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Prakaamy Quasi Shanker Distribution and its Applications

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Abstract: We have introduced Prakaamy Quasi Shanker Distribution which is formulated as a mixture of one parameter Prakaamy distribution (θ) and Quasi Shanker distribution $(\theta,2)$, where 2 is shape parameter and θ is scale parameter. We have then evolved various necessary statistical properties of Prakaamy Quasi Shanker Distribution. The reliability measures of proposed model are also derived and model parameters have been estimated by maximum likelihood estimation method. Flexibility of proposed model is shown in table 1 and hazard rate graph. We have computed Likelihood ratio statistic for testing the significance of mixing parameter. Finally, an application to real data sets is presented to examine the significance of newly introduced model.

Keywords: Quasi Shanker Distribution, Prakaamy Distribution, Mixture Models, Prakaamy Quasi Shanker Distribution, Mixing Parameter, Structural Properties and Maximum Likelihood Estimation, Applications.

1 Introduction

Modeling of data play an important role in various spheres of life. Probability models are fitted to various real life data sets obtained from different fields. Variety of probability models have been fitted by researchers to real life data. In some situations super population (mixture mode *F*) is a genuine mixture of *v* distinct populations $(F_{1,}F_2...F_v)$. On many occasions real life data may come from v (more than 2) different populations with diverse proportions and main aim of data analyst is to estimate the mixing proportions $(p_1, p_2,...p_v)$ in which in which they occur in the super population. Mixture models find greater applicability in diverse fields like cluster analysis, health, etc. Data analysts use mixture models as these models provide flexibility and cover extra variation from the data. Many researchers have developed mixture models. Everitt (1996) gave an introduction to finite mixture distributions [1]. McLachlan& Peel (2000) developed finite mixture models and obtained its properties and applications [2]. Shukla (2018) formulated Prakaamy distribution and studied its vital properties and applications [3]. Shanker (2015) introduced Akash distribution with properties and applications in real life [4]. Shanker (2016) proposed Sujatha distribution and studied its structural properties and applications [5]. Shanker & Shukla (2017) developed a Quasi Shanker distribution and obtained its applications [6].

A continuous random variable X is said to have a mixture distribution if its p.d.f $f(x)$ is obtained as a mixture of *v* distinct populations having density functions $f_1(x), f_2(x),...fv(x)$ and with mixing proportions $p_1, p_2,...p_v$ respectively. Mathematically

$$f(x) = p_1 f_1(x) + p_2 f_2(x) + ... + p_v f_v(x)$$

Where $0 \le p_i \le 1$ and

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$$\sum_{i=1}^{v} p_i = 1$$

In this paper we have obtained Prakaamy Quasi Shanker distribution as a mixture of Prakaamy distribution and Quasi Shanker distribution. The nature of hazard rate increased applications of proposed mixed model in many real life situations along with increased flexibility in respect of moments motivated us to work on this model.

2 Prakaamy Quasi Shanker Distribution

A non-negative random variable X is said to follow a Prakaamy Quasi Shanker Distribution (PQSD) if its probability density function $f(x)$ can be obtained as a mixture of Prakaamy (θ) distribution with p.d.f $f_1(x)$ and Quasi Shanker $(2,\theta)$ distribution with shape parameter $\alpha = 2$ fixed and scale parameter θ with p.d.f $f_2(x)$. Mathematically

$$f(x) = pf_1(x) + (1-p)f_2(x) \tag{2.1}$$

Where p is a mixing parameter and

$$f_1(x) = \frac{\theta^6}{(\theta^5+120)}\left(1+x^5\right)e^{-\theta x} \qquad x>0,\theta>0 \tag{2.2}$$

is the p.d.f of Prakaamy (θ) distribution with the corresponding c.d.f $F_1(x)$ given below

$$F_1(x) = \frac{1}{(\theta^5+120)}\left(\theta^5(1-e^{-\theta x}) + \gamma(6,\theta x)\right) \qquad x>0,\theta>0 \tag{2.3}$$

Where $\gamma(s,x) = \int_0^x t^{s-1}e^{-t}\,dt$ is a lower incomplete gamma function.

And

$$f_2(x) = \frac{\theta^3}{(\theta^3+\theta+4)}\left(\theta+x+2x^2\right)e^{-\theta x} \qquad x>0,\theta>0 \tag{2.4}$$

is the p.d.f of Quasi Shanker $(2,\theta)$ distribution with corresponding c.d.f $F_2(x)$ given below

$$F_2(x) = \left[1-\left\{1+\frac{2\theta^2x^2+\theta x(\theta+4)}{(\theta^3+\theta+4)}\right\}e^{-\theta x}\right] \qquad x>0,\theta>0 \tag{2.5}$$

Putting the values of $f_1(x)$ and $f_2(x)$ in (2.1) we obtain the p.d.f of Prakaamy Quasi Shanker distribution $f(x)$ as

$$f(x) = \left[\frac{p\theta^6}{(\theta^5+120)}\left(1+x^5\right)e^{-\theta x} + \frac{(1-p)\theta^3}{(\theta^3+\theta+4)}\left(\theta+x+2x^2\right)e^{-\theta x}\right], \qquad x>0,\theta>0, 0\le p\le 1 \tag{2.6}$$

The probability density function graphs of Prakaamy Quasi Shanker distribution are given below

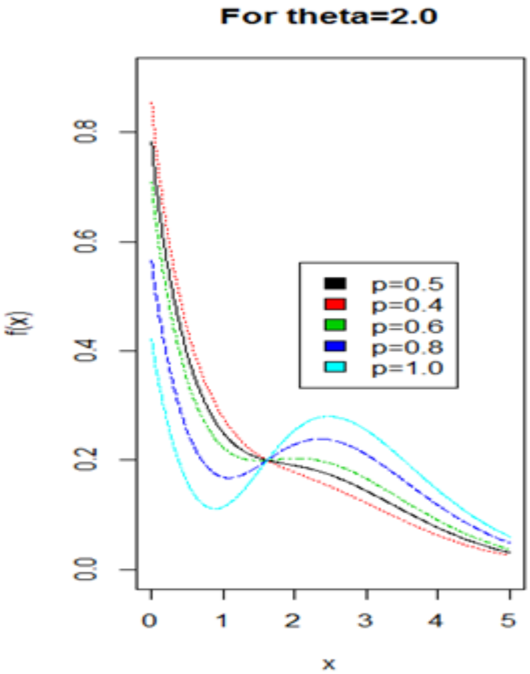

Fig.1(a): Graph of density function.

