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Analysis of Statistical and Structural Properties of Complex networks with Random Networks

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Abstract: Random graphs are extensive, in addition, it is used in several functional areas of research, particularly in the field of complex networks. The study of complex networks is a useful and active research areas in science, such as electrical power grids and telecommunication networks, collaboration and citation networks of scientists,protein interaction networks, World-Wide Web and Internet Social networks, etc. A social network is a graph in which n vertices and m edges are selected at random, the vertices represent people and the edges represent relationships between them. In network analysis, the number of properties is defined and studied in the literature to identify the important vertex in a network. Recent studies have focused on statistical and structural properties such as diameter, small world effect, clustering coefficient, centrality measure, modularity, community structure in social networks like Facebook, YouTube, Twitter, etc. In this paper, we first provide a brief introduction to the complex network properties. We then discuss the complex network properties with values expected for random graphs.

Keywords: Random graph, Complex Network properties, Clustering Coefficient, Centrality measure, Modularity, Community structure.

1 Introduction

In network theory, a complex network is a graph (Figure 1). The graph theory was initiated by the mathematician Leonard Euler, who solved the Knigsberg Bridge problem in 1736. Formally, a network is a collection of nodes and links that connect the pairs of nodes. In complex networks, the interacting elements and their interactions are mapped to network nodes and links. The examples of complex networks include social networks, power grids, World Wide Web, biological networks, protein interactions networks, food webs, and neural networks.

The different complex networks that evolve at different speeds, the World Wide Web is far quicker than other complex networks. Almost one-tenth of the world's population spends more time on Facebook and Twitter than any other social network. The analysis and understand the properties of complex network in the real world such as the Internet [1], World Wide Web [2,3], Social networks of connections among individuals [4] attracted much attention in the research area. One of the most significant implication property for complex network is the small-world effect. A node can be reached to any other node in the network through the small number of hops. In the social network, if the chain of acquaintances about six steps from any node (person) to any other node in the network, then the information will reach the node (people) much faster. D. J. Watts and S. H. Strogatz et al published the first mathematical graph theoretic model based on the small-world property [5]. Albert et al provided evidence for the small-world effect of the network of hyperlinks between documents on the World Wide Web [6]. A fundamental measure in complex networksis the clustering coefficient. Clustering coefficient shows that how well the nodes are embedded in their neighborhood. In [7,8,9] the authors examine structural measures such as degree distribution and clustering coefficient in online social networks. The centrality measure defines how important a node is within a network. Cohn and Marriott et al analyze the centrality measure concept in their attempt to understand political integration in the context of the diversity of Indian social life [10]. In late 1940,the first research application of

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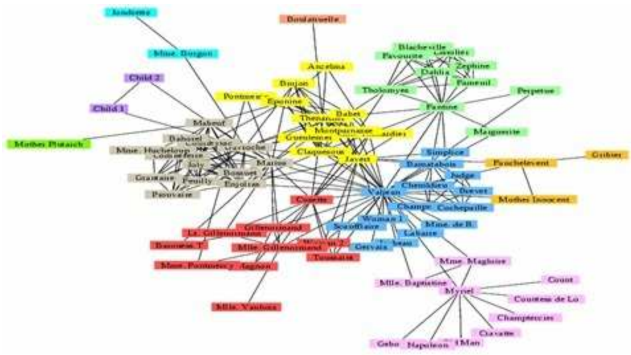

Fig. 1: Les Miserables Novel Complex Network. (M. E. J. Newman1 and M.Girvan, 2003). Nodes represent characters in Les Miserables; Links indicate two characters appeared in the same scene.

centrality measure was made under the guidance of Bavelas at the Group Networks Laboratory, M.I.T. Community structures [11] are an important property of complex networks. In social networks, communities correspond to people with same interest, groups of friends based on similarities in their personal data such as school, location, gender, interest, age etc. In food webs, they may classify compartments. In a citation network communities correspond to related papers on a single topic. In community-aware recommender systems community represents what the users in that community interest or not interested and so on. Many algorithms like Givan-Newman algorithm, modularity maximization, leading eigenvector, walk trap, etc. are used to detect the communities in the networks.

