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Some New Developments and Review on T-X Family of Distributions

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Abstract: A brief review on transformed-transformer {T-X} generated family of distributions is presented in the recent article. There are many ideas for the development of new family of distributions based on {T-X} family of distribution. The related research work of {T-X} family of distribution is listed and comprehensive reviews are provided. In this article we have discussed some {T-X} family of distributions, T transmuted X family, exponentiated {T-X} family of distributions, some generalized classes of distribution and exponential distribution by using the idea of {T-X} family of distributions, also by using quantile function {T-X} family of distributions. A concise review of {T-X} family is present and also proposed some extended forms of these distributions.

Keywords: T-X family, Generalized family, Exponentiated family, Transmuted family, Marshall-Olkin generated family

1 Introduction

The key idea to make a new distribution is addition of one or more parameters to the compounding existing distribution. In the literature there are many methods for generating statistical distribution, even though several distributions have been prolonged. However, in modeling data these established distributions do not give sufficient fit, in many practical areas and there is an obvious requirement for the comprehensive description of these established distributions.

In this analysis, to suggest new families of probability distributions that expand the existing distributions by the induction of one or more parameters in the base line model a serious attempt has been made.

There is no uncertainty that the reputation of gamma functions and Euler-beta in X families have attracted the focus of statisticians, engineers, mathematicians, economists, scientists, demographers and other researchers.

For generating continuous distributions some well-known methods based on differential equations extended by Lee et al. [1] methods of translation based on quantile functions expanded by Johnson et al. [2].

Recently Alzaatreh et al. [3] by using a probability density function as a generator anticipated a new procedure to originate families. After (1980s), the majority of methods developed specified by Lee et al. [1] the methods of combination based on the proposal of combining two existing distributions.

The familiar generated families are Marshall-Olkin generated family introduced by Marshall and Olkin [4]. Nadarajah et al. [5] introduced a simplification referred to as the Beta Gumbel distribution. The beta-G is studied by Eugene et al. [6], beta-normal by Eugene et al. [7], normal and student´s t distribution studied by Ahsanullah et al. [8], a note on a characterization of Gompertz-Verhulst Distribution by Ahsanullah et al. [9], Kumaraswamy-G studied by Cordeiro et al. [10], the Weibull-G studied by Bourguignon et al. [11], the exponentiated half-logistic family studied by Cordeiro et al. [12], log-gamma-G by Amini et al.[13], Gamma-Pareto distribution considered by Alzaatreh et al. [14], Gamma half-Cauchy Alzaatreh et al. [15], Gamma Normal by Alzaatreh et al. [16], Weibull-X family of distributions studied by Alzaatreh et al. [17], Exponentiated generalized class by Cordeiro et al. [18], exponentiated {T-X}by Alzaghal et al. [19], family of gamma-X by Alzaatreh et al. [20], gamma logistic by Alzaatreh et al. [21], logistic-G by Torabi et al. [22], type-I half-logistic family by Cordeiro et al. [23], Kumaraswamy Weibull-generated by Hassan et al. [24], generalized Pearson system of distributions by Shakil et

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al.[25], on a family of product distributions based on the Whittaker functions and generalized Pearson differential equation by Shakil et al. [26], generalized transmuted-G by Nofal et al. [27], T- transmuted X- family of distribution by Jayakumar *and Girish* [28], transmuted exponentiated generalized-G by Yousof et al. [29], new generalized family of distributions by Ahmad et al. [30], some new members of the {T-X} family by Farrukh et al. [31], modified {T-X} family by Aslam et al. [32], properties and applications of new member of {T-X} family of distributions by Handique et al. [33], and exponential {T-X} family of distributions by Zubaira et al. [34]. The details like analytical properties, probabilistic interpretations, simulation algorithms estimation methods and applications are not given, because of the length of this paper. From the cited references, the information can be obtained.

The layout plan of this paper follows Ayman Alzaatreh, Carl Lee and Felix Famoye {T-X} family of distributions and in Section 3 the lists of contributed efforts are presented. We give explanation of the contributed literature on the T-Transmuted X family of distributions in Section 4. Exponentiated {T-X} families of distributions are discussed in Section 5. Generalized classes of exponential distribution using {T-X} family framework is described in Section 6. Section 7 deals with the generating {T-X} family using quantile functions. In Section 8, some new sort of log-Dagum family, transmuted-x family, exponentiated-x family is proposed. Finally, we shall present some concluding remarks.

2 Some Recent Families

In the literature numerous families of distributions are presented which extend probability distributions by adding new parameters. Some of these families of distributions are discussed in the following lines.

2.1The Marshall-Olkin Family

Marshall-Olkin [35] introduced a method for induction of anew parameter to an existing model and applied this method to expand Exponential and Weibull distributions.

The PDF and CDF of the Marshall Olkin family are correspondingly specified by

$$F(x) = \frac{G(x,\varphi)}{\theta + (1-\theta x)G(x,\varphi)} \ , \Theta > 0$$

$$f(x) = \frac{\theta g(x,\varphi)}{[\Theta + (1-\Theta x)G(x,\varphi)]^2} \ .$$

2.2 The Exponentiated Family

The exponentiated family of distributions in general form is developed by Cordeiro et al. [18]. The cumulative distribution function of this family is

$$F_\alpha(x) = [G(x)]^\alpha , \qquad x \in \mathrm{R} \tag{2.1}$$

2.3 McDonald Type Distributions

The upper limit x of the integral withx^c is replaced by McDonald, The CDF of the McDonald type distribution is

$$F(x) = \frac{\beta_{x^c}(\beta,\Theta)}{\Theta(\beta,\Theta)} = \frac{1}{\Theta(\beta,\Theta)} \int_0^{x^c} x^{\beta-1} (1-x)^{\Theta-1} dt \ . \tag{2.2}$$

The Mc distribution includes as sub model the Kumaraswamy and beta type-1. Family of distributions based on generalized beta distribution. These generalized families are called the generalized beta generated distributions

2.4 The Kumaraswamy-G Family

The Kumaraswamy distribution is used as a generator as a replacement of beta by Tahir et al. [36]. The PDF of the Kumaraswamy generalized family is

$$f(x,\alpha,\beta) = \alpha\beta g(x,\varphi) G^{\alpha-1}(x,\varphi)\big(1 - G^\alpha(x,\varphi)\big)^\beta \ . \tag{2.3}$$

2.5 Gamma-G family

New class of Gamma-generated distributions is introduced by Amini et al. [13]. The cumulative distribution function of the gamma-G family is as follows:

$$F(x) = \frac{1}{\Gamma(\alpha)\beta^{\alpha}} \int_{0}^{\frac{F(x,\varphi)}{\bar{F}(x,\varphi)}} e^{-\frac{w}{\beta}} w^{\alpha-1} dw \,. \tag{2.4}$$

They are encouraged to introduced the gamma-uniform distribution by taking G in (2.4) the cumulative distribution function of the gamma uniform distribution becomes

$$F(x) = \int_{0}^{\frac{x-a}{b-x}} \frac{e^{-\frac{w}{\beta}} w^{\alpha-1}}{\Gamma(\alpha)\beta^{\alpha}} dw.$$

They measured a new class of distributions and also discussed the special case of this class. To demonstrate the importance of this distribution by using two real data sets which present better fit than other existing distribution.

