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On the Estimation of $p(y < x < z)$ for Inverse Rayleigh Distribution in the Presence of Outliers

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Abstract: This paper deals with the estimation problem of reliability $R = P(Y < X < Z)$ where X, Y and Z are represent independently inverse Rayleigh distribution with different scale parameters, in presence of k outliers in strength X. The moment, maximum likelihood and mixture estimators of R are derived. An extensive computer simulation is used to compare the performance of the proposed estimators using program Mathcad (14). Simulation study showed that the mixture estimators are better and easier than the maximum likelihood and moment estimators.

Keywords: Inverse Rayleigh distribution, Maximum likelihood estimator, mixture estimator, moment estimator and outliers

1 Introduction

In the field of reliability the stress-strength model describes the life of a component which has a random strength X and is subjected to random stress Y. This problem arises in the classical stress–strength reliability where one is interested in estimating the proportion of the times the random strength X of a component exceeds the random stress Y to which the component is subjected. If X $\leq$ Y, then either the component fails or the system that uses the component may malfunction, and there is no failure when Y<X. The stress-strength models of the types P(Y<X), P(Y<X<Z) have extensive applications in various subareas of engineering, psychology, genetics, clinical trials and so on. [See, Kotz [8]]. The concept of this idea was introduced by Birnbaum [1] and developed by Birnbaum and McCarty [2]. An important particular case is estimation of R= P(Y<X<Z) which represents the situation where the strength X should not only be greater than stress Y but also be smaller than stress Z. For example, many devices cannot function at high temperatures; neither can do at very low ones. Similarly, person's blood pressure has two limits systolic and diastolic and his/her blood pressure should lie within these limits. Chandra and Owen [3] constructed maximum likelihood estimators (MLEs) and uniform minimum unbiased estimators (UMVUEs) for R= P(Y<X< Z). Singh [12] presented the minimum variance unbiased, maximum likelihood and empirical estimators of R= P(Y<X<Z), where X, Y and Z are mutually independent random variables and follow the normal distribution. Hassean, et. al. [7] pressented $R = P(Y < X < Z)$ for Weibull distribution in the presence of outliers. Sinivasa, et. al. [11] presented estimation $R = Z(Y < X)$ for inverse Rayleigh distribution in classical case. Dutta and Sriwastav [4] dealt with the estimation of R when X, Y and Z are exponentially distributed. Ivshin [9] investigated the MLE and UMVUE of R when X, Y and Z are either uniform or exponential random variables with unknown location. The main aim of this article is to focus on the estimation of R= P(Y<X<Z), under the assumption that, X, Y and Z are independent.

The stresses Y and Z have invers Rayleigh distribution with known shape parameters and scale parameters β,γ. While the strength X has inverse Rayleigh distribution with known scale parameter $\sigma_{1,}\ \sigma_2$ in presence of k outliers. Maximum likelihood estimator, moment estimator (ME) and mixture estimator (Mix) are obtained.Monte Carlo simulation is performed for comparing different methods of estimation. The rest of the paper is organized as follows. In Section 2, the estimation of R= P(Y<X< Z) will be derived. The moment estimator of R derived in Section 3. Section 4 discussed MLE of R. The mixture estimator of R is obtained in Section 5.Monte Carlo simulation results are laid out in Section 6. Finally, conclusions are presented in Section 7.

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2 Estimation of $R = P(Y < X < Z)$

This Section deals with estimate the reliability of where Z, Y and X have Inverse Rayleigh distribution in the presence of k outliers in the strength X, such that X, Y and Z are independent. Let $X = (x_1, \cdots, xn_1)$ is the strength of n_1 independent observations such that k of them are distributed inverse Rayleigh with scale parameter(σ) has the following probability density function (pdf),and cumulative distribution (cdf) are respectively given by

$$f(x) = \frac{2\sigma^2}{x^3} e^{-(\frac{\sigma}{x})^2}, x > 0, \sigma > 0 \tag{1}$$

$$F(x) = e^{-(\frac{\sigma}{x})^2}, x > 0, \sigma > 0 \tag{2}$$

Fig. 1 and Fig. 2 gives graphical of (pdf) and (cdf) of inverse Rayleigh distribution for values of $\boldsymbol{\sigma}$ = (1,0.2,0.3,0.4) by using R package program