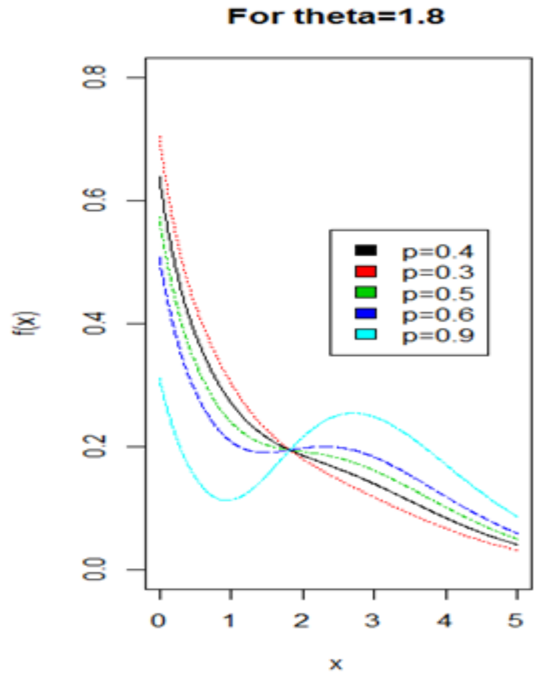

Fig.1(b): Graph of density function.

The corresponding c.d.f of Prakaamy Quasi Shanker distribution is obtained by using (2.3) and (2.5) as

$$F(x)=\left[p\left(\frac{1}{(\theta^{5}+120)}\left(\theta^{5}(1-e^{-\theta x})+\gamma(6,\theta x)\right)\right)+(1-p)\left[1-\left\{1+\frac{2\theta^{2}x^{2}+\theta x(\theta+4)}{(\theta^{3}+\theta+4)}\right\}e^{-\theta x}\right]\right] \tag{2.7}$$

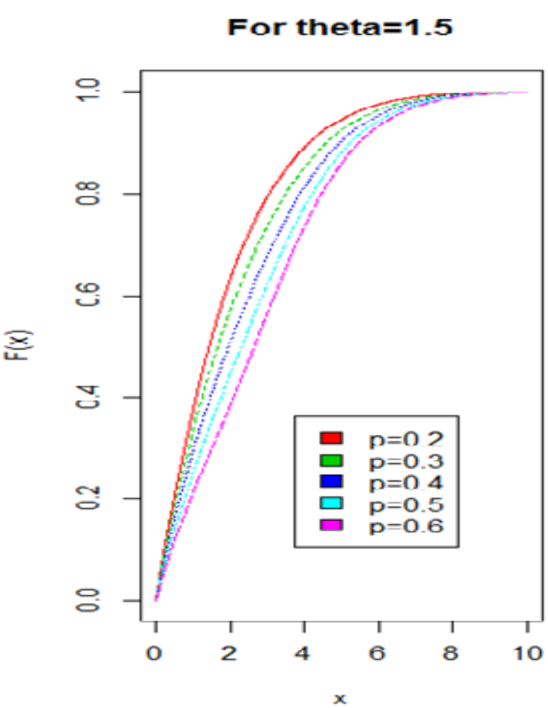

Fig.2(a): Graph of distribution function

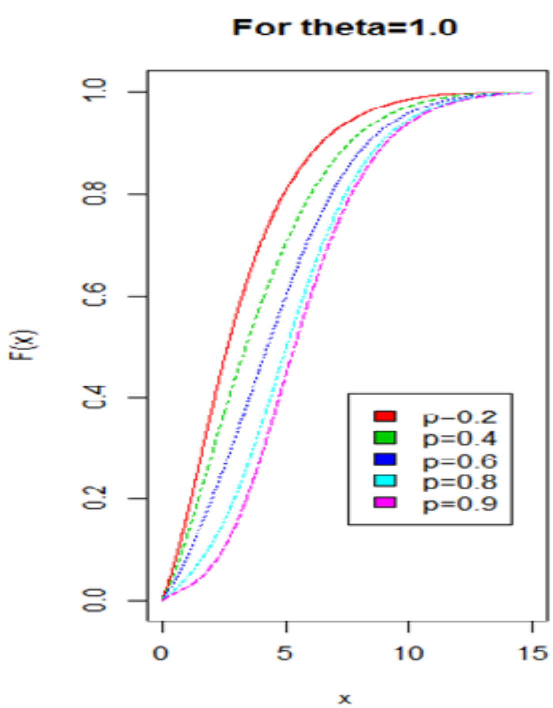

Fig.2(b): Graph of distribution function

The above graph represents the cumulative distribution function of Prakaamy Quasi Shanker distribution.

3 Reliability Analysis

In this particular section of paper survival function, hazard rate, reverse hazard rate of the proposed Prakaamy Quasi Shanker distribution for random variable X , where X denotes the lifetime of a system are obtained.

3.1 Reliability Function R(x)

The reliability function or survival function $R(x,\theta,p)$ gives the numerical value of a probability of surviving of a system or living beings beyond a specified time (t) .

Mathematically

$R(x,\theta,p)=P(X>t)=1-F(x)$

The reliability function or the survival function of Prakaamy Quasi Shanker distribution is calculated as:

$$R(x,\theta,p)=1-\left[\begin{array}{l} p\left(\dfrac{1}{(\theta^5+120)}\left(\theta^5(1-e^{-\theta x})+\gamma(6,\theta x)\right)\right) \\ +(1-p)\left[1-\left\{1+\dfrac{2\theta^2x^2+\theta x(\theta+4)}{(\theta^3+\theta+4)}\right\}e^{-\theta x}\right]\end{array}\right]$$