The random graph is one of the most studied models of a network initiated by Hungarian mathematicians Paul Erdos and Alfred Renvi (ER) (Paul Erdos and Alfred 1959; Paul Erdos and Alfred 1960). The ER model is the first model which has attempted to provide an explanation for the behavior of a large real network.The random graphs are usually used as models for real-world networks. A general way to study complex networks is to compare their characteristics with random networks. The Erdos and Renvi model used to generate the random graphs.The Erdos and Renvi published a series of papers about random graph theory in 1959. They introduced two models such as G (n, p) and G (n, m) place the foundation for the theory of random graph generation [12,13]. The first random graph model G (n, p) with n vertices where an edge exists with independent random probability p, values lie between 0¡p¡1(Refer Figure 2). The second random graph model G (n, m) is obtained by sampling uniformly from all graphs with n vertices and m edges (Refer Figure 3). This paper is structured as follows: Section II explains the properties of complex networks. Conclusion and Future work are illustrated in section III.

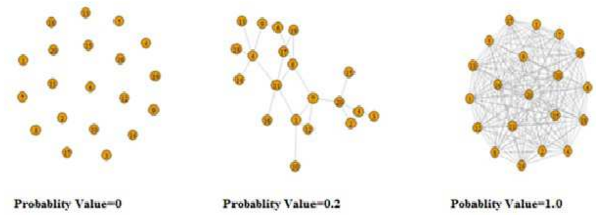

Fig. 2: Schematic illustration of the Erdos-Renyi (ER),G (n, p) model. A random network described by the ER model with probability value 0,0.2 and 1

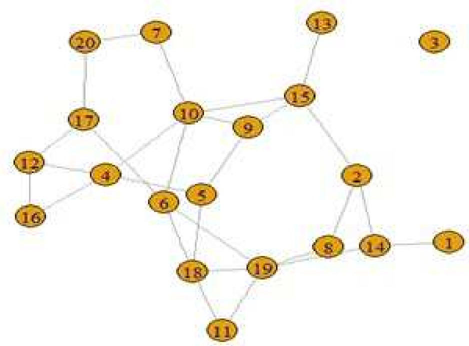

Fig. 3: Schematic illustration of the Erdos-Renyi (ER),G (n, m) model with 28 edges.

2 Properties of Complex Networks

2.1 Small-World Effect property

A network possesses the small-world property in which most nodes are not adjacent to one another, but all nodes can be reached to the other nodes through a small number of hops. In social networks, the average distance from one node to another is small compared to the network size, known as the small-world effect. The size of a network is usually measured by counting the number of nodes in the network. A Small world effect was found in many real-world networks such as biological, technological, and social networks [14], economic networks [15], food web [16], earth sciences [[17] and so on. This property has its roots in experiments carried out by the social psychologist Stanley Milgram in 1960. Milgram decided that the participants to forward a letter to a target person living near Boston, Massachusetts, and distributed them to a randomly selected people in Nebraska with the restriction that each participant could proceed (communicate) the letter only by forwarding it to a single social contact. Over many trials, Milgram found that everyone can be connected by a chain of connections roughly six links long called six degrees of separation [18]. If a person P has n neighbor, and each of Ps neighbor also has n neighbor, then P has about n2 second neighbors. Expand this case P also has n3 third neighbors, n4fourth neighbors and so on. Most people have a hundred and a thousand friends. It is easy to prove that a complex network shows the small-world effect and we can analyze and compare this effect with random graphs.