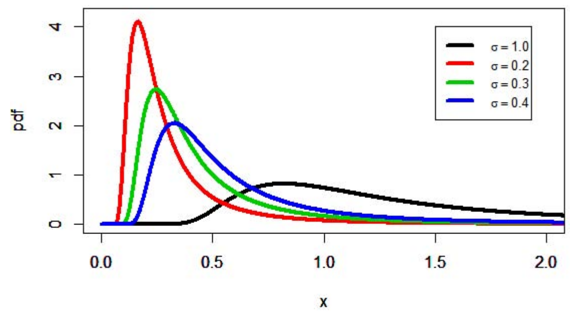

Fig. 1: pdf of inverse Rayleigh distribution for different shapes

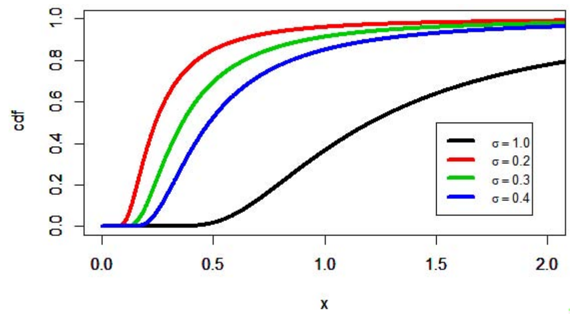

Fig. 2: cdf of inverse Rayleigh distribution for different shapes

Let X represent strength with scale parameter (σ_1) then the probability density function

$$f_1(x) = \frac{2\sigma_1{}^2}{x^3} e^{-(\frac{\sigma_1}{x})^2}, x > 0, \sigma > 0, \tag{3}$$

While the remaining (n_1-k) random variables are distributed inverse Rayleigh with scale parameter (σ_2) and random variable has the distribution

$$f_2(x) = \frac{2\sigma_2{}^2}{x^3} e^{-(\frac{\sigma_2}{x})^2}, x > 0, \sigma > 0 \tag{4}$$

According to Dixit [5] and Dixit and Nasiri [6] The joint distribution of (X_1, X_2,$\cdots$,X_{n1}) in the presence of k outliers can be expressed as:

$$f_x(x, \sigma_1, \sigma_2) = \frac{k}{n} \frac{2\sigma_1^2 e^{-(\frac{\sigma_1}{x})^2}}{x^3} + \frac{n-k}{n} \frac{2\sigma_2^2 e^{-(\frac{\sigma_2}{x})^2}}{x^3}, x, \sigma > 0 \tag{5}$$

The corresponding cumulative distribution function (cdf) represented by:

$$F_X(x;\sigma_1,\sigma_2) = be^{-(\frac{\sigma_1}{x})^2} + \bar{b}e^{-(\frac{\sigma_2}{x})^2}, x>0, \sigma>0,$$
$$where\ b=\frac{k}{n},\ \bar{b}=\frac{n-k}{n}\ and\ b+\bar{b}=1 \tag{6}$$

let Y=($Y_1, Y_2, \ldots, Y_{n2}$) is the stress of independent observations of inverse Rayleigh distribution with scale parameter (γ) then probability density function (pdf):

$$g(y) = \frac{2\gamma^2}{y^3}e^{-(\frac{\gamma}{y})^2};\ y,\gamma>0 \tag{7}$$

In addition, let Z=($Z_1, Z_{2,\ldots} Z_{n3}$) is the stress of n_3 observations of random Z with known scale parameter (β) and has the following pdf distribution probability density function and cumulative distribution are given by

$$h(z) = \frac{2\beta^2}{z^3}e^{-(\frac{\beta}{z})^2}, H(Z) = e^{-(\frac{\beta}{z})^2}; z,\beta>0 \tag{8}$$

and the survival function is:

$$\bar{H} = 1-H(Z) = 1-e^{-(\frac{\beta}{z})^2} \tag{9}$$

According to Singh [12] the reliability R=P(y<x<z) taking the formula:

$$R = p(Y<X<Z) = \int_{-\infty}^{\infty} G_Y(x)\bar{H}_Z(x)f_X(x)\,dx$$

where H_Z(x) is the cdf of Z at X, G_Y(x) is the cdf of Y at X and $\bar{H}_Z$(X) the survival function of Z at X .

Then, the reliability R in the presence of k outliers is given by

$$R = \int_0^{\infty} e^{-(\frac{\gamma}{x})^2}(1-e^{-(\frac{\beta}{x})^2})(b\frac{2\sigma_1{}^2}{x^3}e^{-(\frac{\sigma_1}{x})^2} + \bar{b}\frac{2\sigma_2{}^2}{x^3}e^{-(\frac{\sigma_2}{x})^2})dx$$

$$R = \int_0^{\infty} (e^{-(\frac{\gamma}{x})^2} - e^{-(\frac{\beta}{x})^2-(\frac{\gamma}{x})^2})(b\frac{2\sigma_1{}^2}{x^3}e^{-(\frac{\sigma_1}{x})^2} + \bar{b}\frac{2\sigma_2{}^2}{x^3}e^{-(\frac{\sigma_2}{x})^2})dx$$

