

Energy decay to an abstract coupled system of extensible beams models

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Abstract: In this work we study existence of solutions for an abstract coupled system of nonlinear equations of extensible beams models and present the exponential decay for full energy of the system.

Keywords: Nonlinear beam equation, abstract coupled system, existence and uniqueness of solution, asymptotic behavior.

1. Introduction

A mathematical model for the transverse deflection of an extensible beam of length L whose ends are held at fixed distance apart is equation

$$u_{tt} + \alpha u_{xxxx} + \left(\beta \int_0^L u_\xi^2(\xi, t) d\xi \right) (-u_{xx}) = 0$$

which has been proposed by Woinowsky-Krieger [14], where α is a positive constant, β is a constant, not necessarily positive, and the nonlinear term represents the change in the tension of the beam due to its extensibility. The abstract formulation of this model, that was studied by Medeiros [8] is the equation

$$u'' + \alpha A^2 u + M(|A^{1/2}u|^2) Au = 0$$

where A is a linear operator in a Hilbert space H and M a real function. We use the standard Lebesgue space and Sobolev space with their usual notation and properties as in [6] and in this sense (\cdot, \cdot) and $|\cdot|$ denote respectively the inner product and norm in H . The extensible beam or plate was studied by several authors, like Easley (1964) [5], Dickey (1970) [4], Ball (1973)[1], Menzala(1980)[9], Brito (1984)[3], Biler (1986) [2], Pereira (1989) [11] and more recently, by Rivera (2008) [12] and Ma (2010) [7].

In this paper we prove the existence, uniqueness and exponential decay of solutions for an abstract coupled system of Woinowsky-Krieger model of nonlinear equations of the beam. The importance of this work is to present the abstract formulation of the coupled system and to apply the Theorem of Nakao to obtain the asymptotic behavior. In this direction, for $\delta_1 > 0$ and $\delta_2 > 0$ follows the problem that we consider here,

$$K_1 u'' + A_1^2 u + M \left(|A_1^{1/2} u|^2 + |A_2^{1/2} v|^2 \right) A_1 u + \delta_1 u' = 0 \quad (1)$$

$$K_2 v'' + A_2^2 v + M \left(|A_1^{1/2} u|^2 + |A_2^{1/2} v|^2 \right) A_2 v + \delta_2 v' = 0 \quad (2)$$

$$u(0) = u_0, \quad v(0) = v_0 \quad (3)$$

$$(K_1 u')(0) = K_1^{1/2} u_1, \quad (K_2 v')(0) = K_2^{1/2} v_1 \quad (4)$$

where $M \in C^0[0, +\infty)$, with $M(\lambda) \geq 0 \quad \forall \lambda > 0$ and $K_i, i = 1, 2$ are symmetrical linear operators in H with $(K_i w, w) > 0, \forall w \in H, A_i, i = 1, 2$ are self-adjoint and positive linear operator, with domain $\mathcal{D}(A_i)$ dense in H , that is, there exist positive constants $m_i, i = 1, 2$, such that $(A_1 v, v) \geq m_1 |v|^2, \forall v \in \mathcal{D}(A_1), (A_2 w, w) \geq m_2 |w|^2, \forall w \in \mathcal{D}(A_2)$.

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In section 2 we study the existence of solution by Faedo-Galerkin’s method and in section 3 we prove the stability of system by Nakao’s theorem.

2. Existence of solution

In this section, we will prove that for

$$(u_0, v_0) \in \mathcal{D}(A_1) \times \mathcal{D}(A_2) \text{ and } (u_1, v_1) \in H \times H,$$

there exists a unique solution (u, v) of (1) - (4) in

$$L^\infty([0, T], \mathcal{D}(A_1)) \times L^\infty([0, T], \mathcal{D}(A_2))$$

where the time T depend only of initial data. We make use of Faedo-Galerkin approximation to prove the existence of weak solution and in this direction we consider the following result.

Theorem 1.

