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(3n+1) Conjectures Accuracy is Evident

Fereshteh fazeli

Department of Converging Sciences and Technologies, Science and Research Branch, Islamic Azad University, Tehran, Iran

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Abstract: (3n+1) conjecture was proposed by Lothar collatz in 1937. Although researchers believe that this conjecture is true but no one has ever been able to prove it yet. Many efforts have been made by various scientists all over the world and their actions have led to good results but despite these efforts may be the math world is not ready to prove it yet.
In this paper, I utilize number theory and features in numbers to prove this conjecture. Using powers of number (2) and classification numbers according to their features and finding the location of numbers cause this dangerous conjecture proof. All achievements show that each number we select, finally reaches number (1) and this conjecture is true for all numbers.

Keywords: (3n+1) conjecture, number theory, powers of number (2).

1 Introduction

Collatz conjecture was proposed by Lothar Collatz during the 1930s, and we can find a similar problem to the collatz conjecture in his notebook in 1932. This conjecture says that when we have an even number we have to divide it by (2) and about an odd number first we have to multiply it by (3) and then add it to number (1). You can continue this process finally a number reaches number (1) and it is not important which number you select.
collatz conjecture is also known as the (3n+1) conjecture, the Ulam conjecture, the Kakutani's problem, or the Syracuse problem. Dr.Terence Tao wrote a paper that shows this conjecture is true for almost all numbers [1]. While his result is not full proof of the conjecture, it is a major advance on a problem that does not give up easily. It is one of the problems in the computational math field. Mathematicians all over the world have done efforts to verify this conjecture [2], for example, they changed basic to basic (2) or they used rational numbers, or used of computer they analyzed that this conjecture is true. For example, Wei ren et al. verified (2) power (100000) minus (1) can return to (1). Even graph theory has been used to prove this conjecture [3,4]. But already non of these actions result in proof.
Some researchers are looking forward to finding a paradoxical example against this dangerous conjecture, but some groups are sure that this conjecture is true [5,6]. In this paper with attributes in number theory [7,8,9,10], I prove the collatz conjecture.
First, by natural numbers involving odd and even numbers, I verify how collatz found the $\frac{(3n+1)}{2}$ formula in section[2]. Then I will talk about the main lemma (using of number 4) So the manner of that step by step I prove the conjecture. That number (4) is the best number to prove this problem, so the principle is that we need powers of (2) numbers to verify this conjecture. so we categorize numbers into two groups, even numbers, and odd numbers. Each even number is (4k) or (4k+2) and each odd number is (4k+1)or(4k+3). The accuracy of collatz conjecture for powers of (2) numbers is evident. But for other numbers, we have to consider the base form to define them. As you saw before each natural number is written to one of the shapes (4k,4k+1,4k+2,4k+3), and (k) belongs to Arithmetic numbers. So we can find a specific formula to detect the next number in section [3]. Then we form a table of numbers on page [5] and examine the properties in it, each of which is a key to prove the conjecture.
Next, I will simply explain how each number can be constructed by other numbers. In this paper given that collatz is using the power of (2) to reach the final number (1), I also have the power of (2) to prove this conjecture.
In conclusion in section[4], I say that with all achievements accuracy of the conjecture is evident.
In this paper, I find out that if we select a number how we can find the location of the next number with this lemma

* Corresponding author e-mail: F.Fazeli001@gmail.com

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and the features of numbers we can easily prove that this conjecture is true for all numbers. My paper has a real difference from other papers because all features and formulas that I have written are based on the power of number (2) and by that, I have proved the conjecture for all even and odd numbers with different features and I prove this problem separately for all numbers.

2 Natural numbers:even and odd numbers

We know that numbers are even or odd. First, we start with even numbers. Even numbers, we have dealt with them are powers of (2) or not. If an even number will power of (2), according to collatz conjecture we have to divide it by (2) so finally, we lead up to number (1).
This conjecture is clearly correct for powers of (2), but for other even numbers after dividing by (2) for (n) times, at last we reach an odd number. So if we select an even number,(according to collatz conjecture), finally we reach an odd number. The second group is odd numbers. It is not important which number you select(even or odd number)at last we reach an odd number.
From now on we have to deal with odd numbers. If we prove the conjecture with odd numbers we can verify the conjecture forever.

How collatz reached to $\frac{(3n+1)}{2}$ formula?
Proof:
We know that each odd number was an even number which has divided by (2). $(n = 2m+1 \Rightarrow n = \frac{2(2m+1)}{2})$

If we add the number (1) and then the number (2) to (2n) finally we will have three numbers in the arrow. (2n, 2n+1, 2n+2)

Now we divide (2n+2) by number (4).Then we minus (2n+1) from $\frac{(2n+2)}{4}$

$$(2n+1) - \frac{(2n+2)}{4} = \frac{(6n+2)}{4} = \frac{(3n+1)}{2}$$

I proved how collatz found $\frac{(3n+1)}{2}$ formula.

3 we can find next number from odd number

We said that the foundation of our research is based on powers of (2) and the best power of (2) for proving this conjecture is number (4). According to number theory, we know that each odd number is written in one of the shapes (4k+1) or (4k+3).On the assumption that:m=2n+1,m=4k+1

2(4k+1) 2(4k+1)+1 2(4k+1)+2

$(2(4k+1)+1) - \frac{(2(4k+1)+2)}{4} = 6k+2 \Rightarrow$ $next\ number = 6k+2$

And if m=4k+3

2(4k+3) 2(4k+3)+1 2(4k+3)+2

$(2(4k+3)+1) - \frac{(2(4k+3)+2)}{4} = 6k+5 \Rightarrow$ $next\ number = 6k+5$

According to formulas overhead, we said that if we have an odd number, how we can find the next number.

