Estimation in Step-Stress Accelerated Life Tests for Weibull Distribution with Progressive First-Failure Censoring

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Abstract: Based on progressive first-failure censoring, step-stress partially accelerated life tests are considered when the lifetime of a product follows Weibull distribution. The maximum likelihood estimates (MLEs) are obtained for the distribution parameters and the acceleration factor. In addition, asymptotic variance and covariance matrix of the estimators are given. Furthermore, confidence intervals of the estimators are presented. The optimal stress change time for the step-stress partially accelerated life test is determined by minimizing the asymptotic variance of MLEs of the model parameters and the acceleration factor. Simulation results are carried out to study the precision of the MLEs for the parameters involved.

1 Introduction

In reliability analysis, it is not easy to collect lifetimes on highly reliable products with very long lifetimes since very few or even no failures may occur within a limited testing time under normal operating conditions. To obtain failures quickly an accelerated life test (ALT) or partially accelerated life test (PALT) is often used. If all test units are subjected to higher than usual stress levels, then the test is called ALT. But if only some of them are run under severe condition then the test is called PALT. The information obtained from the test performed in the accelerated or partially accelerated environment is used to estimate the failure behavior of the units under normal conditions. The stress loading in an ALT can be applied in different ways. Commonly used methods are constant-stress and step-stress. Nelson [13] discussed the advantages and disadvantages of each of such methods.
In constant-stress ALTs, each unit is run at constant high stress until either failure occurs or the test is terminated. In step-stress ALTs, the stress on each unit is not constant but is increased step by step at prespecified times or upon the occurrence of a fixed number of failures. When a test involves two levels of stress with the first one as the normal one and has a fixed time point for changing stress referred to as a step-stress partially ALT (SSPALT).
Partially accelerated life tests (PALTs) studied under step-stress scheme by several authors, for example, see Goel [8], DeGroot and Goel [7], Bhattacharyya and Soejoeti [6], Bai and Chung [2], Bai, Chung and Chun [3], Abdel-Ghani [1], Ismail and Sarhan [10] and Ismail and Aly [9].
In ALTs or PALTs, tests are often stopped before all units fail. The estimate from the censored data is less accurate than those from complete data. However, this is more than offset by the reduced test time and expense. The most common censoring schemes is type-II censoring. Consider n units placed on life test the experimenter terminates the experiment after a prespecified number of units $m \leq n$ fail. In this scenario, only the smallest lifetimes are observed. Conventional type-II censoring schemes do not allow removal of units at points other than the terminal point of the experiment. A generalization of type-II censoring is the progressive type-II censoring. It is a method which enables an efficient

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exploitation of the available resources by continual removal of a prespecified number of surviving test units at each failure time. On other hand, the removal of units before failure may be intentional to save time and cost or when some items have to be removed for use in another experiment. A recent account on progressive censoring schemes can be found in the book by Balakrishnan and Aggarwala [4]. Balasooriya [5] indicated that in a situation where the lifetime of a product is quite high and test facilities are scarce but test material is relatively cheap, one can test $k \times n$ units by testing n sets, each containing k units. The life test is then conducted by testing each of these sets of units separately until the occurrence of first failure in each set. Such a censoring scheme is called a first-failure censoring scheme. Note that a first-failure censoring scheme is terminated when the first failure in each set is observed. If an experimenter desires to remove some sets of test units before observing the first failures in these sets, the above described scheme will not be of use to the experimenter. The first-failure censoring does not allow for sets to be removed from the test at the points other than the final termination point. However, this allowance be desirable when some sets of the surviving units in the experiment that are removed early can be used for some other tests. As in the case of accidental breakage of experimental units or loss of contact with individuals under study, the loss of test units at points other than the termination point may also be unavoidable. This paper considers a generalized censoring scheme which is progressive first-failure censoring to save more time and cost associated with testing. It allows for some sets of the surviving units to be removed from the test at each failure time. This type of censoring will be described in the next section.

The paper is organized as follows: in section 2, the progressive first-failure censoring scheme is described. In section 3, a description of the model, test procedure and its assumptions are presented. In section 4, the MLEs of the SSPALT model parameters are derived and the asymptotic confidence bounds for the model parameters are constructed based on the asymptotic distribution of MLEs. In section 5, estimation of optimal stress change time is obtained. Section 6, contains the simulation results. Conclusion is made in section 7.

2 A progressive first-failure censoring scheme

In this section, first-failure censoring is combined with progressive censoring scheme as in Wu and Kus [15]. Suppose that n independent groups with k items within each group are put on a life test. R_1 groups and the group in which the first failure is observed are randomly removed from the test as soon as the first failure $Y_{1;m,n,k}^R$ has occurred, R_2 groups and the group in which the second failure is observed are randomly removed from the test as soon as the second failure $Y_{2;m,n,k}^R$ has occurred, and finally when the m-th failure $Y_{m;m,n,k}^R$ is observed, the remaining groups R_m $(m \leq n)$ are removed from the test. Then $Y_{1;m,n,k}^R < Y_{2;m,n,k}^R < ... < Y_{m;m,n,k}^R$ are called progressively first-failure censored order statistics with the progressive censored scheme $R = (R_1, R_2, ..., R_m)$. It is clear that $n = m + \sum_{i=1}^m R_i$. If the failure times of the $n \times k$ items originally in the test are from a continuous population with distribution function $F(y)$ and probability density function $f(y)$, the joint probability density function for $Y_{1;m,n,k}^R, Y_{2;m,n,k}^R, ..., Y_{m;m,n,k}^R$ is given by Wu and Kus [15] as follows:

$$f_{1,2,...,m}(y_{1;m,n,k}^R, y_{2;m,n,k}^R, ..., y_{m;m,n,k}^R) = A \, k^m \prod_{i=1}^m f(y_{i;m,n,k}^R) \left[1 - F(y_{i;m,n,k}^R) \right]^{k(R_i+1)-1},$$
$$0 < y_{1;m,n,k}^R < y_{2;m,n,k}^R < ... < y_{m;m,n,k}^R < \infty, \tag{1}$$

where

$$A = n(n - R_1 - 1)(n - R_1 - R_2 - 2) \, ... \, (n - R_1 - R_2 - ... - R_{m-1} - m + 1).$$

This censoring scheme has advantages in terms of reducing test time, in which more items are used but only m of $n \times k$ items are failures. Note that using the above notation, some censoring rules can be accommodated such as the first-failure censored order statistics when, $R = (0, 0, ..., 0)$, a progressive type-II censored order statistics when $k = 1$, a usual type-II censored order statistics when $k = 1$ and $R = (0, 0, ..., n - m)$, and complete sample case if $k = 1$ and $R = (0, 0, ..., 0)$, with $n = m$. Also, it should be noted that the progressive first-failure censored sample $Y_{1;m,n,k}^R, Y_{2;m,n,k}^R, ..., Y_{m;m,n,k}^R$ with distribution function $F(y)$, can be viewed as a progressive type-II censored sample from a population with distribution function $1 - (1 - F(y))^k$.