For $(u_0, v_0) \in \mathcal{D}(A_1) \times \mathcal{D}(A_2), (u_1, v_1) \in H \times H,$

there exist functions $u, v : R^+ \rightarrow H,$ such that

$$\begin{aligned} (u, v) &\in L^\infty([0, T], \mathcal{D}(A_1)) \times L^\infty([0, T], \mathcal{D}(A_2)) \\ (K_1 u', K_2 v') &\in [L^\infty([0, T], H)]^2 \\ (u', v') &\in [L^2([0, T], H)]^2 \end{aligned}$$

satisfying in the sense of $\mathcal{D}'(R^+)$

$$\begin{aligned} \forall w \in \mathcal{D}(A_1), \\ \frac{d}{dt}(K_1 u', w) + (A_1 u, A_1 w) + \\ M \left(\left| A_1^{\frac{1}{2}} u \right|^2 + \left| A_2^{\frac{1}{2}} v \right|^2 \right) (A_1 u, w) + \delta_1(u', w) = 0, \end{aligned} \tag{5}$$

and $\forall z \in \mathcal{D}(A_2),$

$$\begin{aligned} \frac{d}{dt}(K_2 v', z) + (A_2 v, A_2 z) + \\ M \left(\left| A_1^{\frac{1}{2}} u \right|^2 + \left| A_2^{\frac{1}{2}} v \right|^2 \right) (A_2 v, z) + \delta_2(v', z) = 0, \end{aligned} \tag{6}$$

with initial conditions

$$\begin{aligned} (u(0), v(0)) &= (u_0, v_0), \\ (K_1 u(0), K_2 v(0)) &= \left(K_1^{\frac{1}{2}} u_1, K_2^{\frac{1}{2}} v_1 \right). \end{aligned}$$

Proof. Approximate problem

Let $(w_\nu)_{\nu \in N}$ and $(z_\nu)_{\nu \in N},$ Hilbert’s basis of $\mathcal{D}(A_1)$ and $\mathcal{D}(A_2)$ respectively. We take

$$V_m = [w_1, \dots, w_m] \text{ and } W_m = [z_1, \dots, z_m]$$

so we have that

$$u_m(t) = \sum_{j=1}^m g_{jm}(t) w_j \in V_m$$

and

$$v_m(t) = \sum_{j=1}^m h_{jm}(t) z_j \in W_m$$

satisfies $\forall w \in V_m$ and $\forall z \in W_m$

$$\begin{aligned} (K_1 u_m''(t), w) + (A_1 u_m(t), A_1 w) \\ + M \left(\left| A_1^{\frac{1}{2}} u_m(t) \right|^2 + \left| A_2^{\frac{1}{2}} v_m(t) \right|^2 \right) (A_1 u_m(t), w) \\ + \delta_1(u_m'(t), w) = 0, \end{aligned} \tag{7}$$

$$\begin{aligned} (K_2 v_m''(t), z) + (A_2 v_m(t), A_2 z) \\ + M \left(\left| A_1^{\frac{1}{2}} u_m(t) \right|^2 + \left| A_2^{\frac{1}{2}} v_m(t) \right|^2 \right) (A_2 v_m(t), z) + \\ \delta_2(v_m'(t), z) = 0, \end{aligned} \tag{8}$$

and

$$(u_m(0), v_m(0)) \rightarrow (u_0, v_0), \tag{9}$$

$$(K_1 u_m'(0), K_2 v_m'(0)) \rightarrow \left(K_1^{\frac{1}{2}} u_1, K_2^{\frac{1}{2}} v_1 \right), \tag{10}$$

in $\mathcal{D}(A_1) \times \mathcal{D}(A_2)$ and $H \times H$ respectively.

Now by the Carathéodory’s theorem, $u_m(t)$ and $v_m(t)$ are defined just in the interval $[0, T_m]$ with $0 < T_m < T$ and we need to prolong for the interval $[0, T], 0 < T < \infty.$

Priori estimates

Taking $w = u_m'(t)$ and $z = v_m'(t)$ in (7) and (8) we respectively obtain

$$\begin{aligned} \frac{1}{2} \frac{d}{dt} \left[\left| K_1^{\frac{1}{2}} u_m' \right|^2 + \left| A_1 u_m \right|^2 \right] \\ + M \left(\left| A_1^{\frac{1}{2}} u_m \right|^2 + \left| A_2^{\frac{1}{2}} v_m \right|^2 \right) (A_1 u_m, u_m') \\ + \delta_1 |u_m'|^2 = 0, \end{aligned} \tag{11}$$

$$\begin{aligned} \frac{1}{2} \frac{d}{dt} \left[\left| K_2^{\frac{1}{2}} v_m' \right|^2 + \left| A_2 v_m \right|^2 \right] \\ + M \left(\left| A_1^{\frac{1}{2}} u_m \right|^2 + \left| A_2^{\frac{1}{2}} v_m \right|^2 \right) (A_2 v_m, v_m') \\ + \delta_2 |v_m'|^2 = 0. \end{aligned} \tag{12}$$

