mean estimation using coefficient of skewness of auxiliary variable. ICICA. Part II, CCIS.,**106**, 103–110 (2010).

[20] Yadav, S.K., Kadilar, C. Efficient family of exponential estimators for the population mean. Hacettepe J. Math. Statist., **42(6)**, 671–677 (2013).

[21] Ahmad, Sohail, Muhammad Arslan, Aamna Khan, and Javid Shabbir. "A generalized exponential-type estimator for population means using auxiliary attributes." Plos one., **16(5)**, (2021).

[22] Sanaullah, Aamir, Muhammad Noor ul Amin, Muhammad Hanif, and Nursel Koyuncu. "Generalized exponential-type estimators for population mean taking two auxiliary variables for unknown means in stratified sampling with sub-sampling the non-respondents." International Journal of Applied and Computational Mathematics., **4(2)**, 1-28. (2018)

[23] Singh, Rajesh, Prabhakar Mishra, Ahmed Auduudu, and Supriya Khare. "Exponential Type Estimator for Estimating Finite Population Mean." International Journal of Computational and Theoretical Statistics ., **07(01)** (2020).

[24] Unal, C. E. R. E. N., and CEM KADILAR. "A New Family of Exponential Type Estimators in the Presence of Non-Response." (2021).