3 Model description

3.1 Weibull distribution

Assume that the random variable Y representing the lifetime of a product has Weibull distribution with shape and scale parameters as α and λ respectively. So, the probability density function (pdf) of Y is

$$f(y) = \frac{\alpha}{\lambda}\left(\frac{y}{\lambda}\right)^{\alpha-1} e^{-(y/\lambda)^{\alpha}}, \quad y > 0, \, \alpha > 0, \, \lambda > 0. \tag{2}$$

Weibull distribution is one of the most common distributions which are used to analyze several lifetime data. Its hazard function can be increasing, decreasing and constant depending on the value of the shape parameter.
The distribution function of the Weibull distribution is given by:

$$F(y) = 1 - e^{-(y/\lambda)^{\alpha}}, \tag{3}$$

and the corresponding failure rate function is given by:

$$h(y) = \frac{\alpha}{\lambda}\left(\frac{y}{\lambda}\right)^{\alpha-1}. \tag{4}$$

3.2 Assumptions and test procedure

The following assumptions are used throughout the paper in the framework of SSPALT:

1. n identical and independent groups with k items within each group are put on a life test.
2. The lifetime of each unit has Weibull distribution.
3. The test is terminated at the m-th failure, where m is prefixed ($m \leq n$).
4. Each of the $n \times k$ units is first run under normal use condition. If it does not fail or remove from the test by a prespecified time τ, it is put under accelerated condition.
5. At the i-th failure a random number of the surviving groups R_i, $i = 1, 2, ..., m-1$, and the group in which the failure $Y^R_{i;m,n,k}$ has occurred are randomly selected and removed from the test. Finally, at the m-th failure the remaining surviving groups $R_m = n - m - \sum_{i=1}^{m-1} R_i$ are all removed from the test and the test is terminated.
6. Let n_1 be the number of failures before time τ at normal condition, and n_2 be the number of failures after time τ at stress condition, with these notations the observed progressive first-failure censored data are

$$y^R_{1;m,n,k} < \ ... \ < y^R_{n_1;m,n,k} < \tau < y^R_{n_1+1;m,n,k} < ... < y^R_{m;m,n,k},$$

where $R = (R_1, R_2, ..., R_m)$ and $\sum_{i=1}^{m} R_i = n - m$.
7. The tampered random variable (TRV) model holds. It was proposed by DeGroot and Goel [7]. According to tampered random variable model the lifetime of a unit under SSPALT can be written as:

$$Y = \begin{cases} T, & if \ T \leq \tau, \\ \tau + (T - \tau)/\beta, & if \ T > \tau. \end{cases} \tag{5}$$

Where T is the lifetime of the unit under normal condition, τ is the stress change time and β is the acceleration factor ($\beta > 1$).
8. From the (TRV) model in (5), the pdf of Y under SSPALT can be given by:

$$f(y) = \begin{cases} 0, & y < 0, \\ f_1(y) = \frac{\alpha}{\lambda}\left(\frac{y}{\lambda}\right)^{\alpha-1} e^{-(y/\lambda)^{\alpha}}, & 0 \leq y \leq \tau, \\ f_2(y) = \frac{\beta\alpha}{\lambda}\left(\frac{\beta(y-\tau)+\tau}{\lambda}\right)^{\alpha-1} e^{-\left(\frac{\beta(y-\tau)+\tau}{\lambda}\right)^{\alpha}}, & \tau < y < \infty. \end{cases} \tag{6}$$

Where $f_1(y)$ as given in equation (2) and $f_2(y)$ is obtained by the transformation variable technique by using $f_1(y)$ and the model in (5).

4 Parameters estimation

The idea behind maximum likelihood parameter estimation is to determine the parameters that maximize the probability (likelihood) of the sample data. From a statistical point of view, the method of maximum likelihood is considered to be more robust and yields estimators with good statistical properties. In other words, maximum likelihood methods are versatile and apply to most models and to different types of data. In addition, they provide efficient methods for quantifying uncertainty through confidence bounds. Although the methodology for maximum likelihood estimation is simple, the implementation is mathematically intense. Using todays computer power, however, mathematical complexity is not a big obstacle. Since these estimators do not always exist in closed form, numerical techniques are used to compute them such as Newton Raphson.

This section discusses the process of obtaining point and interval estimations of the parameters α, λ and β based on progressive first-failure censored data under SSPALT.

4.1 Point estimation

In this subsection, the maximum likelihood estimators of the model parameters are obtained. Let $y_i = y^R_{i;m,n,k}$, $i = 1,2,...,m$, be the observed values of the lifetime Y obtained from a progressive first-failure censoring scheme under SSPALT, with censored scheme $R = (R_1, R_2,...,R_m)$. Then the maximum likelihood function of the observations $y_1 < ... < y_{n_1} < \tau < y_{n_1+1} < ... < y_m$, takes the following form

$$L(\alpha, \lambda, \beta) = Ak^m \prod_{i=1}^{n_1} f_1(y_i)\left[1 - F_1(y_i)\right]^{k(R_i+1)-1} \prod_{i=n_1+1}^{m} f_2(y_i)\left[1 - F_2(y_i)\right]^{k(R_i+1)-1}, \tag{7}$$

where A is given by (1). From (6) in (7), we get

$$L(\alpha, \lambda, \beta) = Ak^m \prod_{i=1}^{n_1} \left\{ \frac{\alpha}{\lambda}\left(\frac{y_i}{\lambda}\right)^{\alpha-1} e^{-k(R_i+1)(y_i/\lambda)^{\alpha}} \right\} \times$$
$$\prod_{i=n_1+1}^{m} \left\{ \frac{\beta\alpha}{\lambda}\left(\frac{\beta(y_i-\tau)+\tau}{\lambda}\right)^{\alpha-1} e^{-k(R_i+1)\left(\frac{\beta(y_i-\tau)+\tau}{\lambda}\right)^{\alpha}} \right\}. \tag{8}$$

