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A Fuzzy EOQ Model with Allowable Shortage under Different Trade Credit Terms

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Abstract: The present study formulates a fuzzy economic order quantity model under conditions of permissible delay in payments by considering price-dependent demand and higher interest earn rate on fixed sales revenue. This research paper scrutinizes all the possible cases which may exist inclusive of those that have not been considered by researchers so far. The proposed models allow fully backlog shortages. The proper mathematical models for various cases are developed to determine the optimal order quantity, the time period in which inventory of the positive stock is finished in addition to the total cycle length by maximizing the total fuzzy profit function. Further, the arithmetic operations for fuzzy demand parameters are defined under the function principle and for defuzzification, signed distance method has been used. Finally, the numerical examples are presented to show the validity of the model followed by the sensitivity analysis.

Keywords: Inventory, shortages, price-dependent demand, trade credit, triangular fuzzy number, function principle and signed distance method

1 Introduction

In the present scenario, it becomes extremely difficult to determine the exact value of the parameters. One way of managing this vagueness is through fuzzy numbers. It is pertinent to discuss the work done in this area before the formulation of the proposed fuzzy economic model. The concept of fuzzy set theory was first introduced by Zadeh [1]. Zimmermann [2] gave a review on applications of fuzzy set theory. Two years later, Park [3] had used fuzzy set concepts to treat the inventory problem with fuzzy inventory cost under the arithmetic operations defined by extension principle. He had examined the EOQ model from the fuzzy set theoretic perspective. Kauffmann and Gupta [4] had provided an introduction to fuzzy arithmetic operations. Subsequently, Vujosevic et al. [5] extended the classical EOQ model by introducing the fuzziness of ordering cost and holding cost. Roy and Maiti [6] had presented a fuzzy EOQ model with demand-dependent unit cost under limited storage capacity, considering different parameters as fuzzy sets with suitable membership function. The ensuing year Chang et al. [7] had presented a fuzzy model for inventory with backorder, where the backorder quantity was fuzzified as triangular fuzzy number.

Further, Yao and Lee [8] presented a fuzzy inventory model with and without backorder for fuzzy order quantity with trapezoidal fuzzy number. Later, Yao et al. [9] proposed the EOQ model in the fuzzy sense, considering the order quantity and demand as triangular fuzzy numbers. Hsieh [10] reflected upon two fuzzy production-inventory models: one for crisp production quantity with fuzzy parameters and the other one for fuzzy production quantity. He used the graded mean integration representation method for defuzzify the fuzzy total inventory cost. After four years, Chen and Ouyang [11] proposed a fuzzy model by fuzzifying the inventory carrying charge, earned and payable interest as interval-valued triangular fuzzy number and used signed distance method to defuzzification this model. In the same year, Mahata and Goswami [12] had developed a fuzzy production-inventory model with permissible delay in payment. The scholars assumed the demand and the production rates as fuzzy numbers and defuzzified the

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associated cost in the fuzzy sense using extension principle.

Recently, Gani and Maheswari [13] debated on the retailer's ordering policy under two levels of delay payments considering the demand and the selling price as triangular fuzzy numbers. The researchers used the graded mean integration representation method for defuzzification. Next year, Uthayakumar and Valliathal [14] developed an economic production model for weibull deteriorating items over an infinite horizon under a fuzzy environment and also introduced some cost component as triangular fuzzy numbers. In their devised model, signed distance method is used to defuzzify the cost function. Sarkar and Chakrabarti [15] formulated an EPQ model with two-component demand under a fuzzy environment and weibull deterioration with shortages. The model uses the α-cut method for defuzzification of the total cost function. Alternately, there are many heuristic approaches to solve the inventory models for which close form solution does not exist. Le et al. [16], Diabat et al. [17, 18, 19] Santibanez-Gonzalez and Diabat [20] have suggested different heuristic approaches.

Demand has always been accepted as one of the most influential factors in decision making related to inventory policy. It is an established fact for all the business firms that the right pricing strategy fetches more customers, which results in an increase in revenues for the firm by increasing its demand. Demand and price are the most fundamental concepts of inventory management and serve as the backbone of a market economy. According to the law of demand, if all of the other factors remain at a constant level, the higher the price, the lower is the demand. As a result, the demand of highly priced products witnesses a decline. Hence, the price of the product plays a very crucial role in inventory analysis. In the field of inventory control theory, the various forms of demand (like constant demand, price dependent demand etc.) have been studied by various scholars such as Cohen [21], Aggarwal and Jaggi [22], Wee [23, 24], Mukhopadhyay et al. [25] and many more.

In today's competitive markets, Trade credit financing has been widely recognized as a very crucial strategy to increase profitability. In practice, a supplier usually provides her/his retailers a permissible delay in payments to stimulate sales and reduce inventory. During the credit period, the retailer can accumulate the revenue and earn interest on the accumulative revenue. However, beyond this credit period, the supplier charges her/his retailers an interest on the unpaid balance. Further, the presence of trade credit reduces the buyer's inventory holding cost, thereby affecting the buyer's economic order quantity (EOQ). In order to deliberate on the possibilities arising out of permissible delay in payments, various scholars have researched in this area. In 1973, Haley and Higgins [26]had developed the economic order quantity model under the condition of permissible delay in payments with deterministic demand, without shortages and zero lead time. Some years later, Goyal [27] extended their model with the exclusion of the penalty cost due to a late payment. Shah [28], and Hwang and Shinn [29] further expanded Goyal's model by incorporating the case of deterioration. The model was further revised by Jamal et al. [30] and Aggarwal and Jaggi [31] include permission of shortages. Also, Jaggi et al. [32] determined a retailer's optimal replenishment decisions with trade credit-linked demand under permissible delay in payments. Further research in this area is summarized by different survey papers (Chang et al. [33], H. Soni et al. [34], D. Seifert et al. [35] and Z. Molamohamadi et al. [36].

In today's financial markets, the retailer may invest his/her money into stock markets or into developing new products, thus, gaining a return on investment that may be higher than the interest charged. If the interest earned is higher than the interest charged, a reasonable retailer may not return the money to the supplier until the end of the replenishment cycle. On the other hand, if the interest earned is less than the interest charged, a reasonable retailer will pay off the total purchase cost to the supplier as soon as possible, following the end of the credit period. Therefore, a more practical option for the retailer would be either to pay off the entire amount owed to the supplier at the end of the credit period or to delay incurring interest charges on the unpaid and overdue balance. Hence, the determination of a retailer's payoff time is affected by the amount of interest income and interest payments.

Considering the above mentioned facts, Cheng et al. [37] had discussed an economic order quantity model with trade credit policy in different financial environment. They had proposed the model under the conditions that the interest earned is higher than the interest charged and the interest earned is lower than the interest charged. Most recently, Bhunia et al. [38] obtained a retailer's optimal ordering and trade credit policy in two-warehouse environment and examined many cases for the calculation of earned and payable interest. Yet, their research does not consider the case where the supplier accepts the partial amount at the end of the credit period and the remaining amount is paid continuously beyond the credit period to a fixed point. Further, most of the financial institutions offer higher rate of interest to the deposits which are fixed in nature than recurring. So, the retailer can earn higher interest on sales revenue which is fixed in nature i.e. revenue generated by fulfilling the shortage, balance amount after having settled the account.

Having substantiated the relevance of the aforementioned facts, the present study is aimed at developing a fuzzy economic order quantity model with price-dependent demand under conditions of permissible delay in payments by considering higher interest earn rate on fixed sales revenue. Additionally, the model also considers fully backlogged shortages. The research further investigates all the possibilities for making the payments in conditions of permissible delay in payment which may exist and are have yet not been considered previously. The main purpose of the present model is to determine the optimal inventory and pricing strategies, so as to maximize the total fuzzy profit function of the system. The arithmetic operations are defined under the function principle and signed distance method has been used for defuzzification,. Finally, the numerical examples and sensitivity analysis have been presented to illustrate the applicability of model in different scenarios. These findings eventually serve as a ready reckoned for the organization to take appropriate decision under the prevailing environment.

2 Preliminaries

In order to treat the fuzzy inventory model, the following definitions have been used:

Definition 2.1.A fuzzy set $\widetilde{k}$ on $R = (-\infty, \infty)$ is called a fuzzy point if its membership function is

$$\mu_{\widetilde{k}}(x) = \begin{cases} 1, & x = k \\ 0, & x \neq k \end{cases}$$

Where the point k is called the support of fuzzy set $\widetilde{k}$.

Definition 2.2.A fuzzy set $[k_\alpha, l_\alpha]$ where $0 \leq \alpha \leq 1$ and $k < l$ defined on R, is called a level of a fuzzy interval if its membership function is

$$\mu_{[k_\alpha, l_\alpha]}(x) = \begin{cases} \alpha, & k \leq x \leq l \\ 0, & \text{otherwise} \end{cases}$$

Definition 2.3.A fuzzy number $\widetilde{K} = (k_1, k_2, k_3)$ where $k_1 < k_2 < k_3$ and defined on R, is called a triangular fuzzy number if its membership function is

$$\mu_{\widetilde{K}}(x) = \begin{cases} \frac{x-k_1}{k_2-k_1}, & k_1 \leq x \leq k_2 \\ \frac{k_3-x}{k_3-k_2}, & k_2 \leq x \leq k_3 \\ 0, & \text{otherwise} \end{cases}$$

When $k_1 = k_2 = k_3 = k$, we have fuzzy point $(k, k, k) = \widetilde{k}$.

The family of all triangular fuzzy numbers on R is denoted as

$$F_N = \{(k_1, k_2, k_3) | k_1 < k_2, < k_3 \ \forall \ k_1, k_2, k_3 \in R\}$$

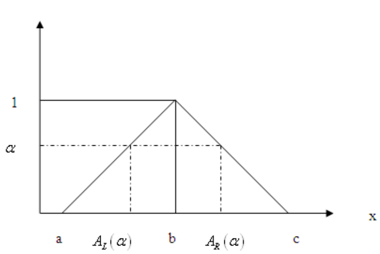

Fig. 1: α-cut of a triangular fuzzy number

The α-cut of $\widetilde{K} = (k_1, k_2, k_3) \in F_N$, $0 \leq \alpha \leq 1$, is $K(\alpha) = [K_L(\alpha), K_R(\alpha)]$.
Where $K_L(\alpha) = k_1 + (k_2 - k_1)\alpha$ and $K_R(\alpha) = k_3 - (k_3 - k_2)\alpha$ are the left and right endpoints of $K(\alpha)$.