A network consists ofa set nodes connected by a set of edges. A path (or walk) in a network is a sequence of alternating nodes and edges that start with a node and end with a node, and which does not visit any point more than once such that adjacent nodes and edges in the sequence are incidents to each other. The length of a walk is represented as the number of edges it contains. The shortest path between two nodes is called a geodesic. Note that there may be and often is more than one geodesic path between two vertices in the network. The distance between two vertices is defined as the length of a geodesic that connects them. If there is no path between two nodes, then the distance between them is infinite. The diameter of a network is the largest distance between any two nodes in the network.The two measures such as diameterand mean distance are related to each other.When two measures are related so that if one measure changes (increase/decrease) the other measure also changes (increase/decrease) in such way that the ratio between the two measures remains constant, then the two measures are said to be in a direct variation. The diameter and mean distance measure direct variations because the diameter of the network increases, then the mean distance also increases. Both complex network and the random network exhibit the same behavior (Refer Table 1, 2 and Figure 4, 5).

Definition 1: For a given graph G= (V, E) Where V= (v1, v2, v3vn) is the set of nodes (users), E is the set of edges (connections).

Definition 2: The diameter of a graph denoted by D (G),is defined by

$$D(G) = max_{u \in V(G)^{d(uv)}},$$

Where d(u,v) is the largest distance between the nodes u and v.

2.2 Clustering Coefficient

In the year 1998, Watts and Strogatz et al published a paper in Nature based on the clustering coefficient measure [17]. The cluster is a measure of the likelihood that two associates of a node are associating themselves. A higher clustering coefficient indicates a greater ’cliquishness’. In most real-world networks, if node A is connected to node B and node B is connected to node C, then node A is also connected to node C as well. In a socialnetwork, the friendship between individuals shows that there is a higher probability that a friend of your friend is also your friend. The clustering coefficient value lies in the range $0 \leq C \leq 1$. This measure can be defined in two different ways, such as local clustering coefficient and global clustering coefficient. The clustering coefficient value is zero for star network and one for the fully connected network.Strogatz, and Watts et al proposed the global clustering coefficient for the complex

Table 1: :Diameter and Mean distancefor a number of different networks

S.N	Complex network dataset	Nodes	Edges	Diameter	Mean Distance
1	Dolphin social,network	62	159	8	3.35
2	Zachary’s karate,club	34	78	5	2.41
3	American College,football	115	613	4	2.51
4	Neural network	297	2359	14	3.90
5	Political blogs	1490	19091	9	3.39
6	Word adjacencies	112	425	5	2.53
7	Co-authorships in,network science	1589	2742	17	5.82
8	High-energy theory,collaborations	8361	15751	19	7.02
9	Books about US,politics	105	441	7	3.07
10	Les,Miserable	77	588	5	2.64

Table 2: Diameter and Mean distancefor a number of random networks

S.N	Nodes	Edges	Diameter	Mean Distance
1	62	159	5	2.68
2	34	78	5	2.39
3	115	613	4	2.24
4	297	2359	4	2.35
5	1490	19091	4	2.61
6	112	425	5	2.55
7	1589	2742	13	6.08
8	8361	15751	15	6.91
9	105	441	4	2.39
10	77	588	3	1.83

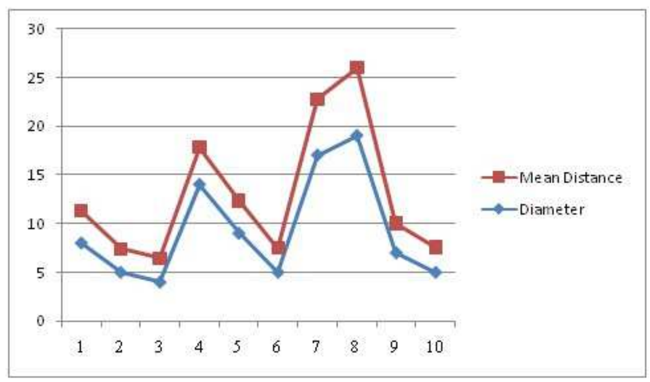

Fig. 4: Comparison of diameter and Mean distancefor a number of different networks