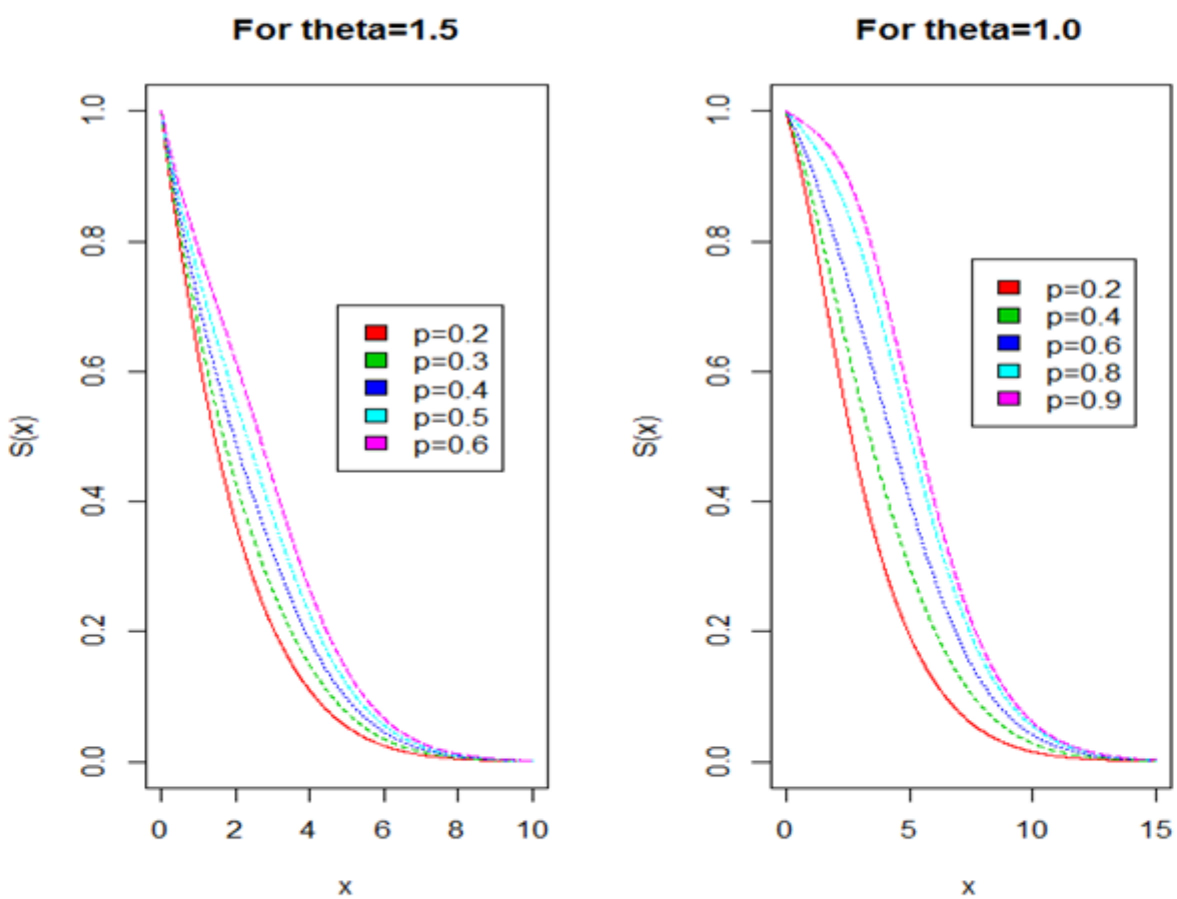

Fig.3(a): Graph of survival function. **Fig.3(b):** Graph of survival function.

The above graph represents the survival function of Prakaamy Quasi Shanker distribution.

3.2 Hazard Function

The hazard function is given as:

$$H.R = h(x;\theta,p) = \frac{f(x,\theta,p)}{R(x,\theta,p)}$$

$$= \frac{\left[\frac{p\theta^6}{(\theta^5+120)}\left(1+x^5\right)e^{-\theta x} + \frac{(1-p)\theta^3}{(\theta^3+\theta+4)}\left(\theta+x+2x^2\right)e^{-\theta x}\right]}{1-\left[\begin{array}{l} p\left(\frac{1}{(\theta^5+120)}\left(\theta^5(1-e^{-\theta x})+\gamma(6,\theta x)\right)\right) \\ +(1-p)\left[1-\left\{1+\frac{2\theta^2x^2+\theta x(\theta+4)}{(\theta^3+\theta+4)}\right\}e^{-\theta x}\right]\end{array}\right]}$$

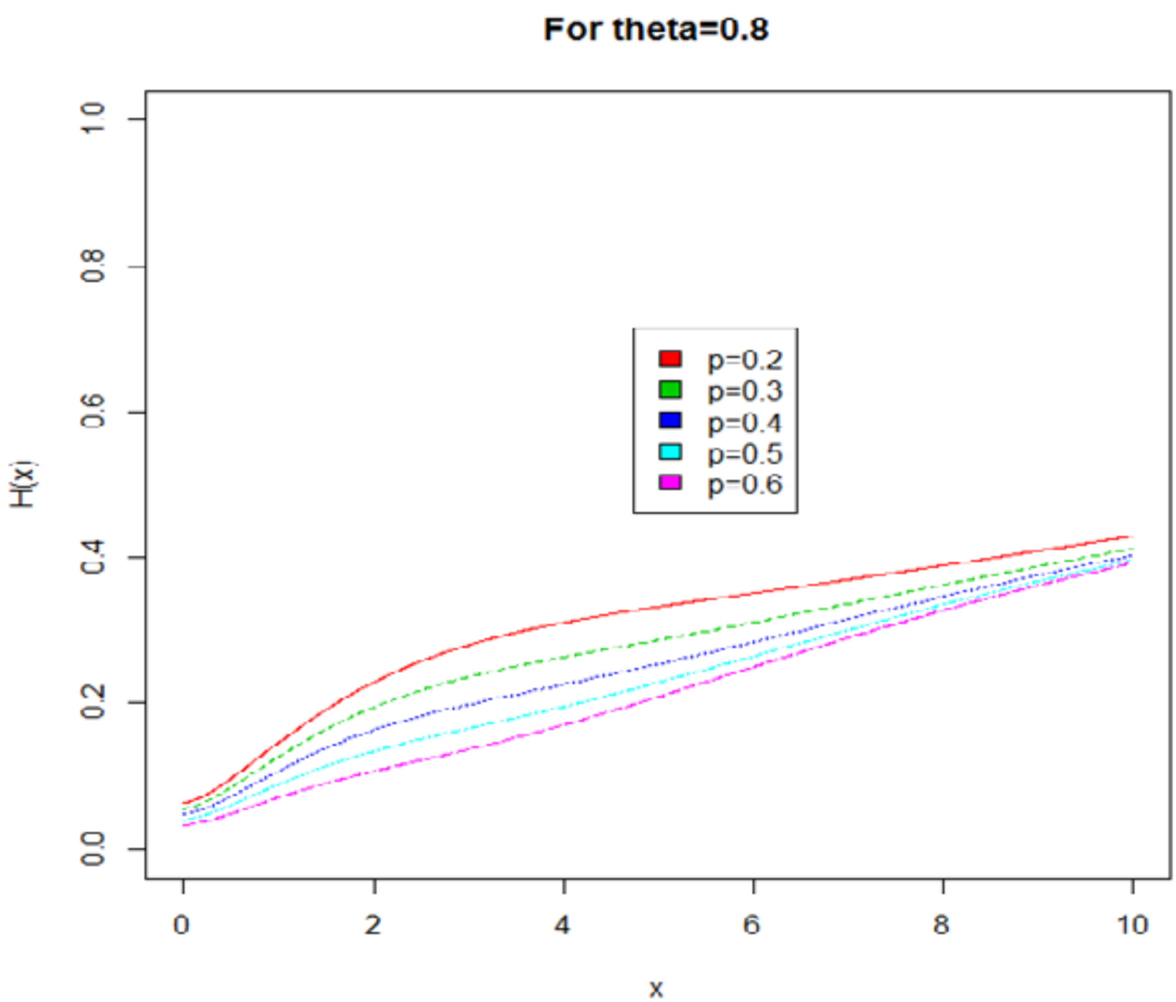

Fig.4: Graph of Hazard function.