Definition 2.4. If $\widetilde{K} = (k_1, k_2, k_3)$ is a triangular fuzzy number then the signed distance of $\widetilde{K}$ is defined as

$$d(\widetilde{K}\widetilde{0}) = \int_0^1 d([k_L(\alpha)_\alpha, K_R(\alpha)_\alpha], \widetilde{0}) = \frac{1}{4}(k_1 + 2k_2 + k_3)$$

3 Assumptions and Notations

This model is developed with the help of the following notations and assumptions:

3.1 Notations

$I(t)$	:	instantaneous inventory level at any time t
Q	:	economic order quantity
$D(p) = D = a - bp$	:	price dependent demand where a and b are positive constant
$\widetilde{D}(p) = \widetilde{D} = \widetilde{a} - \widetilde{b}p$	:	fuzzy price dependent demand
A	:	replenishment cost (ordering cost) for replenishing the items
c	:	unit purchase cost of retailer
π	:	shortage cost per unit per unit time
μ $(\mu > 1)$	:	mark up rate
$p = \mu c$	:	selling price per unit
M	:	credit period offered by the supplier to the retailer
h	:	holding cost per unit per unit time excluding interest charge
I_e	:	interest earned rate on regular sales revenue
I_E	:	interest earned rate on fixed deposit amount
I_p	:	interest payable rate
T	:	replenishment cycle length
t_1	:	length of the period with positive stock of the items
B_i	:	breakeven point, $i = 1, 2, 3$
$AP_{(\cdot)}(\mu, t_1, T)$	:	total profit in case $(\cdot)$
$AP(\cdot)$	:	total profit in combine form
$AP_d(\cdot)$	:	total profit after defuzzification

3.2 Assumptions

1. Demand rate is assumed to be a function of selling price i.e. $D(p) = a - bp$ which is a function of selling price (p), where a, b are positive constants and $0 < b < a/p$. Further, a and b are assumed as triangular fuzzy numbers.
2. The planning horizon of the inventory system is infinite.
3. Unsatisfied demand/shortages are allowed and fully backlogged.
4. Replenishment rate is instantaneous and lead-time is negligible.
5. The entire lot size is delivered in one batch.

4 Model formulation

At $t = 0$, retailer purchased Q units the retail business. In Q units, $Q - S_1$ units are fulfilling the shortages and remaining S_1 unit will be depleted due to demand and exhausted at $t = t_1$.

Let $I(t)$ be the inventory level at any time t $(0 \le t \le T)$. The instantaneous states of $I(t)$ over the period $(0,T)$ with help of some conditions at time $t = 0$, $I(0) = S_1$ and at time $t = t_1$, $I(t_1) = 0$ are given by the following equations

$$I(t) = S_1 - Dt, \quad 0 \le t \le t_1 \tag{1}$$
$$I(t) = -D(t - t_1), \quad t_1 \le t \le T \tag{2}$$

and from (1) at time $t = t_1$, $I(t_1) = 0$

$$S_1 = Dt_1 \tag{3}$$
$$\Rightarrow \quad Q = S_1 + D(T - t_1) \tag{4}$$

The profit function per unit time can be expressed as

$$\begin{aligned} AP(\mu,t_1,T) = \frac{1}{T}[&\langle\text{Total selling revenue}\rangle \\ &+ \langle\text{Interest earned}\rangle - \langle\text{Total purchase cost}\rangle \\ &- \langle\text{Ordering cost}\rangle - \langle\text{Holding cost}\rangle \\ &- \langle\text{Shortage cost}\rangle - \langle\text{Interest paid}\rangle] \end{aligned}$$

where

(a) Ordering cost $= A$ (5)

(b) Stock holding cost $= \frac{1}{2}DT^2h$ (6)

(c) Shortages cost $= \frac{1}{2}D(T - t_1)^2\pi$ (7)

(d) Interest earned

(e) Interest payable

The computation for interest earned and interest payable (i.e. (d) and (e)) will depend on the following three possible cases based on the different situation of I_e, I_E and I_p.

1. $I_e < I_E \le I_p$
2. $I_e \le I_P < I_E$ and
3. $I_p < I_e < I_E$

Further, each case is divided into sub-cases depending upon the values of t_1, M and T. All the possible cases/sub-cases has been shown by the Figure 2.

Case 1: $I_e < I_E \le I_p$

This case situation indicates that the interest earned rate (I_e, I_E), is less than the interest payable rate (I_p). Based on the values t_1, M and T the following three possible sub-cases exist:

1.1:$0 < M \le t_1 < T$, **1.2:** $0 < t_1 \le M < T$ **and 1.3:** $0 < t_1 < T \le M$**.**

Case 1.1: $0 < M \le t_1 < T$

Since both the interest earned rate is less than the interest paid rate, so the retailer would try to pay off the total purchase cost to the supplier as soon as possible. At the expiry of M, the retailer will have a certain amount which is the sum of the sales revenue during the period $[0,M]$ and the interest earned on both regular sales revenue and fixed deposit amount i.e. the amount accumulate after satisfying the shortages during the same time period.

Hence, the total sales revenue during the time period $[0,M] = Dp\{M + (T - t_1)\}$.

The interest earned on regular sales revenue during the time period $[0,M] = \frac{1}{2}DM^2pI_e$.

The interest earned on fixed deposit amount during the time period $[0,M] = D(T - t_1)MpI_E$.

Therefore, the retailer has total amount at M is

$$Dp\left[(T - t_1) + M\left\{1 + (T - t_1)I_E + \frac{1}{2}MI_e\right\}\right] \equiv W_1 \text{ (say)}.$$

However, at time $t = 0$, the retailer owes Qc amounts as the purchase cost to the supplier. Now at M, retailer settling his account with the supplier, but he has only W_1 amount therefore, based on the difference between W_1 and Qc, further, there may be the following two sub cases arise as **1.1.1** $W_1 < Qc$ and **1.1.2** $W_1 \ge Qc$.

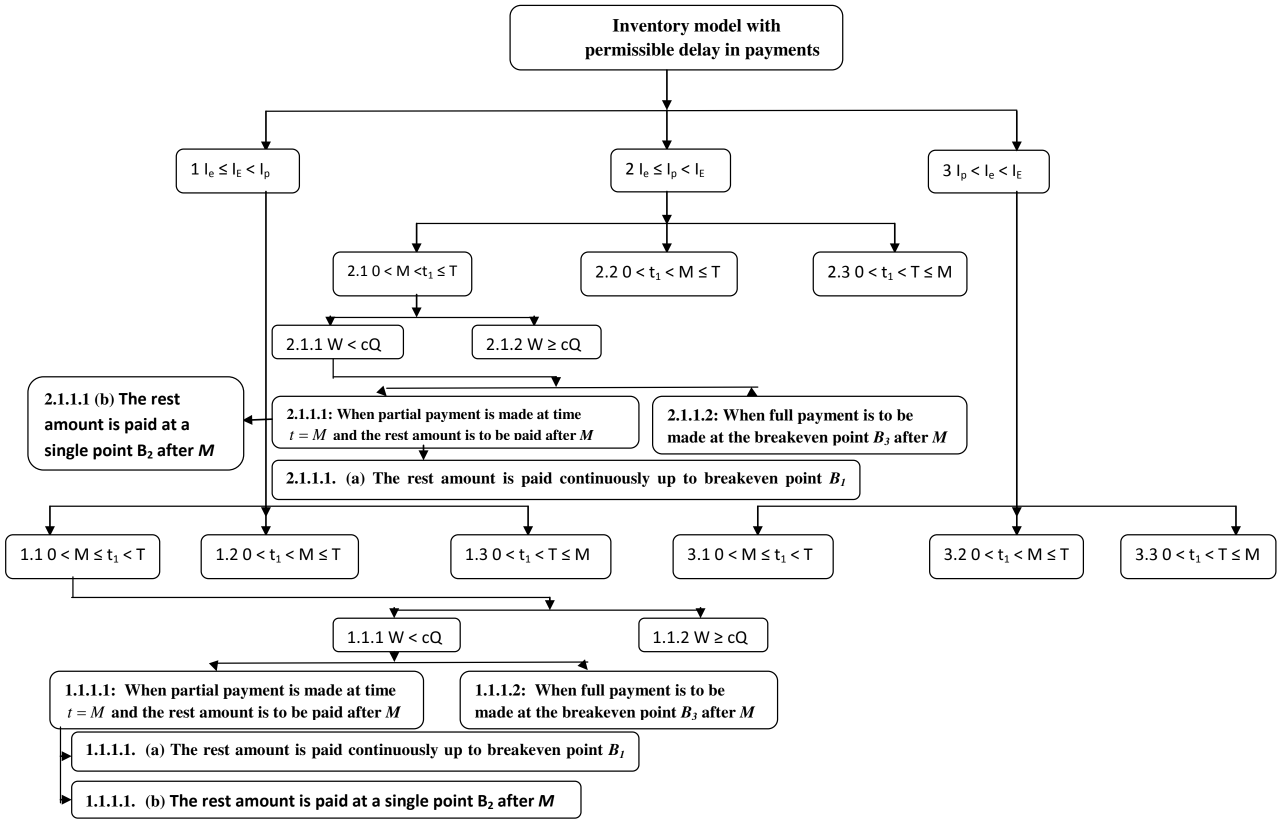

Fig. 2: A Schematic flow of the model

Sub case 1.1.1: $W_1 < Qc$
Here, the retailer's amount (W_1) is less than the amount payable to the supplier. In this situation, the supplier may either agree or disagree to receive the partial payment. As a result, two scenarios may be possible: 1.1.1.1 when partial payment is made at $t = M$ and the rest amount is to be paid after $t = M$ or 1.1.1.2 when full payment is to be made after $t = M$ due to non-willingness of acceptance of partial payment on the part of the supplier.

Scenario 1.1.1.1: When partial payment is made at time $t = M$ and the rest amount is to be paid after the time $t = M$: In this situation, the supplier agrees to accept the partial payment but for the payment overdue, he may agree on two different possible choices: (a) when the rest amount continuously is paid after M and (b) when the rest amount is paid at a single point of time after M.

Scenario 1.1.1.1 (a): When the rest amount is paid continuously up to breakeven point B_1 (say) after M.
In this case, the retailer pays W_1 amount at $t = M$ and the rest amount $(cQ - W_1)$ along with the interest charged is paid continuously from M to some payoff time (says B_1).