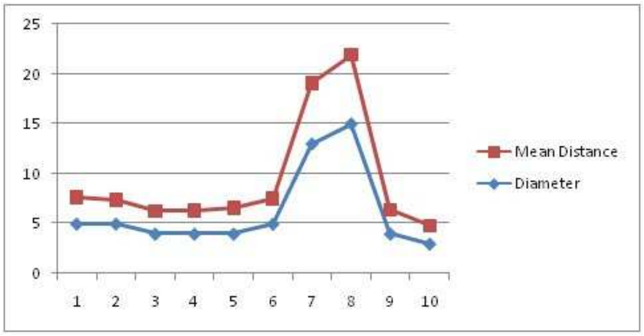

Fig. 5: :Comparison of diameter and mean distancefor a number of random networks

network [5,18].The clustering coefficient values for a number of real-world networks and random graph with the same number of vertices and edges shown in Table 3. The clustering coefficients in the real graphs are significantly larger than in the random graphs (Refer Table-3 and the Figure 7).

Definition 3: A triangle of a graph G is the three node subgraph G_t in which V

$$(G_t) = \{v, v2, v3\} \subset V$$

and

$$E(G_t) = \{\{v1, v2\}, \{v2, v3\}, \{v3, v1\}\} \subset E.$$

A triplet graph G is the three node subgraph Gtr in which in which

$$V(G_{tr} = \{v1, v2, v3\} \subset V$$

and

$$E(G_{tr}) = \{\{v1, v2\}, \{v2, v3\}, \{v3, v1\}\} \subset E.)$$

, where v2 is the center of the triple and dv is the degree of the node v(Refer Figure 6) .

Definition 4:: The local clustering coefficient for a node v denoted by C_v , is defined as the ratio of the number of triangles to the number of connected triplets.Formally,

$$C_v = \frac{2 * G_t}{d_v(d_v - 1).}$$

Definition 5: The average clustering denoted by C_l is defined by

$$C_l = \frac{1}{n} \sum_{v=1}^{n} C_v$$

Definition 6: The global clustering coefficient for a graph G denoted byC_g, is defined by

$$C_g = \frac{2\sum_{v=1}^{n} G_t}{\sum_{v=1}^{n} d_v(d_v - 1)}$$

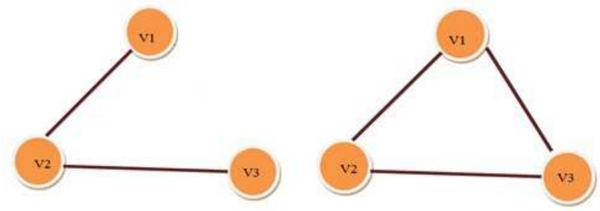

Fig. 6: : A triplet graph and A triangle graph

Table 3: Clustering coefficient for a number of different networks and random networks

S.N	Complex network dataset	Nodes	Edges	$C_{randomnetwork}$	$C_{complexnetwork}$
1	Dolphin social network	62	159	0.068	0.31
2	Zachary's karate club	34	78	0.152	0.26
3	American College football	115	613	0.093	0.41
4	Neural network	297	2359	0.055	0.18
5	Political blogs	1490	19091	0.016	0.23
6	Word adjacencies	112	425	0.072	0.15
7	Co-authorships in network science	1589	2742	0.003	0.69
8	High-energy theory collaborations	8361	15751	0.0003	0.33
9	Books about US politics	105	441	0.089	0.35
10	Les Miserable	77	588	0.190	0.49

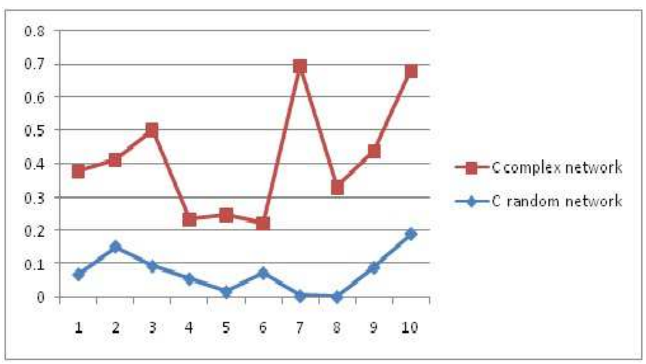

Fig. 7: : Comparison of clustering coefficient for a number of different networks and random networks

2.3 Centrality Measures

Centrality measures are used to analyze the social network and to measure the importance of a node in the network. Many centrality measures proposed to estimate the importance of a vertex or an edge in a network. There are three measures of centrality that are widely used in network analysis: Degree centrality, Closeness, and Betweenness.

