

# Adaptive Successive Detection for OFDM over Doubly Selective Channels

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**Abstract:** This paper studies the problem of detection for orthogonal frequency domain multiplexing (OFDM) systems over rapidly time varying multipath channels. A new adaptive successive detection method, which uses the information provided by all of the received symbol, and the band structure of the frequency domain channel matrix, is proposed. The new scheme can get the similar performance as the maximum likelihood sequence estimation (MLSE) with much lower complexity. It can also make a trade-off between the performance and complexity adaptively. Simulation results demonstrate the merits of the proposed algorithms over traditional ones.

**Keywords:** OFDM, channel equalization, doubly selective channels, time varying channels

## 1 Introduction

Orthogonal frequency domain multiplexing (OFDM) is a promising technique for current and future wireless communication systems. It has been widely used in various wireless systems and standards including IEEE 802.11a, digital audio and video broadcasting DAB and DVB standards. On the other hand, one of the main challenges of the implementation of OFDM in real systems is its sensitivity to large Doppler effect with broadband applications in high-mobility devices. In this scenario, time variation of wireless channels is very fast and severe intercarrier interference (ICI) would decrease the system performance significantly [1]. Therefore, it is crucial to design useful detection algorithms to suppress ICI and exploit the time diversity gain provided by Doppler effect.

Many published works have studied the ICI problem in doubly selective channels. Several authors focus on the properties of ICI power distribution among subcarriers. It has been pointed out that since the channel impulse response does not remain constant during an OFDM symbol, the frequency domain channel matrix is not diagonal matrix any more. The off-diagonal elements demonstrate the interference among subcarriers. The interference power mainly results from the neighbouring subcarriers, so the frequency domain channel matrix is approximately a banded one [1][2]. Meanwhile, various

algorithms have been studied to equalize the effect of doubly selective channels. The maximum likelihood sequence estimation (MLSE) has good performance but the complexity is too high to be used in practical system [3]. The equalizers, such as linear block equalizers like minimum mean square error (MMSE) and zero forcing (ZF), always need to calculate the inverse of frequency domain channel matrix, the banded structure can be used to reduce the complexity [4]. The time domain window is used for preprocessing before using iterative MMSE equalizer [2]. [5] develops a reduced-rate OFDM transmission scheme by transmit and receive processing. Similar to the detection of multiple input and multiple output (MIMO) systems [6], successive detector is proposed in [7] which outperforms the linear block equalizer with the cost of higher complexity. An improved successive detector has been studied in [8], however, this ordering scheme does not make full use of the information provided by the received signal.

This letter studies the successive detector for OFDM using enhanced ordering schemes. Also, by adjusting the band-width of the frequency domain channel matrix, we can get a tradeoff between system performance and complexity. Compared with MLSE, the new scheme has much smaller complexity and similar performance.

Notations: Let  $(\bullet)^T$  and  $(\bullet)^H$  denote matrix transposition and conjugate transposition, respectively. Also,  $\langle \bullet \rangle_N$  denotes the modulo- $N$  operation. The  $n$ th row

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and  $l$ th column element of matrix  $\mathbf{X}$  is denoted by  $\mathbf{X}_{n,l}$  and  $\|\bullet\|$  is the Frobenius norm. Let  $x^*$ ,  $\text{Re}\{x\}$ , and  $|x|^2$  be conjugate, real part and norm, respectively. Let  $\langle \mathbf{x}, \mathbf{y} \rangle$  be the inner product of vector  $\mathbf{x}$  and  $\mathbf{y}$ . Throughout,  $E(\bullet)$  and  $D(\bullet)$  denote the expectation and variance of a random variable, respectively. Lastly,  $\mathbf{F}$  is the  $N \times N$  DFT matrix with  $\mathbf{F}_{k,l} = (1/\sqrt{N})\exp\{-j2\pi kl/N\}$ .

