M/g/c/c State Dependent Queueing Model for Road Traffic Simulation

Nacira Guerrouahane[1,*], *Djamil Aissani*[1], *Louiza Bouallouche-Medjkoune*[1] *and Nadir Farhi*[2]

[1] LaMOS Research Unit, Faculty of Exact Sciences, University of Bejaia, 06000 Bejaia, Algeria.
[2] University Paris-Est, IFSTTAR, GRETTIA, F-77447, Marne-la-Vallée, France.

Abstract: In this paper, we present a stochastic queuing model for the road traffic, which captures the stationary density-flow relationships in both uncongested and congestion conditions. The proposed model is based on the $M/g/c/c$ state dependent queuing model of Jain and Smith, and is inspired from the deterministic Godunov scheme for the road traffic simulation. We first propose a reformulation of the $M/g/c/c$ state dependent model that works with density-flow fundamental diagrams rather than density-speed relationships. We then extend this model in order to consider upstream traffic demand as well as downstream traffic supply. Finally we calculate the speed and travel time distributions for the $M/g/c/c$ state dependent queuing model and for the proposed model and we derive stationary performance measures (expected number of cars, blocking probability, expected travel time and throughput). A comparison with results predicted by the $M/g/c/c$ state dependent queuing model shows that the proposed model correctly represents the dynamic of traffic and gives a good performances measures. The results illustrate the good accuracy of the proposed model.

Keywords: Traffic flow modeling, finite queuing systems, state dependent queue, simulation.

1 Introduction

Traffic flow on freeways is a complex process with many interacting components and random perturbations such as traffic jams, stop-and-go waves, hysteresis phenomena, etc. These perturbations propagate from upstream to downstream sections. During traffic jams, drivers are slowing down when they observe traffic congestion in the downstream section, causing upstream propagation of a traffic density perturbation.

Models for flow on a link have developed from the fundamental diagram, where flow is a function of density via the macroscopic LWR first-order continuum model (i.e., Lighthill-Whitham-Richards theory of kinematic waves) [17,20]. In this paper, we propose a stochastic traffic model based on the queuing model of [21] and on the Godunov scheme [12,16] of the LWR traffic model. We calculate a stationary probability distribution of the $M/g/c/c$ state dependent queuing model [15,21] on a road section, by considering density-flow fundamental diagrams rather than density-speed ones [11]. The model suppose a triangular fundamental diagram which correctly captures the stationary density-flow relationships in both uncongested and congestion

conditions. In this, we use the functions of traffic demand and supply for the section, and derive a model for a road with a downstream supply, we present stationary performance measures (expected travel time, throughput, etc.), and we derive a distributions of speed and travel time. The model we propose here can also be used to the analysis of travel times through road traffic [7,8,9].

The remainder of this paper is organized as follows. In Section 2, we first present a review of the existing works on literature. In this regards, we present a short review of the $M/g/c/c$ state dependent queuing model of Jain and Smith. In Section 3, we rewrite the $M/g/c/c$ state dependent queuing model on a road section by considering density-flow fundamental diagrams rather than density-speed ones (triangular fundamental diagram). In Section 4, we consider the traffic demand and supply functions for the section, and derive a model for a road with a downstream supply. We derive some performance measures (expected number of cars, blocking probability, expected travel time and throughput) and we compared it by $M/g/c/c$ state dependent queuing model . In Section 5 , we calculate the speed and expected

* Corresponding author e-mail: naciraro@hotmail.fr

travel time distributions, and we present our simulation results. In Section 6, we briefly summarize our findings.

2 Literature review

The dynamics of traffic flows in road networks is complex, and is subject to stochastic disturbances. Congested networks involve complex traffic interactions. Providing an analytical description of these intricate interactions is challenging. The study of network congestion is of interest in various fields, ranging from the analysis of spillbacks (i.e. the backwards propagation of congestion) in urban traffic or pedestrian traffic [15, 21].

In the following, we focus on the LWR first order model (Lighthill-Whitham-Richards theory of kinematic waves) [17, 20], for which numerical schemes have been performed since decades [12]. The model is more recently developed [6, 16]. There has been a recent interest in the development of stochastic link models. Most studies have considered stochastic cell-transmission models (CTM) [6], where traffic demand and supply functions are used.

In [19], the authors proposed a stochastic formulation of the link-transmission model, which is an operational instance of Newells theory of kinematic waves [18]. The kinematic wave model (KWM) is more recently developed [22]. In [3], the compositional stochastic model extends the cell transmission model [6] by defining sending (demand) and receiving (supply) functions explicitly as random variables. Several simulation models based on queueing theory have been developed, but few studies have explored the potential of the queueing theory framework to develop analytical traffic models. In [19], the authors proposed an analytical stationary model, which is directly derived from the KWM. A review of stationary queueing models for highway traffic and exact analytical stationary queueing models of signalized and unsignalized intersections are given by several authors [14, 24, 25]. In [10, 13], the authors contributed to the study of signalized intersections and presented a unifying approach to both signalized and unsignalized intersections. These approaches resort to infinite capacity queues, and thus fail to account for the occurrence of breakdown and their effects on upstream links. Calculus of traffic flow breakdown probability remains an important issue when analyzing the stability and reliability of transportation system [2, 26]. Finite capacity queueing network model (FCQN) are of interest for a variety of applications such as the study of manufacturing networks, circulation systems and prison networks [21], etc. FCQN model allows to account for finite lengths, which enables the modeling and analysis of breakdowns. The methods proposed in [1, 15, 21] resort to finite capacity queueing theory and derive stationary performance measures. In [4], the authors describe a

methodology for approximate analysis of open state dependent $M/g/c/c$ queueing networks. The evacuation problem was analyzed using $M/g/c/c$ state dependent queuing networks [5, 27] when an algorithm was proposed to optimize the stairwell case and increase evacuation times towards the upper stories. In [11], the authors proposed a reformulation of the linear case of $M/g/c/c$ state queueing model [15], which uses the density-flow fundamental diagrams and consider upstream traffic demand and downstream traffic supply.