2.3.1 Degree centrality

The importance of degree centrality is that the larger degree node is more important in the network. The degree centrality of a node (V_i) is the number of links directly attached to the node in the network and is defined in terms of the adjacency matrix. A graph G = (V, E) represented by means of its adjacency matrix A, in which a given entry $A_{ij} = 1$ if and only if i and j are connected by an edge, and $A_{ij} = 0$ otherwise. The normalized degree centrality values used to compare the values across networks (e.g., Facebook and Twitter).In order to find the normalized degree centrality, divide the degree of a node by N-1, Where N is the number of nodes in the network. The degree centrality of a node v_i denoted by $C_D(V_i)$ is defined by

$$C_D(V_i) = d(V_i) = \sum_{j}^{N} A_{ij}$$

. The normalized degree centrality denoted by $C_{DN}((V_i)$

$$C_{DN}((V_i) = \frac{d(v_i)}{N-1}$$

2.3.2 Closeness centrality

The closeness can be regarded as a measured efficiency as nodes with high values can reach more nodes, spreading information and have relatively easy and fast access to network resources and information. It can be measured by the sum of the shortest paths between a node and to all the other nodes in a network. The normalized closeness centrality values used to compare the values across networks. In order to find the normalized closeness centrality, divide the closeness centrality of a node by N-1, Where N is the number of nodes in the network andg (i,j) is the shortest distance between these two nodes. The closeness centrality of a node v_i denoted by$C_C(V_i)$ is

$$C_C(V_i) = \left[\sum_{j \neq i}^{N} g(ij)\right]^{-1}$$

The normalized closeness centrality denoted by $C_{CN}(V_i)$ denoted by

$$C_{CN}(V_i) = \left[\frac{N-1}{\sum_{j \neq i}^{N} g(ij)}\right]^{-1}$$

2.3.3 Betweenness centrality

The betweenness centrality used to measure the prominence of nodes in the network. The betweenness centrality of a vertex is that counts the number of shortest paths that run through that vertex. If a node lies on a lot of shortest paths between any two nodes, then it has a high betweenness centrality. The node with high betweenness usually has greater control over communication. The betweenness centrality needs to be normalized to be comparable across networks. In order to find the normalized betweenness centrality, divide the betweennesscentrality of a node by $(N-1)(N-2)/2$, Where σ st is the number of shortest paths between s and t, σ st (i) is the number of shortest paths between s and t, that pass through i. The betweenness centrality of a node v_i is denoted by $C_B(V_i)$, isdefined by

$$C_B(V_i) = \sum_{s \neq i \neq t \in V, s<t} \frac{\sigma_{st}(i)}{\sigma_{st}}$$

normalized betweenness centrality denoted by $C_{BN}(V_i)$,is defined by

$$C_{BN}(V_i) = \frac{2 * C_B(V_i)}{(N-1)(N-2)}$$

The centrality measures values(Refer 7)for complex network and random graph are listed in Table 4 and Table 5.