## 2 Signal Model and Traditional Successive Detector

### 2.1 Signal Model

For simplicity, the description of the remaining text will neglect the index of the transmission block. The length of the cyclic prefix (CP) is assumed to be long enough to eliminate inter-symbol interference (ISI). The number of the subcarriers and the paths of the multipath channel are denoted by  $N$  and  $L$ , respectively. The transmitted signal vector in the frequency domain is denoted by  $\mathbf{d} = [d_0, \dots, d_{N-1}]^T$  where each element of  $\mathbf{d}$  is a quadrature amplitude modulation (QAM) symbol. The QAM constellation set is denoted as  $\mathcal{Q} = \{c_1, c_2, \dots, c_K\}$ . The time domain signal of  $\mathbf{d}$  is written as  $\mathbf{x}$  which is the inverse discrete Fourier transform (IDFT) of  $\mathbf{d}$ , i.e.,  $\mathbf{x} = \mathbf{F}^H \mathbf{d}$ . At the receiver side, after removing cyclic prefix, the time domain and frequency domain received signal are denoted by  $\mathbf{r}$  and  $\mathbf{s}$ , respectively. We have  $\mathbf{s} = \mathbf{F} \mathbf{r}$ . Let  $\mathbf{H}$  and  $\mathbf{H}_f$  be the time domain and frequency domain channel matrix, respectively. We have  $\mathbf{H}_f = \mathbf{F} \mathbf{H} \mathbf{F}^H$ . The relationship between  $\mathbf{s}$  and  $\mathbf{d}$  can be written as

$$\mathbf{s} = \mathbf{H}_f \mathbf{d} + \mathbf{n} \quad (1)$$

where  $\mathbf{n}$  is the complex frequency domain additive white Gaussian noise (AWGN) with variance  $\sigma^2$ . Our aim is to obtain the estimation of  $\mathbf{d}$  given  $\mathbf{H}_f$ .

### 2.2 Traditional Algorithms

#### 2.2.1 Maximum Likelihood Sequence Estimator

Let  $\hat{\mathbf{d}}$  denote the estimation of the symbol vector. The MLSE selects the candidate vector that minimizes the Euclidean distance (ED) as

$$\begin{aligned} \hat{\mathbf{d}} &= \arg \min_{\mathbf{d} \in \mathbf{A}} P(\mathbf{d}|\mathbf{s}) \\ &= \arg \min_{\mathbf{d} \in \mathbf{A}} \|\mathbf{s} - \mathbf{H}_f \mathbf{d}\|^2, \end{aligned} \quad (2)$$

where  $\mathbf{A}$  is the set of the possible transmitted symbol vectors that consists of  $K^N$  elements. Due to the large complexity of MLSE, low-complex equalizers have been studied such as linear equalizers and successive detector [7].

## 3 The New Scheme of Successive Detection

Three steps go into the process of successive detection: interference nulling, ordering and interference cancellation [9].

Firstly, at the step of interference nulling, by applying the equalization matrix  $\mathbf{T}$  into (1), we will get

$$\begin{aligned} \mathbf{p} &= \mathbf{T} \mathbf{s} \\ &= \mathbf{T} \mathbf{H}_f \mathbf{d} + \mathbf{v}, \end{aligned} \quad (3)$$

where  $\mathbf{T} = (\mathbf{H}_f^H \mathbf{H}_f + \frac{\sigma^2}{E_c} \mathbf{I})^{-1} \mathbf{H}_f^H$  and  $\mathbf{T} = (\mathbf{H}_f^H \mathbf{H}_f)^{-1} \mathbf{H}_f^H$  for MMSE and ZF equalizers, respectively. Also,  $E_c$  is the average energy of the QAM symbol. Furthermore, the covariance of  $\mathbf{v}$  for MMSE and ZF equalizer can be written as (4) and (5), respectively.

$$C_v = \sigma^2 (\mathbf{H}_f^H \mathbf{H}_f)^{-1} \mathbf{H}_f^H \mathbf{H}_f (\mathbf{H}_f^H \mathbf{H}_f)^{-1} \quad (4)$$

$$C_v = \sigma^2 (\mathbf{H}_f^H \mathbf{H}_f + \frac{\sigma^2}{E_c})^{-1} \mathbf{H}_f^H \mathbf{H}_f (\mathbf{H}_f^H \mathbf{H}_f + \frac{\sigma^2}{E_c})^{-1} \quad (5)$$

After that, we can get the initial estimation of  $d_i$ , written as

$$\hat{d}_i = J(p_i) \quad (6)$$

where  $J(x)$  is the hard decision function.