3 Review on M/g/c/c state dependent queuing model

In this section, we present a review on the $M/g/c/c$ state dependent queuing model of Jain and Smith. This model was used to model pedestrian and vehicular traffic flow [15, 21].

A link of a road network is modeled with c servers set in parallel, where c is the link capacity (the maximum number of occupants in the link). The model assumed that the average speed v_n depends on the number of occupants n on the road, according to a non-increasing density-speed relationship.

In accordance to Tregenza's empirical studies [23], the average speed that an occupant will move through a link depends on several factors but mainly is a function of the number of occupants in the link. Based on these studies, linear and exponential congestion models are developed for the average pedestrian/vehicles speed in traffic links [15, 27].

The linear congestion model is based on the idea that the service rate is a linear function of the number of occupants in the link and is given as follows.

$$v_n = v_f \frac{(c-n+1)}{c},$$ (1)

The exponential congestion model is based on the idea that the service rate is related to the number of occupants by an exponential function and is given as follows.

$$v_n = v_f \exp\left[-(\frac{n-1}{\beta})^\gamma\right].$$ (2)

in which β and γ are shape and scale parameters respectively. Parameters β and γ are found by fitting points to the curve in Fig. 1. In fact, Fig.1 presents an approximation of empirical vehicular speed-density curves, based on various empirical studies [15]. Fitting the points $(1, v_f)$, (a, v_a) and (b, v_b) gives one the algebraic relationships shown below [27]:

$$\beta = \frac{a-1}{[\ln(v_f/v_a)]^{1/\gamma}} = \frac{b-1}{[\ln(v_f/v_b)]^{1/\gamma}},$$

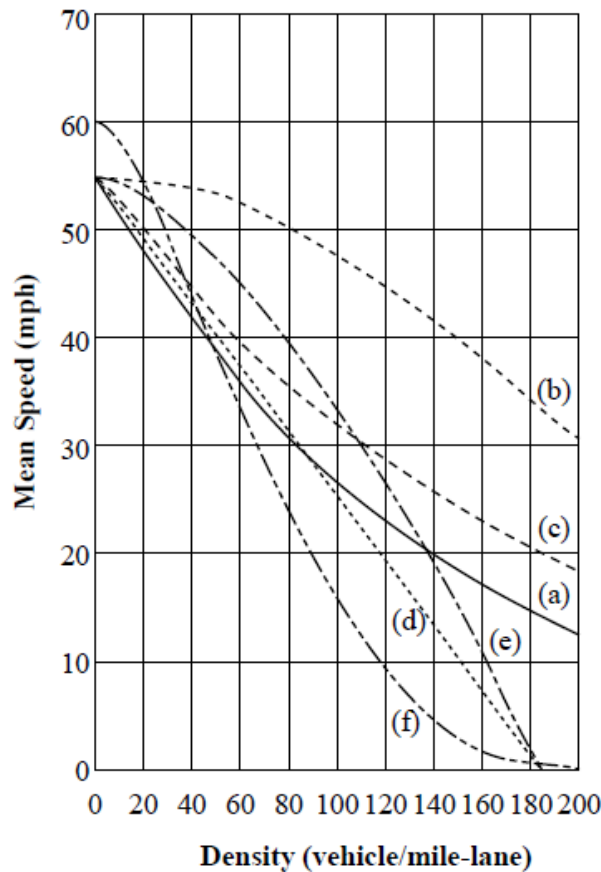$$\gamma = \ln\left[\frac{\ln(v_a/v_f)}{\ln(v_b/v_f)}\right] / \ln\left(\frac{a-1}{b-1}\right).$$

Fig. 1: Exponential approximation of empirical M/g/c/c state-dependent traffic flow model, (a) (Jain and Smith, 1997), and empirical distributions for vehicular traffic flows, (b) (Edie, 1961), (c) (Underwood, 1961), (d) (Greenshields, 1935), (e) (Drake et al., 1967), and (f) (Transportation Research Board, 2000) [15].

The values a and b are arbitrary points used to fit the exponential curve. In vehicular related applications [15], commonly used values are $a = 20Lk$ and $b = 140Lk$ corresponding to densities of 20 and 140 veh/mi-lane respectively. Looking at the curves presented in Fig. 1, reasonable values for such points are $v_a = 48$ miles per hour and $v_b = 20$ miles per hour.
L is the length of the link and k is its width (or number of lanes).
The arrival process of cars into the link is assumed to be Poisson with rate λ, while the service rate is general and depend on the number of occupants n on the link. A normalized service rate $f(n)$ is defined as the ratio of average speed to free speed, in order to capture congestion effects, and is taken $f(n) = v_n/v_f$, $0 \leq f(n) \leq 1$. In the linear case, we have $f(n) = (c-n+1)/c$. In the exponential case, we have

$$f(n) = \exp[-((n-1)/\beta)^\gamma].$$

The stationary probability distribution $P_n = P(N = n)$ of the number of occupants N in the $M/g/c/c$ state dependent model have been developed in [27] and shown in [21] to be stochastic equivalent to a pure Markovian $M/M/c/c$ queueing model. Then, these probabilities can be written as follows.

$$P_n = \frac{(\lambda L/v_f)^n}{\prod_{i=1}^n if(i)}P_0, \qquad n = 1,..,c.$$

$$P_0 = \left(1 + \sum_{n=1}^c \frac{(\lambda L/v_f)^n}{\prod_{i=1}^n if(i)}\right)^{-1}. \tag{3}$$

where L is the length of the link section and v_f is the speed corresponding to one occupant in the link (ie. the free speed).