The above graph represents the hazard rate of Prakaamy Quasi Shanker distribution. Since graph of hazard rate of proposed model is non-decreasing so proposed model finds greater applicability in real life.

3.3 Reverse Hazard Rate

The reverse hazard rate of the Prakaamy Quasi Shanker distribution is given as below

$$R.H.R = h(x,\theta,p) = \frac{f(x,\theta,p)}{F(x,\theta,p)}$$

$$= \frac{\left[\frac{p\theta^6}{(\theta^5+120)}\left(1+x^5\right)e^{-\theta x} + \frac{(1-p)\theta^3}{(\theta^3+\theta+4)}\left(\theta+x+2x^2\right)e^{-\theta x}\right]}{\left[\begin{array}{l} p\left(\frac{1}{(\theta^5+120)}\left(\theta^5(1-e^{-\theta x}) + \gamma(6,\theta x)\right)\right) \\ +(1-p)\left[1-\left\{1+\frac{2\theta^2x^2+\theta x(\theta+4)}{(\theta^3+\theta+4)}\right\}e^{-\theta x}\right] \end{array}\right]}$$

4 Statistical Properties

Moments, characteristic function, generating function characterize probability model. Here we have obtained these statistical properties for our proposed Prakaamy Quasi Shanker model.

4.1 Moments

Assuming X to be a random variable having Prakaamy Quasi Shanker distribution with parameters θ and p .Then the r^{th} moment about origin for a given probability distribution is given by

$$\mu_r' = E\left(X^r\right) = \int_0^\infty x^r f(x,\theta,p)dx \qquad r=1,2,3\ldots$$

$$= \int_0^\infty x^r \left[\frac{p\theta^6}{(\theta^5+120)}\left(1+x^5\right)e^{-\theta x} + \frac{(1-p)\theta^3}{(\theta^3+\theta+4)}\left(\theta+x+2x^2\right)e^{-\theta x}\right]dx$$

$$\mu_r' = \left[\begin{array}{l} \frac{pr!}{(\theta^5+120)\theta^r}\left(\theta^5+(r+5)(r+4)(r+3)(r+2)(r+1)\right)+ \\ (1-p)\left(\frac{r!\left(\theta^3+(r+1)\theta+2(r+1)(r+2)\right)}{\theta^r(\theta^3+\theta+4)}\right) \end{array}\right] \qquad (4.1.1)$$

Put r=1 in equation (4.1.1) we get

$$\mu_1' = \left[\frac{p(\theta^5+720)}{\theta(\theta^5+120)} + \frac{(1-p)(\theta^3+2\theta+12)}{\theta(\theta^3+\theta+4)}\right]$$

Which is mean of the Prakaamy Quasi Shanker distribution

Put r=2 in equation (4.1.1) we get

$$\mu_2' = \left[\frac{2p(\theta^5+2520)}{\theta^2(\theta^5+120)} + \frac{2(1-p)(\theta^3+3\theta+24)}{\theta^2(\theta^3+\theta+4)}\right]$$

Put r=3 in equation (4.1.1) we get

$$\mu_3' = \left[\frac{6p(\theta^5+6720)}{\theta^3(\theta^5+120)} + \frac{6(1-p)(\theta^3+4\theta+40)}{\theta^3(\theta^3+\theta+4)}\right]$$

Put r=4 in equation (4.1.1) we get

$$\mu_4' = \left[\frac{24p(\theta^5+15120)}{\theta^4(\theta^5+120)} + \frac{24(1-p)(\theta^3+5\theta+60)}{\theta^4(\theta^3+\theta+4)}\right]$$

The moments about mean are given as

$$\mu_2 = \left[\frac{p(\theta^{10}+3840\theta^5+86400)}{\theta^2(\theta^5+120)^2} + \frac{(1-p)(\theta^6+4\theta^4+32\theta^3+2\theta^2+24\theta+48)}{\theta^2(\theta^3+\theta+4)^2}\right]$$

Which is the variance of Prakaamy Quasi Shanker distribution.

$$\mu_3 = \left[\frac{2p\left(3(\theta^5+6720)(\theta^5+120)^2-3(\theta^5+720)(\theta^5+2520)(\theta^5+120)+(\theta^5+720)^3\right)}{\theta^3(\theta^5+120)^3} + \frac{2(1-p)\left(\theta^9+6\theta^7+60\theta^6+6\theta^5+84\theta^4+146\theta^3+36\theta^2+144\theta+192\right)}{\theta^3(\theta^3+\theta+4)^3}\right]$$

$$\mu_4 = \left[\frac{3p\begin{pmatrix}8(\theta^5+15120)(\theta^5+120)^3-8(\theta^5+720)(\theta^5+6720)(\theta^5+120)^2\\+4(\theta^5+120)(\theta^5+2520)(\theta^5+720)^2-(\theta^5+720)^4\end{pmatrix}}{\theta^4(\theta^5+120)^4} + \frac{3(1-p)\begin{pmatrix}3\theta^{12}+24\theta^{10}+256\theta^9+44\theta^8+688\theta^7+1664\theta^6+640\theta^5\\+3080\theta^4+4800\theta^3+1344\theta^2+3840\theta+3840\end{pmatrix}}{\theta^4(\theta^3+\theta+4)^4}\right]$$

Table1: Mean, Variance & Index of dispersion for different parameter values.

Properties	$\theta=0.5, p=0.5$	$\theta=2.5, p=0.5$	$\theta=2.0, p=0.8$	$\theta=2.0, p=0.8$
Mean	8.836536	1.046243	2.150376	4.153222
Variance	16.98913	0.891704	1.832	4.896633
Index of Dispersion	1.9226	0.8522912	0.8519441	1.178996

The above table reveals that proposed model is over dispersed as well as under dispersed for different values of parameters.