$$\begin{aligned} R &= \int_0^{\infty} e^{-(\frac{\gamma}{x})^2}\left(1-e^{-(\frac{\beta}{x})^2}\right)\left(\frac{2b\sigma_1^2}{x^3}e^{-(\frac{\sigma_1}{x})^2} + \frac{2\bar{b}\sigma_2^2}{x^3}e^{-(\frac{\sigma_2}{x})^2}\right)dx \\ &= \int_0^{\infty} \frac{2b\sigma_1^2}{x^3}\left(e^{-\frac{\gamma^2+\sigma_1^2}{x^2}} - e^{-\frac{\gamma^2+\beta^2+\sigma_1^2}{x^2}}\right)dx + \int_0^{\infty} \frac{2\bar{b}\sigma_2^2}{x^3}\left(e^{-\frac{\gamma^2+\sigma_2^2}{x^2}} - e^{-\frac{\gamma^2+\beta^2+\sigma_2^2}{x^2}}\right)dx \\ &= b\sigma_1^2\left(\frac{1}{\gamma^2+\sigma_1^2} - \frac{1}{\gamma^2+\beta^2+\sigma_1^2}\right) + \bar{b}\sigma_2^2\left(\frac{1}{\gamma^2+\sigma_2^2} - \frac{1}{\gamma^2+\beta^2+\sigma_2^2}\right). \end{aligned} \tag{10}$$

3 Moment Estimator

In this Section, the moment estimator of R denoted by R_{ME} will be obtained. The moment estimators $\tilde{\sigma}_1, \tilde{\sigma}_2, \tilde{\beta}, \tilde{\gamma}$ of the unknown parameters σ_1, σ_2, β and will be obtained by equating the population moments with the corresponding sample moments.

The population means of random stress Y and Z are given by:

$$\begin{aligned} &\mu_1'(y) = E(y) = \textstyle\int_0^{\infty} yf(y)dy = \int_0^{\infty}\frac{2y\gamma^2}{y^3}e^{-(\frac{\gamma}{y})^2}dy = 2\gamma^2\int_0^{\infty}\frac{1}{y^2}e^{-(\frac{\gamma}{y})^2}dy = \gamma\sqrt{\pi} \\ &m_1 = \mu_1' \to at\ r=1 \therefore E(y) = \bar{y} = \frac{1}{n}\textstyle\sum_{i=1}^n y_i = \gamma\sqrt{\pi} \\ &\tilde{\gamma} = \frac{\bar{y}}{\sqrt{\pi}} \end{aligned} \tag{11}$$

$$\begin{aligned} &\mu_1'(z) = E(z) = \textstyle\int_0^{\infty} zf(z)dz = \int_0^{\infty}\frac{2z\beta^2}{z^3}e^{-(\frac{\beta}{z})^2}dz = 2\beta^2\int_0^{\infty}\frac{1}{z^2}e^{-(\frac{\beta}{z})^2}dz = \beta\sqrt{\pi} \\ &m_1 = \mu_1' \to at\ r=1 \therefore E(z) = \bar{z} = \frac{1}{n}\textstyle\sum_{i=1}^n z_i = \beta\sqrt{\pi} \\ &\tilde{\beta} = \frac{\bar{z}}{\sqrt{\pi}} \end{aligned} \tag{12}$$

The moment estimator of σ_1,σ_2 at X $\sim$ I.R.D ($\sigma_{1,}$ σ_2) given from the population mean by find the first and second population mean

$$\begin{aligned}
&\mu_1'(x) = E(x) = \textstyle\int_0^\infty xf(x)dx = \\
&\textstyle\int_0^\infty (b\frac{2\sigma_1^2}{x^3}e^{-(\frac{\sigma_1}{x})^2} + \bar{b}\frac{2\sigma_2^2}{x^3}e^{-(\frac{\sigma_2}{x})^2})dx = \\
&\textstyle\int_0^\infty b\frac{2\sigma_1^2}{x^3}e^{-(\frac{\sigma_1}{x})^2}dx + \int_0^\infty \bar{b}\frac{2\sigma_2^2}{x^3}e^{-(\frac{\sigma_2}{x})^2}dx \\
&\mu_1'(x) = E(x) = b\sigma_1\sqrt{\Pi} + \bar{b}\sigma_2\sqrt{\Pi}\tilde{\sigma}_1 = \tfrac{\bar{x}}{b\sqrt{\pi}} - \tfrac{\bar{b}}{b}\tilde{\sigma}_2
\end{aligned} \tag{13}$$

Thus it must be first obtain the moment estimator of $\sigma_{1,}$ σ_2 we find the second moment of this formula

$$\begin{aligned}
&m_2' = b\sigma_1^2 + \bar{b}\sigma_2^2 \\
&\tilde{\sigma}_1^2 = \tfrac{\bar{x}^2}{b\pi} - \tfrac{\bar{b}}{b}\tilde{\sigma}_2^2 - 2\tfrac{\bar{x}\bar{b}}{b\sqrt{\pi}}\tilde{\sigma}_2 + \bar{b}\tilde{\sigma}_2^2 \\
&m_2' = b(\tfrac{\bar{x}^2}{b\pi} - \tfrac{\bar{b}}{b}\tilde{\sigma}_2^2 - 2\tfrac{\bar{x}\bar{b}}{b\sqrt{\pi}}\tilde{\sigma}_2) + \bar{b}\sigma_2^2 \\
&\tilde{\sigma}_2 = (\tfrac{\bar{x}^2}{\pi} - m_2'\tfrac{\sqrt{\pi}}{2\bar{b}\bar{x}})
\end{aligned} \tag{14}$$