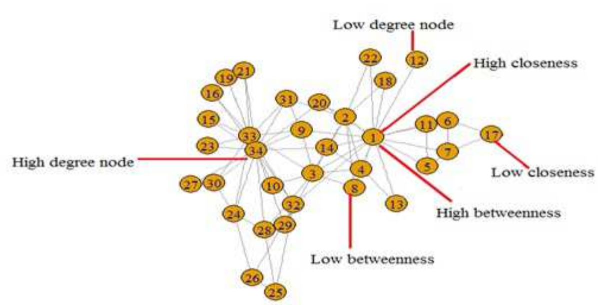

Fig. 8: Centrality Measures for Zacharys network

Table 4: Centrality Measures for a number of different networks

S.N	Network dataset	Nodes	Edges	High C_D	High C_C	High C_B	High C_D Node	High C_C Node	High C_B Node
1	Dolphin social network	62	159	12	0.00684	454.27	15	37	37
2	Zachary's karate club	34	78	17	0.01724	231.07	34	1	1
3	American College football	115	613	12	0.00383	215.99	1	59	83
4	Neural network	297	2359	139	0.00010	9189.27	45	260	178
5	Political blogs	1490	19091	468	1.26*10-6	218480.7	855	293	855
6	Word adjacencies	112	425	49	0.00555	1493.80	18	18	18
7	Co-authorships in network science	1589	2742	34	5.19*10-7	28300.56	34	79	79
8	High-energy theory collaborations	8361	15751	504	4.728*10-8	703646.2	9	31	31
9	Books about US politics	105	441	25	0.00398	747.04	9	31	31
10	Les Miserable	77	254	36	0.00847	1624.46	12	12	12
S.N	**Network dataset**	**Nodes**	**Edges**	**Low CD**	**Low CC**	**Low CB**	**Low CD Node**	**Low CC Node**	**Low CB Node**
1	Dolphin social network	62	159	1	0.00292	0	1	61	**5**
2	Zachary's karate club	34	78	1	0.00862	0	12	17	8
3	American College football	115	613	7	0.00311	19.34	43	109	109
4	Neural network	297	2359	1	1.13*10 -5	0	243	40	40
5	Political blogs	1490	19091	0	4.50*10 -7	0	3	3	3
6	Word adjacencies	112	425	1	0.00240	0	9	96	9
7	Co-authorships in network science	1589	2742	0	3.96*10 -7	0	20	20	1
8	High-energy theory collaborations	8361	15751	2	1.430*10-8	0	11	11	1
9	Books about US politics	105	441	2	0.00231	0	104	35	17
10	Les Miserable	77	254	1	0.00337	0	2	47	2

Table 5: Centrality Measures for a number of Random networks

S.N	Nodes	Edges	High C_D	High C_C	High C_B	High C_D Node	High C_C Node	High C_B Node
1	62	159	10	0.0073	166.15	40	40	40
2	34	78	9	0.0153	83.07	7	32	7
3	115	613	17	0.0044	174.20	1	2	44
4	297	2359	28	0.0016	565.40	122	122	122
5	1490	19091	44	0.0003	3338.64	670	670	1356
6	112	425	14	0.0041	262.57	34	26	34
7	1589	2742	12	1.345*10-5	27299.90	1062	1101	1062
8	8361	15751	14	6.115*10-7	283853	5970	5970	5970
9	105	441	16	0.0046	257.38	94	94	94
10	77	254	21	0.0076	63.57	7	7	18
S.N	**Nodes**	**Edges**	**Low C_D**	**Low C_C**	**Low C_B**	**Low C_D Node**	**Low C_C Node**	**Low C_B Node**
1	62	159	1	0.0043	0	19	19	**14**
2	34	78	1	0.0088	0	5	5	5
3	115	613	3	0.0032	4.64	19	19	19
4	297	2359	7	0.0012	32.45	25	25	25
5	1490	19091	11	0.0002	199.25	1069	1069	1069
6	112	425	2	0.0028	2.21	1	1	1
7	1589	2742	0	3.96*10-7	0	20	20	20
8	8361	15751	0	1.43*10-8	0	27	27	27
9	105	441	1	0.0028	0	38	38	38
10	77	254	8	0.0065	8.15	43	43	43