Family of univariate distributions generated by gamma random variables is developed by Zografos et al. [37] For any baseline $F(x,\varphi)$, and $x \in R$,the gamma-G distribution with cumulative distribution function $F(x)$is specified by

$$F(x) = \frac{1}{\Gamma(\alpha)} \int_{0}^{-log[1-F(x,\varphi)]} r^{\alpha-1} e^{-r} dt.$$

2.6 Topp Leone G family:

Topp Leone distribution is used by several researchers as generator for the development of general families. This generated family of distribution is recommended by Reyad et al. [38] with its PDF and CDF

$$f_{TLG}(x) = 2\alpha\theta g(x)[G(x)]^{\theta\alpha-1}\left[1 - G(x)^{\theta}\right]\left[2 - G(x)^{\theta}\right]^{\alpha-1} ; x \geq 0$$

$$F_{TLG}(x) = \left\{G(x)^{\theta}\left[2 - G(x)^{\theta}\right]^{\alpha-1}\right\}^{\alpha} ; x \geq 0$$

2.7 Odd log logistic G Family

Odd log-logistic-G family is a new transformation of distribution proposed by Alizadeh et al. [39]. The cumulative distribution function

$$F(y) = \frac{G(y,\varphi)^{\alpha}}{G(y,\varphi)^{\alpha} + \bar{G}(y,\varphi)^{\alpha}} \,.$$

2.8 Alpha Power Transformation

The alpha power transform class is proposed byAhmad et al. [40] to obtain a more flexible family introducing an extra parameter to the baseline distribution. The cumulative distribution function of this Transformation class is

$$W(y) = \begin{cases} \dfrac{\Theta^{G(y)} - 1}{\Theta - 1} & \text{if } \Theta > 0, \Theta \neq 1 \\ G(y) & \text{if } \Theta = 1 \end{cases}$$

and the corresponding PDF takes the form

$$w(y) = \begin{cases} \dfrac{\log(\Theta) g(y)\, \Theta^{G(y)}}{\Theta - 1} & \text{if } \Theta > 0, \Theta \neq 1 \\ g(y) & \text{if } \Theta = 1 \end{cases}$$

2.9 Betas-G Family

Generalized family of distributions, known as McDonald-G (Mc-G), by means of logit of the beta random variable is introduced by Alexander et al. [41]. The proposed family is denoted by beta-G and has the cdf

$$F(x) = \frac{\beta_{G(x)}(\alpha,\beta)}{\beta(\alpha,\beta)} = \frac{1}{\beta(\alpha,\beta)} \int_{0}^{G(x)} t^{\alpha-1} (1-t)^{\beta-1} dt.$$

3 Ayman Alzaatreh, Carl Lee and Felix Famoye (2013)'S T-X Family of Distributions

Recently, Alzaatreh et al. [3] extended the beta-generated family namely {T-X} family for generating new distributions. The new class is characterized as

$$F(x) = \int_0^{-\log(1-H(x,\varphi))} r(t)dt = R\{-\log(1-H(x,\varphi)\}. \quad (3.1)$$

The corresponding PDF of the {T-X} distribution is given below

$$f(x) = \frac{h(x,\varphi)}{1-H(x,\varphi)} r\{-\log(1-H(x,\varphi)\}. \quad (3.2)$$

The generated new family of distributions from (3.1) is called {T-X} family of distribution.

Definition1:

Let $v(t)$ be the pdf of a random variable T, where $T \in [m,n]$ for $-\infty \leq m < n < \infty$ and let $U[F(x;\ \varphi)]$ be a function of CDF of a random variable X, depending on the vector parameterφsatisfying the conditions $[F(x,\varphi)] \in [m;\ n]$;

$i.\, U[F(x;\ \varphi)]$ Is differentiable and monotonically increasing, and

$ii.\, U[F(x;\ \varphi)] \to masx \to -\infty$ and $U[F(x;\ \varphi)] \to nasx \to \infty$

Recently, [21] identified the cdf of the T-X family is

$$G(x) = \int_m^{U[F(x;\ \varphi)]} v(t)dt\, x \in R. \quad (3.3)$$

the corresponding pdf is given by

$$g(x) = \left\{\frac{\delta}{\delta(x)} U[F(x,\varphi)]\right\} v\{U[F(x,\varphi)]\}, x \in R. \quad (3.4)$$

By the {T-X} idea, in the literature some new classes of distributions have been introduced.

Several researchers have generated new probability distribution by utilizing recent families of distribution for different choices of $F(x,\varphi)$. Our main interest in this paper is to specify the review of work done on {T-X} family.

3.1 Existing T-X Family of Distributions:

In this section, some existing {T-X} families are discussed. From equation (3.3) Weibull-Pareto distribution is developed . LX families of distributions by using {T-X} family in (3.2) are proposed by Tahir et al. [42] .The PDF of logistic family is specified by

$$r(t,\beta) = \beta e^{-\beta t}\left(1+e^{-\beta t}\right)^{-2} \quad (3.5)$$

By replacing $U[G(x,\varphi)] = \log\{-\log[\bar{G}(x;\ \varphi)]andr(t)$ with (3.5) in (3.3) we obtained the Cdf of the LX family

$$F(x,\beta,\varphi) = \left[1+\{-\log[\bar{G}(x;\ \varphi)]\}^{-\beta}\right]^{-1}. \quad (3.6)$$

The probability density function of the LX family is identified as

$$f(x) = \frac{\beta g(x,\varphi)}{\bar{G}(x;\varphi)}\{-\log[\bar{G}(x;\ \varphi)]\}^{-(\beta+1)}\left[1+\{-\log[\bar{G}(x;\ \varphi)]\}^{-\beta}\right]^{-2} \quad (3.7)$$

The main intention of their work was to pioneer new logistic distributions which are obtained from the concept of the LX families.

The Weibull-Dagum distribution is proposed by Tahir et al. [42].The cumulative distribution function of four-parameter Weibull distribution is specified below

$$F(x) = 1 - e^{-\left[\left(1+\alpha x^{-\delta}\right)^{\beta}-1\right]^{-\theta}} \quad (3.8)$$

In application study to real data, new dagum model is consistently better fit than the other extended models. By considering weibull distribution Alzaatreh et al. [17] proposed new weibull T-X family.

They described the CDF of the new Weibull x family by setting $U[F(x,\varphi)] = \frac{[1-\log\{1-F(x,\varphi)\}]}{1-F(x,\varphi)}$

$$G(x,\alpha,\varphi) = 1 - e^{\left[-\{\frac{[1-\log\{1-F(x,\varphi)\}]}{1-F(x,\varphi)}\}\right]^{\alpha}} \quad \alpha,\varphi > 0, \tag{3.9}$$

They developed the NW-X family, the baseline model is extended by the induction of additional parameter(s) to improve and generate the characteristics of the distributions.

Two new distributions are introduced by Ahmad et al. [43]. Firstly, they introduced the Weibull Rayleigh distribution by using $U(F(x)) = -\log(1 - F(x,\varphi))$ then (3.3) is known as Weibull-Rayleigh distribution. Secondly, they defined Weibull-Exponential distribution by using $W(F(x,\varphi)) = \frac{F(x,\varphi)}{1-F(x,\varphi)}$ the density function and distribution function is obtained. Two data sets are utilized for demonstrating the applicability of these distributions.

Jamal et al. [31] defined some new members of T-X family of probability distributions and checked their existence through graphs of their densities. Properties and applications of a new member of the T-X family are defined by Handique et al. [33]**.** They proposed new density functions of the Jamal Weibull-X family by using the base line function.

$$U[F(x;\varphi)] = \frac{-log[1-F^{\alpha}(x,\varphi)]}{1-F^{\alpha}(x,\varphi)}.$$

They computed the mathematical properties of JW-X family. The linear expansions for the cumulative distribution function and probability density function and are also expressed as a new exponentiated X family.

