Applied Mathematics & Information Sciences
An International Journal

http://dx.doi.org/10.18576/amis/110111

Steady Thermosolutocapillary Instability in Fluid Layer with Nondeformable Free Surface in the Presence of Insoluble Surfactant and Gravity

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Received: 1 Aug. 2016, Revised: 28 Sep. 2016, Accepted: 5 Oct. 2016
Published online: 1 Jan. 2017

Abstract: Steady thermosolutocapillary instabilities in a horizontal thin fluid layer with deformable free surface and uniform temperature at the bottom boundary in the presence of insoluble surfactant and gravity force are examined. The surface tension at the free surface is assumed to be linearly dependent on temperature and concentration gradients. The linear stability theory and the Galerkin method are used to obtain the closed form solutions. The effects of the controlling parameters, namely the Rayleigh number, Biot number, Lewis number, and elasticity parameter on the onset of Marangoni convection are analyzed. The results show that the gravitational force acts as destabilizer while the presence of surfactant delays the onset of convection.

Keywords: Exact analytical solutions, Marangoni Instability, Nondeformable free surface, Surfactant.

1 Introduction

MARANGONI flow is induced by surface tension variation along liquid-liquid and liquid-gas interfaces. The Marangoni behaviours of fluid dynamics in thin liquid films have been extensively investigated by researchers due to the great importance of its applications in industries and material processing, for examples coating, spray painting and moulding. Thin liquid films also exist in a variety of biological context. Variety of waveforms are generated at the interface between the flowing liquid and surrounding gas. These waveforms are determined by the balances of principle driving forces such as gravity, surface tension and viscous effect.

The main subject that always been stressed in the studies of thermocapillary and solutocapillary is their instabilities. The studies of convective instability started with the experimental and theoretical works of [1], [2] and [3]. [3] was the first to investigate theoretically the Marangoni instability in a liquid layer by introducing the linear stability analysis. [4] extensively reviewed and discussed the thermocapillary instability. The investigation of the instabilities is important to avoid striation, dendrites or bubbles in the process of manufacturing due to any disturbances that might occur during the process, for example, due to the external forces and stresses. The disturbances can cause steady flows to change to an undesired oscillatory motion.

Many factors affect the instability of Marangoni convection. The onset of instabilities is proven to delay by the use of feedback control mechanism as discussed in [5] where the Marangoni-Benard convection is altered and maintained by the use of thermal proportional feedback control. The influences on the onset of Marangoni convection such as variable viscosity, free surface deformation, gravity waves, controller gains and heat transfer mechanism are discussed by [6] and [7] through the exact analytical solutions to the system. Magnetic field is also one of the factors that can influence the onset of Marangoni instabilities [7].

The onset Marangoni convection can be influenced by the existence of surface active agent or surfactant. The surface tension in a fluid layer can be affected by the presence of surfactant which produces the additional tangential stresses and therefore influences the convection. The presence of the temperature and solute concentration will affect the surface tension of the fluid layer [8,9]. When the surface is deformable, long-wave instability sets in [10,11]. [12] and [13] studied the long-wavelength Marangoni instability with

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nondeformable and deformable free surface in the presence of insoluble surfactant for both steady and oscillatory convection. They also examined the parametric excitation by periodic heat flux modulation. It was found that the surface active agent has stabilizing effect on the monotonic instability. The ratio of the amplitude of the external heat flux modulation to the stationary mean heat flux and the frequency of the modulation affected the subharmonic instability region.

Numerical investigation has been carried out by [14] on the onset of convection in a horizontal layer of fluid heated from below in the presence of various gravity field. The method of weighted residual (Galerkin type) and collocation employed provide fairly accurate approximation validated by viola's eigenvalue problems.

[15] focused on a weak nonlinear analysis of double diffusive convection with an applied uniform magnetic field with the effect of time periodic gravity field on heat and mass transfer. The derivation of complex Guinzburg-Landau amplitude equation was performed. The range of viscoelastic parameters provides oscillatory behavior. The presence of magnetic field delayed the onset convection whereas the Rayleigh number advanced the onset convection.