4.2 Coefficient of Variation, Skewness, Kurtosis and Index of Dispersion

The coefficient of variation (C.V), coefficient of skewness $\left(\sqrt{\beta_1}\right)$, coefficient of kurtosis (β_2) and index of dispersion (γ) of the PQSD are determined as

$$C.V=\frac{(\mu_2)^{\frac{1}{2}}}{\mu_1'}=\frac{\left\{\left[\dfrac{p(\theta^{10}+3840\theta^5+86400)}{\theta^2(\theta^5+120)^2}+\dfrac{(1-p)(\theta^6+4\theta^4+32\theta^3+2\theta^2+24\theta+48)}{\theta^2(\theta^3+\theta+4)^2}\right]\right\}^{\frac{1}{2}}}{\left\{\left[\dfrac{p(\theta^5+720)}{\theta(\theta^5+120)}+\dfrac{(1-p)(\theta^3+2\theta+12)}{\theta(\theta^3+\theta+4)}\right]\right\}}$$

$$\sqrt{\beta_1}=\frac{\mu_3}{\mu_2^{3/2}}=\left\{\frac{\left[\dfrac{2p\begin{pmatrix}3(\theta^5+6720)(\theta^5+120)^2-3(\theta^5+720)(\theta^5+2520)\\(\theta^5+120)+(\theta^5+720)^3\end{pmatrix}}{\theta^3(\theta^5+120)^3}+\dfrac{2(1-p)\begin{pmatrix}\theta^9+6\theta^7+60\theta^6+6\theta^5+84\theta^4+146\theta^3\\+36\theta^2+144\theta+192\end{pmatrix}}{\theta^3(\theta^3+\theta+4)^3}\right]}{\left\{\left[\dfrac{p(\theta^{10}+3840\theta^5+86400)}{\theta^2(\theta^5+120)^2}+\dfrac{(1-p)(\theta^6+4\theta^4+32\theta^3+2\theta^2+24\theta+48)}{\theta^2(\theta^3+\theta+4)^2}\right]\right\}^{3/2}}\right\}$$

$$\beta_2=\frac{\mu_4}{\mu_2^{2}}=\frac{\left[\dfrac{3p\begin{pmatrix}8(\theta^5+15120)(\theta^5+120)^3-8(\theta^5+720)(\theta^5+6720)(\theta^5+120)^2\\+4(\theta^5+120)(\theta^5+2520)(\theta^5+720)^2-(\theta^5+720)^4\end{pmatrix}}{\theta^4(\theta^5+120)^4}+\dfrac{3(1-p)\begin{pmatrix}3\theta^{12}+24\theta^{10}+256\theta^9+44\theta^8+688\theta^7+1664\theta^6+640\theta^5\\+3080\theta^4+4800\theta^3+1344\theta^2+3840\theta+3840\end{pmatrix}}{\theta^4(\theta^3+\theta+4)^4}\right]}{\left\{\left[\dfrac{p(\theta^{10}+3840\theta^5+86400)}{\theta^2(\theta^5+120)^2}+\dfrac{(1-p)(\theta^6+4\theta^4+32\theta^3+2\theta^2+24\theta+48)}{\theta^2(\theta^3+\theta+4)^2}\right]\right\}^{2}}$$

$$\gamma=\frac{\mu_2}{\mu_1'}=\frac{\left\{\left[\dfrac{p(\theta^{10}+3840\theta^5+86400)}{\theta^2(\theta^5+120)^2}+\dfrac{(1-p)(\theta^6+4\theta^4+32\theta^3+2\theta^2+24\theta+48)}{\theta^2(\theta^3+\theta+4)^2}\right]\right\}}{\left\{\left[\dfrac{p(\theta^5+720)}{\theta(\theta^5+120)}+\dfrac{(1-p)(\theta^3+2\theta+12)}{\theta(\theta^3+\theta+4)}\right]\right\}}$$

4.3 Moment Generating Function and Characteristic Function of PQSD

We will derive moment generating function and characteristic function of PQSD in this segment of paper.

Theorem 1: If $X \sim PQSD(\theta, p)$ then the moment generating function $M_X(t)$ and characteristic generating function $\phi_X(t)$ are

$$M_X(t) = \left[\frac{\theta^6}{(\theta^5 + 120)(\theta - t)^6} \left\{ (\theta - t)^5 + 120 \right\} + \frac{\theta^3}{(\theta^3 + \theta + 4)(\theta - t)^3} \left\{ \theta(\theta - t)^2 + (\theta - t) + 4 \right\} \right]$$

And

$$\phi_X(t) = \left[\frac{\theta^6}{(\theta^5 + 120)(\theta - it)^6} \left\{ (\theta - it)^5 + 120 \right\} + \frac{\theta^3}{(\theta^3 + \theta + 4)(\theta - it)^3} \left\{ \theta(\theta - it)^2 + (\theta - it) + 4 \right\} \right] \quad \text{respectively.}$$

Proof: We begin with the well-known definition of the moment generating function given by

$$M_X(t) = E\left(e^{tx}\right) = \int_0^\infty e^{tx} f(x; \theta, p)\, dx$$

$$= \int_0^\infty e^{tx} \left[\frac{p\theta^6}{(\theta^5 + 120)} \left(1 + x^5\right) e^{-\theta x} + \frac{(1-p)\theta^3}{(\theta^3 + \theta + 4)} \left(\theta + x + 2x^2\right) e^{-\theta x} \right] dx$$

$$M_X(t) = \left[\begin{array}{l} \dfrac{\theta^6}{(\theta^5 + 120)(\theta - t)^6} \left\{ (\theta - t)^5 + 120 \right\} + \\ \dfrac{\theta^3}{(\theta^3 + \theta + 4)(\theta - t)^3} \left\{ \theta(\theta - t)^2 + (\theta - t) + 4 \right\} \end{array} \right] \quad (4.3.1)$$

Which is the m.g.f of Prakaamy Quasi Shanker distribution.

Also we know that $\phi_X(t) = M_X(it)$

Therefore,

$$\phi_X(t) = \left[\begin{array}{l} \dfrac{\theta^6}{(\theta^5 + 120)(\theta - it)^6} \left\{ (\theta - it)^5 + 120 \right\} + \\ \dfrac{\theta^3}{(\theta^3 + \theta + 4)(\theta - it)^3} \left\{ \theta(\theta - it)^2 + (\theta - it) + 4 \right\} \end{array} \right] \quad (4.3.2)$$

Which is the characteristic function of Prakaamy Quasi Shanker distribution.

