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Modified Particle Swarm Algorithm for Radiation Properties of Semi-transparent Rectangular Material

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Abstract: A modified particle swarm optimization algorithm with a stochastic selection is adopted to estimate the radiation property parameters in a two-dimensional absorbing, emitting, and scattering gray material. The radiative heat transfer in the semitransparent rectangular material is solved by discrete ordinate method. The objective function for the inverse radiation problem is established, and the simulated results of radiation intensities on the material's boundaries are served as inputs. The performance of this algorithm and the effect of measurement errors on the accuracy of estimation are examined. The comparison results show that the proposed algorithm can guarantee the convergence of the global optimization solution, all the radiation properties could be estimated accurately even with noisy data. The algorithm is proved to be fast, robust and useful for researching the radiation properties of the semitransparent material such as crystals in the thermal processing.

Keywords: Swarm optimization algorithm, Semi-transparent Rectangular Material

1 Introduction

Inverse radiation analysis is very practical and useful in many different areas such as laser light-scattering flame diagnosing, remote sensing of the atmosphere, optical tomography in medical imaging, the prediction of the temperature distribution in combustion chambers, optimizing the manufacturing and materials processing system and so on. This subject has been reviewed in a series of papers by McCormick extensively [1]. A number of papers has been reported for determining the absorption coefficient, scattering coefficient, single-scattering albedo, scattering phase function, boundary condition, or the optical thickness of a material from various types of radiation measurements [2,3].

On the other hand, a considerable amount of solution techniques has been employed in these inverse radiation problems. The traditional algorithms such as the conjugate gradient (CG) method [4] and Levenberg–Marquardt method [5] depend on the initial value or the derivatives and gradients which are difficult to be solved accurately by numerical simulation in some cases. The random optimal methods have been studied to solve the global optimal problems reliably such as Genetic Algorithm (GA) [6] and

Particle Swarm Optimization (PSO) algorithm [7].

The PSO algorithm, which was introduced by Eberhart and Kennedy [8], is able to find a global optimum solution or a good approximation of the solution, usually without a theoretical proof. This is solely due to its ability to explore the search domain with 'jump' from a local solution to another and therefore, the global optimum solution can be reached step by step. As reported in Ref. [9], many kinds of problems which can be solved by GA are able to be identically solved by PSO, without suffering from the difficulties of GA's. The PSO algorithm has been studied extensively by many researchers in recent years. But it has proved that the PSO can guarantee convergence, but cannot guarantee global optimum [10].

In order to guarantee the convergence of the global optimum solution, we proposed a modified PSO algorithm with a stochastic selection in this paper. Firstly, the discrete ordinate method (DOM) is used for the simulation of a two-dimensional absorbing, emitting, and scattering material with diffusely emitting and reflecting opaque boundaries. And the directional radiative intensities at walls simulated by DOM are served as input for the inverse analysis. A stochastic particle swarm optimization (SPSO) algorithm is adopted to minimize the objective function and estimate the parameters of radiation system including the wall emissivity, the scattering albedo and the scattering asymmetry factor.

2 Methods

2.1. Inverse Problem

Consider the radiative transfer processes in a two-dimensional absorbing, emitting, and scattering gray material with diffusely emitting and reflecting opaque boundaries. Since there are many numerical methods to solve the radiative heat transfer [11-13], here we apply the discrete ordinates method (DOM) to solve the direct problem. In the DOM, the radiative transfer equations are replaced by a discrete set of equations for a finite number of ordinate directions [14]. In the Cartesian ordinate system, the discrete ordinates equation with the boundary conditions can be written as:

$$\mu^m \frac{\partial I^m}{\partial \tau_x} + \eta^m \frac{\partial I^m}{\partial \tau_y} = -I^m + (1-\omega) I_b + \frac{\omega}{4\pi} \sum_{m'=1}^{M} w^{m'} \Phi^{mm'} I^{m'} \tag{2.1}$$