Furthermore, at the stage of ordering, we need to decide which symbol is to be detected first. Assume that the  $l$ th symbol is chosen at the stage of ordering, and then at the stage of interference cancellation, the interference of the estimated symbol  $\hat{d}_l$  on the other symbols is removed. Also,  $\mathbf{s}$  and  $\mathbf{H}_f$  is updated as  $\mathbf{s}_{new}$  and  $\mathbf{H}_f^{new}$  in (7) and (8), respectively.

$$\mathbf{s}_{new} = \mathbf{s} - \hat{d}_l \mathbf{h}_l \quad (7)$$

$$\mathbf{H}_f^{new} = [\mathbf{h}_0, \mathbf{h}_1, \dots, \mathbf{h}_{l-1}, \mathbf{h}_{l+1}, \dots, \mathbf{h}_{N-1}] \quad (8)$$

where  $\mathbf{h}_k$  is the  $k$ th column of  $\mathbf{H}_f$ . The detection process continues until all of the symbols have been detected.

In this section, we propose a new ordering scheme for successive detection.

### 3.1 The New Ordering Scheme

After the statistical variable has been obtained in (3), we propose to choose the symbol with the index calculated through

$$i^* = \arg \max_i P(d_i = \hat{d}_i | \mathbf{p}). \quad (9)$$

Let the log-likelihood ratio (LLR) be

$$L_{i,k} = \ln \frac{P(d_i = \hat{d}_i | \mathbf{p})}{P(d_i = c_k | \mathbf{p})}. \quad (10)$$

Without loss of generality, the symbol of each constellation point is assumed to be transmitted with

equal probability, i.e. for each  $i$ , we have  $P(d_i = c_k) = \frac{1}{K}$ . Then (10) can be written as

$$L_{i,k} = \ln \frac{P(\mathbf{p}|d_i = \hat{d}_i)}{P(\mathbf{p}|d_i = c_k)} = \ln \frac{\sum_{\mathbf{d} \in \mathbf{J}_i^{c_k}} P(\mathbf{p}|\mathbf{d})}{\sum_{\mathbf{d} \in \mathbf{J}_i^{\hat{d}_i}} P(\mathbf{p}|\mathbf{d})} \tag{11}$$

where  $\mathbf{J}_i^c$  is the set of the transmitted vector with the  $i$ th symbol being  $c$  which is an element of  $\mathcal{Q}$ . Furthermore, the closed form of  $P(\mathbf{p}|\mathbf{d})$  is

$$P(\mathbf{p}|\mathbf{d}) = \frac{\exp(-(\mathbf{p} - \mathbf{TH}_f \mathbf{d})^H C_v^{-1} (\mathbf{p} - \mathbf{TH}_f \mathbf{d}))}{\pi^N (\det C_v)} \tag{12}$$

Using the total probability formula, we have  $\sum_k L_{i,k} = 1$ .

And then

$$P(d_i = \hat{d}_i | \mathbf{p}) = \frac{1}{\sum_{k=1}^K e^{-L_{i,k}}} \tag{13}$$

Following (13), equation (9) is equal to

$$i^* = \arg \max_i \sum_{k=1}^K e^{-L_{i,k}} \tag{14}$$

Equation (14) can be simplified based on the band structure of the frequency domain channel matrix  $\mathbf{H}_f$  [2]. In fact, the band structure of  $\mathbf{H}_f$  means that the interference is mainly caused by the neighbour subcarriers. So the complexity of computing (14) can be reduced by only considering the neighbouring subcarriers. Assume that only the  $2B$  neighbour subcarriers are considered for detection. Define the index set  $I_i$  and the signal vector  $\tilde{\mathbf{d}}^i$ , where  $I_i = \{i, \langle i-1 \rangle_N, \langle i+1 \rangle_N, \dots, \langle i-B \rangle_N, \langle i+B \rangle_N\}$  and the  $j$ th element of  $\tilde{\mathbf{d}}^i$  is

$$\tilde{d}_j = \begin{cases} d_j, & j \in I_i \\ 0, & \text{others.} \end{cases}$$

In fact,  $\tilde{\mathbf{d}}^i$  only considers the  $i$ th element and its  $2B$  neighbouring elements of  $\mathbf{d}$ . Furthermore, (11) can be simplified as

$$L_{i,k} = \ln \frac{\sum_{\mathbf{d} \in \mathbf{W}_i^{c_k}} P(\mathbf{p}|\mathbf{d})}{\sum_{\mathbf{d} \in \mathbf{W}_i^{\hat{d}_i}} P(\mathbf{p}|\mathbf{d})} \tag{15}$$

where  $\mathbf{W}_i^x$  is the set of  $\tilde{\mathbf{d}}^i$  with the  $i$ th symbol being  $x$ , which is an element of  $\mathcal{Q}$ .

