Applied Mathematics & Information Sciences
An International Journal

http://dx.doi.org/10.18576/amis/110126

Sumudu Computation of the Transient Magnetic Field in a Lossy Medium

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Received: 10 Apr. 2016, Revised: 2 Nov. 2016, Accepted: 5 Nov. 2016
Published online: 1 Jan. 2017

Abstract: This research article aims at treating the transverse electromagnetic wave propagation in lossy media, labeled TEMP. Following the trail of works by Hussain and Belgacem, and Belgacem et al. towards getting the transient electric field solution of Maxwell's equations, here we seek Sumudu transform based solution for transient magnetic field. Moreover, we feature connected interesting shifting properties of the Sumudu transform, some found useful in solving this very particular problem. Furthermore, we establish new analytico-numerical results, and exhibit graphical profiles of Sumudued ramp, gaussian pulse, and finite sinusoidal functions.

Keywords: TEMP : Transversal Electromagnetic Propagation, Sumudu transform, Maxwell's equations, Magnetic field, Electric field.

1 Introduction

The mutual relation theory between electric and magnetic fields are goverened by Maxwell's equations. The coupled set of simultaneous equations, considered in various media, hones down the relation between electric field with that of conductive current and electric displacement, and the relation between magnetic field with that of magnetic induction and magnetic polarization [1,2,3,4,5]. Applications and treatments of this system of equations abound in the engineering and scientific literature. For instance, the relation of Maxwell's equations to the transmission lines is treated in [6]. Asymptotic methods variations, and analytical models of electromagnetic problems are given in [7,8,9]. Generalized electric signal treatment with Maxwell's equations in [10,11,12]. Determination of the transversal electric and magnetic wave propagations in lossy media, (TEMP) problem solution of Maxwell's equations, is established in [13,14], using Laplace transform. The parallel processing of Maxwells equations in different media and related works in [15,16], and treatments of fractional Maxwell's equations are described in [18,20,30].

Natural and Sumudu transforms based treatments of Maxwell's equations seeking the determination of transient electric and magnetic fields can be found in [17, 19,21,24,25,26,27,28]. Along the same lines, a Sumudu treatment of unsteady fluid flow problems is done in [44].

Distinctive Sumudu treatments for trigonometric functions were recently presented in [40], for Bessels functions (see for instance, [42]) in [21]. Various Sumudu ordinary and partial differential equations resolutions are featured in [32,34,41]. In particular, many fractional differential equations were Sumudu treated in [22,23,26, 29,33,35,36,37,38,39]. Description of various numerical methods for the solutions of partial differential equations along with MATLAB mathematical tool plots and calculations were described in [43].

Like for the Laplace transform (see recent Laplace transform restructure in [31]), the Sumudu transform, simply connoted , "Sumudu", may be applied to piece-wise continuous possibly bilateral functions of exponential order $A = \{f(t)|\exists M,\tau_1,\tau_2 > 0, |f(t)| < Me^{\frac{|t|}{\tau_j}}, \text{if } t \in (-1)^j \times [0,\infty)\}$

For functions in the admissible set A, the sumudu is defined by,

$$S[f(t)] = \int_0^\infty e^{-t} f(ut)dt = \frac{1}{u}\int_0^\infty e^{-\frac{t}{u}} f(t)dt \; ; \; u \in (-\tau_1,\tau_2). \quad (1)$$

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We bring the reader's attention of the Gamma function like exponential kernel and the variable u factoring the argument of the function to be sumudued. As mentioned in various works of the Authors, the Sumudu turns out to be the s-mutiplied Laplace trasform where the s is substituted by $1/u$. Being linear the Sumudu preserves scale, unit and dimension properties.

2 TEMP derivation and Sumudu treatment

Electromagnetism is best summarized through the realtions of interactions among magnetic field, electric displacement and electric field, magnetic induction, given by the of quadruple system of Maxwell vector equations,

(i)$\nabla\times\mathbf{E}(z,t)=-\frac{\partial}{\partial t}\mathbf{B}(z,t)$
(ii)$\nabla\times\mathbf{H}(z,t)=\mathbf{J}(z,t)+\frac{\partial}{\partial t}\mathbf{D}(z,t)$
(iii)$\nabla.\mathbf{B}(z,t)=0$
(iv)$\nabla.\mathbf{D}(z,t)=\rho(z,t)$