3.1 Beautiful features in numbers

Now we want to build a table of numbers and then we will show some features in numbers, each of which is a doorway to prove the conjecture.

Table (1):Making odd numbers from Arithmetic numbers and finding the next numbers from odd numbers.

k	4k+1	6k+2	4k+3	6k+5
0	1	2	3	5
1	5	8	7	11
2	9	14	11	17
3	13	20	15	23
4	17	26	19	29
5	21	32	23	35
6	25	38	27	41
7	29	44	31	47
8	33	50	35	53
9	37	56	39	59
10	41	62	43	65
11	45	68	47	71
12	49	74	51	77
13	53	80	55	83
14	57	86	59	89
15	61	92	63	95
16	65	98	67	101
17	69	104	71	107
18	73	110	75	113
19	77	116	79	119
20	81	122	83	125
21	85	128	87	131
22	89	134	91	137
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
85	341	512	343	515

*(6k+2) is the next number constructed by (4k+1) and (6k+5) is the next number constructed by (4k+3).

We assume that k belongs to Arithmetic numbers.

$$(K \in W)$$

We said before that we can write all numbers with these shapes (4k,4k+1, 4k+2,4k+3) even and odd numbers. First, we talk about even numbers.
According to the collatz conjecture, we have to divide all

even numbers by number (2). Well, at last, if a number was one of the powers of (2) after some dividing by number (2), eventually we reach number (1) and the conjectures accuracy is clear. But if our number was an even number but not the power of (2) after several dividing by number (2), eventually we reach an odd number. Now we can say that we deal with odd numbers. We said before that all odd numbers can write with these shapes (4k+1) and (4k+3). In section (3) we said how we can reach to next number. If we have an odd number,(the next number that we reach to it is exactly the number that we find by the collatz formula). If we start with k=0,1,2,3,... and locate it in (4k+1) and (4k+3) formulas,we have all odd numbers from $(1)$to(∞).
With (6k+2)and(6k+5) formulas we can reach to next numbers from odd numbers. If we pay attention to numbers we see that diversity between two odd numbers arrow is (2) and diversity between two next numbers arrow is(3).
(4k+3)-(4k+1)=2 (6k+5)-(6k+2)=3
From now on we analyze all numbers and then with all results we conclude that according to collatz conjecture it is not important which number you select, each number you select eventually reaches number (1).
I want to show you this conjecture is true for all numbers. In k=0 we have 4k+1=1 and the next number is 6k+2=2. Number (2) is the power of (2).In k=1 we have 4k+1=5 and the next number is 6k+2=8. Number (8) is the power of (2) too. In k=5,4k+1=21 and 6k+2=32 and in k=21,4k+1=85 and 6k+2=128.(128) is power of (2).All these powers of (2) numbers are odd powers of (2).$(2^1,2^3,2^5,2^7)$
As you saw before in k=0,1,5,21 we have odd numbers with the (4k+1) formula and the next number with the (6k+2) formula is surely the power of (2).
Well from this category of numbers we can easily reach number (1) because these numbers are the power of (2) and after several dividing by number (2) at last we reach number(1).
If we find diversity between two ks arrows with this feature, we have 5-1=4,21-5=16. According to the numbers above, we conclude that all numbers with this formula: $k = 4n+1 = 2^{2m}+2^{2m-2}+...+2^2+2^0$

construct next numbers with this shape: $6k+2 = 2^{2m+3}$

Proof:

$k = 2^{2m}+2^{2m-2}+...+2^2+2^0 \Rightarrow 4k+1 = 2^2(2^{2m}+2^{2m-2}+...+2^2+2^0)+1 = 2^{2m+2}+2^{2m}+...+2^4+2^2+1$

We said before that how we reach to (6k+2) formula.
$2(4k+1)+1 = 2(2^{2m+2}+2^{2m}+...+2^4+2^2+1)+1 = 2^{2m+3}+2^{2m+1}+...+2^5+2^3+2+1$

$$2(4k+1)+2 = 2^{2m+3}+2^{2m+1}+...+2^5+2^3+2+2$$

$$(8k+3)-\frac{(8k+4)}{4} = 6k+2$$

$$6k+2 = (2^{2m+3}+2^{2m+1}+...+2^5+2^3+2+1)-\frac{(2^{2m+3}+2^{2m+1}+...+2^5+2^3+2+2)}{4}$$

$$6k+2 = \frac{2^{2m+5}}{2^2} = 2^{2m+3}$$

As you saw before we proved that in all numbers with this shape:
$(k = 2^{2m}+2^{2m-2}+...+2^2+2^0)$
we have (4k+1) an odd number and with the (6k+2) formula we have the next number that number is surely the power of (2) and after (2m+3) times dividing by number (2) eventually we reach number (1) and the conjecture is true for these numbers.
But if k_1=4k+3 and our number will with this shape,4k+3=$2^{2m+1}+2^{2m-1}+\ldots+2^1+2^0$ $(m \geq 0)$ then we lead up from (4k+3)to(6k+5) and next number 6k+5=2^{2m}+1.
proof:

$4k+3 = 2^{2m+1}+\ldots+2^1+2^0 \Rightarrow k = \frac{2^{2m+1}+2^{2m-1}+\ldots+2^5+2^3}{4=2^2} \Rightarrow 6k+5 = (2^3-2)(2^{2m-1}+2^{2m-3}+\ldots+2^3+2^1)+5 = 2^{2m+2}+1 = 2^{2(m+1)}+1 = 2^{2m_1}+1$
As you saw the biggest power of (2) for constructing (4k+3) is (2m+1) and also the biggest power of (2) for constructing (6k+5) is (2m+1)+1=(2m+2).
For constructing (4k+1) we have (4k+3)-2=4k+1=$2^{2m+1}+2^{2m-1}+\ldots+2^3+2^0 \Rightarrow 6k+2 = 2^{2m+2}+1-3 = 2^{2m+2}-2 = 2(2^{2m+1}-1)$.
We have this event for k=0,k=2,k=10,... on the whole in ks with this shape k_1=4k+2,Diversity between two ks arrow is (2^{2n+1}),odd powers of number (2).$k = (4^n)(0)+(4^{n-1})(2)+(4^{n-2})(2)+\ldots+(4^0)(2)$ $(n \geq 0)$.
We reach from (4k+1) to next number (6k+2),if $4k+1 = 2^{2m}+2^{2m-2}+\ldots+2^2+2^0 \Rightarrow k = 2^{2m-2}+2^{2m-4}+\ldots+1 \Rightarrow 6k+2 = (2^3-2)(2^{2m-2}+2^{2m-4}+\ldots+1)+2 = 2^{2m+1}$ $(m \geq 1)$ *and* $6k+2 = 2^{2m+3}$ $(m \geq 0)$.If $k_1 = 4k+1 = 2^{2m}+2^{2m-2}+\ldots+2^2+2^0$ $(m \geq 0)$,in k=0 and m=0 we have 4k+1=1 and in k=1,m=1 we have 4k+1=5.
As you saw ,diversity between two ks arrow is (2^{2n}),even powers of number (2).
For example:$(5-1) = 4 = 2^2$ and for constructing (4k+3) we have$(4k+1)+2 = 4k+3 = 2^{2m}+2^{2m-2}+\ldots+2^2+2^0+2 = 2^{2m}+2^{2m-2}+\ldots+2^4+2^3-2^0$ then we have $6k+5 = 6k+2+3 = 2^{2m+3}+3 = 2^{2m+3}+2^2-2^0 = 2^2(2^{2m+1}+1)-1 = 4(2^{2m+1}+1)-1$.
Proof:

$\frac{(2^{2m}+2^{2m-2}+...+2^2+2^0+2^0)}{2} = 2^{2m-1}+2^{2m-3}+\ldots+2^1+2^0 \Rightarrow$

$(2^{2m-1}+2^{2m-3}+\ldots+2^1+2^0)(2^2-2^0)=2^{2m+1}+1.$

For example,in k=1 we have 6k+5=11=12-1=4(3(1))-1 and in k=5 we have 6k+5=35=36-1=4(9)-1=4(3(3))-1.
As you saw, $1=\frac{(1+1)}{2}$ in k=1 and $3=\frac{(5+1)}{2}$ in k=5.This event is for k=0,k=1,k=5,...,$k=2^{2m}+2^{2m-2}+\ldots+2^2+2^0$ and also we have $k=(4^n)(0)+(4^{n-1})(1)+(4^{n-2})(1)+\ldots+(4^0)(1)\ (n\geq 0)$.
An other point that i want to talk about it is that,we said (k) is belong to Arithmetic numbers and with (4k+1) and (4k+3) formulas we can create odd numbers.
If we pay attention to odd numbers in (4k+1) formula the next number (6k+2) is surely an even number.If we minus (4k+1) from (k) we have (3k+1) and if we divide (6k+2) by (2) we have (3k+1) too.(4k+1)-k=3k+1
$\frac{(6k+2)}{2}=3k+1$
Another interesting point is that in numbers with this shape k=3,k=13,k=53,
k=213,... if we calculate an odd number with the (4k+1) formula and then the next number with the (6k+2) formula we find out that:
Proof:

$$k=10(2^{2m}+2^{2m-2}..+2^2+2^0)+3$$

$$k=(2^3+2^1)(2^{2m}+2^{2m-2}..+2^2+2^0)+(2^2-1)=2^{2m+3}+(2^{2m+2}+2^{2m}...+2^6+2^4+2^2)+2^0=2^{2m+3}+2^2(2^{2m}+2^{2m-2}+...+2^4+2^2+2^0)+2^0=2^{2m+3}+4k_1+1$$

$$6k+2=(2^3-2)(2^{2m+3}+4k_1+1)+2=2^{2m+6}+2^5(2^{2m}+2^{2m-2}+...+2^2+2^0)+2^3-2^{2m+4}-2^3(2^{2m}+2^{2m-2}+...+2^2+2^0)-2+2=2^{2m+6}-2^{2m+4}+2^3+2^{2m+5}+2^{2m+3}+...+2^7+2^5-2^{2m+3}-2^{2m+1}-...-2^5-2^3=2^{2m+6}+2^{2m+5}-2^{2m+4}=2^{2m+4}(2^2+2-1)=2^{2m+4}(5)$$

$$6k+2=2^{2m+4}(5)$$

Well,we proved that in all numbers with this shape,$k=10(2^{2m}+2^{2m-2}+...+2^2+2^0)+3$
we have:$6k+2=2^{2m+4}(5)$
and after (2m+4) times dividing by number (2) at last we reach to number (5).$(5=4k+1\Rightarrow 6k+2=8)$

As you saw always from number (5) we reach number (8). Number (8) is the power of (2) and collatz conjecture has proved for these numbers too.
We said that all numbers with this shape,$(k=2^{2m}+2^{2m-2}+\ldots+2^2+2^0)$ give us the next numbers with the (6k+2) formula that all of them are surely the power of (2) and after several dividing by number (2) at last we reach to number (1) and we prove collatz conjecture easily for these numbers. But for other numbers, we have to find out the new formula with these numbers. Next, we find the next number.