Now we observe that

$$\begin{aligned} M \left(\left| A_1^{\frac{1}{2}} u_m \right|^2 + \left| A_2^{\frac{1}{2}} v_m \right|^2 \right) (A_1 u_m, u_m') \\ = \frac{1}{2} M \left(\left| A_1^{\frac{1}{2}} u_m \right|^2 + \left| A_2^{\frac{1}{2}} v_m \right|^2 \right) \frac{d}{dt} \left| A_1^{\frac{1}{2}} u_m \right|^2, \end{aligned}$$

and

$$\begin{aligned} M \left(\left| A_1^{\frac{1}{2}} u_m \right|^2 + \left| A_2^{\frac{1}{2}} v_m \right|^2 \right) (A_2 v_m, v_m') \\ = \frac{1}{2} M \left(\left| A_1^{\frac{1}{2}} u_m \right|^2 + \left| A_2^{\frac{1}{2}} v_m \right|^2 \right) \frac{d}{dt} \left| A_2^{\frac{1}{2}} v_m \right|^2. \end{aligned}$$

Now adding and using $\widehat{M}(\lambda) = \int_0^\lambda M(s) ds,$ follows from (12), (12) that

$$\begin{aligned} & \frac{1}{2} \frac{d}{dt} [|K_1^{\frac{1}{2}} u'_m|^2 + |A_1 u_m|^2 + |K_2^{\frac{1}{2}} v'_m|^2 \\ & + |A_2 v_m|^2 + \widehat{M} (|A_1^{\frac{1}{2}} u_m|^2 + |A_2^{\frac{1}{2}} v_m|^2)] \\ & + \delta_1 |u'_m|^2 + \delta_2 |v'_m|^2 = 0. \end{aligned}$$

From the equation above, after integration from 0 to t , $0 \leq t \leq T_m$, we obtain

$$\begin{aligned} & |K_1^{\frac{1}{2}} u'_m|^2 + |A_1 u_m|^2 + |K_2^{\frac{1}{2}} v'_m|^2 + |A_2 v_m|^2 \\ & + \widehat{M} (|A_1^{\frac{1}{2}} u_m|^2 + |A_2^{\frac{1}{2}} v_m|^2) \\ & + 2 \int_0^t (\delta_1 |u'_m|^2 + \delta_2 |v'_m|^2) ds \\ & = |K_1^{\frac{1}{2}} u_{1m}|^2 + |A_1 u_{0m}|^2 + |K_2^{\frac{1}{2}} v_{1m}|^2 + |A_2 v_{0m}|^2 \\ & + \widehat{M} (|A_1^{\frac{1}{2}} u_{0m}|^2 + |A_2^{\frac{1}{2}} v_{0m}|^2) \leq C_1, \end{aligned}$$

with $C_1 > 0$, constant independent of m and t . Then we have

$$(K_1^{\frac{1}{2}} u'_m), (K_2^{\frac{1}{2}} v'_m) \text{ limited in } L^\infty([0, T]; H), \quad (13)$$

$$(u_m) \text{ limited in } L^\infty([0, T]; \mathcal{D}(A_1)), \quad (14)$$

$$(v_m) \text{ limited in } L^\infty([0, T]; \mathcal{D}(A_2)). \quad (15)$$

Since that $K_i^{\frac{1}{2}}$, $i = 1, 2$, are limited from H in H using (13) we get

$$(K_1 u'_m), (K_2 v'_m) \text{ limited in } L^\infty([0, T]; H). \quad (16)$$

Passage to the limit

Using the estimates (14) - (15), we can extract subsequences of $(u_m)_{m \in \mathbb{N}}$, $(v_m)_{m \in \mathbb{N}}$, (which we denote with the same symbol) so that

