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Autocorrelation Analysis of Gamma Ray Log of Ras Fanar Oil Field in Egypt

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Abstract: In the current study, multivariate analysis methods of time series are used to simplify the search for correlations among different oil wells. Gamma ray log of potassium, thorium, and uranium isotopes of three wells in Ras Fanar oil field in Gulf of Suez, Egypt are taken as examples. we suppose gamma log as a time series, where time is replaced by depth. Autocorrelation and principal component analysis were applied to such supposed time series to extract stratigraphic structure of the fields under study. The uranium gamma log was the common factor between the three wells and anti-correlated to depth, while thorium log depends on the depth but with less conclusive results. The results clearly show the applicability to model petro-physical data using multivariate analysis.

Keywords: Principal component analysis, autocorrelation function, gamma ray log, time series methods, Ras Fanar oil field

1 Introduction

Earth is a unique and complicated system; its geological structure often fluctuates smoothly over millions of years [1,2]. To put our hands on hidden earth treasures, search for elegant techniques to explore its internal structure is required. Well-logging is a major source of our knowledge about layers of earth's crust. Well-log exploration [3] introduces a standard approach for deducing lithology from wells and many other different properties of rocks, such as permeability, density, and porosity.

Gamma ray log is a common method of using natural radioactive elements to investigate correlation between stratigraphic sections in a borehole [4]. The amount of natural gamma radiation from a rock depends on its internal structure, for example, shales ordinarily radiate more gamma rays than other sedimentary rocks, such as gypsum, sandstone, and limestone [5]. The reason for such behavior depends on two factors, the radiative isotope of potassium (K), which is a basic component of clay, and the cation exchange capacity of clay to absorb uranium (U) and thorium (Th) [6]. Gamma ray logs are used mainly to define and quantify productive intervals by identifying gamma ray energies (E_γ) of each radioactive source [7].

In fact, investigating the correlations between different stratigraphic sections of different oil wells is confronted with the obstacle of huge data logs for each borehole against depth. In order to simplify situation, the depth of a well is treated as similar to time factor, and each log as an event in a time series. In other words, time series methods of multivariate analysis such as autocorrelation and principal components are used to analyze gamma ray logs to explore the correlations between different oil wells. The long-term correlations are explored in the dynamics of many physical, technological, and natural systems [8,9,10,11,12]. They are described by a divergent correlation time [13]. We focus our application in Ras Fanar oil field located on the offshore part of the western side of the Gulf of Suez, Egypt as shown in figure (1).

This study discovers the applicability of gamma well log modeling using multivariate statistics that can be beneficial for geophysicists to develop economical methods for oil and gas discoveries. This study will help

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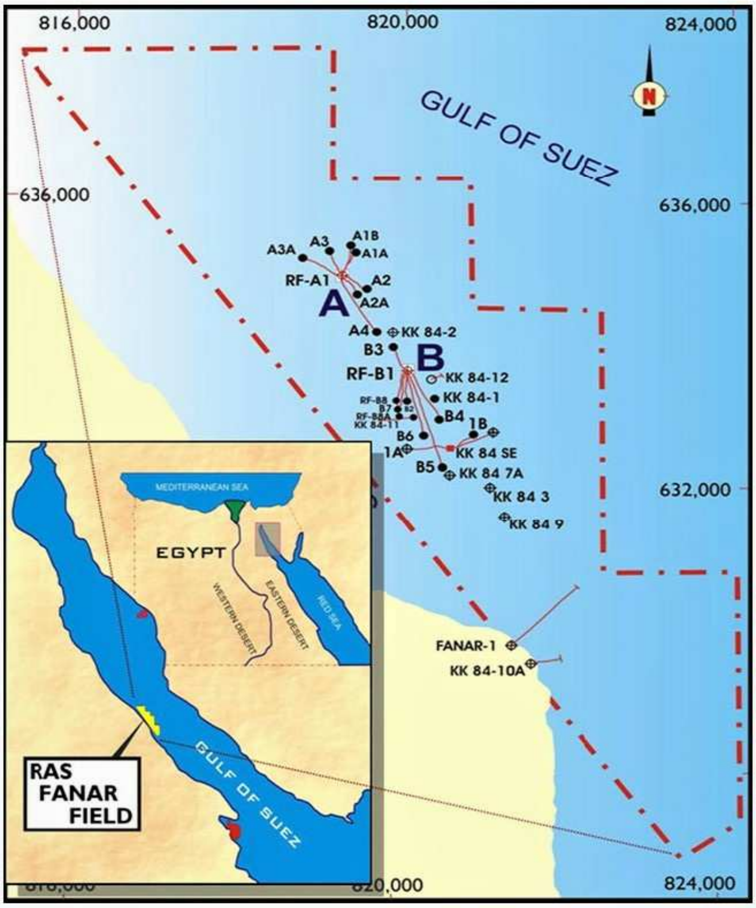