[16] emphasized on the stability analysis of dependent non-linear base concentration profile on the concentration and velocity disturbances in the Rayleigh-Bernard, Benard-Marangoni and Rayleigh-Benard-Marangoni problems. They developed spatial base-profile influenced frozen-time marginal state analysis model (SIFTM) to calculate the critical Rayleigh number and Marangoni number.

Theoretical observation has been made by [17] on the thermalvibrational instabilities of the Marangoni-Bernard and Rayleigh-Marangoni-Bernard by using linear stability analysis in a two layer liquid system. The vertical vibration leads to the stability and horizontal vibration makes the system unstable. The presence of g-disturbances affects the oscillatory region.

In this paper, we consider the steady thermosolutocapillary instability in a fluid layer in the presence of insoluble surfactant and gravity for nondeformable surface. The exact analytical solution is obtained for the critical Marangoni number and some physical parameters are assessed.

2 Problem formulation

Consider a horizontal fluid layer of thickness d bounded by a rigid plate at the lower boundary at $z=0$ and the upper surface is a deformable free surface at $z=d$. The lower boundary has a non slip condition and is maintained at a fixed uniform temperature. The fluid is assumed to be incompressible and the surface tension σ is assumed to depend linearly on temperature and surfactant concentration,

$$\sigma=\sigma_0-\sigma_1(T-T_0)-\sigma_2(\Gamma-\Gamma_0) \tag{1}$$

where T and Γ are the temperature of the liquid and the surfactant concentration, respectively. σ_0 is the reference surface tension corresponding to the reference temperature T_0 and reference concentration Γ_0 and σ_1 and σ_2 are positive constants.

Other physical properties of the liquid such as density, pressure, viscosity etc. are assumed to be constants. When the fluid is at rest, the hydrodynamic pressure p_b with the atmospheric pressure p_a and the gravitational acceleration g at the reference steady state is

$$p_b=p_a-\rho g(d-z). \tag{2}$$

The governing equations of the liquid system are

$$\nabla\cdot\mathbf{v}=0, \tag{3}$$

$$\frac{\partial\mathbf{v}}{\partial t}+(\mathbf{v}\cdot\nabla)\mathbf{v}=-\frac{\nabla p}{\rho}-\nu\nabla^2\mathbf{v}-\mathbf{g}\lambda(T-T_0)\exp_z, \tag{4}$$

$$\frac{\partial T}{\partial t}+(\mathbf{v}\cdot\nabla)T=\chi\nabla^2T, \tag{5}$$

which represent the equations of continuity (mass), momentum and energy, respectively. $\mathbf{v}=(u,v,w)$ is the fluid velocities in the (x,y,z) directions, $\mathbf{g}$ is the gravitational field, χ is the thermal diffusivity, ν is the kinematic viscosity, p is the pressure, ρ is the density, λ is a constant and t is the time. The surfactant distribution at the free surface [12], is

$$\frac{\partial\Gamma}{\partial t}+\nabla_s\cdot(u_\tau\Gamma)+Ku_n\Gamma=D_0\nabla_s^2\Gamma, \tag{6}$$

where u_n and u_τ are normal and tangential velocities, respectively, D_0 is the surfactant diffusivity, K is the local surface curvature and ∇_s is the surface gradient.

At the free nondeformable surface, where the value of surface deflection $Z=0$, the linearized system of equations and boundary conditions become

$$D^4W-2k^2D^2W+k^4W+k^2R\theta=0, \tag{7}$$

$$D^2\theta-k^2\theta+W=0. \tag{8}$$

The no-slip and temperature conditions at the bottom surface $z=0$.

$$W=DW=0, \tag{9}$$

$$\theta=0.\quad \text{(uniform temperature)} \tag{10}$$

At the upper free surface, $z=1$,

$$W=0, \tag{11}$$

$$D\theta = -Bi\theta, \tag{12}$$

$$(D^2+k^2)W + Mk^2\theta + \frac{NDW}{L} = 0, \tag{13}$$

where W and θ are the velocity and temperature amplitudes, respectively, and k is the wave number. The nondimensional parameters are the Biot number Bi, Lewis number L , Marangoni number M and elasticity number N and $D = \frac{d}{dz}$.