2.4 Community Structure

The important property that is found in many networks that is community structure.A community in the real world is represented in a graph as a set of nodes that has interacted more with its members. The nodes in the network are tightly connected within the community and loosely connected between other communities. The community structure exists in realworld graphs such as large social networks, web graphs, network science,physics, applied mathematics, and biological networks. The algorithm proposed by Girvan and Newman [18], called GN algorithm used to find the community structure in the complex networks. This algorithm defines the betweenness of an edge is that the number of geodesic distances between pairs of nodes that run through that edge of the network. The modularity function used to evaluate the community division. Modularity means that the links within a community are higher than the expected links in that community.
The modularity can be either positive or negative, the positive values represent the possible presence of community structure and the negative values represent the non-existence of community structure. If the modularity values between 0.3 to 0.7 on the real world network, then it has the strongest community structure. Let A_{ij} be the links between node i and j, the value of A_{ij} is one if there exists a link between i to j otherwise zero,

$$\frac{K_i K_j}{2M}$$

be the expected number of links between i and j,

$$\partial(C_i, C_j)$$

indicates in which community the node i belongs to, e_{ii} is the actual number of links in the community and a_i is the expected number of links for that community. The modularity function denoted by Q, is defined by

$$Q = \frac{1}{2m} \sum_{ij} \left[A_{ij} - \frac{K_i K_j}{2m} \right] \partial(C_i, C_j)$$

and

$$Q = \sum_i e_{ii} - (a_i)^2$$

The number of communities and modularity values for a number of different networks and random graphs are listed in Table 6and Table 7. The community detected Zacharys network shown in Figure 9. The dendrogram for Zacharys network shown in Figure 10, it consists of five communities such as community 1(5, 6, 7, 11, 17 members), community 2 (12, 13, 18, 1, 22, 2, 4, 8, 14, 20 members), community 3 (24, 30, 27, 19, 21, 16, 23, 15, 34, 9, 31, 33 members), community 4 (25, 26, 28, 29, 32, 3 members) and community 5 (10 member).

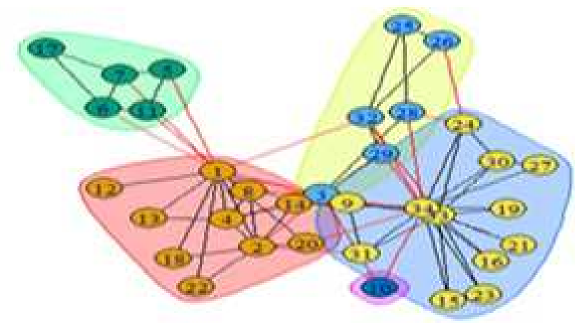

Fig. 9: Community detected inZacharys network

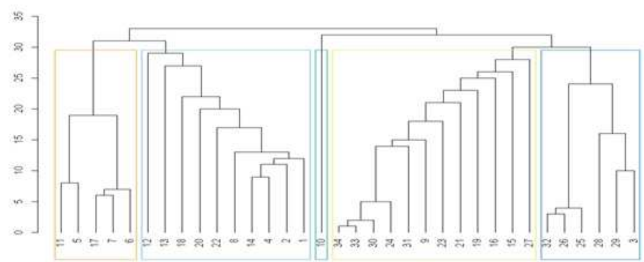

Fig. 10: : Dendrogram(Horizontal (Y-axis) cuts correspond to partitions of the graph in communities,X-axis correspond to the number of nodes in the graph) for Zacharys network

Table 7: Number of communities and modularity for random networks

S.N	Complex network dataset	Nodes	Edges	Number of communities	Modularity value
1	Dolphin social network	62	159	12	0.3
2	Zachary's karate club	34	78	15	0.24
3	American College football	115	613	60	0.11
4	Neural network	297	2359	163	0.075
5	Word adjacencies	112	425	26	0.26
6	Co-authorships in network science	1589	2742	101	0.59
7	Books about US politics	105	441	36	0.19
8	Les Miserable	77	588	40	0.055

2.5 Dataset

2.5.1 Dolphin social network

This dataset contains the network of frequent associations between 62 dolphins in a community living off Doubtful Sound, New Zealand. It contains 159 edges that indicate a frequent association between them.