and by substitute eqs (14) in (13) we get the moment estimator of $\tilde{\sigma}_1$

$$\tilde{\sigma}_1 = \frac{\bar{x}}{b\sqrt{\pi}} - \frac{\bar{b}}{b}\left[\frac{\bar{x}}{\pi} - m'\frac{\sqrt{\pi}}{\bar{b}}\right] \tag{15}$$

Finally, the moment estimator of R , denoted by R_{ME} is obtained by substitute the moment estimators in (10), therefore R_{ME} takes the following form

$$R_{ME} = b\{\frac{\tilde{\sigma}_1^2}{\tilde{\sigma}_1^2 + \tilde{\gamma}^2} - \frac{\tilde{\sigma}_1^2}{\tilde{\gamma}^2 + \tilde{\beta}^2 + \tilde{\sigma}_1^2}\} + \bar{b}\{\frac{\tilde{\sigma}_2^2}{\tilde{\gamma}^2 + +\tilde{\sigma}_2^2} - \frac{\tilde{\sigma}_2^2}{\tilde{\gamma}^2 + \tilde{\beta}^2 + \tilde{\sigma}_2^2}\} \tag{16}$$

4 Maximum likelihood estimator of R

This Section deals with MLE of reliability R=P(Y<X<Z) when X, Y and Z are independent inverse Rayleigh distribution with parameters γ, β,σ_1 and σ_2 . To compute the MLE for R, firstly the MLEs of R$\hat{\gamma}, \hat{\beta}, \hat{\sigma}_1 and\ \hat{\sigma}_{2,}$, must be obtained. The MLEs of the parameters γ, β, σ_1 and σ_2 are the values which maximize the likelihood function. To obtain the maximum Likelihood estimator of $\hat{\gamma}$ let $Y_1, Y_2, \cdots, Y_n$ be a random sample of size n_2 drawn from inverse Rayleigh distribution with parameter γ the likelihood function of observed sample is given by

$$\ell(\underline{y}, \gamma) = \prod_{i=1}^{n}[g(y_i|\gamma_i \ldots \gamma_{n_2})] = \prod_{i=1}^{n}\left[\frac{2\gamma^2}{y^3}e^{-(\frac{\gamma}{y})^2}\right] = (2\gamma)^{2n_2}\sum_i \frac{1}{y_i^3}e^{-\sum_i(\frac{\gamma}{y_i})^2}$$

$$\log\ell = n_2\log 2 + (2n_2\log\gamma) + \sum_i \log\frac{1}{y_i^3} - \sum_i(\frac{\gamma}{y_i})^2 \frac{\partial\log\ell(y|\gamma)}{\partial\gamma|_{\gamma=\hat{\gamma}}} = \frac{2n_2}{\hat{\gamma}} - 2\hat{\gamma}\sum_i \log\frac{1}{y_i^2} = 0$$

$$\frac{2n_2}{\hat{\gamma}} = 2\hat{\gamma}\sum_i \frac{1}{{y_i}^2}\hat{\gamma}^2 = \frac{n_2}{\sum_i \frac{1}{y_i}}\hat{\gamma} = \sqrt(\frac{n_2}{\sum_i \frac{1}{y_i}}) \tag{17}$$

By the similar way, to obtain the maximum Likelihood estimator of β, let $Z_1, \cdots, Z_{n3}$ be a random sample of size n_3 drawn from inverse Rayleigh distribution with parameters β , then takes the following form

$$\ell(\underline{z}, \beta) = \prod_{i=1}^{n}\left[g(z_i|\beta_i \ldots \beta_{n_3})\right] = \prod_{i=1}^{n}\left[\frac{2\beta^2}{z^3}e^{-(\frac{\beta}{z})^2}\right]$$

$$= (2\beta)^{2n_3}\sum_i \frac{1}{z_i^3}e^{-\sum_i(\frac{\beta}{z_i})^2} \log\ell = n_3\log 2 + (2n_3\log\beta) + \sum_i \log\frac{1}{z_i^3} - \sum_i(\frac{\beta}{z_i})^2 \frac{\partial\log\ell(z|\beta)}{\partial\beta|_{\beta=\hat{\beta}}} = \frac{2n_3}{\hat{\beta}} - 2\hat{\beta}\sum_i \log\frac{1}{{z_i}^2} = 0$$

then

$$\hat{\beta}^2 = \frac{n_3}{\sum_i \frac{1}{z_i}}, \hat{\beta} = \sqrt(\frac{n_3}{\sum_i \frac{1}{z_i}}) \tag{18}$$