3.2 Dagum T-X Families of Distributions:

The dagum distribution is proposed by Camilo dagum which has both Type-I and Type-II specification. The Dagum distribution is sub model of generalized beta distribution of the second kind. The PDF and CDF of Dagum distribution are specified by:

$$g_D(x,\lambda,\alpha,\delta) = \alpha\delta\lambda x^{-\delta-1}\left(1+\lambda x^{-\delta}\right)^{-\alpha-1}.$$

and

$$G(x,\lambda,\alpha,\delta) = \left(1+\lambda x^{-\delta}\right)^{-\alpha}.$$

There are many generalizations of the dagum distribution via Kumaraswamy distribution and its exponentiated version, Gumbel dagum distribution, The Odd Dagum Family of distribution.

In the literature by using T-X idea several new classes of distributions have been introduced.

In Table 1 a list of papers on the T-X family of distributions is presented

TABLE I: Contribute work on the T-X family of Distributions for different $U[F(y)]$.

S.No	Distributions	Authors(years)	$W[F(y)]$
1	New method for generating families of continuous distributions	[Alzaatreh et al, 2013b]	$\log[\frac{F(y,\varphi)^{\alpha}}{1-F(y,\varphi)^{\alpha}}]$
2	Weibull-x	[Alzaatreh et al, 2013a]	$-\log[1-F(y,\varphi)]$
3	Weibull-G family	[Bourguignon et al, 2014]	$\frac{F(y,\varphi)}{1-F(y,\varphi)}$
4	Logistic-x family	[Tahir et al, 2015b]	$\log[-\log[1-F(y,\varphi)]$
5	Logistic-*G*	[Torabi and Montazari, 2014]	$\log[\frac{F(y,\varphi)}{1-F(y,\varphi)}]$
6	Lomax-G	[Bozidar et al, 2014]	$-log\,[1-F(y,\varphi)]$
7	Exponential t-x family of distributions	[Ahmed et al, 2021]	$-\log\left[\frac{\sigma F(y,\varphi)}{\sigma-F(y,\varphi)}\right]$
8	Characterization and estimation of weibull rayleigh distribution	[Afaq Ahmad et al, 2017]	$-\log[1-F(y,\varphi)]$
9	Gompertz –G family of distributions	[Alizadeh et al, 2016]	$-log\,[1-F(y,\varphi)]$
10	Generalized odd log-logistic family	[Cordeiro et al, 2017]	$\frac{F(y,\varphi)^{\alpha}}{1-F(y,\varphi)^{\alpha}}$

11	New weibull-x family of distribution	[Ahmad et al, 2018]	$\frac{\log[1-F(y,\varphi)]}{1-F(y,\varphi)}$
12	Extension of rayleigh distribution and applications	[Ateeq et al, 2019]	$-log\,[1-F(y,\varphi)]$
13	Gamma-x distribution	[Alzaatreh et al, 2014]	$-\log\,[1-F(y,\varphi)]$
14	Exponential lindley odd log-logistic family	[Mustafa et al, 2017]	$-\log\,[1-F(y,\varphi)]$
15	Gumbel lomax distribution	[Tahir et al, 2016]	$F(y,\varphi)$
16	New weibulllomax t-x distribution and its application	[Sharqa Hashmi et al, 2019]	$-\log\,[1-F(y,\varphi)]$
17	New T-X family of distributions	[Ampadu et al, 2019]	$-\log\,[1-F(y,\varphi)]$
18	New generalization of the frechetdistribution“	[Jayakumar at al, 201]	$-\log(1-F(y))$
19	New lifetime exponential-x family of distributions	[XiaoyanHuo et al, 2020]	$1-F(y,\varphi)$
20	Odd dagum family of distributions	[Ahmed Z. Afify et al, 2020]	$\frac{F(y,\varphi)}{\bar{F}(y,\varphi)}$
21	Gumbel dagum distribution: a new member of the t-x family of distributions	[Eraikhuemen et al, 2019]	$\log\left[\frac{F(y,\varphi)}{1-F(y,\varphi)}\right]$
22	Jamal weibull-x family	[Jamal and Nasir, 2019]	$\frac{-\log[1-F^{\alpha}(y,\varphi)]}{1-F^{\alpha}(y,\varphi)}$
23	Generalized exponential geometric density function	[Silva et al, 2010]	$-\log\,[1-F(y,\varphi)]$
24	Odd lindley-G family of distributions	[Silva et al, 2017]	$\frac{F(y,\varphi)}{1-F(y,\varphi)}$
25	Poisson weibull-x family of distributions	[Cordeiro et al, 2018]	$-\log(1-F(y))$
26	Weibull rayleigh	[Abouzar Bazyari et al, 2018]	$-\log(1-F(y))$
27	Weibull-exponential	[Abouzar Bazyari et al, 2018]	$\frac{F(y,\varphi)}{1-F(y,\varphi)}$
28	Extended odd frechet-G family of distribution	[Nasiru et al, 2018]	$\frac{F(y,\varphi)^{\alpha}}{1-F(y,\varphi)^{\alpha}}$
29	Modified t-x family of distribution	[Aslam et al, 2020]	$F(y,\varphi)$
30	Nasir logistic-x family	[Farrukh Jamal and Muhammad Nasir, 2019]	$\frac{\ln\,[-log[1-F^{\alpha}(y,\varphi}{1-F^{\alpha}(y,\varphi)}$
31	Nadarajah Topp Leone-G family of distributions	[Reyad et al, 2019]	$\{1-\bar{F}(y;\varphi)^{2}\}^{\theta}$
32	Jamal logistic-x family	[Jamal and Nasir, 2019]	$\log[\frac{F(y,\varphi)^{\alpha}}{1-F(y,\varphi)^{\alpha}}]$

33	New generalized family of distributions	[Mehbooh zaidi et al, 2020]	$\mathrm{Tan}[\frac{\pi}{2}F^{\alpha}(y,\varphi)]$
34	Odd inverse rayleigh family of distribution”	[Hameda et al, 2020]	$\frac{F(y,\varphi)}{1-F(y,\varphi)}$
35	Properties and applications of a new member of the t-x family of distributions	[Handique et al, 2021]	$\frac{-\log[1-F^{\alpha}(y,\varphi)]}{1-F^{\alpha}(y,\varphi)}$
36	Alpha power transformation and exponentiated t-x distributions family	[Klakattawai et al, 2020]	$-\log(1-F^{\alpha}(y,\varphi))$
37	Lindley family of distributions	[Cakmakyapan et al, 2016]	$-\log(1-F^{\alpha}(y,\varphi))$
38	Gumbel-weibull distribution	[Al-Aqtash et al, 2014]	$\log\left[\frac{F(y,\varphi)}{1-F(y,\varphi)}\right]$
39	Novel generalized family of distributions for engineering and life Sciences data applications	[Salahuddin et al, 2021]	$-\log(1-F(y))$

4 T-Transmuted X Family of Distributions

By using the approach of quadratic transmutation map Jayakumar et al. [46] developed a new family which is named as T-transmuted X family of distributions and the {T-X} family method by Alzaatreh et al. [44] then the correlation of cumulative distribution function is given as fellow.

$$G(y)=(1+\lambda)F(y,\varphi)-\lambda[F(y,\varphi)]^2\ . \qquad (4.1)$$

Where $F(y,\varphi)$ is the CDF of the base distribution, Differentiating (4.1) yields

$$g(y)=f(y,\varphi)[1+\lambda-2\lambda F \qquad (4.2)$$

In the literature different research papers have been emerged on transmuted generalization of distributions.