We know that distance between two (6k+2) arrow is always (6).
k=a
4k+1=4a+1
6k+2=6a+2
4k+3=4a+3
6k+5=6a+5
k=a+1
4k+1=4a+5
6k+2=6a+8
4k+3=4a+7
6k+5=6a+11
(6a+11)-(6a+5)=6
(6a+8)-(6a+2)=6

If,$k=2^{2m}+2^{2m-2}+...2^2+2^0\Rightarrow 6k+2=2^{2m+3}$

$k+1=2^{2m}+2^{2m-2}+..+2^2+2^0+1\Rightarrow 6(k+1)+2=6k+2+6=2^{2m+3}+2^3-2$

$k+2=2^{2m}+2^{2m-2}+...+2^2+2^0+2\Rightarrow 6(k+2)+2=2^{2m+3}+12=2^{2m+3}+2^4-2^2$

$k+3=2^{2m}+2^{2m-2}+...+2^2+2^0+3\Rightarrow 6(k+3)+2=2^{2m+3}+18=2^{2m+3}+2^4+2$

We saw that if $(k=2^{2m}+2^{2m-2}\ldots+2^2+2^0)$ for k+1,k+2 and k+3 numbers,we have steady formulas.But for other numbers we have to find a general formula.
For k+4,k+5,k+6,...numbers we have:

In $k+4\Rightarrow 6k+2=2^{2m+3}+2^4-2^2+(2^3-2)(k-3)=2^{2m+4}-8$

In $k+5\Rightarrow 6k+2=2^{2m+3}+2^4-2^2+(2^3-2)(k-2)=2^{2m+4}-2$
If we continue this process we observe that :
$k+4=2(2^{2m+3}-4)$
$k+5=2(2^{2m+3}-1)$
$k+6=2(2^{2m+3}+2)$
As you see above changeable factors in these numbers are (-4,-1,+2,...).
-4=3(0)-4
-1=3(1)-4
2=3(2)-4
Well we can find out a general formula for k+4,k+5,...and next number (6k+2).

$$6k+2=2(2^{2m+3}+3n-4)\ \ and\ (n\geq 0)$$
One thing is important ,from k=1 to k=5 (m=0) and from k=5 to k=21(m=1)
and also from k=21 to k=85 (m=2).We coclude that if we divide the biggest power of (2)in(k) by number (2) we can calculate the (m).
$k=2^{2m}+2^{2m-2}+...+2^2+2^0\Rightarrow m=\frac{2m}{2}$
We know that all numbers are even or odd. Even numbers are with this shape (4k,4k+2) and odd numbers are (4k+1or4k+3). Now I want to analyze all even and odd

numbers and I am going to find the remnant by number (4), then I find the next numbers from odd numbers and again I find the remnant by number (4). The next point is about ks. Assume that our (k) is with these shapes (4k+1or4k+3). We can easily prove how we can reach (6k+2) and what the relation is between (k) and (6k+2).

$k = 2n = 4k_1 + 2 \Rightarrow 4k + 1 = 4(4k_1 + 2) + 1 = 16k_1 + 9 \Rightarrow 6k + 2 = 6(4k_1 + 2) + 2 = 24k_1 + 14$
The remnant of (4k+1) by number (4) is 1 and the remnant of (6k+2) by number(4) is (2).
$k = 4k_1 + 2 \Rightarrow 4k + 3 = 16k_1 + 11 \Rightarrow 6k + 5 = 24k_1 + 17$
In (4k+3), R=3 and in (6k+5), R=1.
$k = 2n = 4k_1 \Rightarrow 4k + 1 = 16k_1 + 1 \Rightarrow 6k + 2 = 24k_1 + 2$
In (4k+1), R=1 and in (6k+2) ,R=2.
$k = 4k_1 \Rightarrow 4k + 3 = 16k_1 + 3 \Rightarrow 6k + 5 = 24k_1 + 5$
In (4k+3) ,R=3 and in (6k+5), R=1.
$k = 2n + 1 = 4k_1 + 1 \Rightarrow 4k + 1 = 16k_1 + 5 \Rightarrow 6k + 2 = 24k_1 + 8$
In (4k+1), R=1 and in (6k+2) ,R=0.
$k = 4k_1 + 1 \Rightarrow 4k + 3 = 16k_1 + 7 \Rightarrow 6k + 5 = 24k_1 + 11$
In (4k+3), R=3 and in (6k+5), R=3.
$k = 2n + 1 = 4k_1 + 3 \Rightarrow 4k + 1 = 16k_1 + 13 \Rightarrow 6k + 2 = 24k_1 + 20$
In (4k+1), R=1 and in (6k+2) ,R=0.
$k = 4k_1 + 3 \Rightarrow 4k + 3 = 16k_1 + 15 \Rightarrow 6k + 5 = 24k_1 + 23$
In (4k+3) ,R=3 and in (6k+5),R=3.
For example,in k=9=4(2)+1 we have 6k+2=56 and as we said before in $k_1 = 4k + 1$ or 4k+3, $6k_1 + 2$ is divisible to number (4). ($\frac{56}{4} = 14$)
If we pay attention we reach to number (14) with an odd number (9) and 4k+1=9 in k=2.Now we can conclude that if $k_1 = 4k + 1$ or 4k+3 , $6k_1 + 2$ numbers are divisible to number (4) and if we divide these numbers by number (4), we reach to (6k+2) in (k).
Proof:

$$k = 4k_1 + 1 \Rightarrow 6k + 2 = 24k_1 + 8 \Rightarrow \frac{24k_1+8}{4} = 6k_1 + 2$$

$$k = 4k_1 + 3 \Rightarrow 6k + 2 = 24k_1 + 20 \Rightarrow \frac{24k_1+20}{4} = 6k_1 + 5$$

Another example is k=11= 4(2)+3.In k=11 we have 6k+2=68 if we divide (68) by the number (4) we reach to number (17) and on the other hand in k=2 we have 6k+5=17 well as you see we reach from k=11=4(2)+3 to 6k+5=17 in k=2.
About ks with this shape (4k) according to this point that (6k+2) is divisible to number (2) if we divide (6k+2)by number (2), we have (3k+1).

$$k_1 = 4k \Rightarrow \frac{4k}{2} = 2k \Rightarrow \frac{2k}{2} = k \Rightarrow 2k + k = 3k$$

We conclude that if $k_1 = 4k$, (3k+1) is locate in $k_2 = 3k$.For example in k=4 we have 4k+1=17 and 6k+2=26 and number (13) is 4(3)+1=13 then $k_2 = 3$ we reach from k=4 to k=3.
In this paper my aim is to show you with some manners that how numbers locate in different ks and at last we reach to a (k) that with that (k) we can reach to number (1) and prove the conjecture.
Now i want to talk about ks with this shape (4k+2).
Assume that $k_1 = 4k + 2$,the number before k_1 is (4k+1) and the number after k_1 is (4k+3)(an odd number).

$$k_1 = 4k + 2 \Rightarrow 6k_1 + 2 = 6(4k + 2) + 2 = 24k + 14 \Rightarrow \frac{24k+14}{2} = 12k + 7$$

As we said before we reach from($k_1 = 4k + 1$)to(6k+2) and from (4k+3)to(6k+5)in(k). (6k+2)+(6k+5)=12k+7
For example: In k=10 we have 6k+2=62 and according to collatz conjecture we have to divide this number by (2) well we have the number (31).
On the other hand we reach to number (14) from number (9) and we reach (17) from number (11) in k=2.
14+17=31 as you see 31=31. Number 31=4k+3=4(7)+3 and it is an odd number in k=7.We reach from k=10 to k=7.
We lead up to (12k+7) from ($k_1 = 4k + 2$), we know that the remnant of (12k+7) by number (4) is (3). Well, we conclude that our number is located in the (4k+3) formula.

$$12k + 7 = 4k_2 + 3 \Rightarrow 12k + 7 - 3 = 4k_2 \Rightarrow 4(3k + 1) = 4k_2 \Rightarrow k_2 = 3k + 1$$

As you saw above we lead up from ($k_1 = 4k + 2$) to ($k_2 = 3k + 1$).It means that if we construct an odd number with the number with this shape ($k_1 = 4k + 2$) then we reach to an odd number in (4k+3) formula in ($k_2 = 3k + 1$) number. Now we can find out that when we have a number how we can construct odd numbers and then next numbers and where the next numbers go.

$$k_1 = 4k + 1 \Rightarrow 6k_1 + 2 = 6(4k + 1) + 2 = 24k + 8 \Rightarrow \frac{(24k+8)}{4} = 6k + 2$$

$$k_1 = 4k + 1 \Rightarrow 6k_1 + 5 = 6(4k + 1) + 5 = 24k + 11 = 24k + 8 + 3 = 4(6k + 2) + 3 = 4k2 + 3$$

$$k_1 = 4k + 3 \Rightarrow 6k_1 + 2 = 6(4k + 3) + 2 = 24k + 20 \Rightarrow \frac{24k+20}{4} = 6k + 5$$

$$k_1 = 4k + 3 \Rightarrow 6k_1 + 5 = 6(4k + 3) + 5 = 24k + 23 = 24k + 20 + 3 = 4(6k + 5) + 3 = 4k2 + 3$$