to obtain the maximum Likelihood estimator of σ_1 ,σ_2 let $X_1,\ldots,X_{n1}$ be a random sample of size n_1 drawn from inverse Rayleigh distribution with presence of k outliers with parameters σ_1,σ_2

$$\ell(x,\sigma_1,\sigma_2) = \prod_{i=1}^{n}[g(x_i|\sigma_i\ldots\sigma_{n_1})]\,\ell(x,\sigma_1,\sigma_2) = (2b)^{n_1}(\sigma_1)^{2n_1}\sum_{i=1}^{n_1}\frac{1}{x_i^3}e^{-\Sigma(\frac{\sigma_1}{x_i})^2}\Psi(x,\sigma_1,\sigma_2)$$

where

$$\Psi(x,\sigma_1,\sigma_2) = 1+(\frac{\bar{b}}{b})(\frac{\sigma_1}{\sigma_2})^2 e^{-\frac{1}{x_i^2}(\sigma_1^2-\sigma_2^2)}$$

$$\log\ell = 2n_1\log b + (2n_1\log\sigma_1) + \sum_i \log\frac{1}{x_i^3} - \sum_i(\frac{\sigma_1}{x_i})^2\sum_{i=1}^{n_1}\log\Psi(x,\sigma_1,\sigma_2)$$

$$\frac{\partial\log\ell}{\partial\sigma_1} = \frac{2n_1}{\sigma_1} - 2\sigma_1\sum_i\log\frac{1}{x_i{}^2} + \frac{(\frac{2\bar{b}}{\sigma_1 b})(\frac{\sigma_2}{\sigma_1})^2 e^{\frac{1}{x^2}(\sigma_2^2-\sigma_1^2)}[(\frac{\sigma_1}{x_i})^2+1]}{\psi(x_i;\sigma_1,\sigma_2)} \tag{19}$$

$$\frac{\partial\ell}{\partial\sigma_2} = \sum_{i=1}^{n}\frac{e^{\frac{1}{x_1^2}(\sigma_2^2-\sigma_1^2)} - (\frac{\sigma_2}{x_i})^2 e^{\frac{1}{x_1^2}(\sigma_2^2-\sigma_1^2)}}{\psi(x_i;\sigma_1,\sigma_2)} \tag{20}$$

MLE's of σ_1 and σ_1 denoted by $\hat{\sigma}_1,\hat{\sigma}_{2,}$ are solution to the system of equations obtained by setting the partial derivatives of the logarithm of likelihood function (19) and (20) to be zero. Obviously, it is not easy to obtain a closed form solution to this system of equations. Therefore, an iterative method must be applied to solve this equation numerically to estimate σ_1,σ_2 . The MLE of R , denoted by R_{MLE} is obtained by substituting $\hat{\gamma},\hat{\beta},\hat{\sigma}_1 and\ \hat{\sigma}_{2,}\ in(10)$

5 Mixture Estimator of R

In statistics, a mixture model is a probabilistic model for representing the presence of subpopulations within an overall population, without requiring that an observed data set should identify the sub-population to which an individual observation belongs. Formally a mixture model corresponds to the mixture distribution that represents the probability distribution of observations in the overall population. However, while problems associated with "mixture distributions" relate to deriving the properties of the overall population from those of the sub-populations, "mixture models" are used to make statistical inferences about the properties of the sub-populations given only observations on the pooled population, without sub-population identity information which on the population overall.

Some ways of implementing mixture models involve steps that attribute postulated sub-population-identities to individual observations (or weights towards such sub-populations), in which case these can be regarded as types of unsupervised learning or clustering procedures. However, not all inference procedures involve such steps.To avoid the difficulty of complicated in the system of likelihood equations; the mixture estimator of R denoted by R_{MIX} will be obtained. Following Read [10], the mixture estimator of γ,β,σ_1 , and σ_2 denoted by R_{MIX} , and will be derived by mixing between moment estimators and MLEs which are obtained previously in Sections 3 and 4. The mixture estimator of γ ,β can be obtained from moment estimator as follows

$$\tilde{\beta} = \frac{\bar{z}}{\sqrt{\pi}} \tag{21}$$

$$\tilde{\gamma} = \frac{\bar{y}}{\sqrt{\pi}} \tag{22}$$

The mixture estimator of σ_1 ,σ_2 can be obtained from likelihood estimator by substitute as follows

$$\frac{\partial\log\ell}{\partial\sigma_1} = \frac{2n_1}{\sigma_1} - 2\sigma_1\sum_i\log\frac{1}{x_i{}^2} + \frac{(\frac{2\bar{b}}{\sigma_1 b})(\frac{\sigma_2}{\sigma_1})^2 e^{\frac{1}{x^2}(\sigma_2^2-\sigma_1^2)}[(\frac{\sigma_1}{x_i})^2+1]}{\psi(x_i;\sigma_1,\sigma_2)} \tag{23}$$

$$\frac{\partial\ell}{\partial\sigma_2} = \sum_{i=1}^{n}\frac{e^{\frac{1}{x_1^2}(\sigma_2^2-\sigma_1^2)} - (\frac{\sigma_2}{x_i})^2 e^{\frac{1}{x^2}(\sigma_2^2-\sigma_1^2)}}{\psi(x_i;\sigma_1,\sigma_2)} \tag{24}$$