A wider class of W (.) functions introduced by Aljarrah et al. [45]

Let $W(F(y))=-ln(1-F(y,\varphi))$ then the CDF of the {T-X} family of distributions becomes

$$J(y)=\int_0^{-ln(1-F(y,\varphi)),} dR(t). \qquad (4.3)$$

In the literature several research papers are appeared based on the {T-X} family introduced by Alzaatreh et al. [3]. Combined family of {T-X} and transmuted distributions are introduced by Jayakumar et al. [46]. A new family T-transmuted X family is established. Various members of T-transmuted X family are recognized.

4.1 Exponential-Transmuted Exponential Distribution

Let $U(F(x))=-ln(1-F(x,\varphi))=-\ln[1-G(x)(1+\lambda G(x))]$where the base distribution is exponential. The consequent CDF of family is given by

$$J(x)=1-e^{-\beta x}\left(1-\lambda+\lambda e^{-\beta x}\right)^{\theta}.$$

The PDF of new family of distributions as exponential-transmuted exponential distribution is

$$f(x)=\quad \theta\beta e^{-\beta x}\ (\frac{(1-\lambda+2\lambda e^{-\beta x})}{(1-\lambda+\lambda e^{-\beta x})^{\theta-1}}\ .$$

4.2 Exponential Transmuted Uniform Distribution

Consider the base distribution is uniform distribution, and then the PDF of ETU distribution is

$$j(x)=\frac{\theta}{\alpha}[\left[1-\frac{x}{\alpha}\left[1+\lambda\left(1-\frac{x}{\alpha}\right)\right]\right]^{\theta-1}\left(1+\lambda\left(1-\frac{2x}{\alpha}\right)\right).$$

4.3 Exponential-transmuted Frechet (ET F) distribution

Considered the base distribution is Frechet distribution with the CDF and PDF are:

$$G(x) = e^{-\left(\frac{\beta}{x}\right)^{\alpha}} \quad \text{and} \quad g(x) = \alpha\beta^{\alpha-1}x^{-(\alpha+1)}e^{-\left(\frac{\beta}{x}\right)^{\alpha}}$$

Then $U[F(x)] = -\ln\left(1 - e^{-\left(\frac{\beta}{x}\right)^{\alpha}}\left[1 + \lambda\left(1 - e^{-\left(\frac{\beta}{x}\right)^{\alpha}}\right)\right]\right)$.

Now the PDF of ETF distribution are given by

$$j(x) = \theta\alpha\beta^{\alpha}x^{-(\alpha+1)}e^{-\left(\frac{\beta}{x}\right)^{\alpha}} \frac{1+\lambda-2\lambda e^{-\left(\frac{\beta}{x}\right)^{\alpha}}}{\left[1-e^{-\left(\frac{\beta}{x}\right)^{\alpha}}\left[1+\lambda\left(1-e^{-\left(\frac{\beta}{x}\right)^{\alpha}}\right)\right]\right]^{1-\Theta}}.$$

4.4 Exponential Transmuted Rayleigh Distribution

The base distribution is considered as the Rayleigh distribution with cumulative distribution function and probability distribution functions are

$$F(x) = 1 - e^{-\frac{x^2}{2\sigma^2}} \quad \text{and} \quad f(x) = \frac{x}{\sigma^2}e^{-\frac{x^2}{2\sigma^2}}$$

Then the probability distribution function of ETR distribution are given by

$$j(x) = \frac{\Theta x e^{-\frac{x^2}{2\sigma^2}}\left[1-\lambda+\lambda e^{-\frac{x^2}{2\sigma^2}}\right]}{\sigma^2\left[1-\lambda+\lambda e^{-\frac{x^2}{2\sigma^2}}\right]^{1-\Theta}}.$$

4.5 Exponential Transmuted Weibull Distribution

Here they consider the weibull distribution is base distribution. Then the CDF and PDF of ETW distribution are

$$J(x) = 1 - e^{-\theta\beta x^{\alpha}}\left[1-\lambda+\lambda e^{-\beta x^{\alpha}}\right]^{\Theta}.$$

$$j(x) = \theta\alpha\beta x^{\alpha-1}e^{-\theta\beta x^{\alpha}}\frac{\left(1-\lambda+2\lambda e^{-\beta x^{\alpha}}\right)}{\left(1-\lambda+2\lambda e^{-\beta x^{\alpha}}\right)^{1-\Theta}}.$$

Exponential transmuted exponential distribution members of the {T-transmuted X family} are studied.

A list of papers on the Transmuted-X family is presented in table III.

TABLE III Contributed work on the Transmuted-X family of distributions.

S.No	Distributions	Authors(Years)	$W[F(x)]$
1	New transmuted family of distributions	[Hassan Bakouch et al. 2017]	$F(x,\varphi)$
2	T-transmuted x family of distribution	[Jayakumar and Girish 2017]	- log [1 - $F^{\alpha}(x,\varphi)$]
3	Generalized transmuted family of distributions	[Morad Aadeh et al. 2000]	$[F(x,\varphi)]^{\alpha}$
4	Transmuted weibull-G family of distributions	[Morad Alizadehy et al. 2017]	$\{\frac{F(x;\varphi)}{\bar{F}(x;\varphi)}\}$
5	Further developments on the {t-transmuted x family} of distribution II	[Clement Boateng Ampadu et al. 2018]	- log [1 - $F(x,\varphi)$]
6	AT-{transmuted X}family of distribution	[Clement and Boateng Ampadu 2019]	$\frac{F(x,\varphi)}{1-F(x,\varphi)}$

7	T-transmuted family of distributions	[Jayakumar et al. 2020]	- log [1 - $F(x,\varphi)[1+\lambda\bar{G}(x)]$]
8	Characterization and estimation of weibull-rayleigh distribution	[Afaq Ahmad et al. 2017]	- log [1 - $F(x,\varphi)$]
9	General transmuted family of distributions	[Mahabuburrehman et al. 2018]	$F(x,\varphi)$
10	Another generalized transmuted family of distributions	[Faton Merovica et al. 2016]	$1-(\bar{F}(x;\varphi))^{\alpha}$
11	Modified T-X family of distributions: classical bayesian analysis	[Aslam et al. 2020]	$F(x,\varphi)$
12	Transmuted geometric-G family of distribution	[Afify et al. 2016]	$\frac{\Theta F(x,\varphi)}{1+(\Theta-1)F(x,\varphi)}$
13	Generalized Odd Lindley-G family	[Afify et al. 2019]	$\frac{F(x,\varphi)^{\alpha}}{1-F(x,\varphi)^{\alpha}}$

5 Ahmad Alzaghal, Felix Famoye and Carl Lee Exponentiated T-X Families of Distributions

The Exponentiated distributions is identify as

$$G(x)=[F(x,\varphi)]^{\alpha}.$$

For the construction of new distributions many researcher developed the class of exponentiated distribution. For example, the exponentiated frechet distribution is studied by Nadarajah et al. [47] exponentiated gumbel distribution is described by [18] and Exponentiated Weibull distribution is proposed by Mudholkar et al. [28].

Alzaghal et al.[45] proposed an extension of {T-X} family in order to improve on some of the drawbacks of the {T-X} method. The new family is called the exponentiated {T-X} family of distributions.

The CDF of this new family is given as below

$$F(x)=\int_0^{-\log(1-G^c(x,\varphi))} r(t)dt = R[-\log(1-G^c(x,\varphi))]. \quad (5.1)$$

Probability density function of the corresponding generalized distribution is given below

$$f(x)=\frac{Cg(x,\varphi)G^{c-1}(x,\varphi)}{1-G^c(x,\varphi)}r[-\log(1-G^c(x,\varphi))], \qquad c>0 \quad (5.2)$$

Every member of the new family exponentiated {T-X} distributions generated from (5.2).