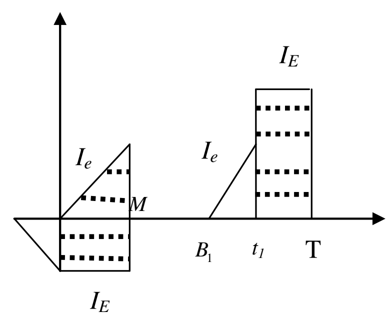

Fig. 3: Interest earned in scenario 1.1.1.1. (a)

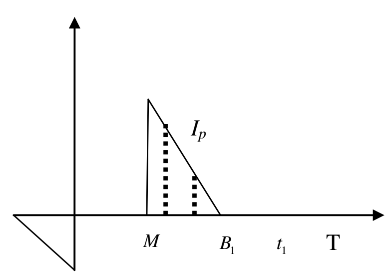

Fig. 4: Interest payable in scenario 1.1.1.1. (a)

So, the interest payable during the time period $[M,B_1] = \frac{1}{2}(cQ-W_1)(B_1-M)I_p$, and

the total amount payable during $[M,B_1]$ is sum of the remaining amount and interest payable during the same period. i.e. $(cQ-W_1)+\frac{1}{2}(cQ-W_1)(B_1-M)I_p$

The sales revenue during the time period $[M,B_1]$ is $Dp(B_1-M)$.

Now at $t = B_1$, the total amount payable to the supplier $=$ the total amount available to the retailer, i.e.

$$(cQ-W_1)+\frac{1}{2}(cQ-W_1)(B_1-M)I_p = D(B_1-M)p \quad (8)$$

$$\Rightarrow B_1 = M + \frac{2(cQ-W_1)}{2Dp-(cQ-W_1)I_p} \quad (9)$$

Now, the retailer starts generating profit from the sales revenue and interest earned on the regular sales revenue during the period $[B_1,t_1]$ is $D(t_1-B_1)p$ and $\frac{1}{2}D(t_1-B_1)^2 pI_e$ respectively. At time $t = t_1$ retailer has $D(t_1-B_1)p+\frac{1}{2}D(t_1-B_1)^2 pI_e$ amount. He uses this revenue to earn interest on fix deposit of this amount during the time period $[t_1,T]$.

The interest earned for time period $[t_1,T]$ is $Dp(t_1-B_1)\left\{1+\frac{1}{2}(t_1-B_1)I_e\right\}(T-t_1)I_E$.

Therefore, the total profit per unit time for this case is given by

$$\begin{aligned} AP_{1.1.1.1.a}(\mu,t_1,T) = \frac{1}{T}[&\langle \text{Total selling revenue during } [B_1,t_1]\rangle \\ &+\langle \text{Interest earned during } [B_1,t_1]\rangle \\ &+\langle \text{Interest earned during } [t_1,T]\rangle \\ &-\langle \text{Ordering cost}\rangle - \langle \text{Holding cost}\rangle \\ &-\langle \text{Shortage cost}\rangle] \end{aligned} \quad (10)$$

$$\begin{aligned} AP_{1.1.1.1.a}(\mu,t_1,T) = \frac{1}{T}\Bigg[&D(t_1-B_1)p+\frac{1}{2}D(t_1-B_1)^2 pI_e \\ &+Dp(t_1-B_1\left\{1+\frac{1}{2}(t_1-B_1)I_e\right\}(T-t_1)I_E \\ &-A-\frac{1}{2}Dt_1^2 h-\frac{\pi D(T-t_1)^2}{2}\Bigg] \end{aligned} \quad (11)$$

Where $B_1 = M + \frac{2(cQ-W_1)}{2Dp-(cQ-W_1)I_p}$.

Scenario 1.1.1.1(b): When the rest amount is paid at a single point after M

In this scenario, the supplier accepts the payment only in two installments, first is at time $t = M$ and second is at time $t = B_2$. The retailer pays amount of W_1 at the time M and the remaining amount $(cQ-W_1)$ along with the interest payable for the period M to B_2 is paid at breakeven point B_2.

Hence, at time $t = B_2$, the total amount payable will be the sum of remaining amount and interest payable during the time period $[M,B_2]$ i.e. $(cQ-W_1)+(cQ-W_1)(B_2-M)I_p$.

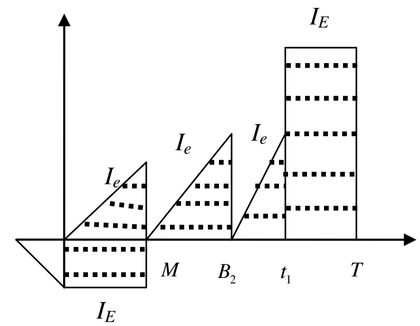

Fig. 5: Interest earned in scenario 1.1.1.1. (b)

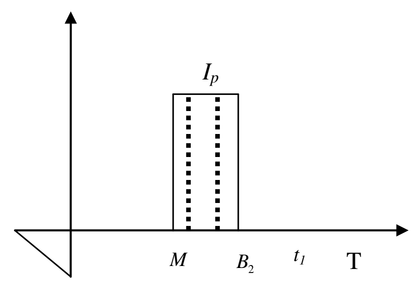

Fig. 6: Interest payable in scenario 1.1.1.1. (b)

The sales revenue during the time period $[M,B_2]$ is $Dp(B_2-M)$ and interest earned on regular sales revenue during the same time period is $\frac{1}{2}D(B_2-M)^2 pI_e$.

Hence, at time $t = B_2$, the total amount in the account of retailer is sum of sales revenue and interest earned during the time period $[M,B_2]$ i.e. $D(B_2-M)p+\frac{1}{2}D(B_2-M)^2 pI_e$.

At time $t = B_2$, retailer wants to settle his account with supplier, so the total amount payable should be equal to total amount generated by retailer i.e.

$$\begin{aligned} &(cQ-W_1)+(cQ-W_1)(B_2-M)I_p \\ &\quad = D(B_2-M)p+\frac{1}{2}D(B_2-M)^2 pI_e \end{aligned}$$

$$\begin{aligned} &(Qc-W_1)+(Qc-W_1)(B_2-M)I_p \\ &\quad = D(B_2-M)p+\frac{1}{2}D(B_2-M)^2 pI_e \\ \Rightarrow B_2 = &\frac{1}{DpI_e}\Bigg\{-Dp+DpI_eM+DTcI_p-I_pW_1 \\ &+\Big(D^2p^2-2D^2TcI_pp+2DpI_pW_1+D^2T^2c^2I_p^2 \\ &-2DTcI_p^2W_1+I_p^2W_1^2+2D^2pI_eTc-2DpI_eW_1\Big)^{\frac{1}{2}}\Bigg\} \end{aligned} \quad (12)$$

After settling the account with this supplier at time $t = B_2$, the retailer's sales revenue would consist of sales revenue $D(t_1 - B_2)p$ for the period $[B_2, t_1]$ and the interest earned on regular sales revenue is $\frac{1}{2}D(t_1 - B_2)^2 pI_e$. At time $t = t_1$ retailer has $D(t_1 - B_2)p + \frac{1}{2}D(t_1 - B_2)^2 pI_e$ amount. He uses this revenue to earn interest on fix deposit of this amount during the time period $[t_1, T]$.

The interest earned on fix deposit amount $= D(t_1 - B_2)p\left(1 + \frac{1}{2}(t_1 - B_2)I_e\right)(T - t_1)I_E$.

Therefore, the total profit per unit time for this case is given by

$$AP_{1.1.1.1.b}(\mu, t_1, T) = \frac{1}{T}[\langle \text{Total selling revenue during } [B_2, t_1] \rangle + \langle \text{Interest earned during } [B_2, t_1] \rangle + \langle \text{Interest earned during } [t_1, T] \rangle - \langle \text{Ordering cost} \rangle - \langle \text{Holding cost} \rangle - \langle \text{Shortage cost} \rangle \quad (13)$$

$$AP_{1.1.1.1.b}(\mu, t_1, T) = \frac{1}{T}\left[D(t_1 - B_2)p + \frac{1}{2}D(t_1 - B_2)^2 pI_e + D(t_1 - B_2)p\left(1 + \frac{1}{2}(t_1 - B_2)I_e\right)(T - t_1)I_E - A - \frac{1}{2}Dt_1^2 h - \frac{\pi D(T - t_1)^2}{2}\right] \quad (14)$$

Where

$$B_2 = \frac{1}{DpI_e}\left\{ -Dp + DpI_eM + DTcI_p - I_pW_1 + \left(D^2p^2 - 2D^2TcI_pp + 2DpI_pW_1 + D^2T^2c^2I_p^2 - 2DTcI_p^2W_1 + I_p^2W_1^2 + 2D^2pI_eTc - 2DpI_eW_1\right)^{\frac{1}{2}} \right\}$$

Scenario 1.1.1.2: When full payment is to be made at the breakeven point B_3 after $t = M$

Supplier wants the full payment at some fixed point B_3 after M when it is possible and he will charge the interest at rate (Ip) on amount Qc for the period $[M, B_3]$. But retailer has W_1 amount at time $t = M$ and he will earn interest at the rate (I_E) on fix deposit of this amount for the period $[M, B_3]$. After M, he also generates the sales revenue as well as earns interest on regular sales revenue during the period $[M, B_3]$.

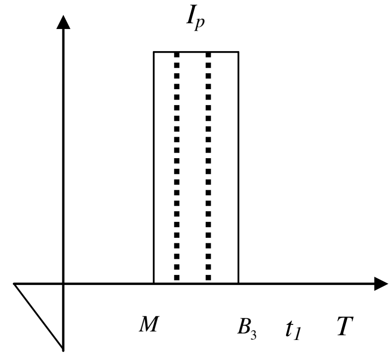

Fig. 7: Interest earned in scenario 1.1.1.2

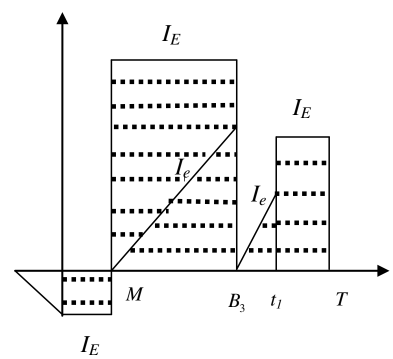

Fig. 8: Interest payable in scenario 1.1.1.2

The interest earned on fix deposit amount W_1 for the time period $[M, B_3]$ is $W_1I_E(B_3 - M)$ and the interest earned on the continuous sales revenue $D(B_3 - M)p$ from time period $[M, B_3]$ is $\frac{1}{2}D(B_3 - M)^2 pI_e$.