The mixture estimator of R denoted by R_{MIX} will be obtained by substitute $\hat{\gamma},\hat{\beta},\hat{\sigma}_1 and\ \hat{\sigma}_{2,}\ in(10)$

6 Numerical Illustration

In this Section, an extensive numerical investigation will be carried out to compare the performance of the different estimators for different sample sizes and parameter values for inverse Rayleigh distribution in the presence of k outliers. The investigated properties are biases and mean square errors (MSEs).All the computation is performed via Mathcad (14) statistical package. The algorithm for the parameter estimation can be summarized in the following steps:

Step (1): Generate 1000 random samples $X_1,\cdots X_{n1}$, $Y_1,\cdots Y_{n2}$ and $Z_1,\cdots,Z_{n3}$ from inverse Rayleigh distribution with the sample sizes (n_1, n_2, n_3) = (15,15,15),(20,20,20),(25,25,25), (15,15,20), (15,15,25), (15,15,20), (15,25,20), (15,25,25), (20,15,15), (20,15,20), (20,15,25), (20,25,20), (20,25,25), (25,15,15), (25,15,20), (25,15,25), (25,20,20), (25,20,25).

Step (2): The parameter values are selected as K=(1,2), γ=(0.5,1.5), β=(2,0.7) ,σ_1=(1,.05), σ_2=(2,2.5)

Step (3): The moment estimator of β and γ are obtained by solving (11),(12). ME of σ_1 is obtained from (13). The ME of σ_2 is obtained by substitute in (14). Once the estimate of these estimators are computed, then R_{ME} will be obtained using (10).

Step (4): The MLE of β,γ and are obtained from (16) and (17). The nonlinear equations (19) and (20) of the MLEs will be solved iteratively using NetwonRaphson method. The MLE of will be obtained by substitute the MLEs of β, γ, σ_1 and σ_2 in (10).

Step (5): The mixture estimator of β,γ , are computed by using (21), (22). NetwonRaphson method is used for solving

(23) and (24), to obtain the e the mixture estimator of R will be substitute $\hat{\gamma}, \hat{\beta}, \hat{\sigma}_1 and\ \hat{\sigma}_2$ in (10)

Step (6): The performance of the estimators can be evaluated through some measures of accuracy which are Biases and MSEs of R Simulation results are summarized in Tables 1-4 From these Tables, the following conclusions can be observed on the properties of estimated parameters from R.

1. The estimated value of R increases as the value of outliers, increases.

2.The estimated values of R based on moment method is the smallest value, on the other hand the estimated values of based on mixture method is the highest one.

3. The biases of R_{MIX} are the smallest relative to the biases of R_{ME} and R_{ML}

4. Comparing the MSEs of all estimators, the mixture estimators perform the best estimator.

5. The biases and MSEs estimators of R based in different estimators increase as the value of outliers increases in almost all cases expect for some few cases.

7 Conclusions

This article considered the problem of estimating the reliability R=P(Y<X<Z) for the invers Rayleigh distribution with presence of k outliers in strength X. Assuming that, X,Y and Z are independent with common known different scale parameters. The moment, maximum likelihood and mixture estimators of are derived. Performance of estimators is usually evaluated through their biases and MSEs. Comparison study revealed that the mixture estimator works the best with respect to biases and MSEs, so the researcher strongly feels that mixture estimator is better and easy to calculate than the maximum likelihood and moment estimators. In general, the mixture method for estimating R=P(Y<X<Z) of the inverse Rayleigh distribution in the presence of k outliers is suggested to be used.

From table 1: we get that in different parameters when the sample size increase reliability estimation R is avoid to be change and it is be increasing and decreasing within the same average on moment and maximum likelihood but it is a variable in Mixture method and the MSE lest possible so mixture estimation is the best method to estimate R

Table 1: Estimates of R, Biases and MSE's of the point estimates from inverse Rayleigh Distribution, when k=1, γ= 0.5 β= 2 , σ_1= 1 and σ_2= 2