$$I_w^m = \varepsilon_w I_b(\tau_w) + \frac{1-\varepsilon_w}{\pi} \frac{\omega}{4\pi} \sum_{\mathbf{n_w} \cdot \mathbf{s}' < 0} w^{m'} I_w^{m'} \left| \mathbf{n_w} \cdot \mathbf{s}' \right| \tag{2.2}$$

where subscripts m and m' denote the discrete directions; w is the quadrature weight; τ_x and τ_y are the optical coordinates; the scattering phase function of material is assumed to be linearly anisotropic, $\Phi^{mm'} = 1 + g\left(\mu^m \mu^{m'} + \eta^m \eta^{m'}\right)$, g is the scattering asymmetry factor.

The direct problem of concern here is to find the radiation intensities exiting the boundaries for the known temperature distribution and radiation properties. Here we consider a rectangle enclosure material, the size is assumed to be 1m in length and 0.5m in width. The gray wall emissivity $\varepsilon_w = 0.7$, the scattering albedo and the scattering asymmetry factor are both set to 0.5, both boundary and material temperatures are set to 1000K.

In the inverse problem, the wall emissivity, the scattering albedo and the scattering asymmetry factor are estimated by the measured data of exit radiation intensities. Here we suppose four measurement positions at every boundary's midpoint. The inverse radiation problem can be formulated as an optimization problem. The objective function of the inverse radiation problem in this paper can be written as:

$$F = \sum_N \left\{ \sum_{\mathbf{n_w} \bullet \mathbf{s}^m < 0} w^m \left\{ \left[Y^m\left(\tau_w\right) - I^m\left(\tau_w ; \mathbf{c}\right) \right] \Big/ Y^m\left(\tau_w\right) \right\}^2 \right\} \tag{2.3}$$

where $Y^m\left(\tau_w\right)$ are measured exit radiative intensities at the boundaries; N is the number of sensor locations; $I^m\left(\tau_w ; \mathbf{c}\right)$ is the radiative intensities at boundaries for an estimated vector $\mathbf{c} = \left(\varepsilon_w, \omega, g\right)_{\text{estimated}}$.

2.2. Optimization Method

According to the simple PSO model which was proposed by Eberhart and Kennedy [8], the velocity V_i for each dimension of the i th particle can be updated as follow:

$$V_{i+1} = wV_i + c_1 r_1 \left(P_i - X_i\right) + c_2 r_2 \left(P_g - X_i\right) \tag{2.4}$$

where w is the inertia weight coefficient; c_1 and c_2 are two positive constants called acceleration coefficients; P_i and P_g are the local individual best location and the global best location; r_1 and r_2 are random numbers in the range of [0,1]. The new position X_i for i th particle can be expressed as:

$$X_{i+1} = X_i + V_{i+1} \tag{2.5}$$

In the standard PSO, if $0 < w < 1$ and $V_{i+1} < V_i$, there exists a certain number that when the generation number is more than it, the current global best position P_g of the swarm does not vary, and consequently, all components of V_i will be smaller than a given error, and the particle will stop evolution. Even if a better solution exists in this direction, the particle swarm may stop evolution before finding this position and fall into premature convergence. This is the reason why the standard PSO may fall into local optimum solution.

To solve the problem of premature convergence, we present the SPSO algorithm. Here set the inertia weight $w = 0$, substituting Eq. (4) into Eq. (5), the following equation can be obtained:

$$X_{i+1} = X_i + c_1 r_1 (P_i - X_i) + c_2 r_2 \left(P_g - X_i\right) \tag{2.6}$$

In order to improve the global searching ability, P_g is maintained to be the historic best position, and an extra particle j with the position X_j is generated randomly in the searching domain. In this way, the following updating procedure is obtained:

$$\begin{cases} P_j = X_j \\ P_g = \arg\min\left(P_i, P_j\right) \end{cases} \tag{2.7}$$

This means that if $P_g = P_j$, the random particle *j* locates at the best position and the new random particle will be searched repeatedly, therefore at least one particle is generated in the searching domain randomly to improve the global searching ability.