From P_n, we can easily derive important performance measures.

–The blocking probability :
$P_c = P_0(\lambda L/v_f)^c / \prod_{i=1}^c if(i)$,
–The throughput : $\theta = \lambda(1 - P_c)$,
–The expected number of cars in the section :
$\bar{N} = \sum_{n=1}^c nP_n$,
–The expected service time : $W = \bar{N}/\theta$.

4 Model of road section

In this section, we slightly modify the $M/g/c/c$ state dependent queueing model of Jain and Smith, by defining the normalized service rate $f(n)$ as the ratio of the average flow (q_n) by the maximum flow (q_{max}), rather than the average speed (v_n) by the free speed (v_f). This modification will permit us to consider the demand and supply functions of a road section, and then to use them in the case where two or many sections are set in tandem. In the following, we present the $M/g/c/c$ state dependent queuing model on one road section, for which we consider a triangular fundamental traffic diagram [11].

$$Q(\rho) = \min(v_f\rho, w(\rho_j - \rho)). \tag{4}$$

The demand and the supply functions $\Delta(\rho)$ and $\Sigma(\rho)$ respectively are given as follows.

$$\Delta(\rho) = \min(v_f\rho, q_{max}), \tag{5}$$
$$\Sigma(\rho) = \min(q_{max}, w(\rho_j - \rho)). \tag{6}$$

where $q_{max} = \rho_j/(1/v_f + 1/w)$, and $\rho = n/L$.
$\rho, Q(\rho), v_f, w, \rho_j, L, q_{max}$ and n denote respectively the car-density in the road section, the car-flow, the free speed, the backward wave speed, the jam-density, the length of the road section, the maximum flow and the number of cars.

We define T as the service time, ie the time needed for a car to pass through one road section. Let us notice here that the service depends on both traffic demand and traffic supply, since we have here a state-dependent service model. The expected service time $E(T)$ depends on the number n of cars on the road and is given by $E(T) = L/v_n$, where $v_n = Q(\rho)/\rho$.

The expected service time $E(T)$ is then given as follows.

$$E(T) = \frac{n}{\min(v_f \frac{n}{L}, w(\frac{c-n}{L}))}.$$

The average service rate μ of one server (one car place) is then given as follows.

$$\mu = \frac{1}{E(T)} = \frac{v_n}{L} = \frac{\min(v_f \frac{n}{L}, w(\frac{c-n}{L}))}{n}.$$

The overall service rate q_n of the road section (the queueing system) with n cars is equivalent to the number of occupied servers multiplied by the rate of each server, and is nothing but the car-flow on the section, given by the fundamental diagram (4).

$$q_n = n\mu = \min\left(v_f \frac{n}{L}, w\left(\frac{c-n}{L}\right)\right).$$

The normalized service rate is then fixed to

$$f(n) = \frac{q_n}{q_{\max}} = \frac{\min(v_f \frac{n}{L}, w(\frac{c-n}{L}))}{q_{\max}}. \tag{7}$$

We have $0 \leq f(n) \leq 1$.

Stationary probabilities of the number of cars on the road section are derived by substituting the expression of q_n, into the Chapman-Kolmogorov equations for solving the probabilities of a single queue [21].

$$P_n = \frac{\lambda^n}{\prod_{i=1}^n \mu_i} P_0, \qquad n = 1, .., c.$$
$$P_0 = \left(1 + \sum_{n=1}^c \frac{\lambda^n}{\prod_{i=1}^n \mu_i}\right)^{-1}. \tag{8}$$

Then, the stationary probability distribution of the number of cars on the road section is given as follows [11]

$$P_n = \frac{(\lambda)^n (\frac{L}{v_f})^{n_{cr}} (\frac{L}{w})^{n-n_{cr}}}{\prod_{i=1}^{n_{cr}} i \prod_{i=n_{cr}+1}^n (c-i)} P_0, \qquad n = 1, .., c.$$
$$P_0 = \left(1 + \sum_{n=1}^c \frac{(\lambda)^n (\frac{L}{v_f})^{n_{cr}} (\frac{L}{w})^{n-n_{cr}}}{\prod_{i=1}^{n_{cr}} i \prod_{i=n_{cr}+1}^n (c-i)}\right)^{-1}. \tag{9}$$

where $n_{cr} = \rho_{cr} L$ is the number of cars corresponding to the critical car-density.

4.1 Model with a downstream supply

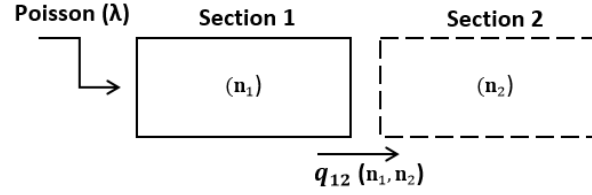We model here a road section with $M/g/c/c$ state dependent queuing model, as presented above, but we

Fig. 2: A road section with a downstream supply constraint.

consider that the service of section 1 is constrained by the supply flow of the downstream section (section 2), as in Fig. 2.