Sample Size	Estimates of R			Bias			MSE		
(n_1,n_2,n_3)	R_{ME}	R_{ML}	R_{Mix}	MLE	Mom	Mix	MLE	Mom	Mix
(15,15,15)	0.362	0.345	0.412	0.025	0.113	-0.003	0.024	0.054	0.006
(20,20,20)	0.345	0.367	0.432	0.086	0.260	-0.031	0.080	0.011	0.005
(25,25,25)	0.328	0.322	0.466	0.084	0.236	-0.035	0.014	0.013	0.018
(15,15,20)	0.365	0.316	0.431	0.019	0.537	-0.095	0.069	0.106	0.009
(15,15,25)	0.311	0.397	0.438	0.066	0.198	-0.014	0.042	0.033	0.007
(15,20,20)	0.311	0.393	0.422	0.075	0.407	-0.080	0.051	0.191	0.005
(15,25,20)	0.333	0.366	0.420	0.073	0.117	-0.001	0.023	0.052	0.002
(15,25,25)	0.373	0.336	0.423	0.084	0.126	-0.061	0.055	0.013	0.022
(20,15,15)	0.365	0.345	0.412	0.024	0.135	-0.010	0.026	0.034	0.007
(20,15,20)	0.356	0.395	0.428	0.086	0.172	-0.019	0.052	0.079	0.063
(20,15,25)	0.323	0.359	0.431	0.049	0.160	-0.013	0.051	0.088	0.042
(20,25,20)	0.367	0.392	0.445	0.098	0.164	-0.063	0.045	0.052	0.005
(20,25,25)	0.342	0.324	0.450	0.002	0.104	-0.051	0.034	0.024	0.011
(25,15,15)	0.327	0.350	0.439	0.061	0.169	-0.039	0.021	0.044	0.002
(25,15,20)	0.311	0.352	0.451	0.016	0.139	-0.018	0.018	0.023	0.010
(25,15,25)	0.333	0.370	0.446	0.006	0.124	-0.077	0.064	0.029	0.001
(25,20,20)	0.356	0.364	0.410	0.033	0.110	-0.019	0.033	0.037	0.034
(25,20,25	0.344	0.385	0.460	0.072	0.149	-0.099	0.034	0.036	0.012

Table 2: Table 2: Estimates of R, Biases and MSE's of the point estimates from inverse Rayleigh Distribution, when k=2 , γ= 0.5 β= 2 , σ_1= 1 and σ_2= 2

Sample Size	Estimates of R			Bias			MSE		
(n_1,n_2,n_3)	R_{ME}	R_{ML}	R_{Mix}	MLE	Mom	Mix	MLE	Mom	Mix
(15,15,15)	0.362	0.335	0.472	0.025	0.123	-0.004	0.034	0.034	0.005
(20,20,20)	0.355	0.377	0.432	0.086	0.240	-0.032	0.050	0.021	0.004
(25,25,25)	0.3248	0.312	0.426	0.084	0.256	-0.034	0.024	0.023	0.013
(15,15,20)	0.325	0.326	0.461	0.019	0.137	-0.093	0.029	0.126	0.002
(15,15,25)	0.391	0.387	0.478	0.067	0.188	-0.012	0.032	0.043	0.001
(15,20,20)	0.371	0.313	0.432	0.074	0.207	-0.080	0.041	0.181	0.004
(15,25,20)	0.343	0.346	0.480	0.078	0.127	-0.021	0.063	0.042	0.001
(15,25,25)	0.393	0.356	0.413	0.086	0.136	-0.051	0.025	0.023	0.032
(20,15,15)	0.355	0.315	0.432	0.023	0.145	-0.050	0.036	0.014	0.007
(20,15,20)	0.326	0.325	0.438	0.082	0.162	-0.089	0.032	0.049	0.073
(20,15,25)	0.363	0.339	0.491	0.049	0.150	-0.023	0.051	0.048	0.042
(20,25,20)	0.387	0.362	0.435	0.096	0.124	-0.043	0.045	0.012	0.005
(20,25,25)	0.362	0.324	0.480	0.032	0.154	-0.021	0.034	0.034	0.011
(25,15,15)	0.347	0.340	0.419	0.051	0.169	-0.049	0.021	0.044	0.002
(25,15,20)	0.331	0.392	0.461	0.046	0.139	-0.018	0.018	0.023	0.010
(25,15,25)	0.323	0.370	0.436	0.036	0.124	-0.077	0.064	0.029	0.001
(25,20,20)	0.356	0.364	0.410	0.033	0.110	-0.019	0.033	0.037	0.034
(25,20,25	0.394	0.385	0.460	0.072	0.149	-0.099	0.034	0.036	0.012

From table 2: we get that in different parameters and changing the outliers observation k when the sample size increase reliability estimation R is avoid to be change and it is be increasing and decreasing within the same average on moment and maximum likelihood but it is a variable in Mixture method and the MSE still possible so mixture estimation is still the best method to estimate R.