Fig. 1: Location Map of Ras Fanar Oil Field Taken from Ref.[14]

the researchers to uncover the critical areas of principal component analysis and autocorrelation function that many researchers were not able to explore.

2 Data Set

The gamma ray log is obtained by recording the gamma radiation in a well at constant intervals of depth. In the current study, three wells are chosen from Ras Fanar oil field namely: RF-A1, RF-A3, and KK 84-11. Three logs are taken into consideration as follows:

–POTA (wt%): radioactive potassium element,
–THOR (ppm): thorium element,
–URAN (ppm): uranium element.

The units used in the estimation of each log is given in brackets. Measurements are recorded in foot interval. We focus only on complete logs for each well. For RF-A1 well, 665 log was obtained in depth 1694 to 2358 ft., while the RF-A3 well a 926 log was reported in the range 1789 to 2714 ft. In case of KK 84-11well, a 1495 record was taken in the range, 1757 to 3251 ft. Figure (2) summarize the variation of the three radioactive elements' concentration with depth for each well.

3 Method of Analysis

3.1 Autocorrelation

The autocorrelation function is used to measure how strongly on average each data point of a series is correlated with another, which is k steps away in the same series. In other words, it is the ratio of the autocovariance to the variance of data [15]. In case of uncorrelated data, the ACF within $\pm 2/\sqrt{N}$ of zero (two standard deviations) for about 95% of the k values, where N is the series length. The ACF could be represented as,

$$ACF = \frac{\sum_{i=1}^{N-K}(x_i - \bar{x})(x_{i+k} - \bar{x})}{\sum_{i=1}^{N}(x_i - \bar{x})^2} \quad (1)$$

The ACF falls from a value of 1 at $k = 0$ to zero at large k. The value of k at which ACF falls to $1/e \approx 37\%$ is called the correlation time τ_c [15], which can be used to