Here $\mathbf{E}$ and $\mathbf{H}$ are respective electric and magnetic field intensity vectors, $\mathbf{B}$, $\mathbf{J}$, $\mathbf{D}$ are respective magnetic induction, current density and electric displacement vectors, ρ , z , t are respective volume, charge density, position vector and time. In a lossy medium, the magnetic induction and electric displacement have the following relations respectively,

(i)$\mathbf{B}(z,t)=\mu\mathbf{H}(z,t)$
(ii)$\mathbf{D}(z,t)=\varepsilon\mathbf{E}(z,t)$

Therefore in the simple medium, with the aid of Ohm's law

(i)$\mathbf{J}(z,t)=\sigma\mathbf{E}(z,t)$

Maxwell's equations are given by,

(i)$\nabla\times\mathbf{E}(z,t)=-\mu\frac{\partial}{\partial t}\mathbf{H}(z,t)$
(ii)$\nabla\times\mathbf{H}(z,t)=\varepsilon\frac{\partial}{\partial t}\mathbf{E}(z,t)+\sigma\mathbf{E}(z,t)$

which by setting $\mathbf{H}(z,t)=\mathbf{H}_y(z,t)$ and $\mathbf{E}(z,t)=\mathbf{E}_x(z,t)$, yields the following PDEs system

$$\frac{\partial\mathbf{E}(z,t)}{\partial z}+\mu\frac{\partial\mathbf{H}(z,t)}{\partial t}=0. \tag{2}$$

$$\frac{\partial\mathbf{H}(z,t)}{\partial z}+\varepsilon\frac{\partial\mathbf{E}(z,t)}{\partial t}+\sigma\mathbf{E}(z,t)=0. \tag{3}$$

where μ,ε,σ being strictly positive constants, are the respective permeability, permittivity and conductivity. Partially differentiating equation (2) with respect to t, equation (3) with respect to z, assuming E is exact, (ie that $\frac{\partial^2\mathbf{E}}{\partial t\partial z}=\frac{\partial^2\mathbf{E}}{\partial z\partial t}$) using equation (2) for $\frac{\partial\mathbf{E}(z,t)}{\partial z}$ leads to linear PDE, for the magnetic field.

$$\frac{\partial^2\mathbf{H}(z,t)}{\partial z^2}=\mu\varepsilon\frac{\partial^2\mathbf{H}(z,t)}{\partial t^2}+\mu\sigma\frac{\partial\mathbf{H}(z,t)}{\partial t}. \tag{4}$$

Sumudu transform application to equation (4) with $S[\mathbf{H}(z,t)]=G(z,u)$ yields the non-homogeneous differential equation,

$$\frac{d^2G(z,u)}{dz^2}-\gamma^2G(z,u)=Q(z,u). \tag{5}$$

where $\gamma^2=\lambda+\psi$, so that $\lambda=\frac{\mu\varepsilon}{u^2}$ and $\psi=\frac{\mu\sigma}{u}$ and $Q(z,u)=-\gamma^2h_0(z)-\lambda uh_0'(z)$, here $h_0(z)=\lim_{t\to0}\mathbf{H}(z,t)$ and $h_0'(z)=\lim_{t\to0}\frac{\partial\mathbf{H}(z,t)}{\partial t}$. The homogeneous solution of equation (5) is obtained by setting $Q(z,u)=0$, so that, $\left(\frac{h_0'}{h_0}\right)\lambda u=-(\lambda+\psi)$, therefore, $\left(\frac{h_0'}{h_0}\right)=-(1+\frac{\psi}{\lambda u})$ and the particular solution of equation (5) obtained by method of variation of parameter are respectively,

$$G_h(z,u)=A(u)e^{-\gamma z}+B(u)e^{\gamma z}. \tag{6}$$

$$G_p(z,u)=\frac{e^{\gamma z}}{2\gamma}\int e^{-\gamma z}Q(z,u)dz+\frac{e^{-\gamma z}}{2\gamma}\int e^{\gamma z}Q(z,u)dz. \tag{7}$$

For the conductivity $\sigma>0$ in the lossy medium, the boundary condition is assumed as $\lim_{z\to0}\mathbf{H}(z,t)=h(t)$; $t\geq0$. Finiteness requirement and boundedness satisfication leads to $B(u)=0$ in equation (6) as $z\to\infty$, and from the boundary condition $A(u)=S[\lim_{z\to0}\mathbf{H}(z,t)]=S[h(t)]=G(u)$. So,

$$G(z,u)=G(u)e^{-\gamma z}. \tag{8}$$

Expanding $e^{-\gamma z}$ (eqn (4), page 416, [42]) with $a=1/\sqrt{\mu\varepsilon}$, $b=\sigma/2\varepsilon$ and $J_0(.)$ is the zeroth order first kind Bessel function.