The log-likelihood function may then be written as:

$$\ell(\alpha, \lambda, \beta) = \log Ak^m + m\log\alpha - \alpha\, m\log\lambda + (m-n_1)\log\beta + (\alpha-1)\sum_{i=1}^{n_1}(R_i+1)y_i^{\alpha}$$
$$+ (\alpha-1)\sum_{i=n_1+1}^{m}\log\left[\beta(y_i-\tau)+\tau\right] - \frac{k}{\lambda^{\alpha}}\sum_{i=n_1+1}^{m}(R_i+1)\left[\beta(y_i-\tau)+\tau\right]^{\alpha}, \tag{9}$$

and thus we have the likelihood equations for α, λ and β respectively as:

$$\frac{\partial\ell(\alpha, \lambda, \beta)}{\partial\alpha} = \frac{m}{\alpha} - m\log\lambda + \sum_{i=1}^{n_1}\log y_i - \frac{k}{\lambda^{\alpha}}\sum_{i=1}^{n_1}(R_i+1)y_i^{\alpha}\log y_i + \frac{k\log\lambda}{\lambda^{\alpha}}\sum_{i=1}^{n_1}(R_i+1)y_i^{\alpha}$$
$$+ \sum_{i=n_1+1}^{m}\log\left[\beta(y_i-\tau)+\tau\right] + \frac{k\log\lambda}{\lambda^{\alpha}}\sum_{i=n_1+1}^{m}(R_i+1)\left[\beta(y_i-\tau)+\tau\right]^{\alpha} \tag{10}$$
$$- \frac{k\log\lambda}{\lambda^{\alpha}}\sum_{i=n_1+1}^{m}(R_i+1)\left[\beta(y_i-\tau)+\tau\right]^{\alpha}\log[\beta(y_i-\tau)+\tau],$$

$$\frac{\partial\ell(\alpha, \lambda, \beta)}{\partial\lambda} = \frac{-\alpha\, m}{\lambda} + \frac{k\,\alpha}{\lambda^{\alpha+1}}\sum_{i=1}^{n_1}(R_i+1)y_i^{\alpha} + \frac{k\alpha}{\lambda^{\alpha+1}}\sum_{i=n_1+1}^{m}(R_i+1)\left[\beta(y_i-\tau)+\tau\right]^{\alpha}, \tag{11}$$

$$\frac{\partial\ell(\alpha, \lambda, \beta)}{\partial\beta} = \frac{(m-n_1)}{\beta} + (\alpha-1)\sum_{i=n_1+1}^{m}\frac{(y_i-\tau)}{\left[\beta(y_i-\tau)+\tau\right]} - \frac{k\,\alpha}{\lambda^{\alpha}}\sum_{i=n_1+1}^{m}(R_i+1)(y_i-\tau)\left[\beta(y_i-\tau)+\tau\right]^{\alpha-1}. \tag{12}$$

Now, we have a system of three nonlinear equations in three unknowns α, λ and β. It is clear that a closed form solution is very difficult to obtain. Therefore, an iterative procedure such as Newton Raphson can be used to find a numerical solution of the above nonlinear system.

4.2 Interval estimation

In this subsection, the approximate confidence intervals of the parameters are derived based on the asymptotic distributions of the MLEs of the elements of the vector of unknown parameters $\Theta = (\alpha, \lambda, \beta)$. It is known that the asymptotic distribution of the MLEs of Θ is given by Miller [12].

$$\left((\hat{\alpha} - \alpha), (\hat{\lambda} - \lambda), (\hat{\beta} - \beta) \right) \mapsto N\left(0, I^{-1}(\alpha, \lambda, \beta)\right),$$

where $I^{-1}(\alpha, \lambda, \beta)$ is the variance-covariance matrix of the unknown parameters $\Theta = (\alpha, \lambda, \beta)$. The elements of the 3×3 matrix I^{-1}, $I_{ij}(\alpha, \lambda, \beta)$, $i, j = 1, 2, 3$; can be approximated by $I_{ij}(\hat{\alpha}, \hat{\lambda}, \hat{\beta})$, where

$$I_{ij}(\hat{\Theta}) = -\frac{\partial^2 \ell(\Theta)}{\partial \theta_i \, \partial \theta_j}\big|_{\theta = \hat{\theta}},$$

from equation (9), we get the following

$$
\begin{aligned}
\frac{\partial^2 \ell(\Theta)}{\partial^2 \alpha} = & \frac{-m}{\alpha^2} - \frac{k}{\lambda^\alpha} \sum_{i=1}^{n_1} (R_i + 1) y_i^\alpha (\log y_i)^2 + \frac{2k \log \lambda}{\lambda^\alpha} \sum_{i=1}^{n_1} (R_i + 1) y_i^\alpha \log y_i - \frac{k (\log \lambda)^2}{\lambda^\alpha} \\
& \times \sum_{i=1}^{n_1} (R_i + 1) y_i^\alpha - \frac{k}{\lambda^\alpha} \sum_{i=n_1+1}^{m} (R_i + 1) \left[\beta(y_i - \tau) + \tau\right]^\alpha \left(\log\left[\beta(y_i - \tau) + \tau\right]\right)^2 \\
& + \frac{2k \log \lambda}{\lambda^\alpha} \sum_{i=n_1+1}^{m} (R_i + 1) \left[\beta(y_i - \tau) + \tau\right]^\alpha \log\left[\beta(y_i - \tau) + \tau\right] - \frac{k}{\lambda^\alpha}(\log \lambda)^2 \\
& \times \sum_{i=n_1+1}^{m} (R_i + 1) \left[\beta(y_i - \tau) + \tau\right]^\alpha,
\end{aligned}
\tag{13}
$$