In (4k+1)and(4k+3) numbers if we find next numbers with (6k+2) formula we see that our next numbers are locate in (k) but if we find next numbers from (6k+5) formula with these odd numbers we see that our next numbers are locate in (6k+2)for(4k+1) numbers and in (6k+5) for (4k+3) numbers.
For example,in $k_1 = 4k + 3$=4(3)+3=15 if we find $6k_1 + 2 = 92$ and divide this number by (4) we have number (23) and as you know we have 6k+5=23 in k=3 but if we find $6k_1 + 5$=95=92+3=4(23)+3 we lead up to an

odd number with this shape $(4(k_2)+3\ )$ in $(k_2=6k+5$=23).
In $(6k_1+2)$ we reach from (k_1) to (k) but in $(6k_1+5)$ we reach from (k_1) to (6k+5).
We talked about $(k_1=4k+1)$ or (4k+3) that if we have these numbers how we can reach to next number and also where the next number goes. We talked about not only (6k+2) but also about (6k+5) too. And also we talked about ks with this shape (4k+2) and we said if we find (6k+2), where we can find this next number. But now I want to say if our (k) will with this shape (4k+2) how and where we can find (6k+5).
$k_1=4k+2\Rightarrow 4k_1+3=4(4k+2)+3=16k+11=16k+8+3\Rightarrow 6k_1+5=6(4k+2)+5$
$6k_1+5=24k+17=(24k+16)+1=4(6k+4)+1=4k_2+1$
As you saw above we lead up from $(k_1=4k+2)$ with an odd number $(4k_1+3)$ to the next number $(6k_1+5)$ that this number is in another number with this shape (4k+1).

In (k_2) we have an odd number ,$(4k_2+1=6k_1+5)$

$4k_2+1=6k_1+5=24k+17\Rightarrow 4k_2-24k=17-1=16\Rightarrow 4k_2=24k+16$

$4k_2=4(6k+4)\Rightarrow k_2=6k+4=(6k+2)+24$

For example, in k=2 we have 6k+5=17 and the number 17=4(4)+1 is an odd number in k_1=4.You see that we reach from k=2 to $k_1=4$.If we continue this process with these methods that I said all along with this paper and other methods that I want to say from now on we find out why we reach number (1) and we answer why the collatz conjecture is true.
Now I want to talk about numbers with this shape $(k_1=4k)$. First, I construct an odd number with the (4k+3) formula then I find the next number with the (6k+5) formula and I say that where we can find this number.

$k_1=4k\rightarrow 4k_1+3=4(4k)+3=16k+3\Rightarrow 6k_1+5=6(4k)+5=24k+5$

$6k_1+5=24k+5=24k+4+1=4(6k+1)+1=4k_2+1$

$4k_2+1=6k_1+5=24k+5\Rightarrow 4k_2+1=24k+5\Rightarrow 4k_2-24k=5-1=4$

$4k_2=24k+4\Rightarrow 4k_2=4(6k+1)\Rightarrow k_2=6k+1$

As you saw we reach from $(k_1=4k)$ and an odd number $(4k_1+3)$ to the next number $(6k_1+5)$, as another odd number with this shape (4k+1). For example, in k=8 we have 4(8)+3=35 and next number 6(8)+5=53=52+1=4(13)+1 and we have an odd number 4(13)+1=4k+1 in k=13.We reach from number (8) to number (13).

We said before that if $k_1=4k+2$,the next number $(6k_1+2=12k+7)$,surely locate in $(k_2=3k+1)$.But if we want to calculate (k_2) with these shapes (4k+1 or 4k+3) we have to attention to some examples.
For example:
$k_1=4k+2=6=4(1)+2\Rightarrow k_2=4=3k+1=3(1)+1$
As you see we have 4=3(1)+1 and 4=4(1)+0 and also (1-0)=1.
In $k=2\Rightarrow k_1=10\Rightarrow k_2=7$
We have 7=3(2)+1=4(1)+3 and (2-1)=1
$k=6\Rightarrow k_1=26\Rightarrow k_2=19$
We have 19=3(6)+3=4(4)+3 and (6-2)=4
In k=0,1 we have 4(k-0) and in k=2,3,4,5 we have 4(k-1) and in k=6,7,8,9 we have 4(k-2) and On the whole, we realize that if:

$$k=4a+2,4a+3,4a+4,4a+5\ (a\geq 0).$$

$$k_1=4k+2\Rightarrow k_2=3k+1=4(k-\frac{(4a+4)}{4})=4(k-(a+1))+(0,1,2,3)$$

$$k_2=3k+1=4(k-(a+1))+(0,1,2,3)$$

One thing is important,(0,1,2,3) are the remnant of number (4) because it is possible we reach to (4k,4k+1 ,4k+2 or4k+3) numbers.
If $k_1=4k+2$, then we have an odd number $4k_1+3$=4(4k+2)+3=16k+11 and the next number is:
$6k_1+5$=6(4k+2)+5=24k+17=24k+16+1=4 (6k+4)+1=$4k_2+1$.
As you see we reach from $(k_1=4k+2)$ to an odd number $(4k_2+1)$ in (k_2).Also if$(k_1=4k)$ we have an odd number $4k_1+3$=4(4k)+3=16k+3 and next number is $6k_1+5$=6(4k)+5=24k+5=24k+4+1=4 (6k+1)+1=$4k_2+1$.
As you see we reach from $(k_1=4k)$ to an odd number $(4k_2+1)$ in (k_2).
For example,in k=10 the next number is (65)=64+1=4(16)+1.Then we lead up from k=10 to k=16 and also in k=8 we have 6k+5=53=52+1=4(13)+1 and we reach from k=8 to k=13.
With these manners, we can find out that if we have a number how and where we can find odd and next numbers and their location in numbers.
We said that we can write numbers with these shapes (4k,4k+1,4k+2, and 4k+3). Now I want to arrange numbers in terms of divisibility to number (3). If k=3n or k=3n+2 (n belongs to Arithmetic numbers), then we construct odd numbers from (k), (4k+1 and4k+3), surely one of these numbers is divisible to number (3).

$$k=3n\Rightarrow 4k+3=4(3n)+3=12n+3$$

$$k=3n+2\Rightarrow 4k+1=4(3n+2)+1=12n+9$$

But if (k=3n+1) we have no odd numbers divisible by the number (3). If we pay attention to odd numbers with this shape, we see organized relations between numbers.