The model assume that the supply flow of the downstream section is stochastic, the stationary probability distribution of the number of cars in the downstream section is given and a triangular fundamental traffic diagrams for the two sections are given (See Fig. 3).
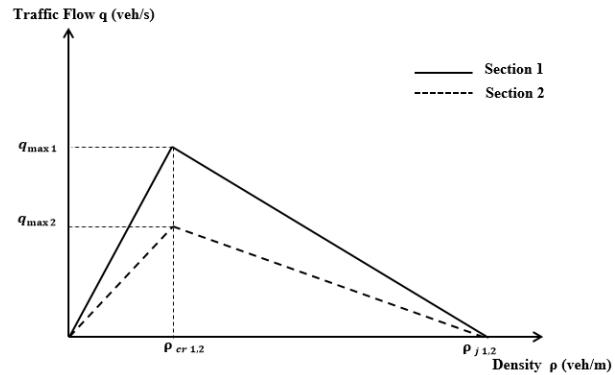

Fig. 3: Triangular fundamental diagrams.

The car-flow outgoing from section 1 and entering to section 2 is assumed to be given by the minimum between the traffic demand on section 1 and the traffic supply of section 2.

$$q_{12}(n_1, n_2) = \min(\Delta_1(\rho_1), \Sigma_2(\rho_2))$$
$$= \min\left(v_{f1} \frac{n_1}{L_1}, q_1^{\max}, q_2^{\max}, w_2\left(\frac{c_2-n_2}{L_2}\right)\right).$$

Therefore, the normalized service rate $f(n_1, n_2)$ of section 1 is given as follows.

$$f(n_1, n_2) = \frac{q_{12}}{q_1^{\max}} = \frac{\min\left(v_{f1} \frac{n_1}{L_1}, q_1^{\max}, q_2^{\max}, w_2(\frac{c_2-n_2}{L_2})\right)}{q_1^{\max}}. \tag{10}$$
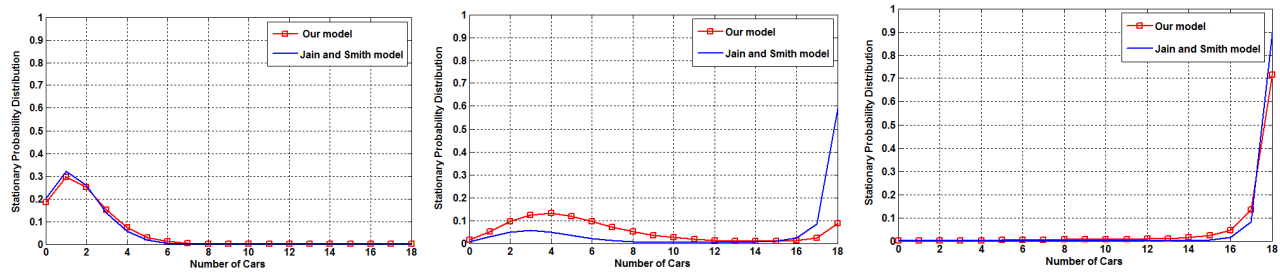
Fig. 4: Stationary probability distributions. $\lambda = 0.5$ veh/s, 1 veh/s and 1.5 veh/s respectively from left to right sides.
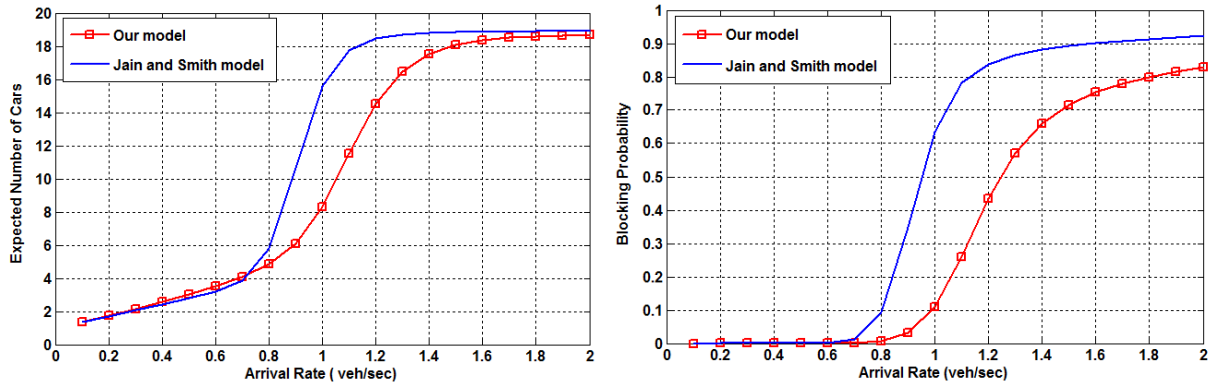

Fig. 5: Comparison of the expected number of cars, in left. Comparison of the blocking probability, in right. Our model with red color. Jain and Smith model with blue color.

1. The stationary probability distribution of the number of cars on section 2, is assumed fixed and given by (9), in function of θ, as follows.

$$P_{n_2}^{(2)}(\theta) = \frac{(\theta)^{n_2} (\frac{L_2}{v_{f2}})^{n_{cr2}} (\frac{L_2}{w_2})^{n_2 - n_{cr2}}}{\prod_{i=1}^{n_{cr2}} i \prod_{i=n_{cr1}+1}^{n_2} (c_2 - i)} P_0(\theta), \qquad n_2 = 1,..,c_2.$$

$$P_0^{(2)}(\theta) = \left(1 + \sum_{n_2=1}^{c_2} \frac{(\theta)^{n_2} (\frac{L_2}{v_{f2}})^{n_{cr2}} (\frac{L_2}{w_2})^{n_2 - n_{cr2}}}{\prod_{i=1}^{n_{cr2}} i \prod_{i=n_{cr1}+1}^{n_2} (c_2 - i)} \right)^{-1}.$$

(11)

2. In section 1, we have an $M/g/c_1/c_1$ system, parameterized by the traffic supply of the downstream section (the number of cars in section 2).