$$\frac{\partial^2 \ell(\Theta)}{\partial^2 \lambda} = \frac{\alpha m}{\lambda^2} - \frac{k\alpha(\alpha+1)}{\lambda^{\alpha+2}} \sum_{i=1}^{n_1} (R_i + 1) y_i^\alpha - \frac{k\alpha(\alpha+1)}{\lambda^{\alpha+2}} (R_i + 1) \left[\beta(y_i - \tau) + \tau\right]^\alpha, \tag{14}$$

$$
\begin{aligned}
\frac{\partial^2 \ell(\Theta)}{\partial^2 \beta} = & -\frac{(m - n_1)}{\beta^2} - \frac{k\alpha(\alpha-1)}{\lambda^\alpha} \sum_{i=n_1+1}^{m} (R_i + 1)(y_i - \tau)^2 \left[\beta(y_i - \tau) + \tau\right]^{\alpha-2} \\
& - (\alpha - 1) \sum_{i=n_1+1}^{m} \frac{(y_i - \tau)^2}{\left[\beta(y_i - \tau) + \tau\right]^2},
\end{aligned}
\tag{15}
$$

$$
\begin{aligned}
\frac{\partial^2 \ell(\Theta)}{\partial \alpha \partial \lambda} = & -\frac{m}{\lambda} + \frac{k\alpha}{\lambda^{\alpha+1}} \sum_{i=1}^{n_1} (R_i + 1) y_i^\alpha \log y_i - \frac{k\alpha \log \lambda}{\lambda^{\alpha+1}} \sum_{i=1}^{n_1} (R_i + 1) y_i^\alpha \\
& + \frac{k}{\lambda^{\alpha+1}} \sum_{i=1}^{n_1} (R_i + 1) y_i^\alpha + \frac{k}{\lambda^{\alpha+1}} (1 - \alpha \log \lambda) \sum_{i=n_1+1}^{m} (R_i + 1) \left[\beta(y_i - \tau) + \tau\right]^\alpha \\
& + \frac{k\alpha}{\lambda^{\alpha+1}} \sum_{i=n_1+1}^{m} (R_i + 1) \left[\beta(y_i - \tau) + \tau\right]^\alpha \log\left[\beta(y_i - \tau) + \tau\right],
\end{aligned}
\tag{16}
$$

$$
\begin{aligned}
\frac{\partial^2 \ell(\Theta)}{\partial \beta \partial \alpha} = & \sum_{i=n_1+1}^{m} \frac{(y_i - \tau)}{\beta(y_i - \tau) + \tau} + \frac{k(\alpha \log \lambda - 1)}{\lambda^\alpha} \sum_{i=n_1+1}^{m} (R_i + 1)(y_i - \tau) \left[\beta(y_i - \tau) + \tau\right]^{\alpha-1} \\
& - \frac{k\alpha}{\lambda^\alpha} \sum_{i=n_1+1}^{m} (R_i + 1)(y_i - \tau) \left[\beta(y_i - \tau) + \tau\right]^{\alpha-1} \log\left[\beta(y_i - \tau) + \tau\right],
\end{aligned}
\tag{17}
$$

and

$$\frac{\partial^2 \ell(\Theta)}{\partial \lambda \partial \beta} = \frac{k\alpha^2}{\lambda^{\alpha+1}} \sum_{i=n_1+1}^{m} (R_i + 1)(y_i - \tau) \left[\beta(y_i - \tau) + \tau\right]^{\alpha-1}. \tag{18}$$

4.2.1 Approximate confidence intervals

For large value of effective sample size m, the approximate $100(1-\gamma)\%$ two sided confidence intervals for α, λ and β are respectively given by

$$\left[\underset{\sim}{\alpha}, \widetilde{\alpha}\right] = \left[\hat{\alpha} \pm Z_{1-\frac{\gamma}{2}} \sqrt{I_{11}^{-1}(\hat{\alpha})}\right], \tag{19}$$

$$\left[\underset{\sim}{\lambda}, \widetilde{\lambda}\right] = \left[\hat{\lambda} \pm Z_{1-\frac{\gamma}{2}} \sqrt{I_{22}^{-1}(\hat{\lambda})}\right], \tag{20}$$

$$\left[\underset{\sim}{\beta}, \widetilde{\beta}\right] = \left[\hat{\beta} \pm Z_{1-\frac{\gamma}{2}} \sqrt{I_{33}^{-1}(\hat{\beta})}\right]. \tag{21}$$

Where Z_q is the $100q-th$ percentile of a standard normal distribution.
The problem with applying normal approximation of the MLE is that when the sample size is small, the normal approximation may be poor. However, a different transformation of the MLE can be used to correct the inadequate performance of the normal approximation. Since the parameters of interest α, λ and β are positive parameters, log-transformation can be considered. Based on the normal approximation of the log-transformed MLE (Meeker and Escobar [11]), the approximate $100(1-\gamma)\%$ confidence interval for α, λ and β are respectively given by

$$\left[\underset{\sim}{\alpha}, \widetilde{\alpha}\right] = \left[\frac{\hat{\alpha}}{exp\left(\frac{Z_{1-\gamma/2}\sqrt{I_{11}^{-1}(\hat{\alpha})}}{\hat{\alpha}}\right)}, \hat{\alpha}\, exp\left(\frac{Z_{1-\gamma/2}\sqrt{I_{11}^{-1}(\hat{\alpha})}}{\hat{\alpha}}\right)\right], \tag{22}$$

$$\left[\underset{\sim}{\lambda}, \widetilde{\lambda}\right] = \left[\frac{\hat{\lambda}}{exp\left(\frac{Z_{1-\gamma/2}\sqrt{I_{11}^{-1}(\hat{\lambda})}}{\hat{\lambda}}\right)}, \hat{\lambda}\, exp\left(\frac{Z_{1-\gamma/2}\sqrt{I_{11}^{-1}(\hat{\lambda})}}{\hat{\lambda}}\right)\right], \tag{23}$$

$$\left[\underset{\sim}{\beta}, \widetilde{\beta}\right] = \left[\frac{\hat{\beta}}{exp\left(\frac{Z_{1-\gamma/2}\sqrt{I_{11}^{-1}(\hat{\beta})}}{\hat{\beta}}\right)}, \hat{\beta}\, exp\left(\frac{Z_{1-\gamma/2}\sqrt{I_{11}^{-1}(\hat{\beta})}}{\hat{\beta}}\right)\right]. \tag{24}$$