3 Results and Discussion

3.1. Algorithm's Performances

The performance of the proposed SPSO is investigated in this section through the comparison with the standard GA and PSO methods. The standard PSO uses a linearly varying inertia weight over the generations, varying from 0.9 to 0.4, and $c_1 = c_2 = 2.05$. The c_1 and c_2 of SPSO are set to 2.0. The probabilities of mutation and crossover of GA are set to 0.3 and 0.6. All three methods use 50 as population size. The criterion of termination is the maximum generation number getting 1000.

As shown in Fig.1, the best fitness objective function of SPSO algorithm converges much faster than standard PSO and GA. Moreover, the SPSO algorithm can get the minimum best fitness values among the three methods after a small number of generations. As shown in Fig.2, the SPSO is less time-consuming than PSO of the same swarm size. So, the SPSO algorithm is superior in term of searching quality and derivation of the results.

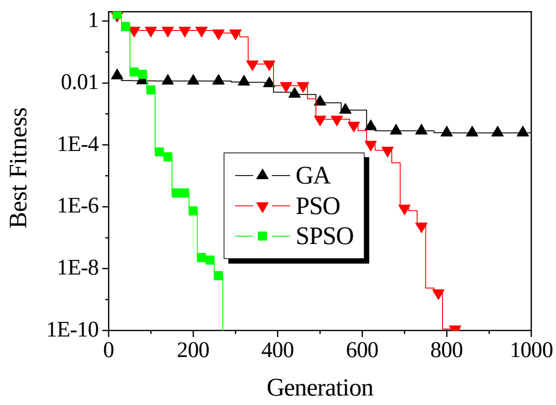

Figure 3.1: Comparison of Best Fitness of GA, PSO and SPSO

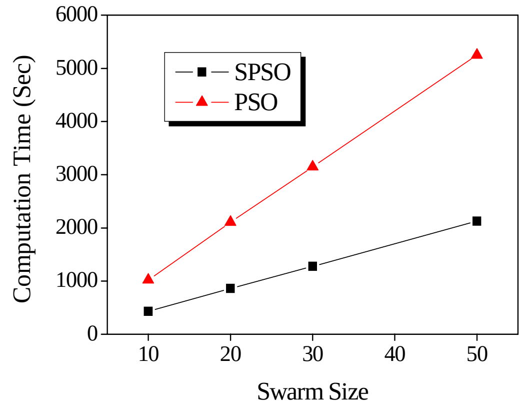

Figure 3.2: Computation Time Comparison of PSO and SPSO for Different Swarm Size

3.2. Estimation Results

The wall emissivity, the scattering albedo and the scattering asymmetry factor are estimated in present study. The search will be terminated if the total number of iterations reaches 10, 000. To demonstrate the effect of measurement errors on the radiative parameters, random standard deviations are added to the exact parameters computed from the direct solution, the following relations have been used in present inverse problem:

$$Y_{measured} = Y_{exact} + \sigma\varsigma \tag{3.1}$$

Here, ς is a normal distribution random variable with zero mean and unit standard deviation. The standard deviations of measured radiative intensities at the boundaries, σ for a γ% measured error at 99% confidence, $\sigma = \dfrac{Y_{exact} \times \gamma\%}{2.576}$. The relative error is defined as follows

$$E_{rel} = 100 \frac{q_{estimated} - q_{exact}}{q_{exact}} \tag{3.2}$$

where q is radiative parameter. Inverse results with different measure errors using SPSO have been shown in *table 1*. Without measurement errors, the relative errors of all parameters become almost negligible. The result indicated that the accuracy in the estimation was sensitive to variations of standard deviations. And the relative error of scattering asymmetry factor increases in measurement errors faster than the other two, this maybe caused by weak positive correlation between the parameter and the radiative intensities.