The onset of steady Marangoni convection can be determined by solving the system of equations (7) – (13). The analytical solutions for the steady Marangoni convection will be determined and the marginal curves will be plotted to assess the effects of the parameters on the critical Marangoni number.

3 Solutions to the linearized problems

The system of equations (7) – (13) are solved using the Galerkin Method. Multiplying (7) and (8) by W and θ, respectively and integrating the resulting equations by parts with respect to z from 0 to 1. The chosen trial functions are

$$W_1 = z^2 - z^4 \quad \text{and} \quad \theta_1 = z^2 - \frac{2+Bi}{1+Bi}, \tag{14}$$

which satisfy all the boundary conditions (9) to (13) and taking $W = AW_1$ and $T = B\theta_1$ where A and B are constants . The expression for the Marangoni number M is

$$M = \frac{C_1 - C_2C_3}{C_4} \tag{15}$$

where

$$C_1 = Rk^2 < W_1\theta_1 >^2, \tag{16}$$

$$C_2 = Rk^2 < W_1\theta_1 >^2, \tag{17}$$

$$C_3 = \frac{N}{L}[DW_1(1))^2 + < (D^2W_1)^2 > -2k^2 < (DW_1)^2 > \tag{18}$$
$$+k^2 < W_1^2 >],$$

$$C_4 = k^2DW_1(1)\theta_1(1) < W\theta_1 >, \tag{19}$$

where the angle brackets $< \cdot >$ denote the integration with respect to z from 0 to 1.

Performing the integration yields

$$C_1 = \frac{Rk^2(46+11Bi)^2}{176400(1+Bi)^2}, \tag{20}$$

$$C_2 = \frac{50Bi + 10Bi^2 + 40 + k^2(16+7Bi+Bi^2)}{30(1+Bi^2)}, \tag{21}$$

$$C_3 = \frac{8N(k^4 - 33k^2 + 819)}{315L}, \tag{22}$$

$$C_4 = \frac{k^2(46+11Bi)}{210(1+Bi)^2}, \tag{23}$$

$$M = \frac{1}{2520(46+11Bi)k^2L}\Big(6348RLk^2 + 3036RLBik^2 \tag{24}$$
$$+363RLk^2Bi^2 - 18345600NBi - 1829184NBik^2$$
$$+81088NBik^4 - 3669120NBi^2 - 219072Nk^2Bi^2$$
$$+10304Nk^2Bi^4 - 14676480N - 5279232Nk^2$$
$$+218624Nk^4 - 7168Nk^6 - 3136NBik^6 - 448NBi^2k^6\Big),$$

4 Result and discussion

The solution to the steady thermocapillary instabilities are shown in the form of marginal curves in the plane (k,M). The values of M and its critical values Mc will be determined. The curves are plotted by accessing the physical parameters such as Rayleigh number, Lewis number, Biot number and elasticity number.