5 Estimation of optimal stress change time

In this section, the optimal change stress time τ^* is found by minimizing the asymptotic variance of MLEs of the model parameters and the acceleration factor. The asymptotic variance of $\hat{\alpha}$, $\hat{\lambda}$ and $\hat{\beta}$ is given by the diagonal entries of the inverse of the Fisher information matrix. FindMinimum option of Mathematica 7 is used to find the time τ^* which minimize the asymptotic variance of MLEs of the model parameters and the acceleration factor. Assume that the true values of the population parameters and the acceleration factor are $\alpha = 0.4$, $\lambda = 0.7$ and $\beta = 1.2$, then for $k = 2$, $n = 25$, $m = 10$ and C.S I, the optimal value of τ is obtained by using FindMinimum option of Mathematica 7 is $\tau^* = 1.1261$.

6 Simulation studies

In this section, simulation studies are conducted to investigate the performances of the maximum likelihood estimators (MLEs) in terms of their biases and mean square errors (MSEs) for different choices of n, m, k and τ values. Also, the 99% and 95% asymptotic confidence intervals based on the asymptotic distribution of the MLEs are computed.
Three progressive censoring schemes are considered:

$$scheme\ I: \ R_1 = n - m, R_2 = 0, ..., R_m = 0,$$

$$scheme\ II:\ R_1 = 0, R_2 = 0, ..., R_m = n - m,$$

$$scheme\ III:\begin{cases} R_{\frac{m+1}{2}} = n - m,\ R_i = 0 \ \ \forall\ i \neq \frac{m+1}{2},\ if\ m\ odd \\ \\ R_{\frac{m}{2}} = n - m,\ R_i = 0 \ \ \forall\ i \neq \frac{m}{2}, \qquad if\ m\ even. \end{cases}$$

The estimation procedure is performed according to the following algorithm:

1. Specify the values of n, m, k and τ.
2. Specify the values of the parameters α, λ and β.
3. Generate a random sample of size $n \times k$ from the random variable Y given by equation (5) and sort it. The Weibull random variable can be easily generated. For example, if U represents a uniform random variable from $[0, 1]$, then $Y = -\lambda [ln(1 - U)]^{1/\alpha}$ has Weibull distribution with pdf given by equation (2) if $y \leq \tau$. But if $y > \tau$ then $Y = \tau + \frac{-\lambda [ln(1-U)]^{1/\alpha} - \tau}{\beta}$ has Weibull distribution with pdf given by equation (6).
4. Use the model given by equation (6) to generate progressively first-failure censored data for given n, m, the set of data can be considered:

$$y^R_{1;m,n,k} < \ ... \ < y^R_{n_1;m,n,k} < \tau < y^R_{n_1+1;m,n,k} < ... < y^R_{m;m,n,k},$$

where $R = (R_1, R_2, ..., R_m)$ and $\sum_{i=1}^{m} R_i = n - m$.
5. Use the progressive first failure censored data to compute the MLEs of the model parameters. The Newton Raphson method is applied for solving the nonlinear system to obtain the MLEs of the parameters.
6. Replicate the steps $3 - 5$, 1000 times.
7. Compute the average values of biases and the mean square errors (MSEs) associated with the MLEs of the parameters.
8. Estimate the asymptotic variances of the estimators of model parameters.
9. Compute the approximate confidence bounds with confidence levels 95% and 99% for the three parameters of the model.
10. Steps 1-9 are done with different values of n, m, k and τ.

Table 1
Average values of the biases of MLES of the parameters, the associated MSEs and the associated approximate confidence intervals based on 1000 simulations, when population parameters values: $\alpha = 0.4$, $\lambda = 0.7$, $\beta = 1.2$ and stress change time $\tau = 2$.