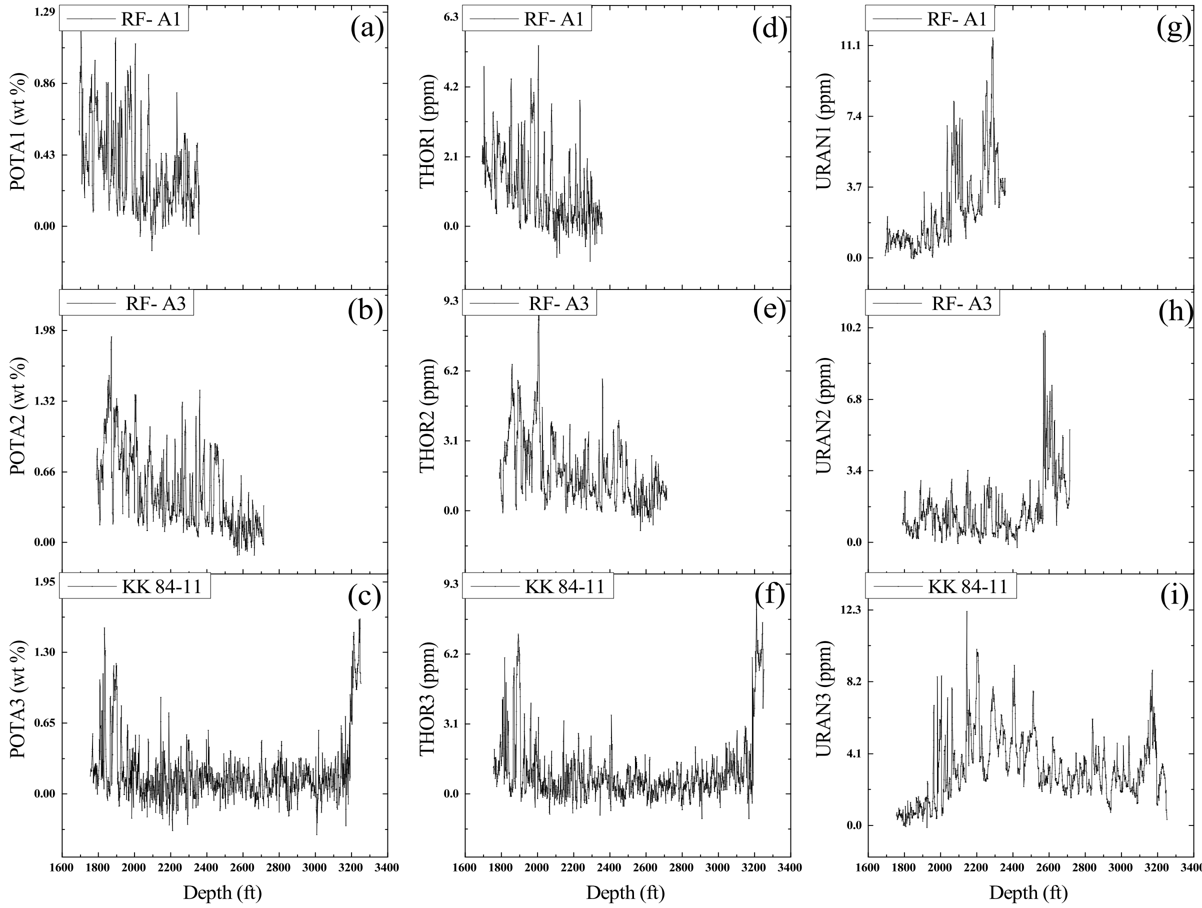

Fig. 2: The Variation of the Radioactive Potassium (POTA), Thorium (THOR), and Uranium (URAN) Concentration with Depth for Each Well

explore long-term correlations within a series of well-log data.

3.2 Principal Component Analysis

Principal Component Analysis (PCA) has a wide range of applications in multivariate statistics [16,17,18,19,20]. PCA is an important tool to compress the size and extract the most important information from the data set. Consequently, it is used to simplify and analyze the observations and the variables. PCA assumes new variables called principal components (PC) which are linearly dependent on the original variables. The new variables are labeled factor scores, and can be represented geometrically as projections of observations onto the PC space [21].

let well depth is represented by I data points described by J geophysical variables, so we have $I \times J$ matrix space, $\mathbf{X}$, whose components $x_{i,j}$. The standardized data matrix say, $\dot{\mathbf{X}}$ is established from $\mathbf{X}$ by subtracting off the mean, and dividing by the standard deviation of each column.

The correlation matrix $\mathbf{C}$ of matrix $\mathbf{X}$, that defines all relations between pairs of measurements is defined as

$$\mathbf{C} = \frac{1}{n-1}\mathbf{D}^{-1/2} \cdot \mathbf{X} \cdot \mathbf{X}^{\mathbf{T}} \cdot \mathbf{D}^{-1/2}, \qquad (2)$$

where $\mathbf{D}^{-1/2}$ is a diagonal matrix equals to $[1/\sigma_{x_j}]$. The eigenvector of matrix $\mathbf{C}$ characterized by the highest eigenvalue is the prevailing PC of the data set and is labeled as (PC1). It represents the most significant connection between the data dimensions. The largest possible variance is associated by The PC1. The second component abbreviated as (PC2) has the second largest variance is considered to be orthogonal to the first component.