$n=0 \Rightarrow k=1 \Rightarrow 4k+1=5=6(1)-1 \quad 4k+3=7=6(1)+1$

$n=1 \Rightarrow k=4 \Rightarrow 4k+1=17=6(3)-1 \quad 4k+3=19=6(3)+1$

On the other hand, we said that where we can find multiples of number (3).

Now if we pay attention to odd numbers and their next numbers, we find out beautiful relations.In k=0 we have 4k+3=3(1)=(0+1)3 and 6k+5=5=3(2)-1 and in k=2 we have 4k+1=9=3(3)=3(2+1) and 6k+2=14=3(5)-1 also in k=3 we have 4k+3=15=3(5)=3(3+2) and 6k+5=23=3(8)-1.

if we continue this process we find out that odd numbers are with this form [3(2n+1)] and according to collatz conjecture, the next numbers are with this form [3(next number)] and the (next number) has constructed with (2n+1). With these details, if we have a number we can easily say the place of the next number and after analyzing with relations we said before we can easily find out that each number we select, eventually reaches number (1).

In this part of this manuscript, I want to say other points each of which is a doorway to realize that collatz conjecture is true.

About ks with this shape (4k+2),if (k) was an even number,the next number from (4k+1) is (6k+2) and after dividing by number (2),(3k+1) is locate in $6(\frac{k}{2})+1$

Proof: $k=2a \Rightarrow 4k+2=4(2a)+2=8a+2 \Rightarrow \frac{6(2a)+2}{2}=3(2a)+1=6a+1$

$a=\frac{k}{2} \Rightarrow 6(\frac{k}{2})+1$

For example,in k_1=18=4k+2=4(4)+2 we have $4k_1$+1=73 and $6k_1$+2=110 and $3k_1$+1=55=4k+3=4(13)+3 ,we have an odd number in k=13=$3k_2+1=3(4)+1$.

But if (k) was an odd number a next number from (4k+1) is (6k+2) and after dividing by (2) is locate in (3k+1) and k=1,3,5,... .

Proof:

$k=2a+1 \Rightarrow 4k+2=4(2a+1)+2=8a+6=4(2a)+6$

$6k+2=6(2a+1)+2=12a+8 \Rightarrow \frac{12a+8}{2}=6a+4$

$a=\frac{(k-1)}{2} \Rightarrow 6(\frac{k-1}{2})+4=3k+1$

For example,in k_1=22=4k+2=4(5)+2 we have $4k_1$+1=89 and $6k_1$+2=134 and $3k_1$+1=67=4k+3=4(16)+3,we have an odd number in k=16=$3(k_2)+1=3(5)+1$.

I want to find a relation between numbers to figure out how odd numbers with the same unity can construct and what is the relation between odd and their next numbers. We have five odd numbers (1,3,5,7,9) and other odd numbers have these numbers as a unity. Number (1) is in k=0 (4k+1=1).In k=2 4k+3=11 and number (21) is in k=5(4k+1=21).We have the number (31) in k=7(4k+3=31). According to collatz conjecture from number (1) we reach number (2) and from number (11), we reach number (17).

In k=0 we have 4k+1=1 and 6k+2=2=3(1)-1 and in k=2,4k+3=11 and 6k+5=17=3(6)-1,in k=5 4k+1=21 and next number 32=3(11)-1.If we continue this procces we see that we have number (1) as a unity in numbers in k=0,2,5,7,10,12,15,17,20,22,...and odd numbers with these formulas (4k+1 or4k+3).As you saw next numbers from these odd numbers are with this shape 3(1)-1,3(6)-1,3(11)-1,3(16)-1,3(21)-1,...3(a1)-1,3(a6)-1.

For other odd numbers (3,5,7,9),we do the same work and we find relations between them and their next numbers.In k=0,3,5,8,10,13,15,18,...we have odd numbers with (4k+1 or4k+3) formulas and all these odd numbers have unity (3) and next numbers are with this shape 3(2)-1,3(7)-1,3(12)-1,3(17)-1,...3(a2)-1,3(a7)-1.In k=1,3,6,8,11,13,16,18,...we have odd numbers with unity (5) and next numbers are with this shape 3(3)-1,3(8)-1,3(13)-1,3(18)-1,...3(a3)-1,3(a8)-1.In k=1,4,6,9,11,14,16,19,...we have odd numbers with unity (7) and next numbers are with this shape 3(4)-1,3(9)-1,3(14)-1,3(19)-1,...3(a4)-1,3(a9)-1.

In k=2,4,7,9,12,14,17,19,...we have odd numbers with unity (9) and next numbers are with this shape 3(5)-1,3(10)-1,3(15)-1,...3(a5)-1,3(a0)-1.

Our odd number is with this shape (2k+1) then the next number that constructs with these numbers are with this shape 3(a[k+1])-1 and 3(a[k+6])-1. All numbers with the same unity plus number (1) and then divide by number (2) show the number that multiplies in number (3) to construct the next number.

For example, in k=11 ,(11)+1=12 and (12) divide by (2) is (6),then we have next number 3(6)-1=17.As you saw we reach from k=(11) to the next number (17).