   – The stationary probability distribution of the number of cars on section 1, parameterized by the number of cars on section 2, is given as follows.

$$P_{n_1|n_2}(\lambda) = \frac{(\lambda/q_1^{\max})^{n_1}}{\prod_{i=1}^{n_1} f(i,n_2)} P_{0|n_2}(\lambda),$$

$$P_{0|n_2}(\lambda) = \left(1 + \sum_{n_1=1}^{c_1} \frac{(\lambda/q_1^{\max})^{n_1}}{\prod_{i=1}^{n_1} f(i,n_2)} \right)^{-1}.$$

(12)

– Then, the stationary probability distribution of the number of cars on section 1 is obtained as follows.

$$P_{n_1}^{(1)}(\lambda,\theta) = \sum_{n_2=0}^{c_2} P_{n_1|n_2}(\lambda) P_{n_2}^{(2)}(\theta), \qquad n_1 = 1,..,c_1.$$

$$P_0^{(1)}(\lambda,\theta) = \sum_{n_2=0}^{c_2} P_{0|n_2}(\lambda) P_{n_2}^{(2)}(\theta).$$

(13)

3. The average outflow from section 1, θ, is given as follows (using little's law).

$$\theta = \lambda \left(1 - P_{c_1}^{(1)}(\lambda,\theta) \right).$$

(14)

Using the data of Table 1, Fig. 4 compares the stationary probability distribution of the number of cars n on the road section for our model with downstream supply (formula 13), with red color and for the linear case of Jain and Smith model (formula 3), with blue color. The arrival rates λ is varied from one illustration to another ($\lambda = 0.5$ veh/s, 1 veh/s and 1.5 veh/s).

Fig. 4 shows that the number of cars n on the road section increases, in term of stationary probability, with the arrival rate λ. The difference between the two stationary distributions in the middle of Fig. 4 ($\lambda = 1$ veh/s) can be explained by using the flow ratio rather than the speed ratio as a measure of the service rate. Moreover, in the case of Jain and Smith model (blue

Table 1: Sections parameters.

Section i	L (m)	v_f (m/s)	w (m/s)	ρ_j (veh/m)	q_{max} (veh/s)	ρ_{cr} (veh/m)	c (veh)
1	100	28	14	0.18	1.6	0.06	18
2	100	14	7	0.18	0.8	0.06	18

color), the probability of saturation of the road section is larger than our model. Fig. 5 shows for an increasing arrival rate λ, the expected number of cars in the road section ($\bar{N} = \sum_{n=1}^{c} nP_n$) and the blocking probability ($P_c = P(N = c)$), for our model with downstream supply (red color) and for the model of Jain and Smith (blue color).

4.2 Performance measures

In the following, we give an illustration example. We consider two road sections as in Fig. 2. We assume that the fundamental diagrams for those roads are triangular, see Fig. 3. Table 1 gives the parameters for the illustrations.

-*Expected travel time (W)*

The travel time through a road section is a random variable and is a function of the number of cars on the road section. Since the road section has a finite length, it can be seen as a queuing system with a finite capacity, for which the travel time W (or, service time) can be derived using the Little's law. The latter law gives the travel time as the average number of cars in the road ($\bar{N}$) divided by the effective arrival rate ($\lambda(1 - P_c)$).
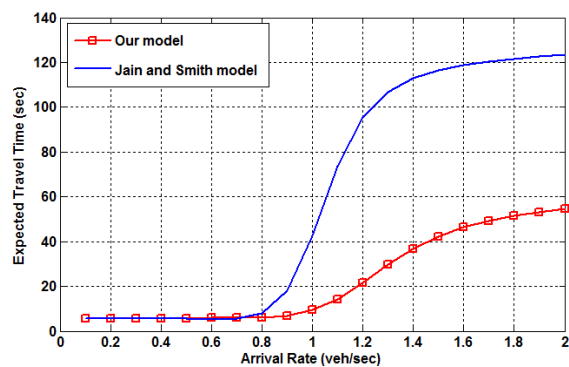

Fig. 6: Comparison of the expected travel time through section 1. In red color, our model. In blue color, model of Jain and Smith.

Fig. 6 compares, for an increasing arrival rate λ, the expected travel time through road section 1 of Fig. 2, for our model with downstream supply and for the linear case

of Jain and Smith model (see Table 1 for the parameters of the road section).

The curves of the expected travel time for the two models are not monotone increasing. There is an upper limit for the arrival rate λ, which is 0.8 veh/s, from there the vehicles begins to slow down and the expected travel time begins to increase. Therefore, the expected travel time before this point ($\lambda = 0.8$ veh/s) is very low (around the free time (L/v_f)) for the linear case of Jain and Smith model and our model. When the traffic volume is large ($\lambda > 0.8$ veh/s) vehicles will slow down and the expected travel time increase in value for the two models of traffic, but still lower in our model with downstream supply. The expected travel time continues to increase with demand beyond capacity.

-*Throughput (θ)*

Throughput θ can be calculated using two methods. Measuring the effective arrival rate of the accepted cars in the system ($\theta = \lambda(1 - P_c)$). Measuring the effective departure rate of the served cars in the system ($\theta = \sum_{n=1}^{c} q_n P_n$).
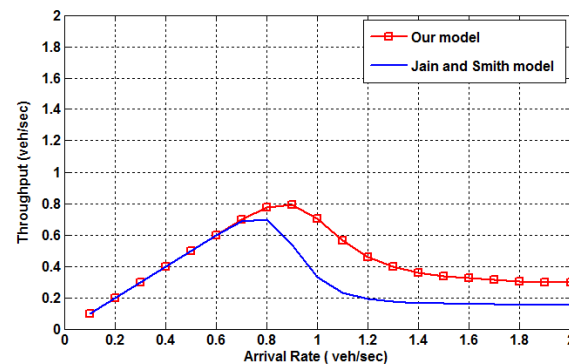

Fig. 7: Comparison of the throughput through section 1. In red color, our model. In blue color, model of Jain and Smith.