5 Order Statistics of Prakaamy Quasi Shanker Distribution

Consider $X_{(1)}, X_{(2)}, X_{(3)}, \ldots, X_{(n)}$ to be the ordered statistics of the random sample $x_1, x_2, x_3, \ldots x_n$ obtained from the Prakaamy Quasi Shanker distribution with cumulative distribution function $F(x)$ and probability density function $f(x, \theta, p)$, then the probability density function of v^{th} order statistics $X_{(v)}$ is given by:

$$f_v(x,\theta,p)=\frac{n!}{(v-1)!(n-v)!}f(x,\theta,p)[F(x)]^{v-1}[1-F(x)]^{n-v} \quad v=1,2,3\ldots n$$

Using the equations (2.6) and (2.7), the probability density function of v^{th} order statistics of Prakaamy Quasi Shanker distribution is given by:

$$f_{(v)}(x,\theta,p)=\left[\begin{array}{l}\frac{n!}{(v-1)!(n-v)!}\left[\frac{p\theta^6}{(\theta^5+120)}\left(1+x^5\right)e^{-\theta x}+\frac{(1-p)\theta^3}{(\theta^3+\theta+4)}\left(\theta+x+2x^2\right)e^{-\theta x}\right]\\ \left[\left[\begin{array}{l}p\left(\frac{1}{(\theta^5+120)}\left(\theta^5(1-e^{-\theta x})+\gamma(6,\theta x)\right)\right)\\ +(1-p)\left[1-\left\{1+\frac{2\theta^2x^2+\theta x(\theta+4)}{(\theta^3+\theta+4)}\right\}e^{-\theta x}\right]\end{array}\right]\right]^{v-1}\\ \left[\left[\begin{array}{l}p\left(\frac{1}{(\theta^5+120)}\left(\theta^5(1-e^{-\theta x})+\gamma(6,\theta x)\right)\right)\\ +(1-p)\left[1-\left\{1+\frac{2\theta^2x^2+\theta x(\theta+4)}{(\theta^3+\theta+4)}\right\}e^{-\theta x}\right]\end{array}\right]\right]^{n-v}.\end{array}\right]$$

Then, the p.d.f of first order statistic $X_{(1)}$ of Prakaamy Quasi Shanker model is given by:

$$f_{(1)}(x,\theta,p)=\left[\begin{array}{l}n\left[\frac{p\theta^6}{(\theta^5+120)}\left(1+x^5\right)e^{-\theta x}+\frac{(1-p)\theta^3}{(\theta^3+\theta+4)}\left(\theta+x+2x^2\right)e^{-\theta x}\right]\\ \left[1-\left[\begin{array}{l}p\left(\frac{1}{(\theta^5+120)}\left(\theta^5(1-e^{-\theta x})+\gamma(6,\theta x)\right)\right)\\ +(1-p)\left[1-\left\{1+\frac{2\theta^2x^2+\theta x(\theta+4)}{(\theta^3+\theta+4)}\right\}e^{-\theta x}\right]\end{array}\right]\right]^{n-1}.\end{array}\right]$$

and the pdf of n^{th} order statistic $X_{(n)}$ of Prakaamy Quasi Shanker model is given as:

$$f_{(n)}(x,\theta,p)=\left[\begin{array}{l}n\left[\frac{p\theta^6}{(\theta^5+120)}\left(1+x^5\right)e^{-\theta x}+\frac{(1-p)\theta^3}{(\theta^3+\theta+4)}\left(\theta+x+2x^2\right)e^{-\theta x}\right]\\ \left[\left[\begin{array}{l}p\left(\frac{1}{(\theta^5+120)}\left(\theta^5(1-e^{-\theta x})+\gamma(6,\theta x)\right)\right)\\ +(1-p)\left[1-\left\{1+\frac{2\theta^2x^2+\theta x(\theta+4)}{(\theta^3+\theta+4)}\right\}e^{-\theta x}\right]\end{array}\right]\right]^{n-1}.\end{array}\right]$$

6 Estimation of Parameters of Prakaamy Quasi Shanker Distribution

We used method of maximum likelihood estimation for estimating parameters of PQSD. Considering $X_1, X_2, X_3.....X_n$ tobe the random sample of size n drawn from Prakaamy Quasi Shanker distribution having density function given by (2.6), then the likelihood function of Prakaamy Quasi Shanker distribution is given as:

$$L(x \mid \theta, p) = \prod_{i=1}^{n} \left[\left[\frac{p\theta^6}{(\theta^5 + 120)} \left(1 + x^5\right) e^{-\theta x} + \frac{(1-p)\theta^3}{(\theta^3 + \theta + 4)} \left(\theta + x + 2x^2\right) e^{-\theta x} \right] \right]$$

Taking log on both sides of likelihood function we get log likelihood function as:

$$\log L = \left\{ \begin{array}{l} 3n \log\theta - n\log(\theta^5 + 120) - n\log(\theta^3 + \theta + 4) - \theta \sum_{i=1}^{n} x_i \\ + \sum_{i=1}^{n} \log\left(p\theta^3(\theta^3 + \theta + 4)(1 + x_i{}^5) + (1-p)(\theta + x_i + 2x_i{}^2)(\theta^5 + 120) \right) \end{array} \right\} \tag{6.1}$$

Differentiating the log-likelihood function with respect to θ & p.This is done by partially differentiate (6.1) with respect to θ & p and equating the result to zero, we obtain the following normal equations,

$$\frac{\partial \log L}{\partial \theta} = \left[\begin{array}{l} \frac{3n}{\theta} - \frac{5\theta^4 n}{(\theta^5 + 120))} - \frac{n(3\theta^2 + 1)}{(\theta^3 + \theta + 4)} - \\ \sum_{i=1}^{n} x_i + \sum_{i=1}^{n} \left[\frac{\left\{ \begin{array}{l} p(1 + x_i{}^5)\theta^2(6\theta^3 + 4\theta + 12) + \\ (1-p)((\theta^5 + 120) + 5\theta^4(\theta + x_i + 2x_i{}^2)) \end{array} \right\}}{p\theta^3(\theta^3 + \theta + 4)(1 + x_i{}^5) + (1-p)(\theta + x_i + 2x_i{}^2)(\theta^5 + 120)} \right] \end{array} \right] = 0 \tag{6.2}$$

$$\frac{\partial \log L}{\partial \theta} = \left[\sum_{i=1}^{n} \left[\frac{\theta^3(\theta^3 + \theta + 4)(1 + x_i{}^5) - (\theta + x_i + 2x_i{}^2)(\theta^5 + 120)}{p\theta^3(\theta^3 + \theta + 4)(1 + x_i{}^5) + (1-p)(\theta + x_i + 2x_i{}^2)(\theta^5 + 120)} \right] \right] = 0 \tag{6.3}$$

MLEs of θ & p cannot be obtained by solving above complex equations as these equations are not in closed form. So we solve above equations by using iteration method through R software.

7 Special Cases

Case I: If we put $p = 0$, then Prakaamy Quasi Shanker distribution (2.6) reduces to Quasi Shanker distribution with shape parameter $\alpha = 2$ and having probability density function as:

$$f_2(x) = \frac{\theta^3}{(\theta^3 + \theta + 4)} \left(\theta + x + 2x^2\right) e^{-\theta x} \quad x > 0, \theta > 0$$

Case II: For $p = 1$, Prakaamy Quasi Shanker distribution (2.6) reduces to Prakaamy distribution with probability density function given as

$$f_1(x) = \frac{\theta^6}{(\theta^5 + 120)} \left(1 + x^5\right) e^{-\theta x} \quad x > 0, \theta > 0$$

8 Applications of Prakaamy Quasi Shanker Distribution

We fitted Prakaamy Quasi Shanker distribution and its related distributions to three real life data sets to check the superiority of proposed model over its related models.