Hence, the total interest earned during the time period $[M, B_3]$ is

$$= W_1I_E(B_3 - M) + \frac{1}{2}D(B_3 - M)^2 pI_e$$

The interest payable during the same time period $= QcI_p(B_3 - M)$.

Again, to determine the value of breakeven point, the total amount payable to the supplier should equal to the total amount available to the retailer i.e.

$$Qc + Qc(B_3 - M)I_p = W_1 + D(B_3 - M)p + W_1I_E(B_3 - M) + \frac{1}{2}D(B_3 - M)^2 pI_e \quad (15)$$

$$\Rightarrow \quad B_3 = \frac{1}{DpI_e}\left\{ -Dp + DpI_eM + DTcI_p - I_eW_1 + \left(W_1^2I_e^2 - 2DTcI_pW_1I_e + D^2T^2c^2I_p^2 - 2D^2pI_pTc + D^2p^2 + 2D^2pI_eTc\right)^{\frac{1}{2}} \right\} \quad (16)$$

Further, the sales revenue during the time period $[B_3, t_1]$ is $D(t_1 - B_3)p$ and the interest earned on regular sales revenue during this period is $\frac{1}{2}D(t_1 - B_3)^2 pI_e$. So that, at time $t = t_1$ retailer has $D(t_1 - B_3)p + \frac{1}{2}D(t_1 - B_3)^2 pI_e$ amount. He uses this revenue to earn interest on fix deposit of this amount during the time period $[t_1, T]$.

The interest earned on fix deposit amount $= D(t_1 - B_3)p\left(1 + \frac{1}{2}(t_1 - B_3)I_e\right)(T - t_1)I_E$.

Accordingly, the total profit per unit time for this case is given by

$$\begin{aligned} AP_{1.1.1.2}(\mu, t_1, T) = \frac{1}{T}[&\langle \text{Total selling revenue during } [B_3, t_1]\rangle \\ &+ \langle \text{Interest earned during } [B_3, t_1]\rangle \\ &+ \langle \text{Interest earned during } [t_1, T]\rangle \\ &- \langle \text{Ordering cost}\rangle - \langle \text{Holding cost}\rangle \\ &- \langle \text{Shortage cost}\rangle] \end{aligned} \tag{17}$$

$$\begin{aligned} AP_{1.1.1.2}(\mu, t_1, T) = \frac{1}{T}\Bigg[&D(t_1 - B_3)p + \frac{1}{2}D(t_1 - B_3)^2 pI_e \\ &+ D(t_1 - B_3)p\left(1 + \frac{1}{2}(t_1 - B_3)I_e\right) \\ &(T - t_1)I_E - A - \frac{1}{2}Dt_1^2 h - \frac{\pi D(T - t_1)^2}{2}\Bigg] \end{aligned} \tag{18}$$

Where

$$\begin{aligned} B_3 = \frac{1}{DpI_e}\Bigg\{&-Dp + DpI_e M + DTcI_p - I_e W_1 \\ &+ \Big(W_1^2 I_e^2 - 2DTcI_p W_1 I_e + D^2T^2c^2I_p^2 \\ &- 2D^2pI_pTc + D^2p^2 + 2D^2pI_eTc\Big)^{\frac{1}{2}}\Bigg\} \end{aligned}$$

Sub case 1.1.2: $W_1 \geq Qc$

In this sub case, the retailer has to pay only Qc amount to the supplier at time $t = M$ and he will fix deposit the excess amount $(W_1 - Qc)$ to earn the interest at the rate of (I_E) for the time period $[M, T]$. The interest earned on this amount is equal to $(W_1 - Qc)(T - M)I_E$. Further, after time $t = M$, the retailer continuously sales the products and uses the revenue to earn interest.

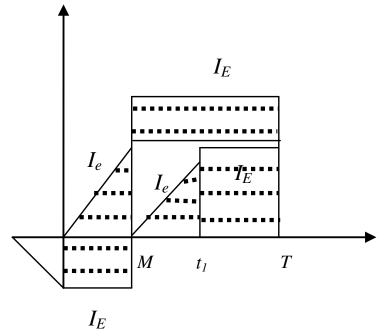

Fig. 9: Interest earned in sub case 1.1.2

The interest earned on the regular sales revenue $D(t_1 - M)p$ during the period $[M, t_1]$ is $\frac{1}{2}D(t_1 - M)^2 pI_e$. At time $t = t_1$ retailer has $D(t_1 - M)p + \frac{1}{2}D(t_1 - M)^2 pI_e$ amount. He uses this revenue to earn interest on fix deposit of this amount during the time period $[t_1, T]$ is $D(t_1 - M)p\left(1 + \frac{1}{2}(t_1 - M)I_e\right)(T - t_1)I_E$.

Therefore, the total profit per unit time for this case is given by

$$\begin{aligned} &AP_{1.1.2}(\mu, t_1, T) \\ &= \frac{1}{T}[\langle \text{Total sales revenue during } [M, t_1]\rangle \\ &+ \langle \text{Interest earned on the sales revenue during } [M, t_1]\rangle \\ &+ \langle \text{Interest earned on the sales revenue during } [t_1, T]\rangle \\ &+ \langle \text{Excess amount after paying the amount to the supplier}\rangle \\ &+ \langle \text{Interest earned on the excess amount during } [M, T]\rangle \\ &- \langle \text{Ordering cost}\rangle - \langle \text{Holding cost}\rangle \\ &- \langle \text{Shortage cost}\rangle \end{aligned} \tag{19}$$

$$\begin{aligned} &AP_{1.1.2}(\mu, t_1, T) \\ &= \frac{1}{T}\Bigg[D(t_1 - M)p + \frac{1}{2}D(t_1 - M)^2 pI_e \\ &+ (W_1 - Qc)\{1 + (T - M)I_E\} \\ &+ \left(D(t_1 - M)p + \frac{1}{2}D(t_1 - M)^2 pI_e\right)(T - t_1)I_E \\ &- A - \frac{1}{2}hDt_1^2 - \frac{\pi D(T - t_1)^2}{2}\Bigg] \end{aligned} \tag{20}$$

Case 1.2: $0 < t_1 \leq M < T$

In this case, the trade credit period M offer by to the supplier is lies between stock out period t_1 and replenishment cycle time T. The retailer will pay off the total amount owed to the supplier at the end of the trade credit period M. Therefore, there is no interest payable to supplier but the retailer uses the sales revenue to earn interest at the rate of I_e and I_E during the period $[0, M]$.

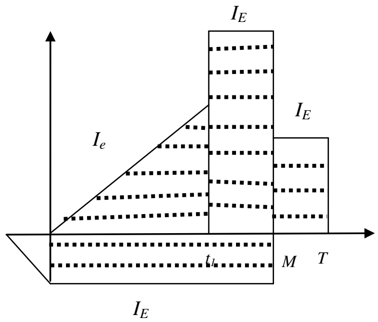

Fig. 10: Interest earned in case 1.2

Hence, the retailer the total interest earned is calculated in three different cases.

1) The interest earned at rate of I_E on the shortages revenue $Dp(T-t_1)$ during the period $[0,M] = Dp(T-t_1)MI_E$.
2) The interest earned interest on continuous sales revenue during the period $[0,t_1] = \frac{1}{2}DpI_e t_1^2$
3) The interest earned during the period

$$[t_1,M] = Dt_1 p\left(1+\frac{1}{2}t_1 I_e\right)I_E(M-t_1)$$

At M, retailer has

$$Dp(T-t_1)(1+MI_E)+Dpt_1\left(1+\tfrac{1}{2}t_1 I_e\right)(1+(M-t_1)I_E) \equiv W_2$$

amount in his account but retailer settled his account with supplier at M. He pays Qc amount to supplier and earned interest on the excess amount $W_2 - Qc$ at the interest rate I_E. The interest earned during the period $[M,T]$ is $(W_2 - Qc)(T-M)I_E$.

Thus, the total profit per unit time for this case is given by

$$\begin{aligned} &AP_{1.2}(\mu,t_1,T)\\ &= \frac{1}{T}[\langle\text{Excess amount}\rangle\\ &+\langle\text{Interest earned on the excess amount during the period } [M,T]\rangle\\ &-\langle\text{Ordering cost}\rangle - \langle\text{Holding cost}\rangle\\ &-\langle\text{Shortage cost}\rangle] \end{aligned} \quad (21)$$

$$AP_{1.2}(\mu,t_1,T) = \frac{1}{T}\Bigg[(W_2-Qc)+(W_2-Qc)(T-M)I_E - A - \frac{1}{2}hDt_1^2 - \frac{\pi D(T-t_1)^2}{2}\Bigg] \quad (22)$$

Case 1.3: $0 < t_1 < T \le M$

In this situation, the trade credit period M offered by the supplier is greater than the replenishment cycle time T. The retailer will pay off the total amount owed to the supplier at the end of the trade credit period M. So, there is no interest payable to supplier but the retailer uses the sales revenue to earn interest at the rate of I_e and I_E during the period $[0,M]$.

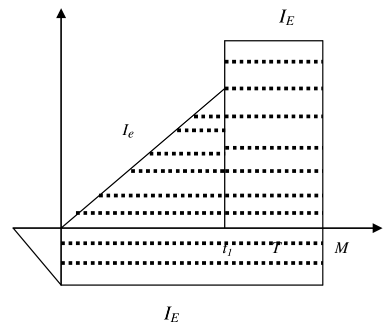

Fig. 11: Interest earned in case 1.3

Hence, the retailer the total interest earned is calculated in three different cases.

1) The interest earned at rate of I_E on the shortages revenue $Dp(T-t_1)$ during the period $[0,M] = Dp(T-t_1)MI_E$.
2) The interest earned interest on continuous sales revenue during the period $[0,t_1] = \frac{1}{2}Dpt_1^2$.
3) The interest earned during the period $[t_1,M] = Dt_1 p\left(1+\frac{1}{2}t_1 I_e\right)I_E(M-t_1)$

At M, retailer has $Dp(T-t_1)(1+MI_E) + Dpt_1\left(1+\frac{1}{2}t_1 I_e\right)(1+(M-t_1)I_E) \equiv W_3$ (say) amount in his account but retailer settled his account with supplier at M. He pays Qc amount to supplier.