$$\frac{e^{-\gamma z}}{\gamma}=a\int_{z/a}^{\infty}e^{-bt}J_0\left(\frac{b}{a}\sqrt{z^2-a^2t^2}\right)e^{-t/u}dt. \tag{9}$$

Differentiating equation (9) with respect to z,

$$e^{-\gamma z}=e^{-\frac{b}{a}z}e^{-\frac{1}{au}z}-a\int_{z/a}^{\infty}e^{-bt}\frac{\partial}{\partial z}J_0\left(\frac{b}{a}\sqrt{z^2-a^2t^2}\right)e^{-t/u}dt. \tag{10}$$

Substituting $t/u=v$ in equation (10),

$$e^{-\gamma z}=e^{-\frac{b}{a}z}e^{-\frac{1}{au}z}-au\times\int_{z/a}^{\infty}\left[e^{-b(uv)}\frac{\partial}{\partial z}J_0\left(\frac{b}{a}\sqrt{z^2-a^2(uv)^2}\right)\right]\times e^{-v}dv. \tag{11}$$

The integral part of equation (11) can be written as,

$$e^{-\gamma z}=e^{-\frac{b}{a}z}e^{-\frac{1}{au}z}-auS[\Phi(z,v)]. \tag{12}$$

where $\Phi(z,v)=e^{-bv}\frac{\partial}{\partial z}J_0\left(\frac{b}{a}\sqrt{z^2-av^2}\right)$ for $v\geq\frac{z}{a}$ and 0 for $0<v<\frac{z}{a}$. Hence,

$$G(z,u)=G(u)e^{-\frac{b}{a}z}e^{-\frac{1}{au}z}-auG(u)S[\Phi(z,v)]. \tag{13}$$

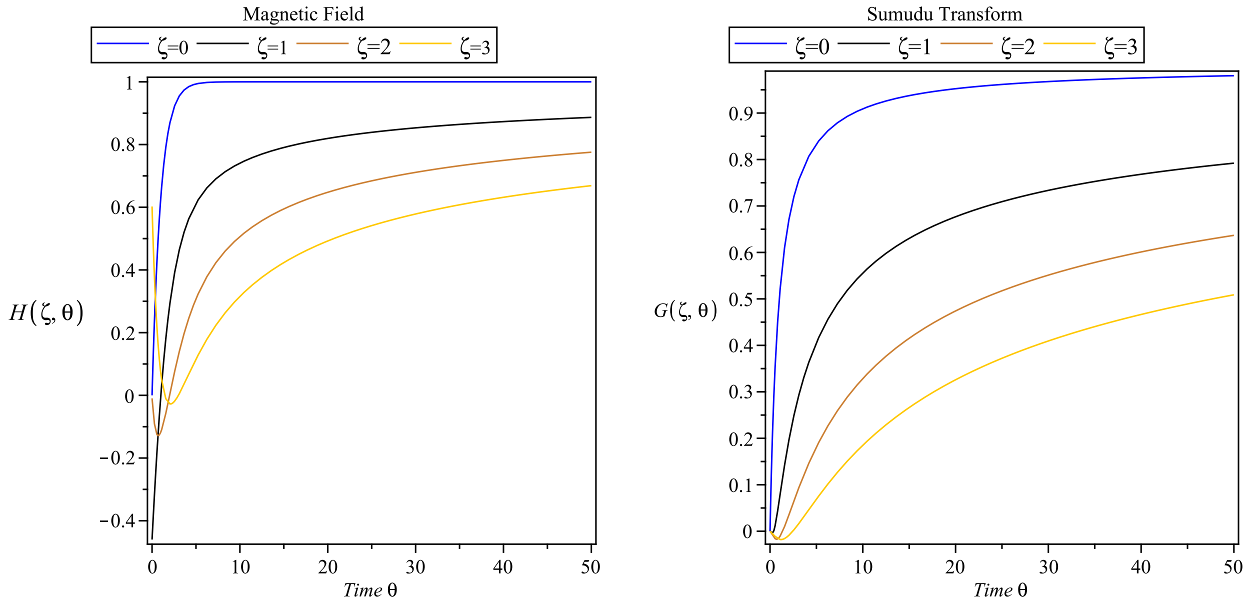

Fig. 1: Magnetic field $\mathbf{H}(\zeta,\theta)$ for the boundary condition $h(t)$ as exponential ramp function and its Sumudu transform $G(\zeta,\theta)$ plotted with distance $\zeta = 0,1,2$ and 3 shown in respective left and right plot.