To sum up, the new successive detection scheme can be demonstrated as follows.

**Step 1:** Obtain the initial estimation of each undetected symbol using (3) and (6).

**Step 2:** Find out the symbol to be detected using (14) and (15), and the information bits can be obtained by demapping.

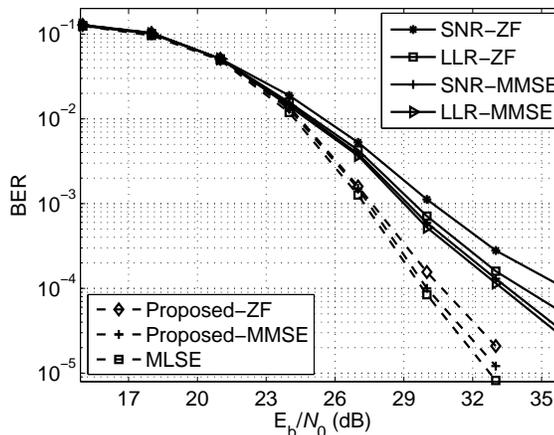


Fig. 1: BER performance comparison of the proposed methods compared with the traditional ones without channel coding,  $f_d = 0.06$ .

**Step 3:** Do interference cancellation and update the parameters according to (7) and (8).

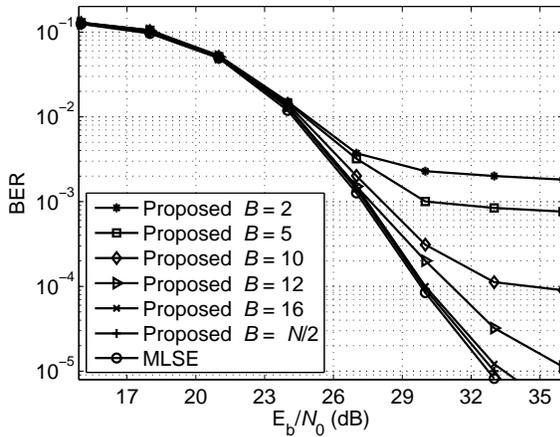
**Step 4:** Go to step 1, until each symbol has been detected.

### 3.2 Comparison Between the New Scheme and the Traditional Methods

The comparison between the proposed scheme and the MLSE is as follows:

1. We need to calculate  $K^{(2B+1)}$  candidates in (14) of the proposed method using (15), and  $K^N$  in (2) of the MLSE. Because  $B$  is always much smaller than  $N$ , the complexity of the new ordering scheme is significantly reduced compared with MLSE.
2. The larger  $B$  is, the more interference are taken into consideration and the more precise the result of ordering is. Therefore, by adjusting  $B$ , we can make a trade-off between system performance and complexity in the proposed approach. In fact,  $B$  can be chosen according to the normalized maximal Doppler frequency offset.
3. The simulations demonstrate that the performance of the new scheme is similar to MLSE even with small  $B$ . Therefore, the new scheme can get the similar performance as MLSE with much lower complexity.

On the other hand, because the new scheme exploits the information provided by the whole vector  $\mathbf{p}$  in (3), it also outperforms the signal to noise ratio (SNR) based and log-likelihood ratio (LLR) based successive detection algorithms in [7] and [8], respectively.



**Fig. 2:** BER performance of the proposed algorithms without channel coding,  $f_d = 0.06$ .

## 4 Simulation

The parameters are as follows. The number of the subcarriers and the paths of the channel is 256 and 12, respectively. The length of the cyclic prefix is 32. The power delay profile obeys exponentially decreasing with decay constant of 5 taps. Each path fading follows Rayleigh fading. The sum of the variance of all of the paths is set to be 1. The modulation is QPSK. The results are obtained after 500000 runs. We use SNR-ZF to denote the SNR based algorithm with  $\mathbf{T}$  in (3) being ZF equalizer. Similarly, we have the notations: SNR-MMSE, LLR-ZF, LLR-MMSE, Proposed-ZF, and Proposed-MMSE. Let  $f_d$  be the normalized maximal Doppler frequency offset.