Therefore, the total profit per unit time for this case is given by

$$\begin{aligned} AP_{1.3}(\mu,t_1,T) = \frac{1}{T}[&\langle\text{Excess amount}\rangle\\ &-\langle\text{Ordering cost}\rangle - \langle\text{Holding cost}\rangle\\ &-\langle\text{Shortage cost}\rangle] \end{aligned} \quad (23)$$

$$\begin{aligned} &AP_{1.3}(\mu,t_1,T)\\ &= \frac{1}{T}\left[(W_3-Qc) - A - \frac{1}{2}hDt_1^2 - \frac{\pi D(T-t_1)^2}{2}\right] \end{aligned} \quad (24)$$

Section 2: $I_e \le I_p < I_E$

This case situation indicates that the interest payable rate (I_p) is lies between both interest rate (I_e, I_E). Further, depending on the values of t_1, M and T the following three sub-cases arises:

2.1: $0 < M \le t_1 < T$**, 2.2:** $0 < t_1 \le M < T$ and **2.3:** $0 < t_1 < T \le M$**.**

Case 2.1: $0 < M \le t_1 < T$

In this case, the retailer would try to pay off the total purchase cost to the supplier as soon as possible. During the period $[0,M]$, the retailer uses the sales revenue to earn interest. Hence, the total sales revenue from the period $[0,M]$ is $Dp\{M+(T-t_1)\}$ and the interest earned during the same time period is $DMp\left\{(T-t_1)I_E+\frac{1}{2}MI_e\right\}$.

Therefore, the retailer has total amount at time $t=M$ is

$$Dp\left[(T-t_1)+M\left\{1+(T-t_1)I_E+\frac{1}{2}MI_e\right\}\right]\equiv W_1 \text{ (say)}.$$

However, the retailer owes Qc amounts as the purchase cost from the supplier at time $t=0$. Now at M, retailer has to settle his account with the supplier. Based on the difference between W_1 and Qc, the following two sub cases may arise: **cases 2.1.1** $W_1<Qc$ and **2.1.2** $W_1\geq Qc$

Sub case 2.1.1: $W_1<Qc$

Here, the fixed amount is less than the amount Qc. This implies that the interest earned on fixed amount is less than the interest paid amount. So the retailer will pay the amount Qc as soon as possible. The mathematical formulation of this sub case is same as that of sub case 1.1.1 $W<Qc$ in case 1.

Sub case 2.1.2: $W_1\geq Qc$

In this sub case, the interest earned on fixed amount is greater than interest payable. So interest on W_1 is greater than interest payable in one cycle. So, the retailer cannot pay any amount before the cycle length. He pays the total amount along with the interest charged at the end of cycle length.

The interest payable during the period $[M,T]=Qc(T-M)I_p$ and

The interest earned on the amount W_1 during the period $=W_1I_E(T-M)$

Further, after time $t=M$, the retailer continuously sales the products and uses the revenue to earn interest.

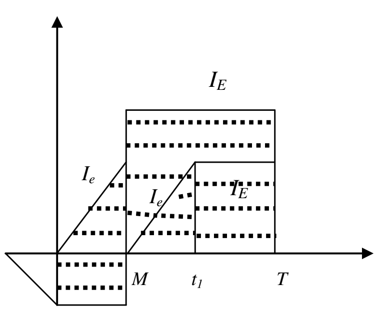

Fig. 12: Interest earned in sub case 2.1.2

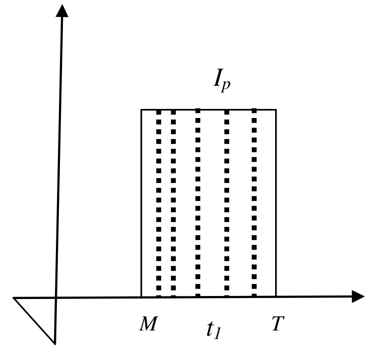

Fig. 13: Interest payable in sub case 2.1.2

So, interest earned on the sales revenue during the period $[M,t_1]$ is $\frac{1}{2}D(t_1-M)^2pI_e$ and also earned the interest during the period $[t_1,T]$ on the revenue $D(t_1-M)p+\frac{1}{2}D(t_1-M)^2pI_e$ is $(D(t_1-M)p+\frac{1}{2}D(t_1-M)^2pI_e)(T-t_1)I_E$.

Therefore, the total profit per unit time for this case is given by

$$\begin{aligned}
&AP_{2.1.2}(\mu,t_1,T)\\
&=\frac{1}{T}[\langle\text{Total sales revenue during }[M,t_1]\rangle\\
&+\langle\text{Interest earned on the sales revenue during }[M,t_1]\rangle\\
&+\langle\text{Interest earned on fixed deposit amount during }[t_1,T]\rangle\\
&+\langle\text{Amount }W_1\rangle\\
&+\langle\text{Interest earned on }W_1\text{ during }[M,T]\rangle\\
&-\langle\text{Purchasing cost}\rangle\\
&-\langle\text{Interest payable during the time period }[M,T]\rangle\\
&-\langle\text{Ordering cost}\rangle-\langle\text{Holding cost}\rangle\\
&-\langle\text{Shortage cost}\rangle]
\end{aligned}\tag{25}$$

$$\begin{aligned}
&AP_{2.1.2}(\mu,t_1,T)\\
&=\frac{1}{T}\left[D(t_1-M)p+\frac{1}{2}D(t_1-M)^2pI_e\right.\\
&+W_1\{1+(T-M)I_E\}-A-\frac{1}{2}hDt_1^2-\frac{\pi D(T-t_1)^2}{2}\\
&+\left(D(t_1-M)p+\frac{1}{2}D(t_1-M)^2pI_e\right)(T-t_1)I_E\\
&\left.-Qc\{1+(T-M)I_p\}\right]
\end{aligned}\tag{26}$$

Case 2.2: $0<t_1\leq M<T$

Here, the retailer sells all his products during the time interval $[0,t_1]$ and the sales revenue is greater than the purchase cost. This implies that the interest on sales revenue is greater than the interest payable in one cycle. So, the retailer cannot pay any amount before the cycle length. He pays the total amount along with the interest charged at the end of cycle length.

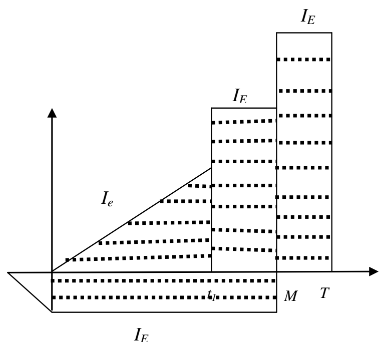

Fig. 14: Interest earned in case 2.2

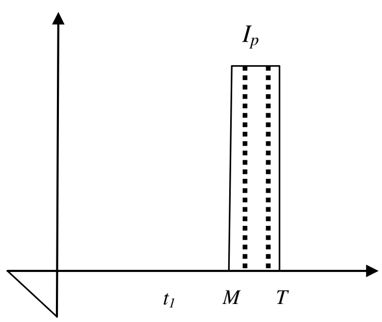

Fig. 15: Interest payable in case 2.2

Hence, the retailer the total interest earned is calculated in three different cases.

1)The interest earned at rate of I_E on the shortages revenue $Dp(T-t_1)$ during the period $[0,M] = Dp(T-t_1)MI_E$.
2)The interest earned interest on continuous sales revenue during the period $[0,t_1] = \frac{1}{2}Dpt_1^2$.
3)The interest earned during the period $[t_1,T] = Dt_1 p\left(1+\frac{1}{2}t_1 I_e\right) I_E (T-t_1)$.

At M, retailer has $Dp(T-t_1)(1+MI_E) + Dpt_1\left(1+\frac{1}{2}t_1 I_e\right)(1+(M-t_1)I_E) \equiv W_2$ amount in his account but W_2 is greater than amount Qc. So, the retailer settles his account with the supplier at the end of cycle length despite at M, since I_E is greater than I_p. He pays Qc amount along with interest payable at the end the cycle length.

The interest earned during the period $[M,T]$ is $W_2(T-M)I_E$.

The interest payable during the period $[M,T]$ is $Qc(T-M)I_p$.

Therefore, the total profit per unit time for this case is given by

$$\begin{aligned} AP_{2.2}(\mu,t_1,T) = \frac{1}{T}[&\langle\text{Total sales revenue}\rangle \\ &+\langle\text{Interest earned on } W_2 \\ &\text{amount during the period } [M,T]\rangle \\ &-\langle\text{Ordering cost}\rangle - \langle\text{Holding cost}\rangle \\ &-\langle\text{Shortage cost}\rangle - \langle\text{Purchasing cost}\rangle \\ &-\langle\text{Interest Payable during } [M,T]\rangle] \quad (27) \end{aligned}$$

$$\begin{aligned} AP_{2.2}(\mu,t_1,T) = \frac{1}{T}\Bigg[&(W_2 - Qc) + W_2(T-M)I_E \\ &-A - \frac{1}{2}hDt_1^2 - \frac{\pi D(T-t_1)^2}{2} \\ &-Qc(T-M)I_p\Bigg] \quad (28) \end{aligned}$$

Case 2.3: $0 < t_1 < T \le M$
In this case, the trade credit period M offered by the supplier is greater than the replenishment cycle time T. The retailer will pay off the total amount owed to the supplier at the end of the trade credit period M. Therefore, there is no interest payable to the supplier but the retailer uses the sales revenue to earn interest at the rate of I_e and I_E during the period $[0,M]$.

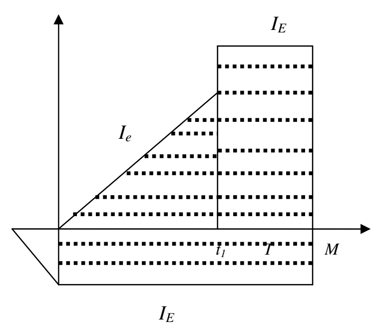

Fig. 16: Interest earned in case 2.3

Consequently, the total interest earned by the retailer is calculated in three different cases:

1) The interest earned at rate of I_E on the shortages revenue $Dp(T-t_1)$ during the period $[0,M] = Dp(T-t_1)MI_E$
2) (2) The interest earned interest on continuous sales revenue during the period $[0,t_1] = \frac{1}{2}DpI_e t_1^2$
3) The interest earned during the period $[t_1,M] = Dt_1 p\left(1+\frac{1}{2}t_1 I_e\right) I_E (M-t_1)$

At M, retailer has $Dp(T-t_1)(1+MI_E) + Dpt_1\left(1+\frac{1}{2}t_1 I_e\right)(1+(M-t_1)I_E) \equiv W_3$ (say) amount in his account but the retailer settles his account with supplier at M and pays Qc amount to the supplier.