The observations (depth) are donated in the principal component space by their factor scores (red points in figure (4)). The inter-correlation between variables are represented as coordinates in the component space. In PCA terms, such correlation is named a loading [21] (blue lines in figure (4)). The circle of correlations is defined as the circle that surround such loadings. As

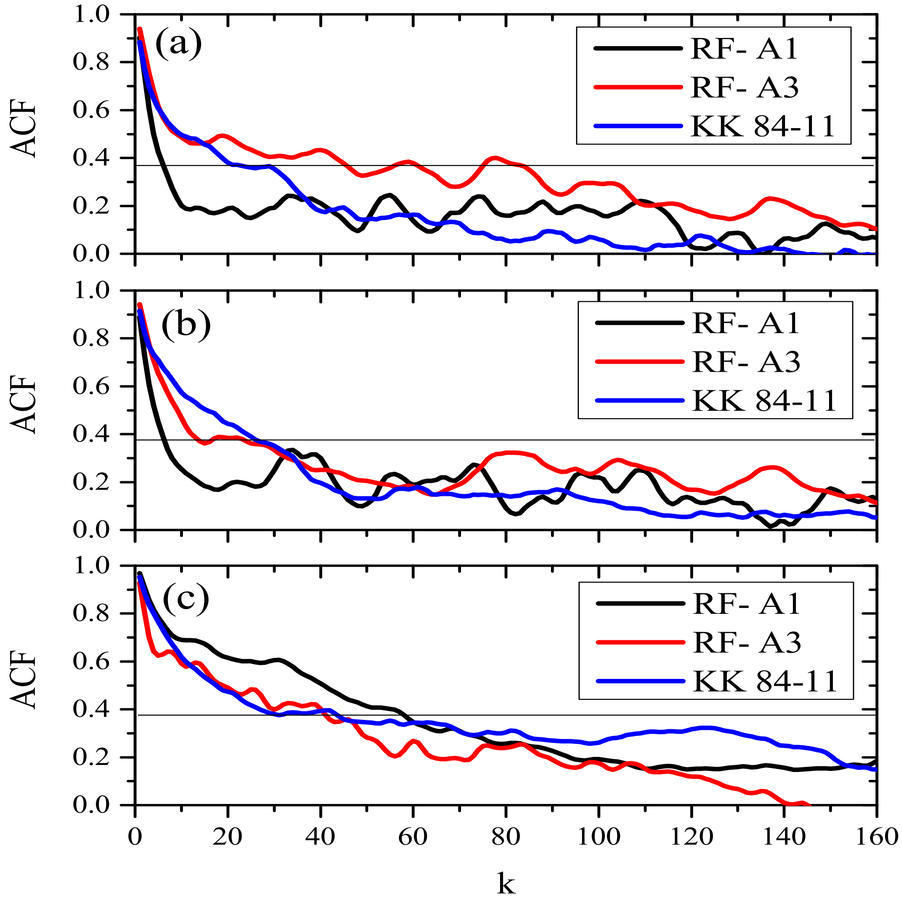

Fig. 3: The ACF of The Three Radioactive Elements for Each Well Calculated by Eq.1

variables get closer to the circle of correlations, it is easier to be interpreted by the given components. On the other hand, variables lose their significance as they approach the center of circle of correlations [21]. The biplot graph is used to interpret the results of PCA, which characterizes the relationships between depth and corresponding variables in the first two PCs.

4 Results and Discussions

Figure (3) shows the ACF for each radioactive element with different color for each well. The additional horizontal line at 0.37 of ACF axis represents the accepted correlation time (τ_c). Consequently, the ACF of higher τ_c with lag number (k) represents higher long term correlation. For radioactive potassium, τ_c is persist up to $k \approx 45$ for RF-A3 well, while for KK 84-11 well τ_c takes an intermediate value of $k \approx 20$. On the other hand, τ_c for RF-A1 well has a small value of $k \approx 6$. This can make us to draw a rough conclusion of a probably higher thickness of clay layers within the RF-A3 borehole than other wells. For thorium, the ACF takes another pattern where τ_c shows that KK 84-11>RF-A3>RF-A1 at $k \approx 26$, 13, and 6 respectively. The small values of lag number k of thorium compared to potassium may indicate to uncommon distribution of thorium within layers of the three wells. On the contrary, of potassium and thorium, uranium shows a different pattern especially the RF-A1 well. The RF-A1 borehole has significant long-term correlation up to $k \approx 58$, followed by RF-A3 of $k \approx 41$, while the KK 84-11 has $k \approx 31$. The higher values of k for uranium may be an indicator for two reasons. The spread distribution of uranium ores in different rock formations, and/or its absorption within different layers as uranium can be dissolved easily in ground water.