Using equation (5.2) the probability density function of exponentiated gamma-x family is defined as

$$f(x)=\frac{c}{\Gamma(\alpha)\beta^{\alpha}}g(x,\varphi)G^{c-1}(x,\varphi)\left(-\log(1-G^c(x,\varphi))\right)^{\alpha-1}\left(1-G^c(x,\varphi)\right)^{\frac{1}{\beta}-1} \quad (5.3)$$

In the exponentiated {T-X}family there are two sub-families. In the first sub-family, the Tdistributions are changed but the Xdistribution is the identical. In the other sub-family$X$distributions are changed but the T-distribution is similar. For different tdistributions the exponentiated {T-X}families are listed in table IV.

TABLE IV Some members of exponentiated T-X family with different t-distributions.

Name	The density $r(t)$	The density $g(x)$ of the T-X random variable
Exponential	$\beta e^{-\beta r}$	$c\beta g(x,\varphi)G^{c-1}(x,\varphi)\left(1-G^c(x,\varphi)\right)^{\beta-1}$

Beta exponential	$\frac{\lambda e^{-\lambda\beta x}\left(1-e^{-\lambda x}\right)^{\alpha-1}}{\beta(\alpha,\beta)}$	$\frac{c\lambda}{B(\alpha,\beta)}g(x,\varphi)G^{c-1}(x,\varphi)\left(1-G^{c}(x,\varphi)\right)^{\lambda\beta-1}\left[1-\left(1-G^{c}(x,\varphi)\right)^{\lambda}\right]^{\alpha-1}$
Exponentiated exponential	$\alpha\lambda\left(1-e^{-\lambda x}\right)^{\alpha-1}e^{-\lambda x}$	$c\alpha\lambda g\,(x,\varphi)G^{c-1}(x,\varphi)\left(1-(1-G^{c}(x,\varphi))^{\lambda}\right)^{\alpha-1}\left(1-G^{c}(x,\varphi)\right)^{\lambda-1}$
Gamma	$\frac{1}{\Gamma(\alpha)}\beta^{\alpha}r^{\alpha-1}e^{-r/\beta}$	$\frac{1}{\Gamma(\alpha)}\beta^{\alpha}g\,(x,\varphi)G^{c-1}(x)((1-G^{c}(x,\varphi))^{\frac{1}{\beta}-1}\left(-log\left(1-G^{c}(x,\varphi)\right)^{\alpha-1}\right.$
Half normal	$\frac{1}{\sigma}\left(\frac{2}{\pi}\right)^{\frac{1}{2}}e^{-r^{2}/2\sigma 2}$	$c/\sigma\left(\frac{2}{\pi}\right)^{\frac{1}{2}}\frac{g(x,\varphi)G^{c-1}(x,\varphi)}{(1-G^{c}(x,\varphi))}\exp\left(\frac{-\{\log(1-G^{c}(x,\varphi))\}^{2}}{2\sigma^{2}}\right)$
Levy	$\left(\frac{\Upsilon}{2\pi}\right)^{\frac{1}{2}}\frac{e^{-\frac{\Upsilon}{2t}}}{r^{3/2}}$	$c\left(\frac{\Upsilon}{2\pi}\right)^{\frac{1}{2}}\frac{g(x,\varphi)G^{c-1}(x,\varphi)}{(1-G(x,\varphi))}\frac{\exp(-(\frac{\Upsilon}{2}(-\log(1-G^{c}(x,\varphi))}{(-\log(1-G^{c}(x,\varphi))^{3/2}}$
Log logistic	$\frac{\beta\left(\frac{r}{\alpha}\right)^{\beta-1}}{\alpha\left(1+\left(\frac{r}{\alpha}\right)^{\beta}\right)^{2}}$	$\frac{c\beta}{\alpha^{\beta}}\frac{g(x,\varphi)G^{c-1}(x,\varphi)}{(1-G^{c}(x,\varphi))}\frac{-log\left(1-G^{c}(x,\varphi)\right)^{\beta-1}}{\left(1+\left\{-log\left(1-\frac{G^{c}(x,\varphi)}{\alpha}\right)\right\}^{\beta}\right)^{2}}$
Rayleigh	$\frac{r}{\sigma 2}e^{-r2/2\sigma 2}$	$-\frac{c}{\sigma 2}\frac{g(x,\varphi)G^{c-1}(x,\varphi)}{(1-G^{c}(x,\varphi))}\log\left(1-G^{c}(x,\varphi)\right)\exp\left(\frac{-\{log(1-G^{c}(x,\varphi))\}^{2}}{2\sigma^{2}}\right)$
Type-2 Gumbel	$\frac{\alpha\beta e^{-\beta r-\alpha}}{r^{\alpha+1}}$	$c\alpha\beta\frac{g(x,\varphi)G^{c-1}(x,\varphi)}{(1-G^{c}(x,\varphi))}\frac{exp\left(-\beta\{-log\left(1-G^{c}(x,\varphi)\right)\}^{-\alpha}\right)}{\{-log\left(1-G^{c}(x,\varphi)\right)\}^{\alpha+1}}$
Lomax	$\frac{\lambda k}{(1+\lambda r)^{k+1}}$	$ck\lambda\frac{g(x,\varphi)G^{c-1}(x,\varphi)}{(1-G^{c}(x,\varphi))}\left(1-\lambda log\left(1-G^{c}(x,\varphi)\right)^{-k-1}\right.$
Inverted beta	$\frac{r^{\beta-1}(1+r)^{-\beta-\gamma}}{B(\beta,\gamma)}$	$\frac{c}{B(\beta,\gamma)}\frac{g(x,\varphi)G^{c-1}(x,\varphi)}{(1-G^{c}(x,\varphi))}\frac{(-log(1-G^{c}(x,\varphi)))^{\beta-1}}{\left(\{1-log\left(1-G^{c}(x,\varphi)\right)\}^{\beta}\right)^{2}}$
Burr	$\frac{\alpha k r^{\alpha-1}}{(1+r^{\alpha})^{k+1}}$	$c\alpha k\frac{g(x,\varphi)G^{c-1}(x,\varphi)}{(1-G^{c}(x,\varphi))}\frac{(-log(1-G^{c}(x,\varphi)))^{\alpha-1}}{(1+\{1-log\left(1-G^{c}(x,\varphi)\right)\}^{\alpha})^{k+1}}$
Weibull	$\frac{\alpha}{\gamma}\left(\frac{r}{\gamma}\right)^{\alpha-1}e^{-\left(\frac{r}{\gamma}\right)^{\alpha}}$	$\frac{c\alpha}{\gamma}\frac{g(x,\varphi)G^{c-1}(x,\varphi)\left\{1-log\left(1-G^{c}(x,\varphi)\right)^{\frac{1}{\gamma}}\right\}^{\alpha-1}\exp\left(-\left\{1-log\left(1-G^{c}(x,\right.\right.\right.}{1-G^{c}(x,\varphi)}$

The CDF of the exponentiated weibull-exponential distribution is classified by

$$G(s)=1-\exp\left[-\left(-\frac{log\{1-(1-e^{-s})^{c}\}}{\Upsilon}\right)^{\alpha}\right] \tag{5.4}$$

The corresponding PDF of the exponentiated weibull-exponential distribution is specified by

$$g(s)=\frac{c\alpha}{\gamma}\frac{e^{-s}(1-e^{-s})^{c-1}}{1-(1-e^{-s})^{c}}\left(-\frac{log\{1-s^{c}\}}{\Upsilon}\right)^{\alpha-1}\exp\left[-\left(-\frac{log\{1-(1-e^{-s})^{c}\}}{\Upsilon}\right)^{\alpha}\right] \tag{5.5}$$

Alzaghal et al. [19] introduced the exponentiated T-Xfamily. For the applicability of the above distribution three real data sets are used.