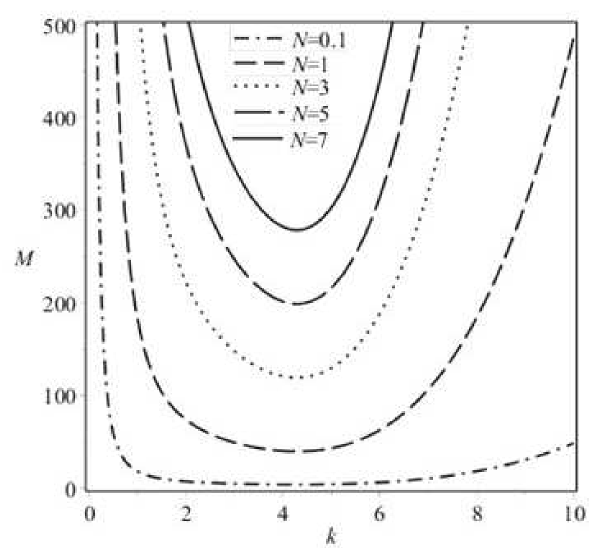

Fig. 1: Stability curves for steady convection with R = 0.1, L = 1 and Bi = 0.1 for various values of elasticity number.

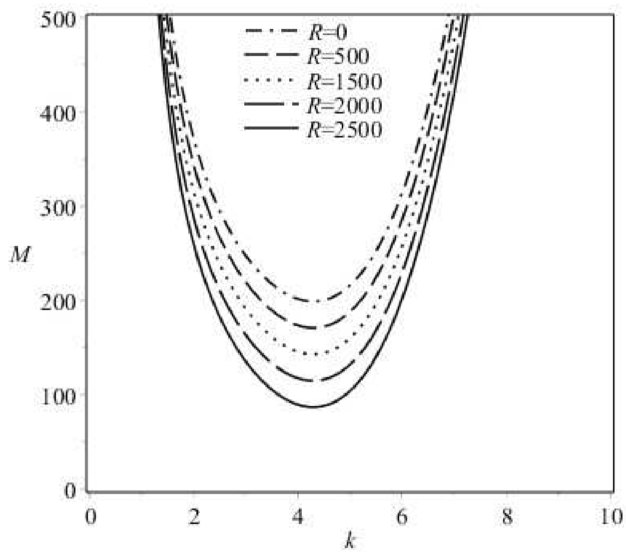

Fig. 2: Stability curves for steady convection with N = 5, L = 1 and Bi = 0.1 for various values of Rayleigh number.

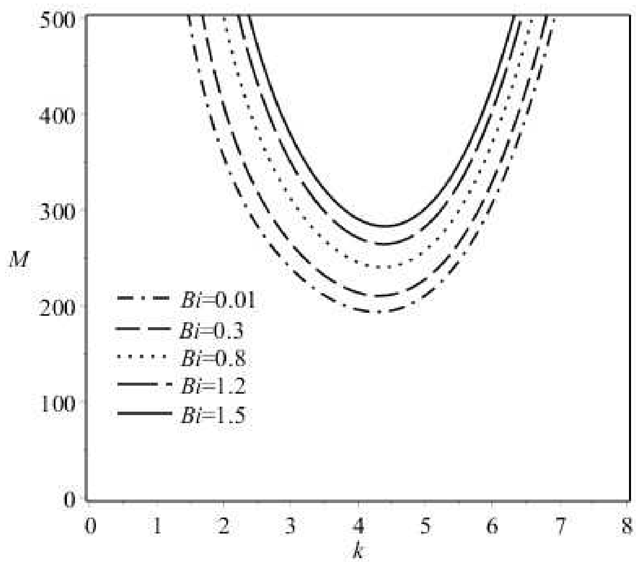

Fig. 3: Stability curves for steady convection with N = 5, L = 1 and R = 0.1 for various values of Biot number.

Figs. 1 – 4 show the behaviour of marginal curves for M in detemining the region of stabilities for various values of the wave number. The values of M are observed corresponding to certain ranges of the parameters Biot number Bi, Rayleigh number R, Lewis number L and elasticity number N. Fig. 1 shows the increasing values of M as the elasticity number increases which means the surfactant stabilizes the system. The same behaviour can