$$u_m \overset{*}{\rightharpoonup} u \text{ weak star in } L^\infty([0, T]; \mathcal{D}(A_1)) \quad (17)$$

$$v_m \overset{*}{\rightharpoonup} v \text{ weak star in } L^\infty([0, T]; \mathcal{D}(A_2)) \quad (18)$$

$$u'_m \rightharpoonup u \text{ weak in } L^\infty([0, T]; H) \quad (19)$$

$$v'_m \rightharpoonup v' \text{ weak in } L^\infty([0, T]; H) \quad (20)$$

$$K_1 u'_m \rightharpoonup K_1 u' \text{ weak in } L^\infty([0, T]; H) \quad (21)$$

$$K_2 v'_m \rightharpoonup K_2 v' \text{ weak in } L^\infty([0, T]; H) \quad (22)$$

From (17)-(20) and Aubin-Lions's compactness theorem, see [6] follows that

$$u_m \longrightarrow u \text{ strongly in } L^2([0, T]; \mathcal{D}(A_1^{\frac{1}{2}})), \quad (23)$$

$$v_m \longrightarrow v \text{ strongly in } L^2([0, T]; \mathcal{D}(A_2^{\frac{1}{2}})). \quad (24)$$

By continuity of M we have the following weak convergence in $L^2([0, T]; H)$,

$$M (|A_1^{\frac{1}{2}} u_m|^2 + |A_2^{\frac{1}{2}} v_m|^2) A_1 u_m$$

$$\rightharpoonup M (|A_1^{\frac{1}{2}} u|^2 + |A_2^{\frac{1}{2}} v|^2) A_1 u, \quad (25)$$

$$\begin{aligned} & M (|A_1^{\frac{1}{2}} u_m|^2 + |A_2^{\frac{1}{2}} v_m|^2) A_2 v_m \\ & \rightharpoonup M (|A_1^{\frac{1}{2}} u|^2 + |A_2^{\frac{1}{2}} v|^2) A_2 v. \end{aligned} \quad (26)$$

These convergence implies that we can take limits in the approximate problem (7)-(10) and then we concludes the existence of weak solution. Finally since that M is locally Lipschitz, the uniqueness of solution can be proved as usual, by Ladyzhenskaya's method [13].

3. Stability of solutions

First we introduce the Nakao's theorem

Theorem 2. Let $E(t)$ be a nonnegative function on $[0, \infty)$ satisfying

$$s \in [t, t + 1] \sup E(s) \leq C_0 (E(t) - E(t + 1))$$

where C_0 is a positive constant. Then there exist C positive constant such that

$$E(t) \leq C e^{-wt} \text{ with } w = \frac{1}{C_0 + 1}.$$

Proof. See page 748 of [10].

Now we will to prove our principal result

Theorem 3. For $t \geq 1$ and $(u_0, v_0) \in \mathcal{D}(A_1) \times \mathcal{D}(A_2)$, $(u_1, v_1) \in H \times H$, the full energy of (1)-(4) defined by

$$\begin{aligned} E(t) = \frac{1}{2} [& |K_1^{\frac{1}{2}} u'|^2 + |A_1 u|^2 + |K_2^{\frac{1}{2}} v'|^2 + |A_2 v|^2 \\ & + \widehat{M} (|A_1^{\frac{1}{2}} u|^2 + |A_2^{\frac{1}{2}} v|^2)] \end{aligned}$$

satisfies

$$E(t) \leq C e^{-wt}.$$

Proof. Using (12) and (12) we obtain after passage to limit

$$\begin{aligned} & \frac{1}{2} \frac{d}{dt} [|K_1^{\frac{1}{2}} u'|^2 + |A_1 u|^2 + |K_2^{\frac{1}{2}} v'|^2 + |A_2 v|^2 \\ & + \widehat{M} (|A_1^{\frac{1}{2}} u|^2 + |A_2^{\frac{1}{2}} v|^2)] + \delta_1 |u'|^2 + \delta_2 |v'|^2 = 0. \end{aligned}$$