About ks with the same unity and odd numbers and next numbers that they construct we can say that:

$k=m \Rightarrow 4k+1=4m+1 \Rightarrow 6k+2=6m+2 \quad 4k+3=4m+3 \Rightarrow 6k+5=6m+5$

$k=10(a)+m \Rightarrow 4k+1=40(a)+4(m)+1 \Rightarrow 6k+2=60(a)+6(m)+2$

$4k+3=40(a)+4(m)+3 \Rightarrow 6k+5=60(a)+6(m)+5$

As you saw above diversity between ks is number (10) and between odd numbers is number (40) and between next numbers is number (60). Also in numbers with the same unity if we calculate the next numbers we observe that diversity between two numbers arrow is number (15).

Proof:

$4k+1=m \Rightarrow 6(\frac{(m-1)}{4})+2=\frac{(3m+1)}{2}$

$4k+3=10(a)+m \Rightarrow 6(\frac{(10(a)+m)-3}{4})+5=\frac{(30(a)+3m+1)}{2}$

$\frac{3(10(a)+m)+1}{2}-\frac{3m+1}{2}=15(a)=(2^4-1)(a)$

In this part of the paper, I want to write numbers according to the power of number (2).
We know that we have,(2^n-1) numbers between $(2^n) and (2^{n+1})$ and for writing numbers between these two powers of (2), we need numbers (0) to (n).

For example:

$1=2^0, 2=2^1, 3=2^1+2^0, 4=2^2, 5=2^2+2^0, 6=2^2+2^1, 7=2^2+2^1+2^0, 8=2^3$

We can write all numbers in the form of a sum of several powers of (2). If we pay attention to numbers (1,2,3,..) and numbers in powers of (2) (0,1,(1,0),2,...) we see that there is a relation between these numbers and powers of (2). Now I want to write natural numbers in the form of powers of (2), but I write only numbers as a power of number (2).

Table (2):Writing natural numbers in terms of powers of number (2) and the relation between powers of number (2).

natural numbers	In terms of powers of number 2	only powers of number2
1	1	0
2	2	1
3	2+1	1,0
4	4	2
5	4+1	2,0
6	4+2	2,1
7	4+2+1	2,1,0
8	8	3
9	8+1	3,0
10	8+2	3,1
11	8+2+1	3,1,0
12	8+4	3,2
13	8+4+1	3,2,0
14	8+4+2	3,2,1
15	8+4+2+1	3,2,1,0
.	.	.
.	.	.

Between (1) and (2) we have no number and between (2) and (4) we have one number (3) and between (4) and (8) we have three numbers (5,6,7).
As you saw above we have a regular relation between powers of (2) and natural numbers. We can write each number we want in the form of powers of two and with relation, we found before we can write numbers of powers of two.
On the whole we have this relationship between two powers of number two:

Table (3):Writing natural numbers according to the powers of number (2).

natural numbers	only powers of number2
n	0
n	1
n	1,0
.	.
.	.

As you saw above we have one (n) and we have,(2^{n-1}) combination with (n-1) and (2^{n-2}) combination with (n-2), and if we continue this process, at last, we have,(2^1) combination with the number (1) and ane combination with the number (0).
Each odd number we select is between two powers of number two and also we have, (2^{n-1}) odd numbers between two powers of two,$(2^n) and (2^{n+1})$.
With these points, we can write each odd number in the form of a sum of the power of two and other odd numbers.
For example:
In k_1=17=4(4)+1=4k+1 in k=4 the next number is (6k+2)=26.We can write 17=16+1 and we know that we reach from 4k+1=1 in k=0 to next number (6k+2)=2, now we can write,16+8+2=26 and in k_1=19=4(4)+3=4k+3 in k=4 the next number is 6k+5=29,number (19) is between two powers of two (16,32).we have 19=16+3 and as we know we reach from 4k+3=3 in k=0 to next number from our odd number,(6k+5)=5, now we can write,16+8+5=29.
On the whole, we have:

$a=4k+1 \Rightarrow 4k+1=2^n+(2m+1) \quad 2m+1=4k_1+1 \Rightarrow 4k_1=2m \Rightarrow m=2k_1$

$6k_1+2=6(\frac{m}{2})+2=3m+2 \quad 2^n+2^{n-1}+3m+2=6k+2 \ (m \geq 0)$

$a=4k+3 \Rightarrow 4k+3=2^n+(2m+1) \quad 2m+1=4k_1+3 \Rightarrow 4k_1+2=2m \Rightarrow m=2k_1+1$
$6k_1+5=6(\frac{m-1}{2})+5=3m+2 \quad 2^n+2^{n-1}+3m+2=6k+5 \ (m \geq 0)$

With these methods, we can write and find the location of the next number and if we follow the sequence of all points that I said all along with this paper, at last, we reach number (1) and we find out why the collatz conjecture is true.

4 Conclusion

In this paper, I talked about numbers and I said we can write all numbers according to numbers (4),(4k,4k+1,4k+2, and 4k+3). In this manner, I classify all numbers and I said some features in numbers, each of which is a doorway to verify that the collatz conjecture is true. When we choose a number (it is not important which number we select), if we follow the sequence of features in numbers at last we lead up to number (1).
At the end of this paper I want to say that because of logical relevance between numbers (i said all of them in this manuscript), each number with collatz manner at last reaches to number (1) and accuracy of this conjecture is obvious.

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