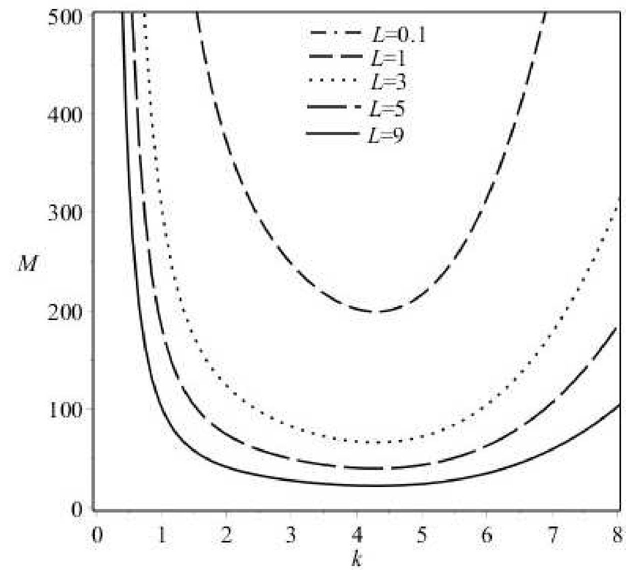

Fig. 4: Stability curves for steady convection with R = 0.1, N = 5 and Bi = 0.1 for various values of Lewis number.

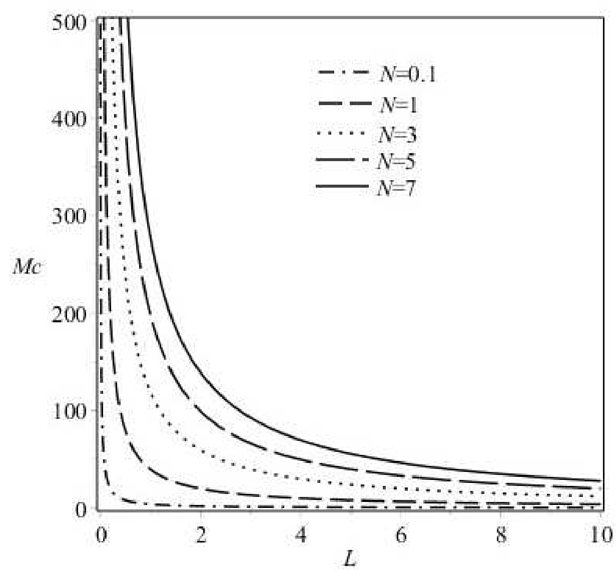

Fig. 5: The effect of Lewis number on the critical Marangoni number for various values of elasticity number with R = 0.1 and Bi = 0.1.

be seen in Fig. 3 where the system is more stable with larger values of Biot number. The Rayleigh number and Lewis number have destabilizing effects on the system as shown in Fig. 2 and Fig. 4 respectively. The value of M reaches the same minimal value in all cases. This means

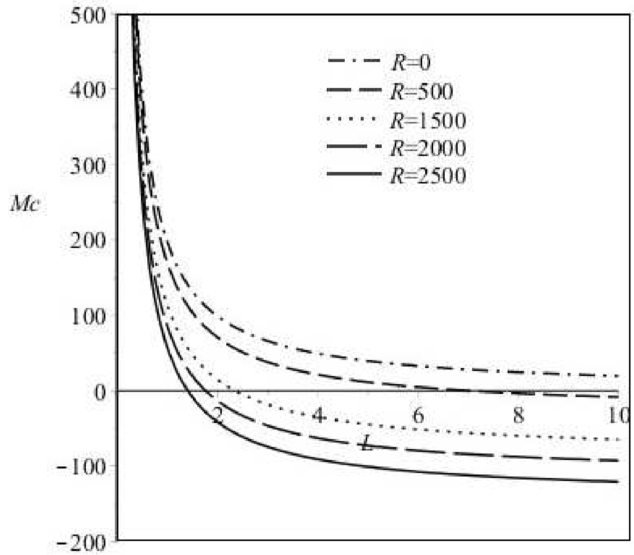

Fig. 6: The effect of Lewis number on the critical Marangoni number for various values of Rayleigh number with N = 5 and Bi = 0.1.