k	n	m	C.S	Bias of $\hat{\alpha}$	MSE of $\hat{\alpha}$	Bias of $\hat{\lambda}$	MSE of $\hat{\lambda}$	Bias of $\hat{\beta}$	MSE of $\hat{\beta}$	99% CI of $\hat{\alpha}$	95% CI of $\hat{\alpha}$	99% CI of $\hat{\lambda}$	95% CI of $\hat{\lambda}$	99% CI of $\hat{\beta}$	95% CI of $\hat{\beta}$
1	25	15	I	0.2190	0.0480	0.4998	0.2498	0.1116	0.0191	0.324, 1.179	0.379, 1.011	0.654, 2.113	0.779, 1.846	0.483, 3.558	0.613, 2.803
			II	0.1205	0.0344	0.4279	0.2471	−0.0764	0.1885	0.197, 1.370	0.249, 1.087	0.502, 2.395	0.634, 2.000	0.285, 4.426	0.395, 3.190
			III	0.1885	0.0386	0.4806	0.2360	−0.0757	0.0395	0.292, 1.183	0.345, 1.001	0.617, 2.164	0.744, 1.872	0.438, 2.883	0.548, 2.302
	50	20	I	0.0157	0.0112	0.1332	0.0774	0.0490	0.0102	0.107, 1.612	0.148, 1.166	0.206, 3.069	0.308, 2.248	0.447, 3.487	0.571, 2.729
			II	−0.1071	0.0158	0.3733	0.1467	0.1923	0.1112	0.131, 0.651	0.159, 0.538	0.648, 1.717	0.750, 1.535	0.726, 2.669	0.848, 2.285
			III	−0.0525	0.0056	0.0504	0.0371	0.1939	0.0384	0.106, 1.131	0.141, 0.853	0.205, 2.510	0.299, 1.881	0.875, 2.219	0.978, 1.986
	75	30	I	0.0274	0.0078	0.1179	0.0392	0.0480	0.0084	0.014, 0.869	0.090, 0.764	0.016, 1.618	0.208, 1.427	0.245, 2.250	0.485, 2.010
			II	−0.0297	0.0107	0.3106	0.1062	0.1617	0.0297	0.065, 0.558	0.123, 0.499	0.421, 1.599	0.562, 1.458	0.541, 2.182	0.737, 1.986
			III	−0.0536	0.0054	0.0406	0.0327	0.1847	0.0343	−0.076, 0.768	0.024, 0.667	−0.191, 1.673	0.030, 1.450	0.834, 1.935	0.965, 1.803
		40	I	0.0499	0.0025	0.1098	0.0120	−0.0214	0.0016	0.199, 0.700	0.259, 0.640	0.331, 1.288	0.445, 1.174	0.350, 2.00	0.548, 1.808
			II	−0.0618	0.0058	0.2080	0.0560	0.1567	0.0257	0.190, 0.485	0.225, 0.450	0.723, 1.092	0.767, 1.048	0.588, 2.125	0.771, 1.941
			III	0.0326	0.0020	0.1096	0.0120	0.0273	0.0076	0.085, 0.779	0.168, 0.696	0.148, 1.471	0.306, 1.313	0.203, 2.251	0.447, 2.006
		50	I	0.0437	0.0019	0.1080	0.0116	0.0104	0.0005	0.264, 0.623	0.307, 0.580	0.402, 1.213	0.499, 1.116	0.511, 1.909	0.678, 1.742
			II	−0.0254	0.0020	0.1032	0.0202	0.1171	0.0229	0.176, 0.572	0.224, 0.524	0.592, 1.013	0.643, 0.963	0.245, 2.389	0.501, 2.133
			III	0.0389	0.0015	0.0978	0.0096	−0.0129	0.0012	0.192, 0.685	0.251, 0.626	0.308, 1.286	0.425, 1.169	0.211, 2.162	0.444, 1.929
2	25	10	I	−0.1984	0.0399	0.2202	0.0805	1.1431	1.3207	0.040, 1.007	0.059, 0.686	0.171, 4.412	0.279, 3.034	0.817, 6.717	1.051, 5.223
			II	−0.0468	0.0035	0.2950	0.0883	0.0027	0.1088	0.166, 0.748	0.199, 0.625	0.588, 1.624	0.685, 1.444	0.480, 3.007	0.598, 2.416
			III	−0.1806	0.0344	0.2144	0.0610	1.2128	1.5402	0.189, 0.253	0.196, 0.245	0.377, 2.090	0.487, 1.715	0.901, 6.455	1.140, 5.103
		20	I	−0.1103	0.0172	0.0929	0.0700	0.4978	0.2722	0.048, 1.723	0.074, 1.125	0.085, 6.323	0.163, 3.851	0.605, 4.764	0.771, 3.723
			II	−0.0685	0.0063	0.2810	0.0862	0.3542	0.1795	0.129, 0.848	0.162, 0.677	0.454, 2.014	0.567, 1.696	0.790, 3.056	0.928, 2.600
			III	−0.1093	0.0146	0.1648	0.0689	0.5328	0.2951	0.055, 1.536	0.081, 1.032	0.115, 5.661	0.206, 3.614	0.690, 4.348	0.860, 3.490

7 Conclusion

Censoring is a common phenomenon in life testing and reliability studies. The subject of progressive censoring has received considerable attention in the past few years, due in part to the availability of high speed computing resources,

Table 1 (continued)

k	n	m	C.S	Bias of $\hat{\alpha}$	MSE of $\hat{\alpha}$	Bias of $\hat{\lambda}$	MSE of $\hat{\lambda}$	Bias of $\hat{\beta}$	MSE of $\hat{\beta}$	99% CI of $\hat{\alpha}$	95% CI of $\hat{\alpha}$	99% CI of $\hat{\lambda}$	95% CI of $\hat{\lambda}$	99% CI of $\hat{\beta}$	95% CI of $\hat{\beta}$
2	50	30	I	−0.1411	0.0256	0.0186	0.0653	0.4746	0.2277	0.036, 0.480	0.089, 0.427	0.062, 1.374	0.219, 1.217	0.670, 2.678	0.910, 2.438
			II	−0.0533	0.0036	0.2914	0.0857	0.2614	0.1180	0.225, 0.467	0.254, 0.439	0.794, 1.187	0.841, 1.140	1.028, 1.894	1.131, 1.791
			III	−0.1047	0.0121	0.1333	0.0431	0.5208	0.2721	0.092, 0.498	0.140, 0.449	0.307, 1.359	0.432, 1.233	0.826, 2.615	1.039, 2.401
		35	I	−0.1239	0.0198	0.0358	0.0519	0.4570	0.2138	0.002, 0.554	0.064, 0.488	0.075, 1.547	0.118, 1.353	0.988, 2.325	1.148, 2.165
			II	−0.0515	0.0032	0.2916	0.0885	0.2852	0.1179	0.266, 0.430	0.286, 0.410	0.668, 1.314	0.745, 1.237	1.387, 1.582	1.410, 1.559
			III	−0.0971	0.0101	0.1284	0.0342	0.4836	0.2351	0.053, 0.552	0.112, 0.492	0.156, 1.500	0.316, 1.340	1.161, 2.205	1.286, 2.080
	75	40	I	−0.1120	0.0142	0.0120	0.0193	0.3999	0.1599	0.114, 0.461	0.155, 0.420	0.304, 1.119	0.401, 1.022	0.738, 2.461	0.944, 2.255
			II	−0.0480	0.0024	0.2975	0.0887	0.2417	0.0971	0.183, 0.520	0.223, 0.480	0.677, 1.318	0.753, 1.241	0.877, 2.006	1.012, 1.871
			III	−0.1045	0.0111	0.0395	0.0040	0.4700	0.2209	0.145, 0.445	0.181, 0.409	0.386, 1.092	0.470, 1.008	0.783, 2.556	0.995, 2.344
		50	I	−0.0634	0.0050	−0.0072	0.0105	0.1995	0.0398	0.064, 0.608	0.129, 0.543	0.144, 1.240	0.275, 1.109	0.782, 2.016	0.929, 1.869
			II	−0.0329	0.0011	0.1144	0.0181	0.1954	0.0385	0.223, 0.511	0.257, 0.476	0.591, 1.036	0.644, 0.983	0.960, 1.830	1.064, 1.726
			III	−0.1066	0.0121	0.0065	0.0079	0.3996	0.1597	0.126, 0.460	0.166, 0.420	0.292, 1.120	0.391, 1.021	0.869, 2.329	1.043, 2.155
		60	I	−0.0502	0.0031	0.0182	0.0087	0.1901	0.0363	0.137, 0.562	0.188, 0.511	0.286, 1.149	0.389, 1.046	1.197, 1.583	1.243, 1.537
			II	−0.0295	0.0009	0.0806	0.0106	0.1884	0.0356	0.238, 0.502	0.269, 0.471	0.602, 0.958	0.645, 0.915	1.185, 1.590	1.275, 1.501
			III	−0.0930	0.0096	−0.0108	0.0071	0.3599	0.1319	0.130, 0.483	0.172, 0.441	0.265, 1.112	0.365, 1.011	1.078, 2.040	1.193, 1.926