The four parameter exponentiated weibull exponential distributions are proposed by Elgarhy et al. [48] the cumulative density function of EWE distribution takes the following sort

$$F(t,\psi) = \left[1 - e^{\left(-\alpha\left(e^{\lambda t}-1\right)^{\beta}\right)}\right]^{\alpha} \quad (a,\alpha,\lambda,\beta\ ) > 0 \tag{5.6}$$

In comparison study the EWE distribution provided a better fitting distribution to several other lifetime distributions.

The exponentiated weibull-generated family is introduced by Mudholkar et al. [28]. The cumulative distribution function of exponentiated weibull generated family is given by

$$F(x) = [1 - e(^{-\alpha\left[\frac{G(x)}{1-G(x)}\right]^{\beta}})]^{\alpha}. \tag{5.7}$$

A new four parameter model is originated in this article as a competitive extension for the weibull distribution using the EW-G distribution.

The generalized exponential geometric density function, and using $U[G(x)] = -\log[1 - \mathrm{G}(y,\varphi)$ the CDF of a new family is defined by

$$F(x) = \int_{\alpha}^{-\log[1-G(y,\varphi)],} \frac{\alpha\lambda(1-p)\mathrm{e}^{-\lambda t}[1-\mathrm{e}^{-\lambda t}]^{\alpha-1}}{[1-p\mathrm{e}^{-\lambda t}]^{\alpha+1}}dt. \tag{5.8}$$

The density function of the corresponding equation can be obtained by differentiating (5.8). Some special EMO distributions such as the gamma, normal, Fréchet, Gumbel and beta baselines respectively are defined by Marshall and Olkin [35]. To check the potentiality of some EMOG family is by fitting two real data sets.

A record of papers on the exponentiated -x family is presented in Table IV.

TABLE IV Work on the Exponentiated-X family of distributions.

S.No	Distributions	Author(s) (Year	U[F(y)]
1	Exponentiated weibull-x	[Alzaghal et al. 2013]	$-\log[1-\mathrm{F}^{\alpha}(y,\varphi)]$
2	Exponentiated half-logistic-G	[Cordeiro et al. 2014]	$-\log[1-\mathrm{F}(y,\varphi)]$
3	Exponentiated (t-x) family of distribution	[Alzaghal Famoye and Lee. 2013]	$-\log[1-\mathrm{F}^{\alpha}(y,\varphi)]$
4	Exponentiated-G {Kw-G type 2}	[Cordeiro et al. 2013]	$\mathrm{F}(y,\varphi)$
5	Exponentiated gamma-x	[Alzaghal et al. 2013]	$-\log[1-\mathrm{F}^{\alpha}(y,\varphi)]$
6	New exponentiated (t-x) class of distributions	[Ahmad et al. 2019]	$\frac{-\log[1-\mathrm{F}(y,\varphi)^{\alpha}]}{1-\mathrm{F}(y,\varphi)^{\alpha}}$
7	Arcsine exponentiated-x family	[Ahmad et al. 2020]	$\mathrm{F}(y,\varphi)^{\alpha}$
8	Exponentiated Marshall Olkin family of distributions	[Cicero et al. 2016]	$-\log[1-\mathrm{F}(y,\varphi)]$
9	Exponentiated exponential weibull distribution	[Dawlah Al-Sulami. 2020]	$-\log[1-\mathrm{F}(y,\varphi)]$
10	Exponentiated generalized transformed-transformer family of distribution	[Nasiru et al. 2017]	$-\log[1-\mathrm{F}^{\alpha}(y,\varphi)]$
11	New technique for generating distributions Alpha power transformation and exponentiated t-x distributions family	[Klakattawai and Aljuhani 2021]	$-\log[-\mathrm{F}^{\alpha}(y,\varphi)]$
12	Exponentiated gumbel exponential distribution	[Uwadi et al. 2019]	$\log\left(\frac{\mathrm{F}(y,\varphi)}{1-\mathrm{F}(y,\varphi)}\right)$
13	Exponentiated frechet-G of distributions	[Baharith et al. 2021]	$-\log[1-\mathrm{F}(y,\varphi)]$

6 On Generalized Classes of Distributions Using T-X Family Framework

Some generalized new classes of exponential distribution introduced by Zubaira et al. [34] are called t-exponential {y} class by using the quantile functions of renowned distributions. Some generalized families of this class are investigated and derived.

The T-R{Y} Family

The CDF of this class of distribution is given by

$$G_X(x) = \int_{\alpha}^{Q_Y(F_R(x,\varphi))} f_T(t)\,dt = F_T\left(Q_Y\left(F_R(x,\varphi)\right)\right). \tag{6.1}$$

PDF can be obtained by differentiating (6.1).

The T-Exponential [Y] Class

The t-exponential [y] class of distributions is defined as

$$G_X(x) = \int_{\alpha}^{Q_Y(1-e^{-\frac{x}{\beta}})} f_T(t)\,dt = F_T\left(Q_Y[1-e^{-\frac{x}{\beta}}]\right). \tag{6.2}$$

The PDF corresponding to Eq. (6.2) can be expressed as

$$f_x(x) = \frac{1}{\beta} e^{-\frac{x}{\beta}} \frac{f_T\left(Q_Y[1-e^{-\frac{x}{\beta}}]\right)}{f_Y\left(Q_Y[1-e^{-\frac{x}{\beta}}]\right)}$$

The T-E (Log-Logistic) Family

Let the random variable $T \in (0,\infty)$. by using the quantile function (c) in $QY(p) = \alpha\left(\frac{p}{1-p}\right)^{1/b}$ the CDF and PDF follow from Eq. (6.1) and Eq. (6.2) as

$$F_X(x) = F_T\left(\alpha\left[e^{\frac{x}{\beta}}-1\right]^{\frac{1}{b}}\right)$$

And

$$f(x) = \frac{\alpha e^{\frac{x}{\beta}}}{b\beta}\left[e^{\frac{x}{\beta}}-1\right]^{\frac{1}{b}-1} f_T\left(\alpha\left[e^{\frac{x}{\beta}}-1\right]^{\frac{1}{b}}\right)$$

The T-E {Extreme Value} Family

Let the random variable $T \in (-\infty,\infty)$: by using the quantile function

$Q_Y(p) = a + b\log[-\log(1-p)],\ b > 0$, the CDF follow from Eq (6.1) as

$$F_X(x) = F_T\left(a + b\log\left(\frac{x}{\beta}\right)\right).$$

A recent class of the t-exponential [y] family and generalized exponential family is defined and study. Some sub models of t-exponential {y} family are studied in some aspect. These models show the great flexibility of t-exponential {y} family in terms of the shapes of the density and hazard rate functions.

Generalized Transmuted Family of Distributions

A new generator of distributions which is known as generalized transmuted family is introduced by Alizadehy et al.[49]. The CDF of the generalized transmuted family is specified by

$$F(y,\beta,\alpha,\varphi) = \int_0^{[G(y,\varphi)]^{\alpha}} (1+\beta-2\beta t)\,dt.$$

By differentiating the above equation we get the corresponding PDF.

They introduced and study the generalized transmuted- G family; they demonstrate the importance of the new family with an application to real data set.

A new family which is named as modified {T-X} family is proposed by Aslam et al. [32]. The estimation studies are conducted and for the goodness of fit criteria compare the distribution with five existing distributions.

The CDF of the proposed family of distribution is given below

$$F(y;\, \varphi) = \int_0^{G(y;\, \varphi)} \{(1 - \lambda)h_1(t;\, \varphi) + \lambda h_2(t;\, \varphi)\}dt.$$

The characteristics are mathematically verified through simulation.