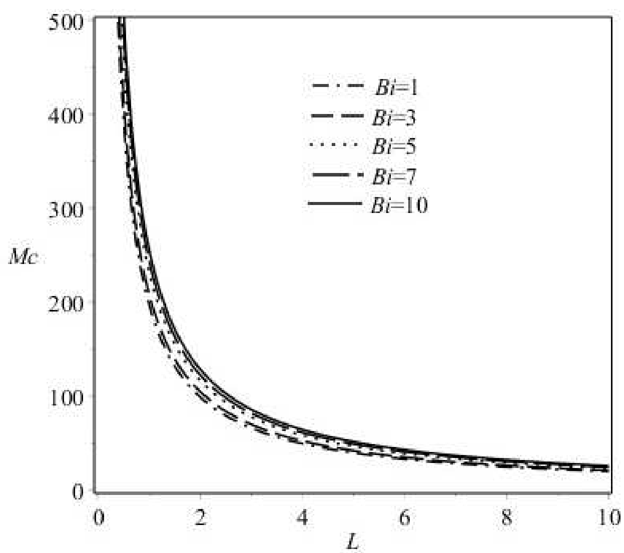

Fig. 7: The effect of Lewis number on the critical Marangoni number for various values of Biot number with R = 1 and N = 5.

that the values of the parameters does not affect the wave number at the point where M is minimal.

Figs. 5 – 13 illustrate the characteristics of critical Marangoni number for several values of parameters N, Bi, L and R. Figs. 5 – 7 show the influences of Lewis number L, Figs. 8 – 10 give the effect of Biot number Bi and the impact of elasticity number N is shown in Figs. 11 – 13.

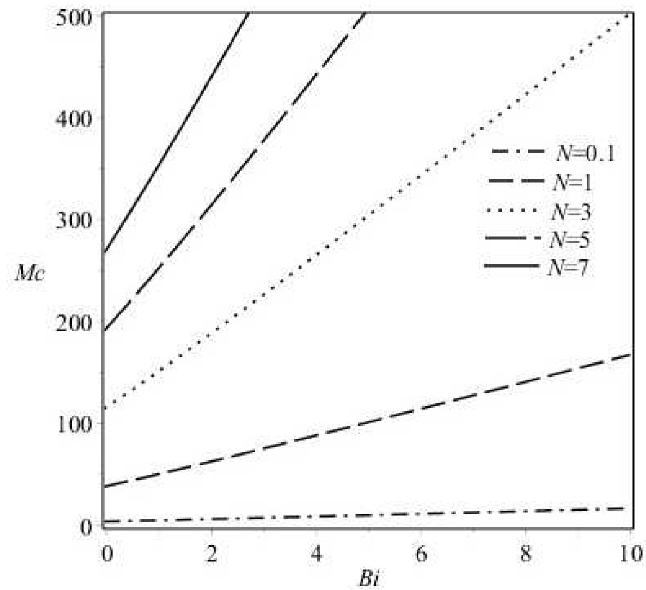

Fig. 8: The effect of Biot number on the critical Marangoni number for various values of elasticity number with L = 1 and R = 0.1.

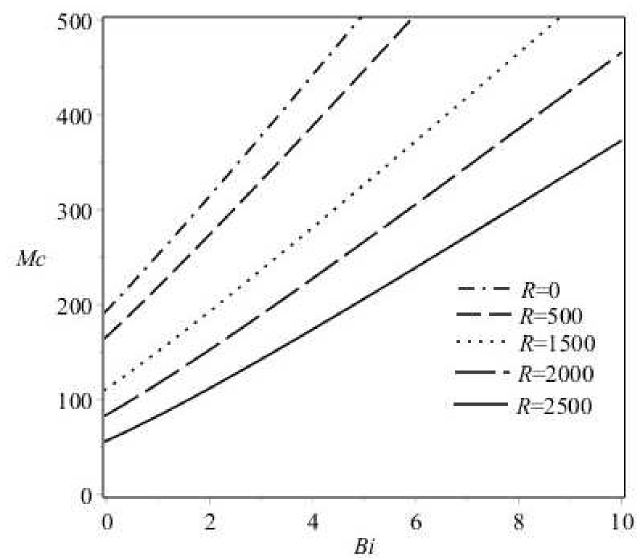

Fig. 9: The effect of Biot number on the critical Marangoni number for various values of Rayleigh number with L = 1 and N = 5.