Table 2
Average values of the biases of MLES of the parameters, the associated MSEs and the associated approximate confidence intervals based on 1000 simulations, when population parameters values: $\alpha = 0.4$, $\lambda = 0.7$, $\beta = 1.2$ and stress change time $\tau = 3.5$.

k	n	m	C.S	Bias of $\hat{\alpha}$	MSE of $\hat{\alpha}$	Bias of $\hat{\lambda}$	MSE of $\hat{\lambda}$	Bias of $\hat{\beta}$	MSE of $\hat{\beta}$	99% CI of $\hat{\alpha}$	95% CI of $\hat{\alpha}$	99% CI of $\hat{\lambda}$	95% CI of $\hat{\lambda}$	99% CI of $\hat{\beta}$	95% CI of $\hat{\beta}$
1	25	15	I	0.0518	0.0130	0.2158	0.0801	0.2066	0.0767	0.199, 1.024	0.242, 0.842	0.326, 2.398	0.440, 1.905	0.569, 3.474	0.706, 2.799
			II	−0.0678	0.0114	0.2934	0.1514	0.4871	0.2910	0.126, 0.868	0.159, 0.690	0.253, 3.562	0.375, 2.625	0.632, 4.499	0.799, 3.559
			III	−0.0876	0.0175	0.1673	0.0808	0.5231	0.2965	0.108, 0.901	0.139, 0.699	0.172, 3.924	0.274, 2.736	0.695, 4.266	0.864, 3.435
	50	20	I	−0.0167	0.0028	0.0173	0.0184	0.1033	0.0106	0.129, 1.131	0.168, 0.873	0.190, 2.477	0.279, 1.842	0.670, 2.532	0.786, 2.160
			II	0.0151	0.0019	0.3061	0.1122	−0.3422	0.1263	0.268, 0.641	0.298, 0.577	0.894, 1.122	0.925, 1.093	0.625, 1.175	0.674, 1.090
			III	0.0182	0.0017	0.2726	0.0783	0.2544	0.1080	0.209, 0.835	0.246, 0.708	0.519, 1.748	0.622, 1.519	1.010, 2.093	1.102, 1.918
	75	30	I	−0.0269	0.0016	−0.0102	0.0057	0.102	0.0104	0.056, 0.689	0.132, 0.613	0.017, 1.361	0.178, 1.201	0.541, 2.062	0.723, 1.880
			II	0.0130	0.0010	0.2769	0.0805	−0.2993	0.1023	0.249, 0.576	0.288, 0.537	0.776, 1.177	0.824, 1.129	0.680, 1.120	0.733, 1.068
			III	0.0088	0.0007	0.1814	0.0469	0.1649	0.0313	0.026, 0.844	0.077, 0.740	0.117, 1.880	0.120, 1.642	1.107, 1.621	1.168, 1.559
	75	40	I	0.0232	0.0009	0.0612	0.0054	−0.0015	0.0012	−0.413, 1.259	−0.213, 1.060	−0.908, 2.430	−0.509, 2.031	0.295, 2.101	0.510, 1.886
			II	−0.0024	0.0006	0.1982	0.0395	−0.1484	0.0372	0.169, 0.625	0.223, 0.571	0.453, 1.342	0.560, 1.236	0.398, 1.704	0.554, 1.548
			III	−0.0240	0.0030	−0.0431	0.0136	0.0520	0.0088	0.326, 0.960	0.401, 0.884	1.555, 2.784	1.702, 2.637	0.087, 3.387	0.481, 2.993
		50	I	0.0202	0.0004	0.0580	0.0033	0.0065	0.00008	0.008, 1.615	0.185, 1.421	0.437, 4.647	0.939, 4.144	−0.272, 2.926	0.109, 2.544
			II	−0.0067	0.0003	0.1764	0.0337	0.0293	0.0114	1.012, 1.270	1.042, 1.240	2.192, 2.839	2.269, 2.762	0.839, 1.670	0.938, 1.571
			III	0.0312	0.0010	0.0979	0.0095	0.0054	0.0007	0.0006, 0.752	0.089, 0.662	−0.177, 1.491	0.021, 1.292	0.729, 1.774	0.854, 1.649
2	25	10	I	0.0445	0.0023	0.3477	0.1210	0.4259	0.1957	0.226, 0.873	0.265, 0.743	0.587, 1.799	0.694, 1.581	0.607, 4.351	0.768, 3.439
			II	0.0527	0.0039	0.3465	0.1319	−0.3064	0.1419	0.254, 0.805	0.291, 0.702	0.649, 1.633	0.745, 1.468	0.376, 2.119	0.463, 1.724
			III	0.0747	0.0056	0.3998	0.1598	0.2414	0.0999	0.270, 0.832	0.309, 0.728	0.714, 1.642	0.809, 1.492	0.629, 3.300	0.767, 2.707
		20	I	0.0174	0.0022	0.2986	0.0929	0.2526	0.1292	0.088, 1.963	0.128, 1.356	0.197, 4.542	0.315, 3.163	0.544, 3.871	0.688, 3.063
			II	0.04247	0.0024	0.3165	0.1006	−0.1696	0.0612	0.239, 0.818	0.276, 0.706	0.630, 1.587	0.723, 1.427	0.618, 1.716	0.698, 1.519
			III	0.0430	0.0023	0.3161	0.1001	0.0586	0.0516	0.163, 1.202	0.207, 0.947	0.387, 2.502	0.511, 2.017	0.642, 2.464	0.754, 2.098
	50	30	I	0.0077	0.0004	0.2097	0.0499	0.2953	0.0879	0.155, 0.659	0.216, 0.599	0.366, 1.453	0.496, 1.323	0.650, 2.339	0.852, 2.137
			II	0.0440	0.0021	0.2993	0.0897	−0.2082	0.0648	0.211, 0.676	0.266, 0.621	0.544, 1.454	0.653, 1.345	0.586, 1.396	0.683, 1.300
			III	0.0448	0.0021	0.2745	0.0780	0.1356	0.0258	0.159, 0.729	0.227, 0.661	0.392, 1.556	0.531, 1.417	0.647, 2.023	0.812, 1.859
		35	I	0.0081	0.0004	0.1967	0.0398	0.2458	0.0686	0.114, 0.702	0.184, 0.631	0.251, 1.541	0.405, 1.387	0.894, 1.997	1.023, 1.865
			II	0.0310	0.0010	0.2099	0.0440	−0.1320	0.0278	0.266, 0.595	0.305, 0.556	0.642, 1.177	0.706, 1.113	0.914, 1.221	0.951, 1.184
			III	0.0428	0.0019	0.2610	0.0688	0.0792	0.0155	0.142, 0.743	0.214, 0.671	0.344, 1.578	0.491, 1.430	0.981, 1.576	1.052, 1.505
	75	40	I	0.0031	0.00007	0.1379	0.0198	0.2286	0.0523	0.234, 0.571	0.274, 0.531	0.507, 1.168	0.586, 1.089	0.696, 2.161	0.871, 1.986
			II	0.0298	0.0010	0.1954	0.0404	−0.1496	0.0330	0.300, 0.558	0.331, 0.528	0.688, 1.102	0.737, 1.053	0.659, 1.441	0.752, 1.348
			III	0.0243	0.0007	0.1483	0.0252	0.0923	0.0092	0.270, 0.578	0.306, 0.541	0.559, 1.136	0.628, 1.067	0.695, 1.889	0.838, 1.746
		50	I	−0.0166	0.0003	0.0746	0.0067	0.1999	0.0399	0.194, 0.572	0.239, 0.527	0.371, 1.177	0.468, 1.081	0.744, 2.054	0.901, 1.898
			II	0.0291	0.0008	0.2000	0.0400	−0.0684	0.0120	0.283, 0.574	0.318, 0.539	0.651, 1.148	0.710, 1.089	0.863, 1.399	0.927, 1.335
			III	0.0098	0.0001	0.0987	0.0106	0.0939	0.0090	0.231, 0.588	0.273, 0.545	0.441, 1.155	0.527, 1.070	0.768, 1.819	0.89, 1.693
		60	I	−0.0054	0.00008	0.0392	0.0017	0.0800	0.0070	0.194, 0.594	0.242, 0.547	0.325, 1.152	0.424, 1.054	0.924, 1.635	1.009, 1.550
			II	0.0068	0.00004	0.04500	0.0020	−0.0082	0.0002	0.293, 0.519	0.320, 0.492	0.576, 0.913	0.616, 0.873	1.064, 1.318	1.095, 1.288
			III	0.0052	0.00004	0.0582	0.0034	0.0467	0.0028	0.169, 0.640	0.226, 0.584	0.267, 1.249	0.384, 1.132	0.844, 1.647	0.940, 1.551