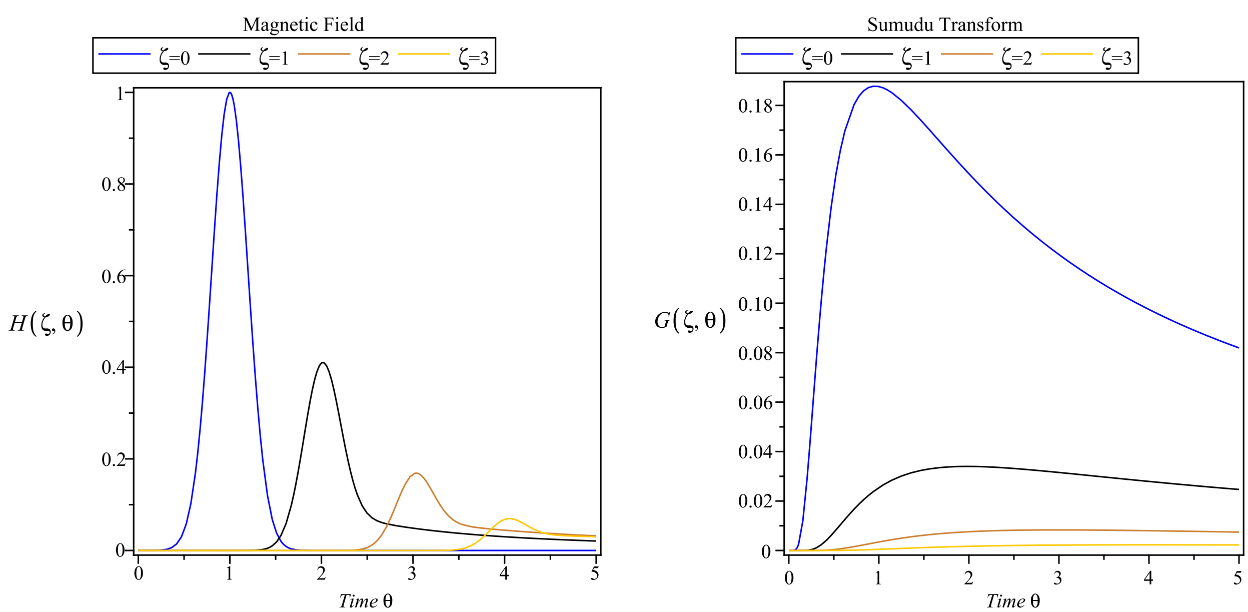

Fig. 2: Magnetic field $\mathbf{H}(\zeta,\theta)$ for the boundary condition $h(t)$ as gaussian pulse function and its Sumudu transform $G(\zeta,\theta)$ plotted with distance $\zeta = 0,1,2$ and 3 shown in respective left and right plot.