Therefore, the total profit per unit time for this case is given by

$$AP_{2.3}(\mu,t_1,T) = \frac{1}{T}[\langle \text{Excess amount}\rangle - \langle \text{Ordering cost}\rangle - \langle \text{Holding cost}\rangle - \langle \text{Shortage cost}\rangle] \quad (29)$$

$$AP_{2.3}(\mu,t_1,T) = \frac{1}{T}\left[(W_3 - Qc) - A - \frac{1}{2}hDt_1^2 - \frac{\pi D(T-t_1)^2}{2}\right] \quad (30)$$

Section 3: $I_p < I_e < I_E$
In this scene, both the interest earned I_e and I_p are greater than the interest payable I_p. In this case, the retailer cannot pay any amount at the end of permissible delay in payments. He settles his account at the end of cycle length if permissible delay period is less than cycle length. If the permissible delay period is greater than cycle length, then he settles his account at M. Further, depending on the values of T and M, two sub-cases may arise:

3.1: $M \le T$ **and 3.2:** $T < M$

Case 3.1: $M \le T$
This case situation indicates that the replenishment cycle time T is greater than or equals to the permissible delay in payments M.

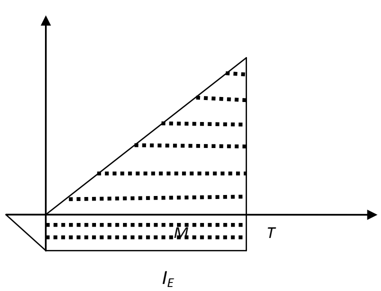

Fig. 17: Interest earned in case 3.1

In this case, interest earned is calculated in three parts:

1) Interest earned on shortages revenue during the period $[0,T] = D(T-t_1)pTI_E$
2) Interest earned on the sale revenue proceeds during the period $[0,t_1] = \frac{1}{2}Dt_1^2 pI_e$
3) Interest earned during the period $[t_1,T] = \left(Dt_1p + \frac{1}{2}Dt_1^2pI_e\right)(T-t_1)I_E$ and the total interest earned in one cycle

$$= D(T-t_1)pTI_E + \frac{1}{2}Dt_1^2pI_e + \left(Dt_1p + \frac{1}{2}Dt_1^2pI_e\right)(T-t_1)I_E$$

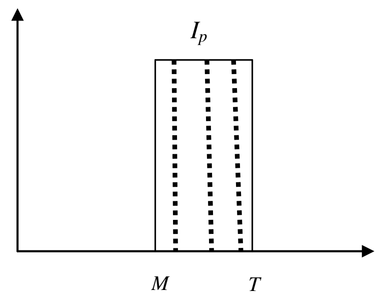

Fig. 18: Interest payable in case 3.1

In this case, the retailer paid total amount along with interest payable at end of cycle length T is

$$= DTc(1 + (T-M)I_p).$$

Therefore, the total profit per unit time for this case is given by

$$AP_{3.1}(\mu,t_1,T) = \frac{1}{T}[\langle \text{Total sale revenue during } [0,T]\rangle + \langle \text{Interest earned on the sales revenue during } [0,t_1]\rangle + \langle \text{Interest earned on shortages revenue during } [0,T] - \langle \text{Total amount paid as well as interest payable at } T\rangle - \langle \text{Ordering cost}\rangle - \langle \text{Holding cost}\rangle - \langle \text{Shortages cost}\rangle] \quad (31)$$

$$AP_{3.1}(\mu,t_1,T) = \frac{1}{T}\Bigg[DT(p-c) + \frac{1}{2}Dt_1^2pI_e + D(T-t_1)pTI_E + \left(Dt_1p + \frac{1}{2}Dt_1^2pI_e\right)(T-t_1)I_E + D(T-M)p\left\{1 + \frac{1}{2}(T-M)I_e\right\}(T-M)I_e - DTc(T-M)I_p - A - \frac{1}{2}hDT^2 - \frac{\pi D(T-t_1)^2}{2}\Bigg] \quad (32)$$

Case 3.2: $T < M$
In this case, the replenishment cycle time T is less than or equal to the permissible delay period M. In this situation the retailer will pay off the total amount owed to the supplier at the end of the trade credit period M. Therefore, there is no interest payable to the supplier but the retailer uses the sales revenue to earn interest at the rate of I_e and I_E during the period $[0,M]$.

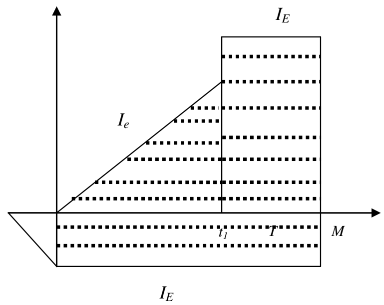

Fig. 19: Interest earned in case 3.2

The interest earned is calculated in three parts.

1) Interest earned on shortages revenue during the period $[0,M] = D(T-t_1)pMI_E$
2) Interest earned on the sale revenue proceeds during the period $[0,t_1] = \frac{1}{2}Dt_1^2 pI_e$
3) Interest earned during the period $[t_1,M] = \left(Dt_1 p + \frac{1}{2}Dt_1^2 pI_e\right)(M-t_1)I_E$

Hence, the interest earned in one cycle

$$= D(T-t_1)pMI_E + \frac{1}{2}Dt_1^2 pI_e + \left(Dt_1 p + \frac{1}{2}Dt_1^2 pI_e\right)(M-t_1)I_E$$

Therefore, the total profit per unit time for this case is given by

$$\begin{aligned} &AP_{3.2}(\mu,t_1,T) \\ &= \frac{1}{T}[\langle \text{Excess sales revenue} \rangle \\ &+ \langle \text{Interest earned in one cycle} \rangle \\ &- \langle \text{Ordering cost} \rangle - \langle \text{Holding cost} \rangle \\ &- \langle \text{Shortages cost} \rangle] \end{aligned} \quad (33)$$

$$\begin{aligned} AP_{3.2}(\mu,t_1,T) = \frac{1}{T}\Bigg[&(DTp - DTc) + D(T-t_1)pMI_E \\ &+ \frac{1}{2}Dt_1^2 pI_e + \left(Dt_1 p + \frac{1}{2}Dt_1^2 pI_e\right) \\ &(M-t_1)I_E - A - \frac{1}{2}hDT^2 - \frac{\pi D(T-t_1)^2}{2}\Bigg] \end{aligned} \quad (34)$$

Hence, the total profit per unit time $AP(\mu,t_1,T)$ for the inventory system can be expressed as

$$AP(\mu,t_1,T) = \begin{cases} AP_{1.1.1.1.(a)}(\cdot) \\ \quad \text{if } 0 < M \le t_1 < T,\ W < Qc,\ I_e < I_E \le I_p, \\ \quad \text{partially and rest amount paid continuously} \\ AP_{1.1.1.1.(b)}(\cdot) \\ \quad \text{if } 0 < M \le t_1 < T,\ W < Qc,\ I_e < I_E \le I_p, \\ \quad \text{partially and rest amount second shipment} \\ AP_{1.1.1.2}(\cdot) \\ \quad \text{if } 0 < M \le t_1 < T\ W < Qc,\ I_e < I_E \le I_p, \\ \quad \text{and full amount made after } t = M \\ AP_{1.1.2}(\cdot) \\ \quad \text{if } 0 < M \le t_1 < T,\ W \ge Qc, \text{ and } I_e < I_E \le I_p \\ AP_{1.2}(\cdot) \\ \quad \text{if } 0 < t_1 \le M < T, \text{ and } I_e < I_E \le I_p \\ AP_{1.3}(\cdot) \\ \quad \text{if } 0 < t_1 < T \le M, and I_e < I_E \le I_p \\ AP_{2.1.1.1.(a)}(\cdot) \\ \quad \text{if } 0 < M \le t_1 < T,\ W < Qc,\ I_e \le I_p < I_E, \\ \quad \text{partially and rest amount paid continuously} \\ AP_{2.1.1.1.(b)}(\cdot) \\ \quad \text{if } 0 < M \le t_1 < T,\ W < Qc,\ I_e \le I_p < I_E, \\ \quad \text{partially and rest amount second shipment} \\ AP_{2.1.1.2}(\cdot) \\ \quad \text{if } 0 < M \le t_1 < T,\ W < Qc,\ I_e \le I_p < I_E, \\ \quad \text{and full amount made after } t = M \\ AP_{2.1.2}(\cdot) \\ \quad \text{if } 0 < M \le t_1 < T,\ W \ge Qc, \text{ and } I_e \le I_p < I_E \\ AP_{2.2}(\cdot) \\ \quad \text{if } 0 < t_1 \le M < T, \text{ and } I_e \le I_p < I_E \\ AP_{2.3}(\cdot) \\ \quad \text{if } 0 < t_1 < T \le M, \text{ and } I_e \le I_p < I_E \\ AP_{3.1}(\cdot) \\ \quad \text{if } M < T \text{ and } I_p < I_e < I_E \\ AP_{3.2}(\cdot) \\ \quad \text{if } T \le M \text{ and } I_p < I_e < I_E \end{cases} \quad (35)$$