Performing integration from t_1 to t_2 , with $0 < t_1 < t_2$ we get

$$E(t_2) + \int_{t_1}^{t_2} [\delta_1 |u'(s)|^2 + \delta_2 |v'(s)|^2] ds = E(t_1) \quad (27)$$

and for all $t > 0$

$$E(t + 1) + \int_t^{t+1} [\delta_1 |u'(s)|^2 + \delta_2 |v'(s)|^2] ds = E(t).$$

Now, defining $C_2 [E(t) - E(t + 1)] \equiv F^2(t)$ we obtain

$$\int_t^{t+1} [|u'(s)|^2 + |v'(s)|^2] ds \leq F^2(t), \quad (28)$$

with

$$C_2 = \frac{1}{\min\{\delta_1, \delta_2\}}.$$

Then, we can choose

$t_1 \in [t, t + \frac{1}{4}]$ and $t_2 \in [t + \frac{3}{4}, t + 1]$, such that

$$|u'(t_1)|^2 + |v'(t_1)|^2 \leq 4F^2(t),$$

and so

$$|u'(t_1)| + |v'(t_1)| \leq 4F(t). \tag{29}$$

Similarly we get

$$|u'(t_2)| + |v'(t_2)| \leq 4F(t). \tag{30}$$

Making $w = u(t)$ and $z = v(t)$ in (7) and (8) we obtain respectively

$$\begin{aligned} & (K_1 u''(t), u(t)) + |A_1 u(t)|^2 \\ & + M \left(\left| A_1^{\frac{1}{2}} u(t) \right|^2 + \left| A_2^{\frac{1}{2}} v(t) \right|^2 \right) \left| A_1^{\frac{1}{2}} u(t) \right|^2 \\ & + \delta_1 (u'(t), u(t)) = 0, \end{aligned}$$

$$\begin{aligned} & (K_2 v''(t), v(t)) + |A_2 v(t)|^2 + \\ & M \left(\left| A_1^{\frac{1}{2}} u(t) \right|^2 + \left| A_2^{\frac{1}{2}} v(t) \right|^2 \right) \left| A_2^{\frac{1}{2}} v(t) \right|^2 \\ & + \delta_2 (v'(t), v(t)) = 0, \end{aligned}$$

and adding

$$\begin{aligned} & \frac{d}{dt} (K_1 u'(t), u(t)) - \left| K_1^{\frac{1}{2}} u'(t) \right|^2 + |A_1 u(t)|^2 \\ & + M \left(\left| A_1^{\frac{1}{2}} u(t) \right|^2 + \left| A_2^{\frac{1}{2}} v(t) \right|^2 \right) \left| A_1^{\frac{1}{2}} u(t) \right|^2 \\ & + \frac{d}{dt} (K_2 v'(t), v(t)) - \left| K_2^{\frac{1}{2}} v'(t) \right|^2 + |A_2 v(t)|^2 \\ & + M \left(\left| A_1^{\frac{1}{2}} u(t) \right|^2 + \left| A_2^{\frac{1}{2}} v(t) \right|^2 \right) \left| A_2^{\frac{1}{2}} v(t) \right|^2 \\ & + \delta_1 (u'(t), u(t)) + \delta_2 (v'(t), v(t)) = 0. \end{aligned}$$

Integrating from t_1 to t_2 we have

$$\begin{aligned} & \int_{t_1}^{t_2} \left[|A_1 u(t)|^2 + |A_2 v(t)|^2 \right. \\ & + M \left(\left| A_1^{\frac{1}{2}} u(t) \right|^2 + \left| A_2^{\frac{1}{2}} v(t) \right|^2 \right) \left| A_1^{1/2} u(t) \right|^2 \\ & + M \left(\left| A_1^{\frac{1}{2}} u(t) \right|^2 + \left| A_2^{1/2} v(t) \right|^2 \right) \left| A_2^{\frac{1}{2}} v(t) \right|^2 \Big] dt \\ & = (K_1 u'(t_1), u(t_1)) - (K_1 u'(t_2), u(t_2)) \\ & + (K_2 v'(t_1), v(t_1)) - (K_2 v'(t_2), v(t_2)) \\ & + \int_{t_1}^{t_2} \left[\left| K_1 u'(t) \right|^2 + \left| K_2 v'(t) \right|^2 \right. \\ & \left. - \delta_1 (u'(t), u(t)) - \delta_2 (v'(t), v(t)) \right] dt. \end{aligned}$$