Fig. 7 compares, for an increasing arrival rate λ, the throughput through road section 1 of Fig. 2, for our model with downstream supply and for the linear case of Jain and Smith model (see Table 1 for the parameters of the road section).

The curves of throughput increases linearly with arrival rate λ, but there is a halt to the monotonic increase. The throughput decreases with the arrival rate from the value

$\lambda = 0.8$ veh/s, which corresponds to the flow capacity of the second section (minimum between flow capacities of sections 1 and 2).

When the blocking probability is low, the throughput is linear up to 0.8 veh/s. From that arrival rate ($\lambda = 0.8$ veh/s), the blocking probability increases (see Fig.5 in right), the throughput decreases, the expected number of cars also increases up to the system capacity c (see Fig.5 in left), and the expected travel time worsens around 10 times. Arrival rates above 0.8 cannot improve the system throughput that reaches its limit around $\theta = 0.32$ veh/s for our model and $\theta = 0.17$ veh/s for Jain and Smith model. Then, the system would be able to give a higher throughput under an arrival rate of 0.8 veh/s. That seems to be the flow capacity of the second section.

5 Speed and travel time distributions

One of the most basic formula in traffic flow theory is the one expressing the interdependence of the average car-flow (q), the average car-density (ρ) and the average car-speed (v). The formula tells that $q = \rho v$. When two of the three variables are known, the third variable can easily be obtained.

The average car-speed v_n through a road section is a random variable because the number of cars n on the road section is random. Using the expression of the linear speed in the model of Jain and Smith (equation (1)), the car-speed probability distribution is given by:

$$P(v_n = v) = P\left(n = \left\lfloor 1 + c\left(1 - \frac{v}{v_f}\right)\right\rfloor\right).$$

in which $\lfloor x \rfloor$ is the largest integer not superior to x.
Then, the car-speed distribution is given as follows.

$$P_v = P(v_n = v) = \frac{(\lambda L/v_f)^{\lfloor 1+c(1-v/v_f)\rfloor}}{\prod_{i=1}^{\lfloor 1+c(1-v/v_f)\rfloor} i(c-i+1)/c} P_0, \qquad v = 1,..,v_f.$$

$$P_0 = \left(1 + \sum_{v=1}^{v_f} \frac{(\lambda L/v_f)^{\lfloor 1+c(1-v/v_f)\rfloor}}{\prod_{i=1}^{\lfloor 1+c(1-v/v_f)\rfloor} i(c-i+1)/c}\right)^{-1}. \tag{15}$$

The average travel time τ through a road section can be evaluated given the road section length L and the average car-speed v. Basically, we have $\tau = L/v$. By this, the travel time probability distribution is given by:

$$P(\tau = t) = P(v = \frac{L}{t}) = P\left(n = \left\lfloor 1 + c\left(1 - \frac{L}{tv_f}\right)\right\rfloor\right).$$

Then, the average travel time distribution is given as follows.

$$P_t = P(\tau = t) = \frac{(\lambda L/v_f)^{\lfloor 1+c(1-L/(tv_f))\rfloor}}{\prod_{i=1}^{\lfloor 1+c(1-L/(tv_f))\rfloor} i(c-i+1)/c} P_0, \qquad t = \lfloor L/v_f \rfloor,..,L.$$

$$P_0 = \left(1 + \sum_{t=\lfloor L/v_f \rfloor}^{L} \frac{(\lambda L/v_f)^{\lfloor 1+c(1-L/(tv_f))\rfloor}}{\prod_{i=1}^{\lfloor 1+c(1-L/(tv_f))\rfloor} i(c-i+1)/c}\right)^{-1}. \tag{16}$$

Using the parameters of section 1 in Table 1. Fig. 8 shows the histograms for the probability distribution of the average car-speed and the average travel time through the road section, for the linear case of the model of Jain and Smith. The arrival rate considered is $\lambda = 0.8$ veh/s.

For our road section model with a triangular fundamental diagram, the average car-speed v_n is given by the car-flow ($Q(\rho)$) in the road divided by the car-density (ρ).

The car-speed probability distribution satisfies.

$$P(v_n = v) = P\left(\frac{\min(v_f\frac{n}{L}, w(\frac{c-n}{L}))}{\frac{n}{L}} = v\right).$$

Then, two cases are distinguished:

1. $\rho \leq \rho_{cr} \Longrightarrow v_n = v_f$. Then, $P(v_n = v_f) = \sum_{n=0}^{n_{cr}} P(N = n)$,
2. $\rho > \rho_{cr} \Longrightarrow v_n < v_f$. Then, $P(v_n = v) = P\left(N = \lfloor \frac{wc}{v+w} \rfloor\right)$.

The car-speed probability distribution is then given as follows.

$$P(v_n = v) = \begin{cases} 0 & \text{if } v > v_f, \\ \sum_{n=0}^{n_{cr}} P(N = n) & \text{if } v = v_f, \\ P\left(N = \lfloor \frac{wc}{v+w} \rfloor\right) & \text{if } v < v_f. \end{cases} \tag{17}$$

Similarly, we get the following formula for the probability distribution of the average travel time τ through a road section. We use the formula $\tau = n/q$.

$$P(\tau = t) = P\left(\frac{n}{\min(v_f n/L, w((c-n)/L))} = t\right).$$

Then, two cases are distinguished:

1. $\rho \leq \rho_{cr} \Longrightarrow \tau = L/v_f$. Then, $P(\tau = L/v_f) = \sum_{n=0}^{n_{cr}} P(N = n)$,
2. $\rho > \rho_{cr} \Longrightarrow \tau > L/v_f$. Then, $P(\tau = t) = P\left(N = \lfloor \frac{wct}{L+wt} \rfloor\right)$.