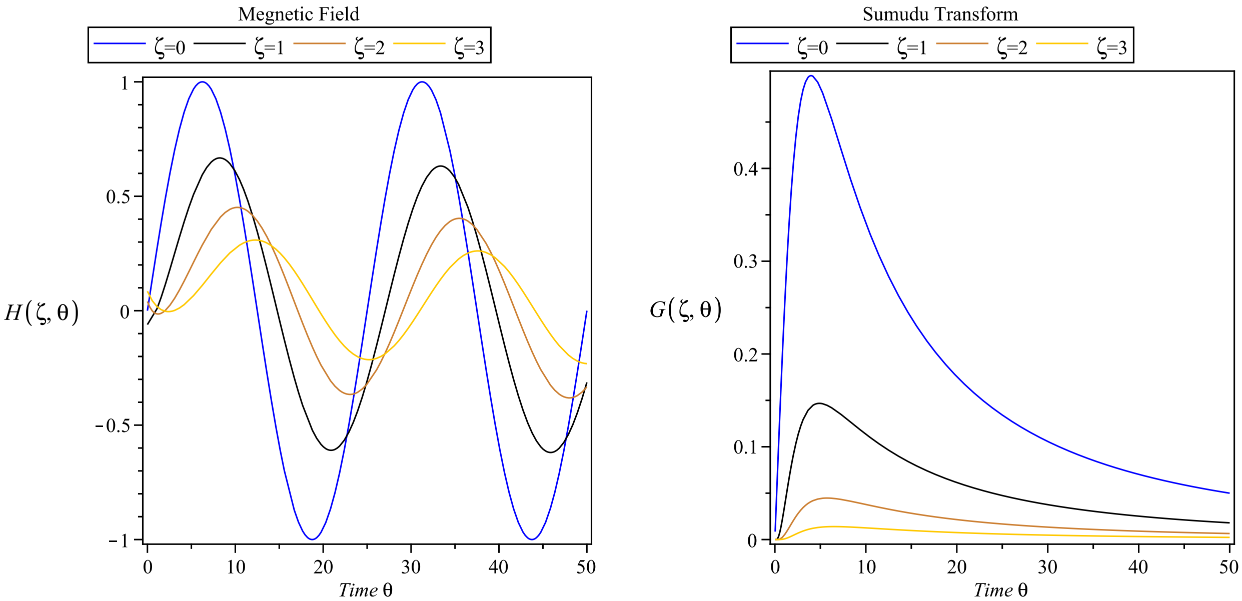

Fig. 3: Magnetic field $\mathbf{H}(\zeta,\theta)$ for the boundary condition $h(t)$ as finite sinusoidal function and its Sumudu transform $G(\zeta,\theta)$ plotted with distance $\zeta = 0, 1, 2$ and 3 shown in respective left and right plot.

The inverse Sumudu transform of equation (13) (application of second shifting and convolution theorem equations (50) and (51) in [17]) yields the magnetic field solution.

$$\mathbf{H}(z,t) = e^{-\frac{b}{a}z}h(t-z/a) - a \times \int_{z/a}^{\infty} h(t-\tau)e^{-b\tau} \times \frac{\partial}{\partial z}J_0\left(\frac{b}{a}\sqrt{z^2-(a\tau)^2}\right)d\tau. \quad (14)$$

Introducing the variable $t-\tau=\beta$ in (14),

$$\mathbf{H}(z,t) = e^{-\frac{b}{a}z}h(t-z/a) - ae^{-bt} \times \int_{0}^{t-z/a} h(\beta)e^{b\beta} \times \frac{\partial}{\partial z}J_0\left(\frac{b}{a}\sqrt{z^2-a^2(t-\beta)^2}\right)d\beta. \quad (15)$$

Considering the assigned and related permeability, permittivity and conductivity to a and b earlier and further substituting for the normalised time θ and space ζ variables, $\theta = bt = \frac{\sigma t}{2\varepsilon}$; $\zeta = \frac{bz}{a} = \frac{\sigma z}{2}\sqrt{\frac{\mu}{\varepsilon}}$; $\eta = b\beta = \frac{\sigma\beta}{2\varepsilon}$, so that $(t-z/a) = \frac{2\varepsilon}{\sigma}(\theta-\zeta)$, $\left(z^2-a^2(t-\beta)^2\right) = \frac{4\varepsilon}{\mu\sigma^2}\left(\zeta^2-(\theta-\eta)^2\right)$, $d\beta = \frac{2\varepsilon}{\sigma}d\eta$. In consideration of Bessel functions properties, The normalised magnetic field in the lossy but conducting media $(\sigma > 0)$ is,

$$\mathbf{H}(\zeta,\theta) = e^{-\zeta}h\left(\frac{2\varepsilon}{\sigma}(\theta-\zeta)\right) + \zeta e^{-\theta} \times \int_{0}^{\theta-\zeta} e^{\eta}h\left(\frac{2\varepsilon}{\sigma}\eta\right) \times \frac{I_1\left(\sqrt{(\theta-\eta)^2-\zeta^2}\right)}{\left(\sqrt{(\theta-\eta)^2-\zeta^2}\right)}d\eta. \quad (16)$$

For the normalized time variable $\theta = bu$ and space variable $\zeta = bz/a$ the Sumudu transform of magnetic field is given by,

$$G(\zeta,\theta) = S[\mathbf{H}(\zeta,\theta)] = e^{-\zeta}G\left(\frac{2\varepsilon}{\sigma}\theta\right)e^{-\zeta/\theta} + \zeta e^{-\theta} \times \int_{0}^{\theta-\zeta} e^{\eta}G\left(\frac{2\varepsilon}{\sigma}\theta\right)e^{-(\theta-\eta)/\theta} \times \frac{I_1\left(\sqrt{(\theta-\eta)^2-\zeta^2}\right)}{\left(\sqrt{(\theta-\eta)^2-\zeta^2}\right)}d\eta. \quad (17)$$