For the convenience, we let the twelve events as

$$\begin{aligned} E_1 = \{t | 0 < M \le t_1 < T, W < Qc,\ I_e < I_E \le I_p, \\ \text{partially and rest amount paid continuously}\} \\ E_2 = \{t | 0 < M \le t_1 < T, W < Qc,\ I_e < I_E \le I_p, \\ \text{partially and rest amount second shipment}\} \\ E_3 = \{t | 0 < M \le t_1 < T, W < Qc,\ I_e < I_E \le I_p, \\ \text{and full amount made after } t = M\} \\ E_4 = \{t | 0 < M \le t_1 < T, f0 < M \le t_1 < T,\ W \ge Qc, \\ \text{and } I_e < I_E \le I_p\} \end{aligned}$$

$$E_5 = \{t|0 < M \leq t_1 < T, \text{ if } 0 < t_1 \leq M < T \text{ and } I_e < I_E \leq I_p\}$$
$$E_6 = \{t|0 < t_1 < T \leq M \text{ and } I_e < I_E \leq I_p\}$$
$$E_7 = \{t|0 < M \leq t_1 < T, W < Qc, I_e \leq I_p < I_E, \text{ partially and rest amount paid continuously}\}$$
$$E_8 = \{t|0 < M \leq t_1 < T, W < Qc, I_e \leq I_p < I_E, \text{ partially and rest amount second shipment}\}$$
$$E_9 = \{t|0 < M \leq t_1 < T, W < Qc, I_e \leq I_p < I_E, \text{ and full amount made after } t = M\}$$
$$E_{10} = \{t|0 < M \leq t_1 < T, W \geq Qc, I_e \leq I_p < I_E\}$$
$$E_{11} = \{t|0 < t_1 \leq M < T, W \geq Qc, I_e \leq I_p < I_E\}$$
$$E_{12} = \{t|0 < t_1 < T \leq M, W \geq Qc, I_e \leq I_p < I_E\}$$
$$E_{13} = \{t|M < T \text{ and } I_p < I_e < I_E\}$$

$$E_{14} = \{t|\, T \leq M \text{ and } I_p < I_e < I_E\} \quad (36)$$

and define the characteristic functions as

$$\phi_j(t) = \begin{cases} 1 & t \in E_j \\ 0 & t \in E_j^c \end{cases}, \quad j = 1, \ldots, 14, \quad (37)$$

and also let

$$H_1 = \frac{1}{T}\left(A + \frac{hDt_1^2}{2} + \frac{\pi D(T-t_1)^2}{2}\right) \quad (38)$$

$$H_{k+1} = X_k \phi_k(t), \quad k = 1, \ldots, 14 \quad (39)$$

where

$$\begin{aligned}
&X_1 = AP_{1.1.1.1.(a)}(\cdot) - H_1, X_2 = AP_{1.1.1.1.(b)}(\cdot) - H_1,\\
&X_3 = AP_{1.1.1.2}(\cdot) - H_1, X_4 = AP_{1.1.2}(\cdot) - H_1,\\
&X_5 = AP_{1.2}(\cdot) - H_1, X_6 = AP_{1.3}(\mu, T) - H_1,\\
&X_7 = AP_{2.1.1.1.(a)}(\cdot) - H_1, X_8 = AP_{2.1.1.1.(b)}(\cdot) - H_1,\\
&X_9 = AP_{2.1.1.2}(\cdot) - H_1, X_{10} = AP_{2.1.2}(\cdot) - H_1,\\
&X_{11} = AP_{2.2}(\cdot) - H_1, X_{12} = AP_{2.3}(\mu, T) - H_1,\\
&X_{13} = AP_{3.1}(\cdot) - H_1, X_{14} = AP_{3.2}(\cdot) - H_1
\end{aligned} \quad (40)$$

Now, using the above notations (36) to (39), we can obtain collective form of the total profit per

$$AP(\mu, t_1, T) = \left(\left(\sum_{k=1}^{14} H_{k+1}\right) - H_1\right) \quad (41)$$

5 Fuzzy Model

In real business environments, the exact value of the parameters by decision maker would not be easily determined. Thus, the decision maker assumes the approximate values. In this model, demand rate $D = \widetilde{D} = \widetilde{a} - \widetilde{b}p$ is considered in fuzzy environment. By substituting $D = \widetilde{D} = \widetilde{a} - \widetilde{b}p$ in equation (41), the crisp model converts into fuzzy model i.e.

$$\widetilde{AP}(\mu, t_1, T) = \left(\sum_{k=1}^{14} \widetilde{H}_{k+1}\right) - \widetilde{H}_1 \quad (42)$$

Since the demand function is taken to be a triangular fuzzy number, $\widetilde{AP}(\mu, t_1, T)$ is also triangular fuzzy number i.e.

$$\widetilde{AP}(\mu, t_1, T) = (AP_1, AP_2, AP_3) \quad (43)$$

where $AP_i = \left(\sum_{k=1}^{14} H_{(k+1)_i}\right) - H_{1_{4-i}}, i = 1, 2 and 3$

$$\widetilde{H}_k = (H_{k_1}, H_{k_2}, H_{k_3}) \text{ and } k = 1, \ldots, 13 \quad (44)$$

$$H_{1_i} = \frac{1}{T}\left(A + \frac{hD_i t_1^2}{2} + \frac{\pi D_i (T-t_1)^2}{2}\right) \quad (45)$$

$$\begin{aligned}
H_{2_i} = \frac{1}{T}\Bigg[&D_i(t_1 - B_{1_{(4-i)}})p + \frac{1}{2}D_i(t_1 - B_{1_{(4-i)}})^2 pI_e\\
&+ D_i p(t_1 - B_{1_{(4-i)}})\left\{1 + \frac{1}{2}(t_1 - B_{1_{(4-i)}})I_e\right\}\\
&(T-t_1)I_E\Bigg]\phi_1(t)
\end{aligned} \quad (46)$$

$$\begin{aligned}
H_{3_i} = \frac{1}{T}\Bigg[&D_i(t_1 - B_{2_{(4-i)}})p + \frac{1}{2}D_i(t_1 - B_{2_{(4-i)}})^2 pI_e\\
&+ D_i(t_1 - B_{2_{(4-i)}})p\left(1 + \frac{1}{2}(t_1 - B_{2_{(4-i)}})I_e\right)(T-t_1)I_e\Bigg]\\
&\phi_2(t)
\end{aligned} \quad (47)$$

$$\begin{aligned}
H_{4_i} = \frac{1}{T}\Bigg[&D_i(t_1 - B_{3_{(4-i)}})p + \frac{1}{2}D_i(t_1 - B_{3_{(4-i)}})^2 pI_e\\
&+ \left(D_i(t_1 - B_{3_{(4-i)}})p + \frac{1}{2}D_i(t_1 - B_{3_{(4-i)}})^2 pI_e\right)(T-t_1)I_E\Bigg]\\
&\phi_3(t)
\end{aligned} \quad (48)$$

$$\begin{aligned}
H_{5_i} = \frac{1}{T}\Bigg[&D_i(t_1 - M)p + \frac{1}{2}D_i(t_1 - M)^2 pI_e\\
&+ (W_{1_i} - Q_{(4-i)}c)\{1 + (T-M)I_E\}\\
&\left(D_i(t_1 - M)p + \frac{1}{2}D_i(t_1 - M)^2 pI_e\right)\\
&(T-t_1)I_E\Bigg]\varphi_4(t)
\end{aligned} \quad (49)$$

$$H_{6_i} = \frac{1}{T}[(W_{2_i} - Q_{(4-i)}c) + (W_{2_i} - Q_{(4-i)}c)(T-M)I_E]\phi_5(t) \quad (50)$$

$$H_{7_i} = \frac{1}{T}[(W_{3_i} - Q_{(4-i)}c)]\phi_6(t) \quad (51)$$

$$H_{8_i} = \frac{1}{T}\Bigg[D_i(t_1 - B_{1_{(4-i)}})p + \frac{1}{2}D_i(t_1 - B_{1_{(4-i)}})^2 pI_e + D_i p(t_1 - B_{1_{(4-i)}}) \left\{1 + \frac{1}{2}(t_1 - B_{1_{(4-i)}})I_e\right\}(T - t_1)I_E\Bigg]\phi_7(t) \quad (52)$$

$$H_{9_i} = \frac{1}{T}\Bigg[D_i(t_1 - B_{2_{(4-i)}})p + \frac{1}{2}D_i(t_1 - B_{2_{(4-i)}})^2 pI_e + D_i(t_1 - B_{2_{(4-i)}}) p\left(1 + \frac{1}{2}(t_1 - B_{2_{(4-i)}})I_e\right)(T - t_1)I_E\Bigg]\phi_8(t) \quad (53)$$

$$H_{10_i} = \frac{1}{T}\Bigg[D_i(t_1 - B_{3_{(4-i)}})p + \frac{1}{2}D_i(t_1 - B_{3_{(4-i)}})^2 pI_e + \left(D_i(t_1 - B_{3_{(4-i)}})p + \frac{1}{2}D_i(t_1 - B_{3_{(4-i)}})^2 pI_e\right)(T - t_1)I_E\Bigg]\phi_9(t) \quad (54)$$

$$H_{11_i} = \frac{1}{T}\Bigg(D_i(t_1 - M)p + \frac{1}{2}D_i(t_1 - M)^2 pI_e + (W_{1_i} - Q_{(4-i)}c)\{1 + (T - M)I_E\} + \left(D_i(t_1 - M)p + \frac{1}{2}D_i(t_1 - M)^2 pI_e\right)(T - t_1)I_E\Bigg)\phi_{10}(t) \quad (55)$$

$$H_{12_i} = \frac{1}{T}(W_{2_i} - Q_{(4-i)}c) + (W_{2_i} - Q_{(4-i)}c)(T - M)I_E - Q_{(4-i)}c)(T - M)I_p\varphi_{11}(t) \quad (56)$$

$$H_{13_i} = \frac{1}{T}\left(W_{3_i} - Q_i c\right)\varphi_{12}(t) \quad (57)$$

$$H_{14_i} = \frac{1}{T}\Bigg[D_i T(p - c) + \frac{1}{2}D_i t_1^2 pI_e + D_i(T - t_1)pTI_E + \left(D_i t_1 p + \frac{1}{2}D_i t_1^2 pI_e\right)(T - t_1)I_E + D_i(T - M)p\left\{1 + \frac{1}{2}(T - M)I_e\right\} (T - M)I_e - D_i Tc(T - M)I_p\Bigg]\phi_{13}(t) \quad (58)$$

$$H_{15_i} = \frac{1}{T}\Bigg[(D_i Tp - D_i Tc) + D_i(T - t_1)pMI_E + \frac{1}{2}D_i t_1^2 pI_e + \Bigg(D_i t_1 p + \frac{1}{2}D_i t_1^2 pI_e\Bigg)(M - t_1)I_E\Bigg]\varphi_{14}(t) \quad (59)$$

$$B_{1_i} = M + \frac{2(cQ_i - W_{1_{(4-i)}})}{2D_{(4-i)}p - (cQ_i - W_{1_{(4-i)}})I_p} \quad (60)$$

$$B_{2_i} = \frac{1}{D_{(4-i)}pI_e}\Bigg\{-D_{(4-i)}p + D_i pI_e M + D_i TcI_p - I_p W_{1_{(4-i)}} + \Big(D_i^2 p^2 - 2D_{(4-i)}^2 TcI_p p + 2D_i pI_p W_{1_i} + D_i^2 T^2 c^2 I_p^2 - 2D_{(4-i)}TcI_p^2 W_{1_{(4-i)}} + I_p^2 W_{1_i}^2 + 2D_i^2 pI_e Tc - 2D_{(4-i)}pI_e W_{1_{(4-i)}}\Big)^{\frac{1}{2}}\Bigg\} \quad (61)$$