Data Set 1: The first real data set is a subset of the data reported by Bekker et al. (2000), which corresponds to the survival times (in years) of a group of patients given chemotherapy treatment alone. The data consisting of survival times (in years) for 45 patients is given in table 2.

Table 2: Survival times (in years) for 45 patient's chemotherapy treatment alone.

0.047	0.115	0.121	0.132	0.164	0.197	0.203	0.260	0.282
0.296	0.334	0.395	0.458	0.466	0.501	0.507	0.529	0.534
0.540	0.641	0.644	0.696	0.841	0.863	1.099	1.219	1.271
1.326	1.447	1.485	1.553	1.581	1.589	2.178	2.343	2.416
2.444	2.825	2.830	3.578	3.658	3.743	3.978	4.033	4.033

Data set 2: The data set is from Kundu & Howlader (2010), the data set given in table 3 represents the survival times (in days) of guinea pigs injected with different doses of tubercle bacilli. This data set was recently used by Shanker, Hagos & Sujatha (2015).

Table 3: Survival times (in days) of 72 guinea pigs injected with different doses of tubercle bacilli.

12	15	22	24	24	32	32	33	34
38	38	43	44	48	52	53	54	54
55	56	57	58	58	59	60	60	60
60	61	62	63	65	65	67	68	70
70	72	73	75	76	76	81	83	84
85	87	91	95	96	98	99	109	110
121	127	129	131	143	146	146	175	175
211	233	258	258	263	297	341	341	376

Data Set 3: Here we consider a data set initially analysed by Chhikara & Folks (1977). The maintenance data set given in table 4 represents active repair times (in hours) for an airborne communication transceiver.

Table 4: Active repair times (in hours) of 46 transceiver.

0.2	0.3	0.5	0.5	0.5	0.5	0.6	0.6
0.7	0.7	0.7	0.8	0.8	1.0	1.0	1.0
1.0	1.1	1.3	1.5	1.5	1.5	1.5	2.0
2.0	2.2	2.5	2.7	3.0	3.0	3.3	3.3
4.0	4.0	4.5	4.7	5.0	5.4	5.4	7.0
7.5	8.8	9.0	10.3	22.0	24.5		

For analyzing data sets 1, 2 & 3 we used R software version 3. 5. 3 [9]

Table 5: Summary statistics of data sets 1, 2 & 3.

Data Set	No. of observations	Min.	First quartile	median	mean	Third quartile	Max.
1	45	0.047	0.395	0.841	1.341	2.178	4.033
2	72	12.00	54.75	70.00	99.82	112.75	376.00
3	46	0.200	0.800	1.750	3.607	4.375	24.500

Table 6: ML Estimates with standard errors in parenthesis, model function of proposed model and its related models for data set 1.

Distribution	**Parameter Estimates**	**Model function**
Quasi Shanker (QSD)	$\hat{\theta}=1.5427$ (0.1177)	$\frac{\theta^3}{(\theta^3+\theta+4)}\left(\theta+x+2x^2\right)e^{-\theta x}$
Prakaamy (PD)	$\hat{\theta}=2.6065$ (0.1244)	$\frac{\theta^6}{(\theta^5+120)}\left(1+x^5\right)e^{-\theta x}$
Prakaamy Quasi Shanker (PQSD)	$\hat{\theta}=2.0698$ (0.2670) $\hat{p}=0.3531$ (0.1995)	$\left[\frac{p\theta^6}{(\theta^5+120)}\left(1+x^5\right)e^{-\theta x}+\frac{(1-p)\theta^3}{(\theta^3+\theta+4)}\left(\theta+x+2x^2\right)e^{-\theta x}\right]$

Table 7: ML Estimates with standard errors in parenthesis of fitted models to data sets 2 and 3.

Distribution	**Data set**	
	2	**3**
QSD	$\hat{\theta}=0.0299$ (0.0020)	$\hat{\theta}=0.7646$ (0.0610)
PD	$\hat{\theta}=0.0601$ (0.0028)	$\hat{\theta}=1.563$ (0.085)
PQSD	$\hat{\theta}=0.0346$ (0.0029) $\hat{p}=0.1537$ (0.0790)	$\hat{\theta}=0.880$ (0.081) $\hat{p}=0.151$ (0.082)

Table 8: Model comparison and Likelihood ratio statistic of proposed model and its related models for data set 1.

Distribution	$-\log L$	AIC	BIC	AICC	HQIC	Shannon entropy $H(X)$	Likelihood Ratio
PQSD	57.15003	118.3001	121.9134	118.5858	119.6471	1.27	3.92
QSD	59.11046	120.220	122.027	120.3139	120.8944	1.31	
PD	59.6851	121.370	123.176	121.4633	122.0438	1.32	

Table 9: Model comparison and Likelihood ratio statistic of proposed model and its related models for data set 2.

Distribution	$-\log L$	AIC	BIC	AICC	HQIC	Shannon entropy $H(X)$	Likelihood Ratio
PQGD	394.2350	792.4699	797.0233	792.2826	794.2826	5.47	6.119
QSD	397.2945	796.589	798.865	796.6462	797.4954	5.51	
PD	427.358	856.716	858.993	856.773	857.62	5.93	

Table 10: Model comparison and Likelihood ratio statistic of proposed model and its related models for data set 3.

Distribution	$-\log L$	AIC	BIC	AICC	HQIC	Shannon entropy $H(X)$	Likelihood Ratio
PQGD	117.977	239.9545	243.6122	240.234	241.325	2.564	9.91
QSD	122.9342	247.8684	249.697	247.9593	248.553	2.67	
PD	162.243	326.487	328.3163	326.578	327.1727	3.52	

For testing statistical significance of mixing parameter p and for determining superiority of Prakaamy Quasi Shanker distribution over Quasi Shanker distribution and Prakaamy distribution for data sets 1, 2 & 3 we computed likelihood ratio (LR) statistic. To test $H_0 : p = 0$ versus $H_1 : p \neq 0$ the LR statistic for testing H_0 is $\omega_1 = 2\{L(\hat{\Theta}) - L(\hat{\Theta}_0)\}$ =3.92, for data set 1, $\omega_2 = 2\{L(\hat{\Theta}) - L(\hat{\Theta}_0)\} = 6.119$ for data set 2 and $\omega_3 = 2\{L(\hat{\Theta}) - L(\hat{\Theta}_0)\} = 9.91$ for data set 3, where $\hat{\Theta}$ and $\hat{\Theta}_0$ are MLEs under H_1 and H_0. LR statistic $\omega \sim (\chi_{(1)}{}^2(\alpha = 0.05) = 3.841)$ as $n \to \infty$, where 1= degrees of freedom is the difference in dimensionality. From tables 8, 9, 10 $\omega_1 = 3.92 > 3.841$, $\omega_2 = 6.119 > 3.841$, $\omega_3 = 9.91 > 3.841$ at 5% level of significance, so we reject H_0 and conclude that mixing parameter p plays statistically a significant role.