Using $M \in C^0[0, +\infty)$, with $M(\lambda) \geq 0$ we get

$$\begin{aligned} & \int_{t_1}^{t_2} |A_1 u(t)|^2 + |A_2 v(t)|^2 dt \\ & \leq M_1 \int_{t_1}^{t_2} |u'(t)|^2 dt + M_2 \int_{t_1}^{t_2} |v'(t)|^2 dt \\ & + C_3 \delta_1 \int_{t_1}^{t_2} |u'(t)| |A_1 u(t)| dt + C_4 \delta_2 \int_{t_1}^{t_2} |v'(t)| |A_2 v(t)| dt \\ & + M_1 |u'(t_1)| |u(t_1)| + M_1 |u'(t_2)| |u(t_2)| \\ & + M_2 |v'(t_1)| |v(t_1)| + M_2 |v'(t_2)| |v(t_2)|, \end{aligned}$$

where $M_i = \|K_i\|_{\mathcal{L}(H)}$, $i = 1, 2$ and C_3, C_4 are constants such that $|u(t)| \leq C_3 |A_1 u(t)|$ and $|v(t)| \leq C_4 |A_2 v(t)|$.

Using (29) and (30) we obtain for $s \in [t, t + 1]$

$$\begin{aligned} & \int_{t_1}^{t_2} [|A_1 u(t)|^2 + |A_2 v(t)|^2] dt \leq M_1 F^2(t) + M_2 F^2(t) \\ & + \frac{C_3^2 \delta_1^2}{2} \int_{t_1}^{t_2} |u'(t)|^2 dt + \frac{1}{2} \int_{t_1}^{t_2} |A_1 u(t)|^2 dt \\ & + \frac{C_4^2 \delta_2^2}{2} \int_{t_1}^{t_2} |v'(t)|^2 dt + \frac{1}{2} \int_{t_1}^{t_2} |A_2 v(t)|^2 dt \\ & + M_1 C_3 \sup |A_1 u(s)| (|u'(t_1)| + |u'(t_2)|) \\ & + M_2 C_4 \sup |A_2 v(s)| (|v'(t_1)| + |v'(t_2)|). \end{aligned}$$

Defining

$$C_4 [F^2(t) + \sup |A_1 u(s)| (|u'(t_1)| + |u'(t_2)|) + \sup |A_2 v(s)| (|v'(t_1)| + |v'(t_2)|)] \equiv G^2(t)$$

we get

$$\int_{t_1}^{t_2} [|A_1 u(t)|^2 + |A_2 v(t)|^2] \leq G^2(t), \tag{31}$$

where

$$C_4 = \max \{2M_0, 2M_1 C_3, 2M_2 C_4\},$$

with

$$M_0 = \left(M_1 + M_2 + \frac{C_3^2 \delta_1^2}{2} + \frac{C_4^2 \delta_2^2}{2} \right).$$

From (28) and (31) we obtain

$$\begin{aligned} & \int_{t_1}^{t_2} \left[(|A_1 u(t)|^2 + |A_2 v(t)|^2) \right. \\ & \left. + \delta_1 |u'(t)|^2 + \delta_2 |v'(t)|^2 \right] dt \leq C_5 F^2(t) + G^2(t), \end{aligned}$$

where $C_5 = \delta_1 + \delta_2$.

Then, there exist $t^* \in [t_1, t_2]$ such that

$$|A_1 u(t^*)|^2 + |A_2 v(t^*)|^2 + \delta_1 |u'(t^*)|^2 + \delta_2 |v'(t^*)|^2 \leq 2[C_5 F^2(t) + G^2(t)]. \tag{32}$$

Observe that

$$\begin{aligned} & \widehat{M} \left(|A_1^{1/2} u(t^*)|^2 + |A_2^{1/2} v(t^*)|^2 \right) \\ &= \int_0^{|A_1^{1/2} u(t^*)|^2 + |A_2^{1/2} v(t^*)|^2} M(s) ds \\ &\leq m_0 \left(|A_1^{1/2} u(t^*)|^2 + |A_2^{1/2} v(t^*)|^2 \right) \\ &\leq m_0 C_6 \left(|A_1 u(t^*)|^2 + |A_2 v(t^*)|^2 \right), \end{aligned} \tag{33}$$

where $m_0 = \max M(s)$ that is finite and C_6 is a constant such that

$$|A_1^{1/2} u(t)|^2 \leq C_6 |A_1 u(t)|^2 \quad \text{and} \quad |A_2^{1/2} v(t)|^2 \leq C_6 |A_2 v(t)|^2.$$