3 Boundary conditions and numerical results

For the exponential ramp function $h(t) = 1-e^{-1/\tau}$ in equation (16) with the ratio [17] $2\varepsilon/\sigma = \tau$, $\mathbf{H}(\zeta,\theta)$ is

Table 1: Numerical calculations of magnetic field $\mathbf{H}(\zeta,\theta)$ due to $h(t)$ as exponential ramp function with the ratio [17] $2\varepsilon/\sigma=\tau$ for the different distance ζ and time θ.

$H(\zeta,\theta)$	$\theta=10$	$\theta=20$	$\theta=30$	$\theta=40$	$\theta=50$
$\zeta=0$	0	0.99995	1	1	1
$\zeta=1$	0.75453	0.82413	0.85571	0.87476	0.88780
$\zeta=2$	0.53103	0.65638	0.71600	0.75252	0.77778
$\zeta=3$	0.34354	0.50387	0.58499	0.63608	0.67196
$\zeta=4$	0.20241	0.37175	0.46608	0.52788	0.57224
$\zeta=5$	0.10656	0.26284	0.36166	0.42976	0.48004
$\zeta=6$	0.048812	0.17751	0.27294	0.34296	0.39648

Table 2: Numerical calculations of Sumudu transform of magnetic field $G(\zeta,\theta)$ due to $G(u)$ as Sumudu transform of exponential ramp function with the ratio [17] $2\varepsilon/\sigma=\tau$ for the different distance ζ and time θ.

$G(\zeta,\theta)$	$\theta=10$	$\theta=20$	$\theta=30$	$\theta=40$	$\theta=50$
$\zeta=0$	0.90909	0.95238	0.96774	0.97561	0.98039
$\zeta=1$	0.55564	0.67680	0.73362	0.76820	0.79210
$\zeta=2$	0.32852	0.47411	0.55117	0.60104	0.63678
$\zeta=3$	0.18502	0.32595	0.40948	0.46650	0.50881
$\zeta=4$	0.097442	0.21889	0.30006	0.35869	0.40366
$\zeta=5$	0.046886	0.14288	0.21638	0.27279	0.31763
$\zeta=6$	0.019986	0.090159	0.15317	0.20489	0.24763

Table 3: Numerical calculations of magnetic field $\mathbf{H}(\zeta,\theta)$ due to $h(t)$ as gaussian pulse function with the ratio [17] $2\varepsilon/\sigma=\Delta\tau$ and $t_0=\Delta\tau$ where $\Delta\tau=0.354$ ns [17] for the different distance ζ and time θ.

$H(\zeta,\theta)$	$\theta=1$	$\theta=2$	$\theta=3$	$\theta=4$	$\theta=5$
$\zeta=0$	1	0.0000034886	1.4783×10^{-22}	7.6084×10^{-50}	4.7946×10^{-88}
$\zeta=1$	0.0000012834	0.40902	0.048668	0.029978	0.020639
$\zeta=2$	-3.2092×10^{-8}	4.7215×10^{-7}	0.16673	0.044262	0.031858
$\zeta=3$	-2.0463×10^{-8}	-1.3208×10^{-8}	1.7369×10^{-7}	0.067757	0.029866
$\zeta=4$	6.2332×10^{-10}	-3.2728×10^{-9}	-4.6344×10^{-9}	6.3897×10^{-8}	0.027460
$\zeta=5$	1.5204×10^{-8}	4.7466×10^{-9}	4.4811×10^{-10}	-1.4303×10^{-9}	2.3506×10^{-8}
$\zeta=6$	1.4138×10^{-8}	6.1290×10^{-9}	2.5462×10^{-9}	6.8364×10^{-10}	-3.7871×10^{-10}