The average travel time probability distribution is then given as follows.

$$P(\tau = t) = \begin{cases} 0 & \text{if } t < L/v_f, \\ \sum_{n=0}^{n_{cr}} P(N = n) & \text{if } t = L/v_f, \\ P\left(N = \lfloor \frac{wct}{L+wt} \rfloor\right) & \text{if } t > L/v_f. \end{cases} \tag{18}$$

Fig. 9 displays the histograms for the probability distribution of the average car-speed and the average travel time through the road section, for our model with downstream supply. The arrival rate is fixed to $\lambda = 0.8$ veh/s. Fig. 9 shows that when the arrival rate is low, speed distribution corresponds to the free speed ($v_f = 28$ m/s) with a high probability, and the average travel time distribution corresponds to the free time
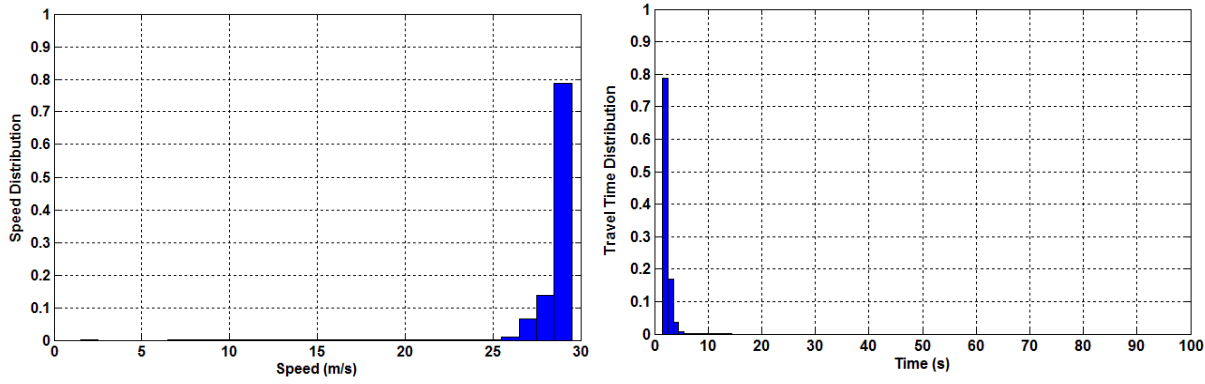
Fig. 8: Car-speed probability distribution histogram, in left. Average travel time distribution histogram, in right. Linear case of Jain and Smith model. The parameters of the road section are those of road section 1 in Table 1, and the arrival rate is $\lambda = 0.8$ veh/s.
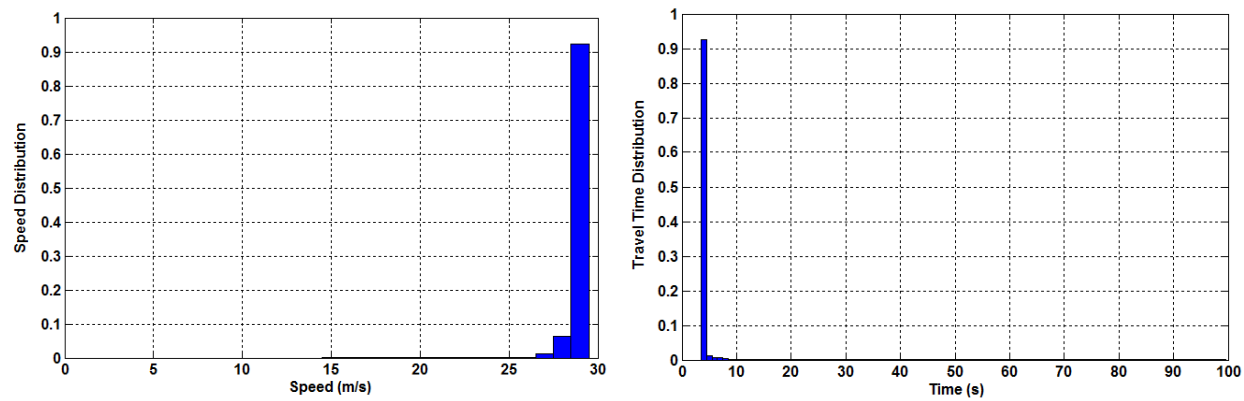

Fig. 9: Car-speed probability distribution histogram, in left. Average travel time distribution histogram, in right. Model with downstream supply. The parameters of the road section are those of road section 1 in Table 1, and the arrival rate is $\lambda = 0.8$ veh/s.
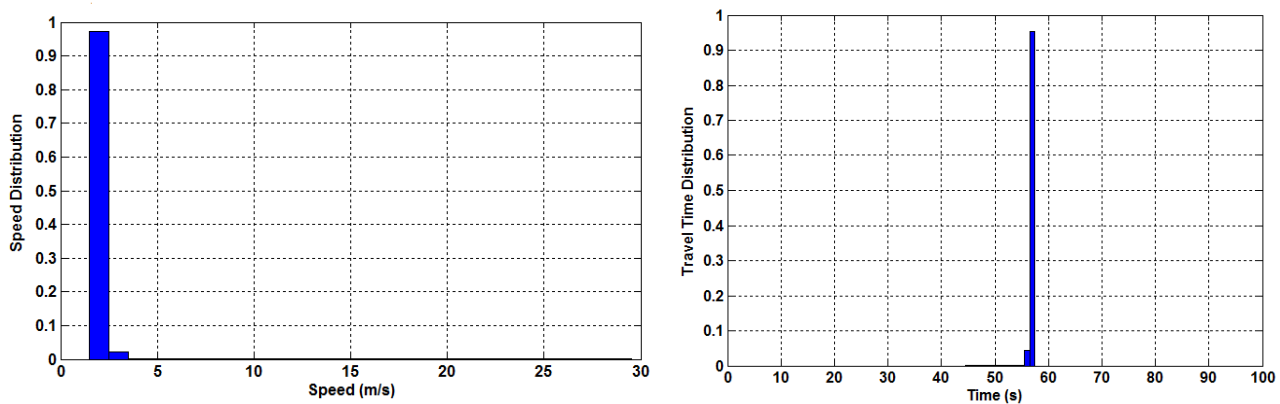