In order to confirm the ACF results, we give the PCA of gamma ray log for the three wells together with depth in figure (4). The PCA relies on singular value decomposition of the correlation matrix C as given in table (1). For statistical significance, two test are applied to the correlation matrix [22]:

1. Kaiser-Meyer-Olkin test: measures the quality of data to be studied by factor analysis. The minimum accepted value is 0.5.

Table 1: The correlation matrix C of the three wells calculated by eq.2.

	Depth	POTA1	THOR1	URAN1	POTA2	THOR2	URAN2	POTA3	THOR3	URAN3
Depth	1	-0.8372	0.16685	-0.6785	-0.2458	0.26632	-0.2516	-0.0088	0.46219	-0.4306
POTA1	-0.8372	1	-0.019	0.58212	0.33754	-0.1876	0.15479	-0.0115	-0.4517	0.50151
THOR1	0.16685	-0.019	1	0.09602	-0.1426	-0.1326	0.09764	-0.1794	0.20001	0.25035
URAN1	-0.6785	0.58212	0.09602	1	-0.1391	-0.1651	0.25639	-0.1775	-0.3154	0.38055
POTA2	-0.2458	0.33754	-0.1426	-0.1391	1	-0.1663	-0.1862	0.29779	-0.2002	0.23285
THOR2	0.26632	-0.1876	-0.1326	-0.1651	-0.1663	1	-0.0603	-0.0579	-0.161	-0.1733
URAN2	-0.2516	0.15479	0.09764	0.25639	-0.1862	-0.0603	1	-0.1108	-0.0592	0.09085
POTA3	-0.0088	-0.0115	-0.1794	-0.1775	0.29779	-0.0579	-0.1108	1	-0.1216	-0.1374
THOR3	0.46219	-0.4517	0.20001	-0.3154	-0.2002	-0.161	-0.0592	-0.1216	1	-0.1898
URAN3	-0.4306	0.50151	0.25035	0.38055	0.23285	-0.1733	0.09085	-0.1374	-0.1898	1