For comparison of two or more models when fitted on real life data sets we consider model to be better for those data sets based on criteria's like AIC (Akaike information criterion), AICC (corrected Akaike information criterion), BIC (Bayesian information criterion) and HQIC which give the measure of loss of information by fitting probability model to the data. The better distribution corresponds to lesser AIC, AICC, BIC and HQIC values. We also compute the Shannon's entropy *H(X)* which is the measure of average uncertainty. The model which possesses the least *H(X)* value is considered to be best.

AIC = 2k-2log$L$ AICC = AIC+$\dfrac{2k(k+1)}{n-k-1}$

BIC = k log$n$-2log$L$ HQIC=2k log(log(n))+2 log L

$$H(X) = -\frac{\log L}{n}$$

where k is the number of parameters in the statistical model, n is the sample size and -2log$L$ is the maximized value of the log-likelihood function under the considered model. From Tables 8, 9, 10 it has been observed that the Prakaamy Quasi Shanker distribution possesses the lesser AIC, AICC BIC, HQIC & *H(X)* values as compared to Quasi Shanker distribution and Prakaamy distribution for data sets 1, 2 and 3. Hence we can conclude that the Prakaamy Quasi Shanker distribution leads to a better fit than the Quasi Shanker distribution and Prakaamy distribution for data sets 1, 2 and 3.

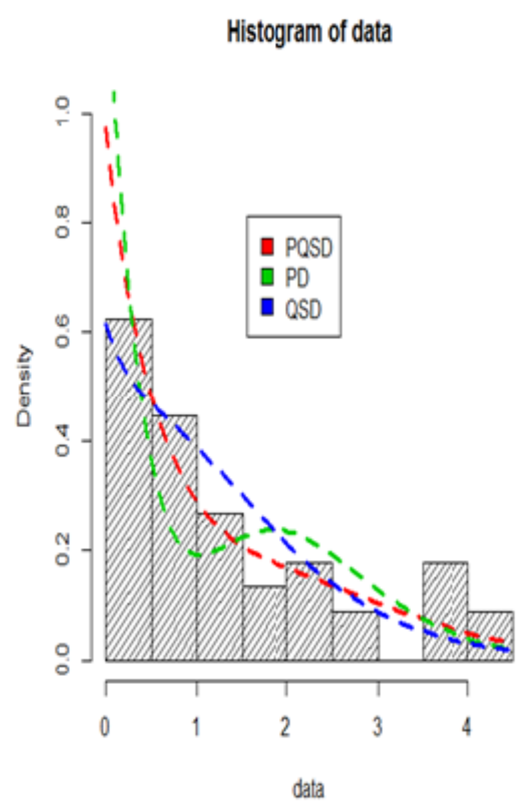

Fig.5: Curve fitting of data of set 1.

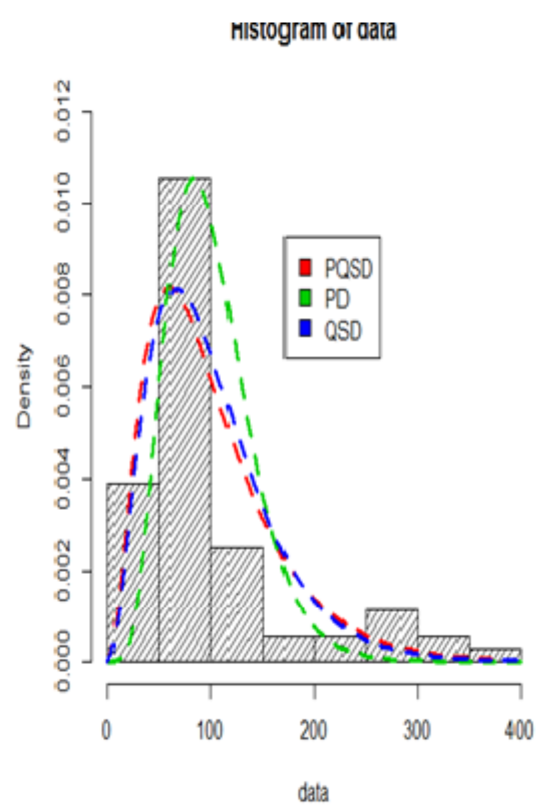

Fig.6: Curve fitting of data of set 2.

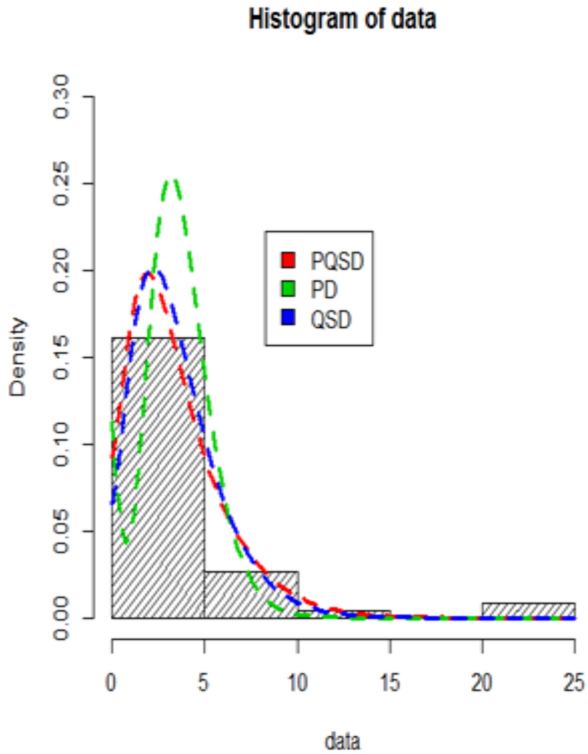

Fig.7: Curve fitting of data of set 2.

9 Conclusions

We developed Prakaamy Quasi Shanker distribution as a genuine mixture of Prakaamy distribution with parameter θ and Quasi Shanker distribution with scale parameter θ and shape parameter 2. We obtained the important statistical properties like moments, reliability, moment generating function, order statistics etc., of our formulated model. We also obtained the estimates of unknown parameters of our proposed model by method of maximum likelihood estimation. For testing the supremacy of our model over its competitive models and for testing the significance of mixing parameter we fitted our model to three real life data sets and concluded that our model fits better to these data sets, so our model finds applicability in real life especially for lifetime data.

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