Table 3.1: Estimated Parameters and Relative Errors for Different Standard Deviations

Para-meter	True value	$\gamma = 0$		$\gamma = 1$		$\gamma = 2$		$\gamma = 3$	
		SPSO	E_{rel}	SPSO	E_{rel}	SPSO	E_{rel}	SPSO	E_{rel}
ε_w	0.7	0.7000	5.91e-6	0.6980	0.30	0.6928	1.03	0.6847	2.19
ω	0.5	0.5000	1.10e-5	0.5008	0.16	0.5025	0.50	0.5042	0.84
g	0.5	0.5000	3.35e-5	0.5058	1.16	0.5175	3.50	0.5290	5.80

4 Conclusions

An inverse radiation problem for estimating the radiative parameters for an absorbing, emitting and scattering gray material with diffusely emitting and reflecting opaque boundaries was investeged. The discrete ordinates method was employed to solve the radiative transfer equation for two-dimensional enclosure. A stochastic particle swarm optimization (SPSO) algorithm is adopted to estimate the wall emissivity, the scattering albedo and the scattering asymmetry factor by minimizing an objective function. For the inverse problem in this paper, it is found that SPSO has fast convergence speed and high computation accuracy. By using SPSO, the wall emissivity, the scattering albedo and the scattering asymmetry factor can be estimated properly even with noisy data. In conclusion, the optimization algorithm SPSO has the potential to be implemented in the field of various inverse radiation problems.

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References

[1] N.J. McCormick, Recent developments in inverse scattering transport method, *Transport. Theor. Stat.*, ***13*** (1984), 15-28.

[2] S. Subramaniam and M.P.Menguc, Solution of the inverse radiation problem for inhomogeneous and anisotropically scattering media using a Monte Carlo technique, *Int. J. Heat Mass Tran.* ***34*** (1991), 253-266.

[3] L.H. Liu, H.P. Tan and Z.H. He, Inverse radiation problem of source term in three-dimensional complicated geometric semitransparent media, *Int. J. Therm. Sci.* ***40*** (2001), 528-538.

[4] H.Y. Li, Inverse radiation problem in two-dimensional rectangular Media, *J. Thermophys. Heat Tr.* ***11***(1997), 556-561.

[5] M.P. Menguc and S. Manickavasagam, Inverse radiation problem in axisymmetric cylindrical scattering media, *J. Thermophys. Heat Tr.* ***7*** (1993), 479-486.

[6] K.W. Kim, S.W. Baek, M.Y. Kim, H.S. Ryou, Estimation of emissivities in a two-dimensional irregular geometry by inverse radiation analysis using hybrid genetic algorithm, *J. Quant. Spectrosc. Ra.* ***87***(2004), 1-14.

[7] Y. Yuan, H.L. Yi, Y. Shuai, F.Q. Wang and H.P. Tan, Inverse problem for particle size distributions of atmospheric aerosols using stochastic particle swarm optimization, *J. Quant. Spectrosc. Ra.* ***111*** (2010), 2106-2114.

[8] J. Kennedy, Review of Engelbrecht's fundamentals of computational swarm intelligence, *Genetic Programming and Evolvable Machines* ***8***(2007), 107-109.

[9] M. Clerc and J. Kennedy, The particle swarm-explosion, stability, and convergence in a multidimensional complex space, *IEEE T Evolut. Comput.* ***6***(2002), 58-73.

[10] F. Van den Bergh, *An analysis of particle swarm optimizers*, PhD thesis, Department of Computer Science, University of Pretoria, South Africa, 2002.

[11] M.F. Modest, *Radiative heat transfer*, Second Edition, Academic Press, Amsterdam, 2003.

[12] Y. Shuai, H.C. Zhang and H.P. Tan, Radiation symmetry test and uncertainty analysis of Monte Carlo method based on radiative exchange factor, *J. Quant. Spectrosc. Ra.* **109** (2008), 1281-1296

[13] Y. Shuai, S.K. Dong and H.P. Tan, Simulation of the infrared radiation characteristics of high temperature exhaust plume including particles using the backward Monte Carlo method, *J. Quant. Spectrosc. Ra.* **95** (2005), 231-240

[14] H. P. Tan, *Numerical calculation of infrared radiation properties and transfer*, Harbin Institute of Technology Press, China, 2006.

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