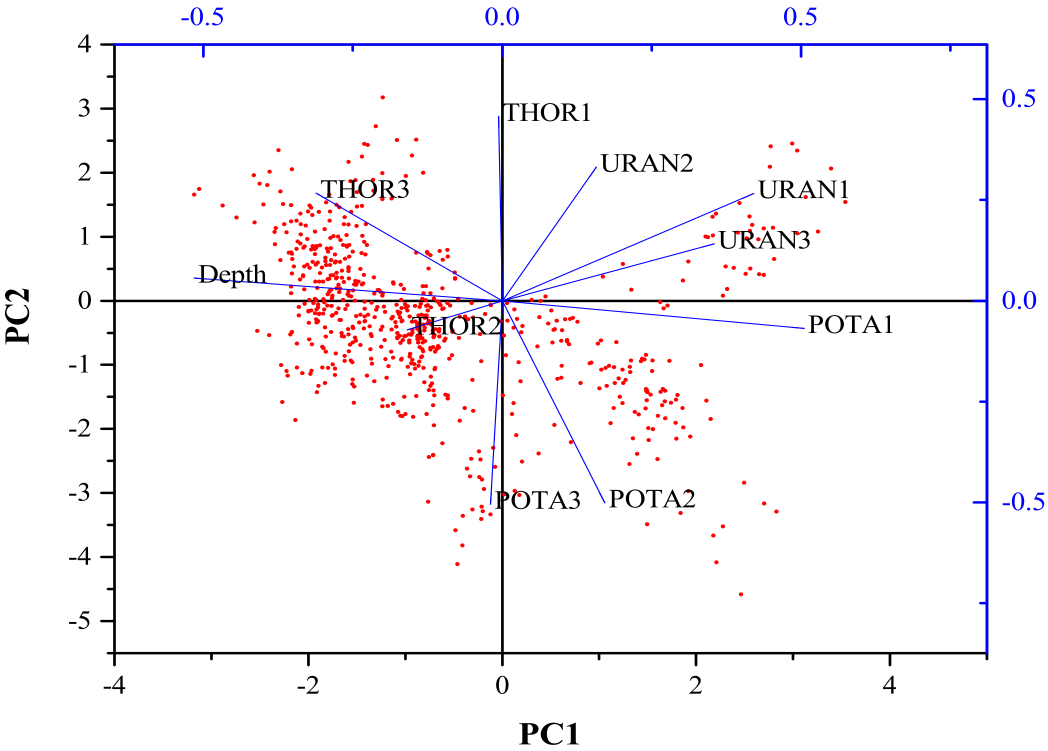

Fig. 4: PCA of Correlation Matrix of Table (1) Between the Three Wells

2.Bartlett's Test of Sphericity: examines the null hypothesis that there are no correlations between variables. We have applied such two tests to our correlation matrix and it passed them.

The calculated PC1, and PC2 of the correlation matrix between the three wells are given in figure (4). Abbreviations are given for each well to facilitate their location on the figure, i.e. numbers 1, 2, and 3 for RF-A1, RF-A3, and KK 84-11 well respectively. The following points can be extracted:

–As a general trend, the radioactive elements of the same type are grouped together independently on the well location. This is clear in potassium on the lower right side, uranium on the upper right side, and thorium on the left side.

–POTA1, URAN1, and URAN3 are negative (anti)correlated with depth, as depth of well increase their concentration in the borehole layers decreases. This may be inferred as soil structure depends on depth, as depth increases the soil density increase and consequently the shale layers – which are less dense– disappear.

–POTA3, and THOR1 are independent on depth, in other words they are not positive or negative correlated to depth.

–The distribution of data points (red points) are more correlated to depth.

– THOR2 is the shortest one of the loadings, accordingly we cannot estimate its values by PC1, and PC2 with acceptable accuracy if we lost some of its logs. On the other hand, POTA1, and URAN1 have higher values of loadings (longest), therefore could be easily estimated with good accuracy.

To the best of our knowledge, there are no sufficient studies that use autocorrelation, and PCA together to explore well-log data as a time series. Subsequently, comparison with previous studies have not been conducted here. Different statistical methods are used recently in literature. The authors of Ref. [23] used non-parametric regression with multivariate analysis approach to predict permeability with different well logs. Guevara *et al*. [24] extended a data-driven sweet spotting technique for shale to predict horizontal well production using vertical well logs using functional Principal Component Analysis. The authors of Ref. [25] used wavelet transform for identification of the main sequence boundaries from well-log data. While Srivardhan [26] applied discrete wavelet transform and multi-scale analysis for detecting stratigraphic interfaces and correlating them between wells. Principal component and cluster analysis are used to detect electrofacies of the Kareem Formation in the Southern Gulf of Suez, Egypt. The identified electrofacies exhibited good correlation with the lithofacies that were elicited from core analysis [27].

5 Perspective

In the current study, two methods of multivariate analysis namely autocorrelation, and PCA are used to inspect gamma ray log for long term correlation, and their cross correlation. The results show that, the uranium gamma log was the common factor among the boreholes under study, and anti-correlated to depth, and can be used as a benchmark in stratigraphic mapping. We would like to stress that, the results obtained here are not self-conclusive, but should be taken into consideration with other petro-physical methods such as Seismic, Neutron porosity, Density, Sonic, photoelectrical absorption, resistivity logs etc. These logs together give us a complete picture of the stratigraphic structure of a field under study.

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