Firstly, the performance of the proposed algorithm, SNR based and LLR based algorithm of [8] is shown when ZF or MMSE estimators is used. In figure 1, the bit error rate (BER) versus  $E_b/N_0$  is shown. The maximal normalized Doppler frequency offset is 0.06. It can be found that the performance is similar when SNR is low. When SNR increases, the proposed schemes outperform the other ones. The proposed algorithm provide 3.5 dB gain over LLR based ones at BER of  $10^{-4}$  when MMSE is used. In this figure, the parameter  $B$  is 16. We can also see that the proposed algorithm with  $B = 16$  has the similar performance as MLSE, but has much lower complexity.

Figure 2 demonstrates the system performance of the proposed algorithm as  $B$  changes when MMSE is used. It can be seen that the the performance of the proposed algorithm increases with  $B$ . When  $B$  is larger than 16, the performance can not be further improved notably. The complexity also grows with the increase of  $B$ . The larger  $B$  is, the better the system performance is. Therefore, it is necessary to choose a proper  $B$  to make a trade-off

between performance and complexity. Generally speaking, the larger  $f_d$  is, the larger  $B$  should be chosen.

## 5 Conclusion

This letter studies the successive detection algorithms for OFDM systems in doubly selective wireless channels. A new ordering approach is proposed. The new approach can exploit more information provided by the whole received signal vector compared with SNR and LLR ordering schemes which only use the information of a single received symbol. So the symbol to be detected based on the proposed algorithm is more reliable and leads to a better performance. Based on the band structure of frequency domain channel matrix, the new scheme is simplified by only considering the neighbour subcarriers. The simplified approach can reduce the complexity significantly. By adjusting the bandwidth of the frequency domain channel matrix, we can make a trade-off between system performance and complexity. Also, the algorithm can achieve the similar performance as MLSE with much lower complexity. The effectiveness of the new scheme is verified by simulations.

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## References

- [1] A. Stamoulis, S. N. Diggavi, and N. Al-Dhahir, "Inter-carrier interference in MIMO OFDM," *IEEE Trans. Signal Process.*, vol. 50, no. 10, pp. 2451-2464, Oct. 2002.
- [2] P. Schniter, "Low-complexity equalization of OFDM in doubly selective channels," *IEEE Trans. Signal Process.*, vol. 52, no. 4, pp. 1002-1011, April 2004.
- [3] A. Goldsmith, *Wireless Communications*, Cambridge University Press, 2005.
- [4] E. Panayirci, H. Dogan, and H. V. Poor, "Low-complexity MAP-Based successive data detection for coded OFDM systems over highly mobile wireless channels," *IEEE Trans. Veh. Tech.*, vol. 60, no. 6, pp. 2849-2857, July 2011.
- [5] J. Ma, P. V. Orlik, J. Zhang, and G. Li, "Reduced-rate OFDM transmission for inter-subchannel interference self-cancellation over high-mobility fading channels," *IEEE Trans. Wirel. Commun.*, vol. 11, no. 6, pp. 2013-2023, June 2012.
- [6] S. Lee, S. Park, S. Kim, and I. Lee, "Enhanced detection with new ordering schemes for V-BLAST systems," *IEEE Trans. on Commun.*, vol. 57, no. 6, pp. 1648-1651, June 2009.

- [7] X. D. Cai and G. B. Giannakis, "Bounding performance and suppressing intercarrier interference in wireless mobile OFDM," *IEEE Trans. Commun.*, vol 51, no. 12, 2047-2056, Dec. 2003.
  - [8] F. Li, S. Zhu and M. Rong, "Improved successive detector for OFDM in time-variant multipath channels," *IEICE Trans. Commun.*, vol. E92-B, no. 5, pp. 1892-1896, May 2009.
  - [9] Y. Soo, "On channel estimation and detection for multicarrier signals in fast and selective Rayleigh fading channels," *IEEE Trans. Commun.*, vol. 49, no. 8, pp. 1375-1387, Aug. 2001.
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