which makes it both a feasible topic for simulation studies for researchers and a feasible method of gathering lifetime data for practitioners. It has been illustrated by Viveros and Balakrishnan [14] that the inference is feasible and practical when the sample data are gathered according to a type-II progressively censored experimental scheme. Balasooriya [5] indicated that in a situation where the lifetime of a product is quite high and test facilities are scarce but test material is relatively cheap, one can test $k \times n$ units by testing n sets, each containing k units. Such a censoring scheme is called a first-failure censoring scheme. Wu and Kus [15] combined the first-failure censoring with progressive censoring to develop a new life test plan called a progressive first-failure censoring plan.

In this paper, we considered the classical inference procedure for the unknown parameters of the Weibull distribution

(α, λ) and the acceleration factor (β) when the data are progressive first-failure censored from step-stress partially accelerated life tests. It is observed that the maximum likelihood estimators can not be obtained in closed form and we proposed to use the Newton Raphson as an iterative method to compute them. The approximate confidence intervals of the model parameters are also constructed. The calculations are worked out based on different sample sizes $(n \times k)$, different stress change time (τ) and three different progressive censoring schemes (I, II, III). The performances of the estimators are investigated by Monte Carlo simulations and it is observed that they are quite satisfactory. The results shown that the MSEs of the three estimators $\hat{\alpha}$, $\hat{\lambda}$ and $\hat{\beta}$ are decreasing when the sample size is increasing. The MSEs of $\hat{\alpha}$ are less than those of both $\hat{\lambda}$ and $\hat{\beta}$. We also see that as τ increases the MSEs for $\hat{\alpha}$ decrease. On the other hand, for $k = 1$, the MSEs for $\hat{\beta}$ increase as τ increases and the MSEs for $\hat{\lambda}$ decrease as τ increases. But for $k = 2$, the MSEs for $\hat{\beta}$ decrease as τ increases and the MSEs for $\hat{\lambda}$ increase as τ increases. It is hard to decide on which censoring scheme is the beast. In Table 1, by comparing the values of the MSEs of the estimators $\hat{\alpha}$, $\hat{\lambda}$ and $\hat{\beta}$ for each censoring scheme, we conclude that for $k = 1$, the beast censoring scheme for both α and λ is III, and the beast censoring scheme for β is I. For $k = 2$, the beast censoring scheme for both α and β is II, and the beast censoring scheme for λ is III. In Table 2, we conclude that for $k = 1$, the beast censoring scheme for both λ and β is I, and the beast censoring scheme for α is III. For $k = 2$, the beast censoring scheme for both α and λ is I, and the beast censoring scheme for β is III. Finally, for the interval estimation of the three parameters the second scheme (II), in which censoring occurs after the last observed failures, gives lower lengths than the other two schemes except for some few cases. This may be due to fluctuation in data.

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