Table 3: Estimates of R, Biases and MSE's of the point estimates from inverse Rayleigh Distribution when k=1 , γ= 1.5 β= 0.7 , σ_1= 0.5 and σ_2= 2.5

Sample Size	Estimates of R			Bias			MSE		
(n_1,n_2,n_3)	R_{ME}	R_{ML}	R_{Mix}	MLE	Mom	Mix	MLE	Mom	Mix
(15,15,15)	0.462	0.335	0.672	0.075	0.133	-0.015	0.084	0.074	0.015
(20,20,20)	0.455	0.477	0.632	0.076	0.230	-0.022	0.070	0.081	0.004
(25,25,25)	0.448	0.412	0.626	0.074	0.226	-0.014	0.084	0.093	0.013
(15,15,20)	0.425	0.326	0.661	0.089	0.127	-0.023	0.089	0.176	0.037
(15,15,25)	0.491	0.487	0.678	0.067	0.138	-0.022	0.072	0.083	0.003
(15,20,20)	0.471	0.313	0.632	0.084	0.227	-0.010	0.081	0.191	0.004
(15,25,20)	0.443	0.346	0.680	0.078	0.137	-0.031	0.073	0.072	0.001
(15,25,25)	0.493	0.456	0.613	0.086	0.126	-0.031	0.085	0.083	0.032
(20,15,15)	0.455	0.315	0.632	0.083	0.125	-0.050	0.076	0.084	0.007
(20,15,20)	0.426	0.325	0.638	0.082	0.132	-0.089	0.072	0.079	0.073
(20,15,25)	0.463	0.439	0.691	0.089	0.130	-0.023	0.081	0.088	0.042
(20,25,20)	0.487	0.362	0.635	0.096	0.124	-0.043	0.075	0.092	0.005
(20,25,25)	0.462	0.324	0.680	0.082	0.134	-0.021	0.084	0.074	0.011
(25,15,15)	0.447	0.440	0.619	0.071	0.139	-0.049	0.071	0.084	0.002
(25,15,20)	0.431	0.492	0.661	0.086	0.129	-0.018	0.078	0.093	0.010
(25,15,25)	0.423	0.370	0.636	0.086	0.124	-0.077	0.084	0.079	0.001
(25,20,20)	0.456	0.464	0.610	0.083	0.130	-0.019	0.073	0.087	0.034
(25,20,25	0.494	0.385	0.660	0.072	0.129	-0.099	0.084	0.096	0.012

From table 3: we get within small initial parameters and increasing samples size with first outliers observation k=1 reliability R is increasing with method of moment and mixture but still within average on maximum likelihood method and sill the MSE smallest on mixture method.

Table 4: Estimates of R, Biases and MSE's of the point estimates for inverse Rayleigh Distribution, when k=2 , γ= 1.5 β= 0.7 , σ_1= 0.5 and σ_2= 2.5

Sample Size	Estimates of R			Bias			MSE		
(n_1,n_2,n_3)	R_{ME}	R_{ML}	R_{Mix}	MLE	Mom	Mix	MLE	Mom	Mix
(15,15,15)	0.462	0.435	0.572	0.065	0.143	-0.025	0.064	0.084	0.018
(20,20,20)	0.455	0.477	0.532	0.076	0.230	-0.032	0.080	0.071	0.014
(25,25,25)	0.448	0.412	0.526	0.084	0.236	-0.024	0.074	0.073	0.015
(15,15,20)	0.425	0.426	0.561	0.089	0.147	-0.023	0.069	0.186	0.017
(15,15,25)	0.491	0.487	0.578	0.077	0.148	-0.032	0.062	0.073	0.013
(15,20,20)	0.471	0.413	0.532	0.074	0.237	-0.020	0.081	0.161	0.014
(15,25,20)	0.443	0.446	0.580	0.088	0.127	-0.021	0.063	0.082	0.014
(15,25,25)	0.493	0.456	0.513	0.076	0.136	-0.021	0.065	0.073	0.012
(20,15,15)	0.455	0.415	0.532	0.073	0.135	-0.030	0.076	0.074	0.017
(20,15,20)	0.426	0.425	0.538	0.082	0.122	-0.039	0.062	0.089	0.013
(20,15,25)	0.463	0.439	0.591	0.079	0.120	-0.033	0.081	0.078	0.012
(20,25,20)	0.487	0.462	0.535	0.086	0.134	-0.023	0.075	0.082	0.015
(20,25,25)	0.462	0.424	0.580	0.072	0.124	-0.011	0.084	0.064	0.011
(25,15,15)	0.447	0.440	0.519	0.081	0.129	-0.049	0.071	0.064	0.012
(25,15,20)	0.431	0.492	0.561	0.086	0.139	-0.018	0.078	0.083	0.010
(25,15,25)	0.423	0.470	0.536	0.086	0.134	-0.037	0.084	0.089	0.011
(25,20,20)	0.456	0.464	0.510	0.073	0.120	-0.019	0.073	0.067	0.014
(25,20,25	0.494	0.485	0.560	0.072	0.139	-0.039	0.084	0.066	0.013

From table 4 : we get that using the second observation outliers k=2 and different values of parameters the reliability R is completely changed on three method of estimation with increasing samples size and still the MSE of mixture method is the smallest value between three method.

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