When L is very small, the values of critical Marangoni Mc is very high and as L increases, the values of Mc is exponentially decreasing as shown in Fig. 5. As the values of elasticity number getting larger, there is slight increment in Mc. Elasticity number stabilize the fluid

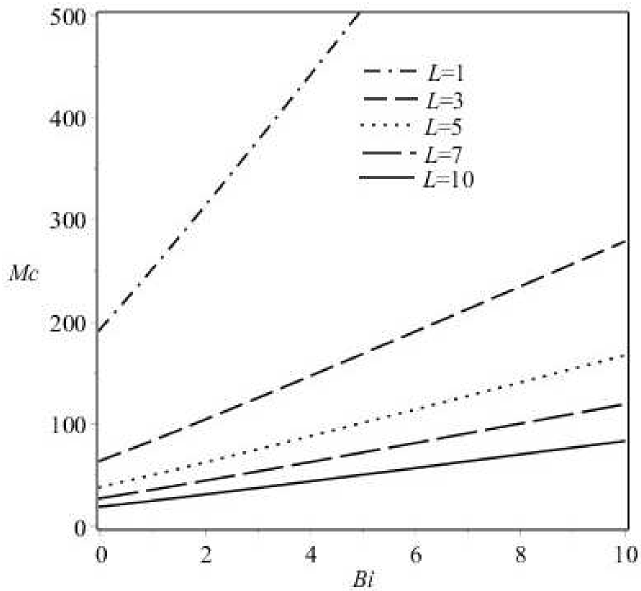

Fig. 10: The effect of Biot number on the critical Marangoni number for various values of Lewis number with N = 5 and R = 0.1.

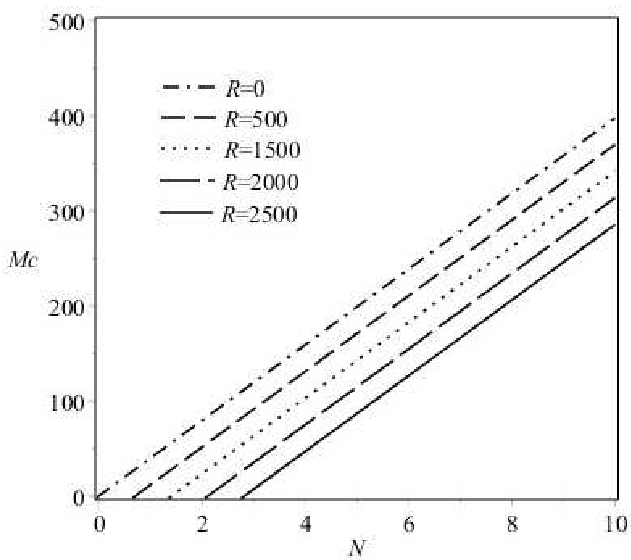

Fig. 11: The effect of elasticity number on the critical Marangoni number for various values of Rayleigh number with L = 1 and Bi = 0.1.