Table 6: Number of communities and modularity for a number of different networks

S.N	Complex network dataset	Nodes	Edges	Number of communities	Modularity value
1	Dolphin social network	62	159	5	0.52
2	Zachary's karate club	34	78	5	0.4
3	American College football	115	613	10	0.6
4	Neural network	297	2359	194	0.081
5	Word adjacencies	112	425	69	0.081
6	Co-authorships in network science	1589	2742	405	0.96
7	Books about US politics	105	441	5	0.52
8	Les Miserable	77	588	11	0.54

2.5.2 Zachary's karate club

This dataset contains 34 members of a karate club at a US university, as described by Wayne Zachary in 1977. This network contains 78 pairwise links between members.

2.5.3 American College football

This dataset consists of 115 teams considered as a nodesand 613edges correspond to games played by the teams against each other during the regular season of fall 2000.

2.5.4 Neural network

The original neural network experimental data taken from J. G. White, E. Southgate, J. N. Thompson, and S. Brenner, Phil. Trains. R. Soc. London 314, 1-340 (1986).It consists of 297 neurons considered as nodes, and 2359synaptic connections between them are represented by directed edges.

2.5.5 Political blogs

This data set consists of hyperlinks between weblogs on US politics, recorded in 2005 by Adamic and Glance. It consists of 1490 weblogs considered as nodes and 19091hyperlinks between them.

2.5.6 Word adjacencies

The adjacency network of common adjectives and nouns in the novel David Copperfield by Charles Dickens. It consists of 112 nodes that represent the most commonly occurring adjectives and nouns in the book and 425 edges connect any pair of words that occur in adjacent positions in the text of the book.

2.5.7 Co-authorships in network science

The co-authorship network of scientists working on network theory and experiment, as compiled by M. Newman onMay 2006. It contains 1589 scientists and 2742 communication links between them.

2.5.8 High-energy theory collaborations

This data set covers scientific collaborationsbetween scientists posting preprints on the High-Energy Theory E-Print Archive in 1999. It consists of 8361 scientists and 15751collaborations between them.

2.5.9 Books about US politics

A set of books about US politics published around the time of the 2004 presidential election. It consists of 105 books that represent the nodes and 441 edges between books represent frequent co-purchasing of books by the customer from the same merchant.

2.5.10 Les Miserables

The co-occurrences of characters in Victor Hugo's novel 'Les Miserables'. It contains 77charactersthat represent the nodes and the two characters appeared in the same chapter of the book representsan edge between two nodes.

3 Conclusion and Future Work

In this paper, we evaluated several classes of properties of complex networks, namely diameter, small world effect, clustering coefficient, centrality measure, modularity, and community structure. All these properties of complex network are compared with values expected for random graphs. The diameter of the network increases, then the mean distance also increases. The clustering coefficients in the real network are significantly larger than in the random graphs. The low average distance (or diameter) and high clustering have been observed in complex network and random network.Thus, the networks with the largest possible average clustering coefficient are found to have a modular structure.The networks with the largest possible average clustering coefficient have the smallest possible average distance among the different nodes.

A node with a number of neighbors is the most popular node in the network. The degree centrality is the appropriate measure to find the popular node in the network. The most popular person should have the highest number of friends. To obtain information, one should be near from everyone. In this sense, the node at the nearest position on average can most efficiently obtain information. The closeness centrality is anappropriate measure to find the node that is near to all nodes in the network. To control the information flow, a node should be between other nodes because the node can interrupt information flow between them. Thus, the betweenness centrality measure is used to control the information flow in the network. The nodes which have high betweenness centrality are not necessarily the ones that have the most connections (high degree). The highest betweenness node has more influence the node in the network. The low degree and low closeness node almost same in the random graph.

To find the communities within the network is a powerful tool for understanding the function of the network.Only less information available about each user in recommender systems, which results in an inability to describe to recommend items to users.Then the recommender system runs into the cold start problem. The cold start is one of the challenging problems in recommender systems. In future, the present work may be extended to solve the cold-start problem based on the community structure- property.

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