Now, using the energy functional and (32), (33), we obtain

$$E(t^*) \leq C_7 [F^2(t) + G^2(t)]. \tag{34}$$

From (27), (28) and (34) we have

$$E(t^{t_1}) = E(t^*) + \int_{t_1}^{t^*} [\delta_1 |u'(s)|^2 + \delta_2 |v'(s)|^2] ds,$$

so

$$E(t) \leq \int_1^{t+1} [\delta_1 |u'(s)|^2 + \delta_2 |v'(s)|^2] ds + E(t^*),$$

and then for $s \in [t, t + 1]$

$$\begin{aligned} \sup E(s) &\leq E(t^*) + \int_t^{t+1} [\delta_1 |u'(s)|^2 + \delta_2 |v'(s)|^2] ds \\ &\leq C_7 [F^2(t) + G^2(t)] + C_5 F^2(t), \end{aligned}$$

so we obtain

$$\sup E(s) \leq C_8 F^2(t) + \frac{1}{2} \sup E(s).$$

From last inequality we have

$$\sup E(s) \leq 2C_8 F^2(t) = C_0 [E(t) - E(t + 1)],$$

and then

$$E(t) \leq C_0 [E(t) - E(t + 1)].$$

Finally, by theorem 3.1 we concludes that

$$E(t) \leq C e^{-w t} \quad \text{with} \quad w = \frac{1}{C_0 + 1}.$$

4. Conclusion

In this paper, we employed the Nakao's method to analyze the asymptotic behaviour for an abstract coupled system of nonlinear equations of extensible beams models. We prove that when the time t goes to infinity the system has a exponential decay.

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References

- [1] J. M. Ball, Initial-boundary value problem for an extensible beam, *J. Math. Anal. Appl.* 42:61-90 (1973).
- [2] P. Biler, Remark on the decay for damped string and beam equations, *Nonlinear Analysis, Theory, Methods and Applications* 10(9):839-842 (1986).
- [3] E. H. Brito, Decay estimates for generalized damped extensible string and beam equation, *Nonlinear Analysis, Theory, Methods and Applications* 8(12):1489-1496 (1984).
- [4] R. W. Dickey, Free vibrations and dynamic buckling of extensible beam, *Journal Math. Anal. Appl.* 129:443-454 (1970).
- [5] J. G. Eisley, Nonlinear vibrations of beams and rectangular plates, *Journa Angew. Math. Phys.* 15:167-175 (1964).
- [6] J. L. Lions, *Quelques méthodes de résolution des problèmes aux limites non linéaires.* (Dunod, Paris, 1969).
- [7] T. F. Ma, V. Narciso, Global attractor for a model of extensible beam with nonlinear damping and source terms, *Nonlinear Anal.* 73 3402-3412 (2010).
- [8] L. A. Medeiros, On a new class of a nonlinear wave equations, *Journal Math. Appl.* 69:252-262 (1979).
- [9] G. P. Menzala, On global classical solution of a nonlinear wave equation, *Journal Math. Appl.* 10:179-195 (1980).
- [10] M. Nakao, A difference inequality and its application to nonlinear evolution equation, *J. Math. Soc. Japan.* 30(4):747-762 (1978).
- [11] D. C. Pereira. Existence, uniqueness and asymptotic behaviour of solutions of the nonlinear beam equation, *Nonlinear Analysis. Theory, Methods and Applications* 14(08):613-623 (1990).
- [12] J. E. M. Rivera, Existence and asymptotic behaviour in a class of nonlinear wave equation with thermal dissipation, *Nonlinear Analysis, Theory, Methods and Applications* 23(10):1255-1271 (1994).
- [13] M. I. Visik, O. A. Ladyzenskaya, On boundary value problems for PDE and certain class of operators equations, *A.M.S. Transl* 2(10), (1958).
- [14] S. Woinowsky-Krieger, The effect of axial force on the vibration of hinged bars, *Journal Appl. Mech.* 17:35-36 (1950).



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