New class of continuous distribution is called the Kumaraswamy odd log logistic-G family is proposed by Alizadeh et al. [50] by taking

$$U[G(x)] = \frac{G(x;\, \varphi)^{\alpha}}{G(x;\, \varphi)^{\alpha} + \bar{G}(x;\, \varphi)^{\alpha}} \quad \text{and} \quad r(t) = abt^{\alpha-1}(1-t^{\alpha})^{b-1}.$$

Its CDF is given by

$$F(x) = \int_0^{\frac{G(x;\, \varphi)^{\alpha}}{G(x;\, \varphi)^{\alpha} + \bar{G}(x;\, \varphi)^{\alpha}}} abt^{\alpha-1}(1-t^{\alpha})^{b-1}dt.$$

For each baseline G distribution provide more flexibility to the extended model with the addition of three extra shape parameters. The significance of this family is demonstrated by means of two applications to real data.

New family is called the transmuted geometric-G family which extends the transmuted family initiated by Shaw et al. [51]. They build a new generator by using

$$\mathrm{U[G(y)]} = \frac{\Theta \mathrm{G}(y,\varphi)}{1+(\Theta-1)G(y,\varphi)} \text{ and} p(t) = 1 + \lambda - 2\lambda t.$$

Then, the TG-G family CDF is given by

$$F(y) = \int_0^{\frac{\Theta \mathrm{G}(y,\varphi)}{1+(\Theta-1)G(y,\varphi)}} (1+\lambda-2\lambda t)dt \;= \frac{\Theta \mathrm{G}(y,\varphi)}{1+(\Theta-1)G(y,\varphi)}\left[1 + \frac{\lambda \bar{\mathrm{G}}(y,\varphi)}{1+(\Theta-1)G(y,\varphi)}\right].$$

They presented a transmuted geometric-G family of distributions by the addition of parameter to the transmuted family.

7 On Generating T-X Family of Distribution Using Quantile Functions

By using quantile function Q_Y, T-X[Y] family is defined by Aljarrah et al. [19]. The CDF is given as below

$$\mathrm{G(s)} = \int_0^{\mathrm{Q_Y(F(s,\varphi))}} \mathrm{r(t)dt} \quad = \mathrm{R[Q_Y(F(s,\varphi))]}. \tag{7.1}$$

The corresponding PDF is

$$g(s) = \frac{f(s,\varphi)}{P\{\mathrm{Q_Y(F(s,\varphi))}\}} r[\mathrm{Q_Y(F(s,\varphi)}]. \tag{7.2}$$

New method is introduced b Alzaatreh et al. [9] for generating the T-X (U) families.

8 Future Directions of Research

There are several approaches to extend the {T-X} family of distributions by using different weight function some of these are as follow.

By using Log Dagum approach we can extend the Pareto, Logistic, Frechet, Rayleigh, Weibull Rayleigh distributions. One can find the Log Dagum Pareto distribution by considering

The CDF of Log Dagum distribution is given as below

$$\pi(t) = (1 + e^{-\lambda x})^{-\beta} t \in R, \quad \beta > 0, \quad \lambda > 0 \tag{8.1}$$

and their corresponding PDF is:

$$r(t) = \beta\lambda e^{-\lambda x}(1 + e^{-\lambda x})^{-\beta-1}.\, t \in R, \beta > 0, \quad \lambda > 0 \tag{8.2}$$

Let $G(y)$ and $\vec{G}(y) = 1 - G(y, \varphi)$ be the baseline CDF and survival function by replacing U[G(y)] by $\log(\frac{G(y,\varphi)}{1-G(y,\varphi)})$ and $r(t)$ with (8.2) in equation (3.5) we characterize the CDF of the Log Dagum-x family by

$$F(y) = \left[1 + \left(\frac{G(y,\varphi)}{1-G(y,\varphi)}\right)^{-\lambda}\right]^{-\beta}. \tag{8.3}$$

The log Dagum family PDF is expressed as

$$f(y) = \left[1 + \left(\frac{G(y,\varphi)}{1-G(y,\varphi)}\right)^{-\lambda}\right]^{-\beta-1} \left(\frac{G(y,\varphi)}{1-G(y,\varphi)}\right)^{-\lambda-1} \frac{\lambda\beta g(y,\varphi)}{[1-G(y,\varphi)]^2}. \tag{8.4}$$

In practice the fundamental incentives for using the log dagum-x family are to assemble heavy tailed distributions for modeling real data. The CDF of the Pareto distribution is

$$F(y) = 1 - \left(\frac{l}{y}\right)^{\alpha}.$$

The CDF of the corresponding distribution is obtained as

$$F(y) = \left[1 + \left(\left(\frac{l}{y}\right)^{-\alpha} - 1\right)^{-\lambda}\right]^{-\beta}.$$

PDF of log Dagum Pareto distribution can be obtained by differentiating the CDF.

$$f(y) = \frac{(\alpha\lambda\beta)}{y^2}\left[1 + \left(\left(\frac{l}{y}\right)^{-\alpha} - 1\right)^{-\lambda}\right]^{-\beta-1} \left(\left(\frac{l}{y}\right)^{-\alpha} - 1\right)^{-\lambda-1} \left(\frac{l}{y}\right)^{-\alpha-1}.$$

Survival function

$$S(y) = 1 - F(y)$$

$$S(y) = 1 - \left[1 + \left(\left(\frac{l}{y}\right)^{-\alpha} - 1\right)^{-\lambda}\right]^{-\beta}.$$

Hazard Function

$$H(y) = \frac{f(y)}{s(y)}$$

$$H(y) = \frac{\frac{(\alpha\lambda\beta)}{y^2}\left[1 + \left(\left(\frac{l}{y}\right)^{-\alpha} - 1\right)^{-\lambda}\right]^{-\beta-1} \left(\left(\frac{l}{y}\right)^{-\alpha} - 1\right)^{-\lambda-1} \left(\frac{l}{y}\right)^{-\alpha-1}}{1 - \left[1 + \left(\left(\frac{l}{y}\right)^{-\alpha} - 1\right)^{-\lambda}\right]^{-\beta}}$$

Logistic Distribution

The CDF of logistic distribution is $F(y) = (1 + e^{-y})^{-1}$

By using baseline function (6) the CDF of the log dagum logistic distribution is

$$F(y) = [1 - (e^{-y} + e^{-2y})^{\lambda}]^{-\beta}.$$

PDF of log dagum logistic distribution can be obtained by differentiating the CDF.

$$f(y) = \beta\lambda[1 - (e^{-y} + e^{-2y})^{\lambda}]^{-\beta-1}(e^{-y} + e^{-2y})^{\lambda-1}](-e^{-y} - 2e^{-2y})$$

Survival function

$$S(y) = 1 - F(y)$$

$$S(y) = 1 - \left[1 - \left(e^{-y} + e^{-2y}\right)^{\lambda}\right]^{-\beta}.$$

Hazard Function

$$H(y) = \frac{f(y)}{s(y)}$$

$$H(y) = \frac{\beta\lambda\left[1 - \left(e^{-y} + e^{-2y}\right)^{\lambda}\right]^{-\beta-1} . \left(e^{-y} + e^{-2y}\right)^{\lambda-1}\right]\left(-e^{-y} - 2e^{-2y}\right)}{1 - \left[1 - \left(e^{-y} + e^{-2y}\right)^{\lambda}\right]^{-\beta}}$$

Frechet Distribution

The CDF of Frechet distribution is $F(y) = e^{-\left(\frac{\beta}{y}\right)^{\alpha}}$.