shown in Fig. 1 the first plot and its Sumudu transform $G(u)=\frac{u/\tau}{1+(u/\tau)}$ in equation (17) with the same ratio $G(\zeta,\theta)$ is shown in Fig. 1 the second plot. For both the plots the normalized distance considered as $\zeta=0,1,2$ and 3 and the time $\theta\in[0,50]$. The numerical calculations of magnetic field $\mathbf{H}(\zeta,\theta)$ for the exponential ramp function is given in Table 1. From Table 1, for the fixed distance ζ, when time θ is increasing, magnetic field $\mathbf{H}(\zeta,\theta)$ is increasing, while for the fixed time θ, when distance ζ is increasing magnetic field $\mathbf{H}(\zeta,\theta)$ is decreasing. This holds true for the Sumudu transform of magnetic field $G(\zeta,\theta)$ also, whose numerical calculations are shown in Table 2 thus our result coincides with the result in (first paragraph, page 287, [17]). Therefore when distance ζ while increasing for the fixed time θ magnetic field $\mathbf{H}(\zeta,\theta)$ and its Sumudu transform $G(\zeta,\theta)$ slowly

Table 4: Numerical calculations of Sumudu transform of magnetic field $G(\zeta,\theta)$ due to $G(u)$ as Sumudu transform of gaussian pulse function with the ratio [17] $2\varepsilon/\sigma=\Delta\tau$ and $t_0=\Delta\tau$ where $\Delta\tau=0.354$ ns [17] for the different distance ζ and time θ.

$G(\zeta,\theta)$	$\theta=1$	$\theta=2$	$\theta=3$	$\theta=4$	$\theta=5$
$\zeta=0$	0.18764	0.15239	0.11968	0.097469	0.081940
$\zeta=1$	0.025395	0.043960	0.045454	0.043029	0.039885
$\zeta=2$	-0.0048998	0.0075871	0.014073	0.016809	0.017814
$\zeta=3$	-0.0065423	-0.0021483	0.0021922	0.0050194	0.0067787
$\zeta=4$	-0.00099165	-0.0018952	-0.00076673	0.00065675	0.001884
$\zeta=5$	0.0037114	0.00063773	-0.00035708	-0.00025265	0.00020310
$\zeta=6$	0.0040575	0.0017812	0.00043775	-0.000032921	-0.000079335

Table 5: Numerical calculations of magnetic field $\mathbf{H}(\zeta,\theta)$ due to $h(t)$ as finite sinusoidal function with the ratio [14] $2\varepsilon/\sigma=\tau/25$ for the different distance ζ and time θ.

$H(\zeta,\theta)$	$\theta=10$	$\theta=20$	$\theta=30$	$\theta=40$	$\theta=50$
$\zeta=0$	0.58779	-0.95106	0.95106	-0.58779	0
$\zeta=1$	0.60835	-0.59414	0.42201	-0.053014	-0.31339
$\zeta=2$	0.45068	-0.25561	0.090338	0.17729	-0.33318
$\zeta=3$	0.27304	-0.029227	-0.058575	0.21752	-0.23138
$\zeta=4$	0.13997	0.082407	-0.088605	0.17172	-0.11350
$\zeta=5$	0.060783	0.11322	-0.062743	0.10716	-0.026044
$\zeta=6$	0.021879	0.10107	-0.023689	0.055585	0.022194

Table 6: Numerical calculations of Sumudu transform of magnetic field $G(\zeta,\theta)$ due to $G(u)$ as Sumudu transform of finite sinusoidal function with the ratio [14] $2\varepsilon/\sigma=\tau/25$ for the different distance ζ and time θ.

$G(\zeta,\theta)$	$\theta=10$	$\theta=20$	$\theta=30$	$\theta=40$	$\theta=50$
$\zeta=0$	0.34118	0.17565	0.10572	0.070274	0.049988
$\zeta=1$	0.20853	0.12483	0.080143	0.055335	0.040386
$\zeta=2$	0.12329	0.087443	0.060213	0.043295	0.032466
$\zeta=3$	0.069440	0.060120	0.044729	0.033603	0.025941
$\zeta=4$	0.036570	0.040373	0.032780	0.025838	0.020580
$\zeta=5$	0.017596	0.026351	0.023639	0.019650	0.016195
$\zeta=6$	0.0075009	0.016630	0.016734	0.014759	0.012625

tends to zero. Now for the fixed distance ζ when the time θ is increasing, magnetic field $\mathbf{H}(\zeta,\theta)$ and its Sumudu transform $G(\zeta,\theta)$ slowly tend to 1 which can be seen in both the plots of Fig. 1 and the entries of Tables 1 and 2.