Fig. 10: Car-speed probability distribution histogram, in left. Average travel time distribution histogram, in right. Model with downstream supply. The parameters of the road section are those of road section 1 in Table 1, and the arrival rate is $\lambda = 2$ veh/s.

$(t_f = L/v_f = 3.5$ s), with the same probability. In this case, traffic is fluid because the section is not occupied (or blocked) by the cars.

Fig. 10 displays the histograms for the probability distribution of the average car-speed and the average travel time through the road section, for our model with downstream supply. The arrival rate is fixed to $\lambda = 2$ veh/s.

Fig. 10 shows that when the arrival rate λ is large, the average car-speed is very low, and the average travel time is very large (more than 3.5 s). Note that the average travel time in the road section is almost fifteen times larger than the free speed (about 60 seconds).

6 Conclusion and future work

This paper presents a queuing model for road traffic that preserves the finite capacity property of the real system. Based on the $M/g/c/c$ state dependent queuing model of Jain and Smith, we have proposed a stochastic queuing model for the road traffic which captures the stationary density-flow relationships in both uncongested and congestion conditions.

Experimental investigations of the proposed model are presented. Performance measures have been validated by comparison with $M/g/c/c$ state dependent queuing model of Jain and Smith. Car-speed and average travel time probability distributions are derived for two case of arrival rate. The curves of those distributions shows that the proposed model correctly captures the interaction between upstream traffic demand and downstream traffic supply. Future work shall include the extension of the model to more than two sections in tandem to tree-topologies (complex series, merge, and split networks), and consider the case where traffic demand, traffic supply and fundamental diagrams are stochastic.

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Nacira Guerrouahane is a PhD student at the university of Bejaia, faculty of exact sciences. Bejaia, Algeria. Member at the Research Unit LaMOS (Modeling and Optimization of Systems). His research interests are: stochastic processes, Markov chains, probability, modeling and optimization of transportation systems, dynamical systems, and traffic flow.

Djamil Assani was born in 1956 in Biarritz (Basque Country, France). He started his career at the University of Constantine in 1978. He received his Ph.D. in 1983 from Kiev State University (Soviet Union). He is at the University of Bejaia since its opened in 1983/1984. Director of Research, Head of the Faculty of Science and Engineering Science (1999?2000), Director of the Research Unit **LAMOS** (Modeling and Optimization of Systems, http://www.lamos.org), Scienti?c Head of the Doctoral Computer School (2004 - 2011), he has taught in many universities (U.S.T.H.B. Algiers, University of Annaba, University of Montpellier, University of Rouen, University of Tizi Ouzou, I.N.H. Boumerds, University of Bourgogne, University of Stif, ENITA Bordj-el-Bahri, E.H.E.S.S. Paris, I.N.P.S. Ben Aknoun, Algiers Polytechnical School, C.N.A.M. Paris,...). He has published many papers on Markov chains, queueing systems, reliability theory, performance evaluation and their applications in such industrial areas as electrical networks and computer systems. Prof. Aissani was the president of the National Mathematical Committee (Algerian Ministry of Higher Education and Scienti?c Research, 1995-2005).

Louiza BOUALLOUCHE-MEDJKOUNE works as a teacher at the Department of Computer Science of University of Bjaa and as a researcher at the Research Unit LAMOS (Modeling and Optimization of Systems). She is head of the research team EPSIRT (Evaluation de Performances des Systmes Informatiques et Rseaux de Tlcommunication) since 2005 and Chair of the Scientific Committee since January 2016. Her publications have appeared in various publishing houses: Taylor&Francis, Elsevier, Springer, AMS, BCS, etc. Her research interests are: Performance evaluation (Markov chains, Queuing networks, Simulation, etc.), Quality of service, Communication standards and protocols, Stability, and Security of Computer Systems and Telecommunication Networks (Wireless, mobiles, Ad hoc, Sensors,...).

Nadir Farhi is a researcher (charg de recherche) at IFSTTAR (the French institute of science and technology for transport, spatial planning, development and networks) within GRETTIA team (Engineering of surface transportation and advanced computing) since 2011. He obtained his PhD degree in applied mathematics at the University of Paris 1 Panthon-Sorbonne in 2008. Since then, and before joining IFSTTAR, he worked as a postdoctoral researcher for one year at the University of Texas at Dallas (USA), and then at INRIA (The French Institute for Research in Computer Science and Automation) and at ENS (Ecole Normale Suprieure) Paris. His research interests cover traffic flow theory and control, and modeling and optimization of transportation systems in general, with over 30 publications. He has also been involved in teaching activities, giving courses in applied mathematics and in traffic flow theory and control, at the University of Paris 1, and at the French schools of engineering ENPC, ENTPE, ESIEE, EFREI, and ECE.