CDF of Log Dagum Frechet distribution can be obtain by using equation (6)

$$F(y) = \left[1 + \left(e^{-\left(\frac{\beta}{y}\right)^{\alpha}} - 1\right)^{\lambda}\right]^{-\beta}.$$

PDF of log dagum logistic distribution can be obtained by differentiating the CDF.

$$f(y) = \frac{\alpha\beta^2\lambda}{y^2}\left[1 + \left(e^{-\left(\frac{\beta}{y}\right)^{\alpha}} - 1\right)^{\lambda}\right]^{-\beta-1}\left(e^{-\left(\frac{\beta}{y}\right)^{\alpha}} - 1\right)^{\lambda-1} e^{-\left(\frac{\beta}{y}\right)^{\alpha}}\left(\frac{\beta}{y}\right)^{\alpha-1}.$$

Survival function

$$S(y) = 1 - F(y)$$

$$S(y) = 1 - \left[1 + \left(e^{-\left(\frac{\beta}{y}\right)^{\alpha}} - 1\right)^{\lambda}\right]^{-\beta}.$$

Hazard Function

$$H(y) = \frac{f(y)}{s(y)}$$

$$\mathrm{H(y)} = \frac{\frac{\alpha\beta^2\lambda}{y^2}\left[1 + \left(e^{-\left(\frac{\beta}{y}\right)^{\alpha}} - 1\right)^{\lambda}\right]^{-\beta-1}\left(e^{-\left(\frac{\beta}{y}\right)^{\alpha}} - 1\right)^{\lambda-1} e^{-\left(\frac{\beta}{y}\right)^{\alpha}}\left(\frac{\beta}{y}\right)^{\alpha-1}}{1 - \left[1 + \left(e^{-\left(\frac{\beta}{y}\right)^{\alpha}} - 1\right)^{\lambda}\right]^{-\beta}}.$$

Lomax Distribution

The CDF of Lomax distribution is $F(y) = 1 - \left[1 + \left(\frac{y}{\lambda}\right)\right]^{-\alpha}$.

CDF of log dagum Lomax distribution can be obtain by using equation (6).

$$F(y) = \left[1 + \left[\left(1 + \frac{y}{\lambda}\right)^{\alpha} - 1\right]^{-\lambda}\right]^{-\beta}.$$

PDF of log dagum Lomax distribution can be obtained by differentiating the above equation.

$$f(y)=\ \alpha\beta\left[1+\left[\left(1+\frac{y}{\lambda}\right)^{\alpha}-1\right]^{-\lambda}\right]^{-\beta-1}\left[\left(1+\frac{y}{\lambda}\right)^{\alpha}-1\right]^{-\lambda-1}\left(1+\frac{y}{\lambda}\right)^{\alpha-1}.$$

Survival function

$$S(y)=1-F(y)$$

$$S(y)=1-\left[1+\left[\left(1+\frac{y}{\lambda}\right)^{\alpha}-1\right]^{-\lambda}\right]^{-\beta}.$$

Hazard Function

$$H(y)=\frac{f(y)}{s(y)}$$

$$H(x)=\frac{\alpha\beta\left[1+\left[\left(1+\frac{y}{\lambda}\right)^{\alpha}-1\right]^{-\lambda}\right]^{-\beta-1}\left[\left(1+\frac{y}{\lambda}\right)^{\alpha}-1\right]^{-\lambda-1}\left(1+\frac{y}{\lambda}\right)^{\alpha-1}}{1-\left[1+\left[\left(1+\frac{y}{\lambda}\right)^{\alpha}-1\right]^{-\lambda}\right]^{-\beta}.}$$

Weibull Rayleigh distribution

CDF of the Weibull Rayleigh distribution is

$$F(y)=1-e^{-\alpha\left(e^{\frac{\Theta}{2}y^{2}}-1\right)^{\beta}}.$$

The CDF of Log Dagum Weibull Rayleigh distribution is

$$F(y)=\left[1+\left(e^{\alpha\left(e^{\frac{\Theta}{2}y^{2}}-1\right)^{\beta}}-1\right)^{-\lambda}\right]^{-\beta}.$$

PDF of log dagum weibull Rayleigh distribution can be obtain by using equation (6).

$$f(y)=\alpha\beta^{2}\lambda\Theta y\left[1+\left(e^{\alpha\left(e^{\frac{\Theta}{2}y^{2}}-1\right)^{\beta}}-1\right)^{-\lambda}\right]^{-\beta-1}\left(e^{\alpha\left(e^{\frac{\Theta}{2}y^{2}}-1\right)^{\beta}}-1\right)^{-\lambda-1}e^{\alpha\left(e^{\frac{\Theta}{2}y^{2}}-1\right)^{\beta}}\left(e^{\frac{\Theta}{2}y^{2}}-1\right)^{\beta-1}e^{\frac{\Theta}{2}y^{2}}$$

Survival function

$$S(y)=1-F(y)$$

$$S(y)=1-\left[1+\left(e^{\alpha\left(e^{\frac{\Theta}{2}y^{2}}-1\right)^{\beta}}-1\right)^{-\lambda}\right]^{-\beta}.$$

Hazard Function

$$H(y)=\frac{f(y)}{s(y)}$$

$$H(x)=\frac{\alpha\beta^2\lambda\Theta y\left[1+\left(e^{\alpha\left(e^{\frac{\Theta}{2}y^2}-1\right)^{\beta}}-1\right)^{-\lambda}\right]^{-\beta-1}\left(e^{\alpha\left(e^{\frac{\Theta}{2}y^2}-1\right)^{\beta}}-1\right)^{-\lambda-1}e^{\alpha\left(e^{\frac{\Theta}{2}y^2}-1\right)^{\beta}}\left(e^{\frac{\Theta}{2}y^2}-1\right)^{\beta-1}e^{\frac{\Theta}{2}y^2}}{\mathbf{1}-\left[1+\left(e^{\alpha\left(e^{\frac{\Theta}{2}y^2}-1\right)^{\beta}}-1\right)^{-\lambda}\right]^{-\beta}}.$$

For generating new class of {T-X} distributions one can classify a different upper limit$U[G(x)]$. Log Dagum Gompertz distribution Sine Burr family of distribution can also be formulated.

T-Transmuted X-Family of Distributions

In t-transmuted x-family of distributions, exponential distribution, exponentiated exponential distribution, beta exponential distribution, gamma distribution, half normal distribution, Levy distribution, log logistic distribution, Rayleigh distribution, Lomax distribution, Weibull distribution can be developed for modeling of data. One can also develop exponential transmuted logistic and exponential transmuted frechet distribution.

Exponentiated (T-X) Families of Distributions

In exponentiated {T-X} family of distribution, exponentiated frechet exponential distribution, exponentiated frechet Lomax distribution, exponentiated frechet Rayleigh distribution and exponentiated frechet gompertz distribution for modeling of data can be developed. Generalized classes of exponential distribution can also be developed.

On generating {T-X} family of distributions using quantile functions Rayleigh, exponential and frechet distributions can be expanded as far as we gathered, no studies have been carried out in this region.

9 Concluding Remarks

In this article we discussed the brief review of {T-X} family, t transmuted -x family, exponentiated {T-X}family and generalized family of distributions. The extensions of new models are attracted to several statisticians to expand the new models. In probability and mathematical statistics, the study of applications and construction of {T-X} family of distributions and their other related categories are one of the active fields of research. In recent years various research papers containing theory about the {T-X} family of distributions have been published. The main objective of the proposed model is giving a variety of some classes to check the flexibility of the models with the data. The addition of parameter might be helpful for the generated phenomenon from real life data sets. We recommend to the learners and practitioners to contrast models and demonstrate the usefulness of old and new class of distribution. By numerous authors, exhaustive studies on the structure of various {T-X} families of distributions are confabulated. This area is a dynamic field of research and new models are frequently being discovered.

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