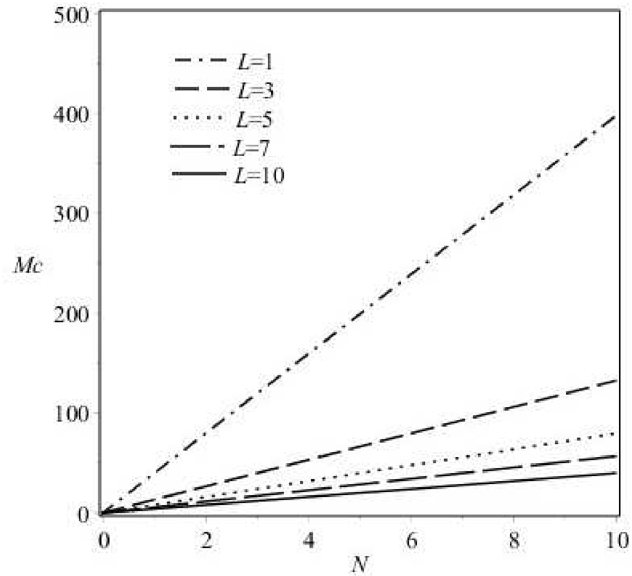

Fig. 12: The effect of elasticity number on the critical Marangoni number for various values of Lewis number with R = 0.1 and Bi = 0.1.

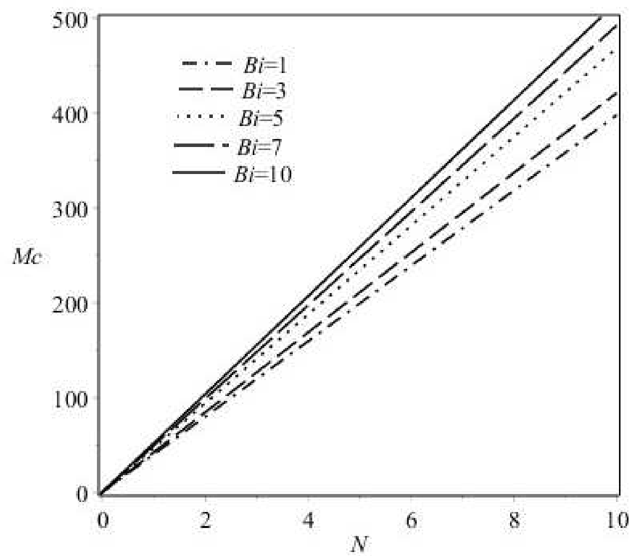

Fig. 13: The effect of elasticity number on the critical Marangoni number for various values of Biot number with L = 1 and R = 0.1.

system. In Fig. 6, Mc reaches negative values even with small L but with large value of R. Rayleigh number destabilizes the system as the values of R increases, Mc decreases. The buoyancy effect driven by gravity force destabilizes the liquid layer system. Fig. 7 shows similar behaviour as in Fig. 5 for various values of heat transfer coefficient Bi under the influence of L . As Bi increases, Mc increases. The fluid system is stable when more heat is allowed to transfer between the fluid and the gas phases (i.e as Bi increases).

Figs. 8 – 10 show linear increment for critical values Mc as Biot number Bi becomes larger as more heat is allowed to escape from the liquid to the gas phases. The marginal stability curves shift to a higher position when N increases which results in the increase of the values of Mc. The values of R and L decrease the values of Mc. N stabilizes the fluid system whereas R and L have destabilizing effect. Figs. 11 – 13 also illustrate the linearly increasing of the values of Mc as N increasing. When N=0, Mc=0 for all values of Bi and L as shown in Fig. 12 and Fig. 13.

5 Conclusion

In this paper, the steady thermosolutocapillary instability in a liquid layer is discussed with the presence of insoluble surfactant and gravity for nondeformable surface. Analysis is done by using the linear stability analysis and the analytical solution for Marangoni number is obtained by using the Galerkin method. The effects of physical parameters on the onset of steady instability has been analysed. In conclusion we found that the elasticity number and Biot number have stabilizing effect as more heat is transferred between the fluid to the gas phases whereas Lewis number and Rayleigh number destabilizing the system. The increasing value of Rayleigh number will increase the buoyancy driven flow of the fluid. Gravitational force creates the destabilization enviroment for thermocapillary convection in a liquid layer.

Acknowledgement

The authors gratefully acknowledge the financial support received from the Ministry of Higher Education of Malaysia and the Universiti Teknologi MARA.

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