Secondly, the gaussian pulse function $h(t)=\exp(-a^2(t-t_0)^2)$ in equation (16) with the ratio [17] $2\varepsilon/\sigma=\Delta T$, $a=2\sqrt{\pi}/\Delta T$, $\Delta T=0.354$ ns with relative permittivity $\varepsilon_r=80$, permittivity $\varepsilon=705\times10^{-12}$ F/m and conductivity $\sigma=4$ S/m [17], $\mathbf{H}(\zeta,\theta)$ is shown in Fig. 2 the first plot and its Sumudu transform $G(u)=\frac{\sqrt{\pi}}{2au}\operatorname{erfc}\left(\frac{1}{2au}-at_0\right)\exp\left(\left(\frac{1}{2au}\right)^2-\frac{t_0}{u}\right)$ in equation (17) with the same ratio $G(\zeta,\theta)$ is shown in Fig. 2 the second plot. For both plots the normalized distance considered as $\zeta=0,1,2$ and 3 and the time $\theta\in[0,5]$. The numerical calculations of magnetic field $\mathbf{H}(\zeta,\theta)$ for the gaussian pulse function is given in Table 3.

Finally the boundary condition finite sinusoidal function defined by $h(t) = \sin(2\pi t/\tau)$ for $0 \le t \le \tau$ and 0 for $t > \tau$ in equation (16) with the ratio [14] $2\varepsilon/\sigma = \tau/25$, $\mathbf{H}(\zeta,\theta)$ is shown in Fig. 3 the first plot and its Sumudu transform $G(u) = \frac{2u\tau\pi\left(\exp(\frac{2\tau}{u})-1\right)\exp(-\frac{2\tau}{u})}{\tau^2+4\pi^2u^2}$ in equation (17) with same ratio $G(\zeta,\theta)$ is shown in Fig. 3 the second plot. For both plots the normalized distance considered as $\zeta = 0,1,2$ and 3 and the time $\theta \in [0,50]$. The corresponding numerical values are given in Tables 5 and 6.

4 Conclusion

In this work., in Sumudu treating the TEMP problem with aim to ascertain the related magnetic field behavior, we obtained analytical expressions, numerical tables and graphical profiles for both the magnetic field and its Sumudu under various initial conditions functions, namely exponential ramp, gaussian pulse, and finite sinusoidal initiations. We observe that both the magnetic field, $\mathbf{H}(\zeta,\theta)$, and its Sumudu transform, $G(\zeta,\theta)$ emanate at the value for, $\theta = 0$,.which confirms the intial value theorem for the Sumudu transform. In the case of the exponential ramp function, both the magnetic field $\mathbf{H}(\zeta,\theta)$ and its Sumudu $G(\zeta,\theta)$ increase with increasing θ,and decrease with increasing, ζ. This same behavior is also observed with gaussian pulse function initiation. However, in this case also the moving peaks are decreasing with increasing ζ, and θ. In fact, we can extrapolate that the peaks tend to decay fast and are expected to vanish as both distance and time tend to infinity, which corroborates the final value theorem for the Sumudu. In the case of finite sinusoidal function, periodicity is observed in the magnetic field, $\mathbf{H}(\zeta,\theta)$ and the time location of the peak in its Sumudu transform $G(\zeta,\theta)$ is maintained for various values of, ζ, albeit diminishing in height with increasing values of, ζ. Unlike for the Laplace transform (and even other transforms like Fourier or Mellin see for instance [45, 46, 47]), we were able to mirror the magnetic field results and profiles with their Sumudu counterparts, simply because the Sumudu transform does preserve scale and units. We note however that the Sumudu does not preserve periodicity as is observed from Fig.3 and as is expected from previous works by the authors et al. Moreover this is clearly expected due to domain reduction caused by convergence requirements prescribed on the transformed function by the constants τ_1 and τ_2. We feel that there may be much more to be deduced and deducted from the collective works and treatments so far by the authors et al. and others regarding the interactions of the electric and magnetic field and the TEMP problem as a physical application. Furthermore, we particularly feel that the Sumudu has a lot more to give both on the theoretical basis and the applied ones, and that there may be more to be observed on both ends in this very work and in its relation to previous ones. Hence, while we feel specially lucky that we extended the Sumudu theory and applications thus far, we remain completely open for readers feedbacks and comments, which regardless of sways, we would always and continuously appreciate.

Acknowledgement

The Authors Acknowledge the support of the Editorial Board of AMIS.

Fethi Bin Muhammad Belgacem is pleased to acknowledge the Kuwait Public Authority for Applied Education and Training Research Support Department, PAAET RSD.

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