$$B_{3_i} = \frac{1}{D_{(4-i)}pI_e}\Bigg\{-D_{(4-i)}p + D_i pI_e M + D_i TcI_p - I_e W_{1_{(4-i)}} + \Big(W_{1_i}^2 I_e^2 - 2D_{(4-i)}TcI_p W_{1_{(4-i)}}I_e + D_i^2 T^2 c^2 I_p^2 - 2D_{(4-i)}^2 pI_p Tc + D_i^2 p^2 + 2D_i^2 pI_e Tc\Big)^{\frac{1}{2}}\Bigg\} \quad (62)$$

$$W_{1_i} = D_i p\left[(T - t_1) + M\left\{1 + (T - t_1)I_E + \frac{1}{2}MI_e\right\}\right] \quad (63)$$

$$W_{2_i} = D_i p(T - t_1)(1 + MI_E) + D_i pt_1\left(1 + \frac{1}{2}t_1 I_e\right)(1 + (M - t_1)I_e) \quad (64)$$

$$W_{3_i} = D_i p(T - t_1)(1 + MI_E) + D_i pt_1\left(1 + \frac{1}{2}t_1 I_e\right)(1 + (M - t_1)I_e) \quad (65)$$

$$Q_i = D_i T, \quad D_i = a_i - pb_{(4-i)} \text{ and } p = \mu c \quad (66)$$

Now, let us defuzzify the fuzzy profit function by, using signed distance method, measured from $\widetilde{AP}(\mu, t_1, T)$ to $\widetilde{0}$

$$AP_d(\mu, t_1, T) = \frac{1}{4}\{AP_1 + 2AP_2 + AP_3\} \quad (67)$$

6 Solution Procedure

To maximize the total profit per unit time in the fuzzy sense, taking the first derivative of $AP_d(\mu, t_1, T)$ with respect to μ, t_1 and T and equate it to zero.

$$\frac{\partial AP_d(\mu, t_1, T)}{\partial \mu} = 0 \quad (68)$$

$$\frac{\partial AP_d(\mu,t_1,T)}{\partial t_1}=0 \tag{69}$$

and

$$\frac{\partial AP_d(\mu,t_1,T)}{\partial T}=0 \tag{70}$$

Equations (68), (69), and (70) can be solved simultaneously for the optimal values of μ, t_1 and T (say μ^*, t_1^* and T^*) and also satisfies the following sufficient conditions.

The sufficient conditions for maximizing $AP_d(\mu,t_1,T)$ using the Hessian matrix H, which is a the matrix of second order partial derivatives are

$$H=\begin{bmatrix} \frac{\partial^2 AP_d(\mu,t_1,T)}{\partial \mu^2} & \frac{\partial^2 AP_d(t_r,T,p)}{\partial \mu \partial t_1} & \frac{\partial^2 AP_d(\mu,t_1,T)}{\partial \mu \partial T} \\ \frac{\partial^2 AP_d(\mu,t_1,T)}{\partial t_1 \partial \mu} & \frac{\partial^2 AP_d(\mu,t_1,T)}{\partial t_1^2} & \frac{\partial^2 AP_d(\mu,t_1,T)}{\partial t_1 \partial T} \\ \frac{\partial^2 AP_d(\mu,t_1,T)}{\partial T \partial \mu} & \frac{\partial^2 AP_d(\mu,t_1,T)}{\partial T \partial t_1} & \frac{\partial^2 AP_d(\mu,t_1,T)}{\partial T^2} \end{bmatrix}$$

$$D_1=\frac{\partial^2 AP_d(\mu,t_1,T)}{\partial \mu^2}<0,$$

$$D_2=\begin{vmatrix} \frac{\partial^2 AP_d(\mu,t_1,T)}{\partial \mu^2} & \frac{\partial^2 AP_d(\mu,t_1,T)}{\partial \mu \partial t_1} \\ \frac{\partial^2 TP_{ij}(t_r,T,p)}{\partial t_1 \partial \mu} & \frac{\partial^2 AP_d(\mu,t_1,T)}{\partial t_1^2} \end{vmatrix}>0$$

and

$$D_3=\det H$$
$$=\begin{vmatrix} \frac{\partial^2 AP_d(\mu,t_1,T)}{\partial \mu^2} & \frac{\partial^2 AP_d(t_r,T,p)}{\partial \mu \partial t_1} & \frac{\partial^2 AP_d(\mu,t_1,T)}{\partial \mu \partial T} \\ \frac{\partial^2 AP_d(\mu,t_1,T)}{\partial t_1 \partial \mu} & \frac{\partial^2 AP_d(\mu,t_1,T)}{\partial t_1^2} & \frac{\partial^2 AP_d(\mu,t_1,T)}{\partial t_1 \partial T} \\ \frac{\partial^2 AP_d(\mu,t_1,T)}{\partial T \partial \mu} & \frac{\partial^2 AP_d(\mu,t_1,T)}{\partial T \partial t_1} & \frac{\partial^2 AP_d(\mu,t_1,T)}{\partial T^2} \end{vmatrix}$$
$$<0$$

WhereD_1, D_2 and D_3 are the minors of the Hessian matrix H.

Mathematically, it is very difficult to prove the sufficient conditions as the profit function is highly non-linear, so concavities of profit function are shown graphically in Figures 20-22.

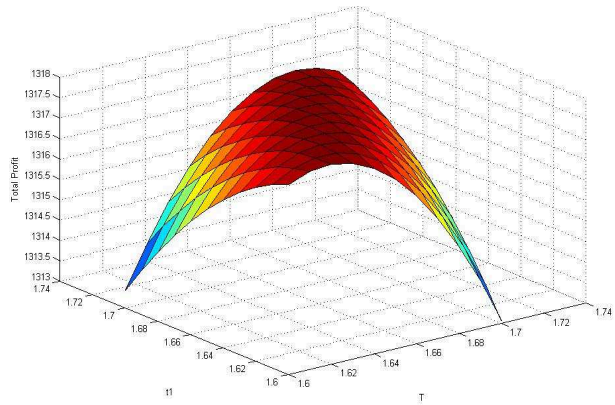

Fig. 20: Total profit versus t_1 and T

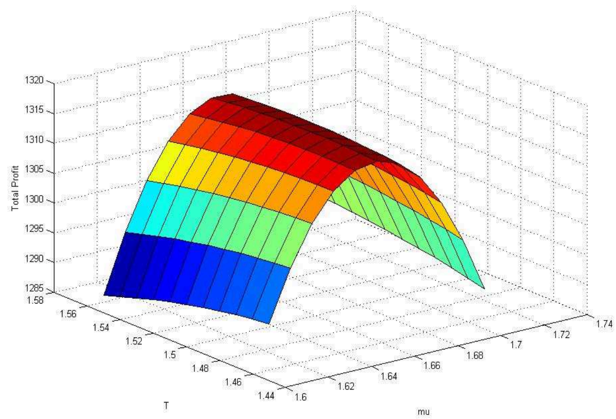

Fig. 21: Total profit versus T and μ

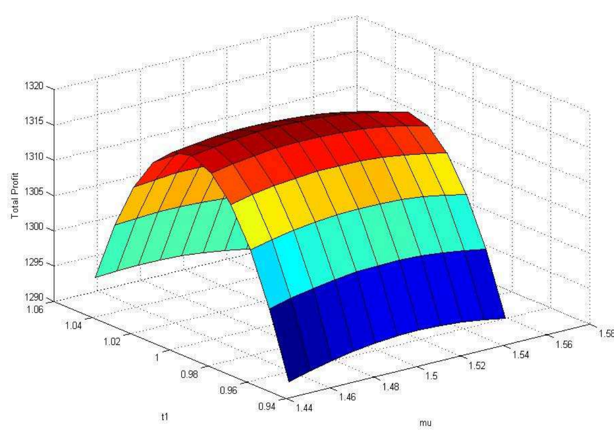

Fig. 22: Total profit versus t_1 and μ

Special case: If we assume $a_1 = a_2 = a_3 = a$, and $b_1 = b_2 = b_3 = b$ and substitute in equation (42), then $AP_d(\mu, t_1, T)$ becomes the crisp total profit function

$$AP_d(\mu, t_1, T) = \left(\sum_{k=1}^{14} H_{k+1} \right) - H_1 = AP(\mu, t_1, T) \quad (71)$$

That is, the fuzzy case becomes the crisp case.

7 Algorithm

The procedure for finding the economic ordering policy in section1 i.e. $(I_e < I_E \le I_p)$ is as follows:

For $I_e < I_E \le I_p$

Step 1:For event E_1, determine μ^*, t_1^* and T^* from equation (68), (69) and (70). If μ^*, t_1^* and T^* are in E_1 then calculate $AP_d(\mu^*, t_1^*, T^*)$ from (67), this gives $AP_{(d)1.1.1.1.(a)}(\mu^*, t_1^*, T^*)$. Otherwise go to Step 2.

Step 2:For event E_2, determine μ^*, t_1^* and T^* from equation (68), (69) and (70). If μ^*, t_1^* and T^* are in E_2 then calculate AP_d (μ^*, t_1^*, T^*) from (67), this gives $AP_{(d)1.1.1.1.(b)}(\mu^*, t_1^*, T^*)$. Otherwise go to Step 3.

Step 3:For event E_3, determine μ^*, t_1^* and T^* from equation (68), (69) and (70). If μ^*, t_1^* and T^* are in E_3 then calculate AP_d (μ^*, t_1^*, T^*) from (67), this gives $AP_{(d)1.1.1.2}(\mu^*, t_1^*, T^*)$. Otherwise go to Step 4.

Step 4:For event E_4, determine μ^*, t_1^* and T^* from equation (68), (69) and (70). If μ^*, t_1^* and T^* are in E_4 then calculate AP_d (μ^*, t_1^*, T^*) from (67), this gives $AP_{(d)1.1.2}(\mu^*, t_1^*, T^*)$. Otherwise go to Step 5.

Step 5:For event E_5, determine μ^*, t_1^* and T^* from equation (68), (69) and (70). If μ^*, t_1^* and T^* are in E_5 then calculate $AP_d(\mu^*, t_1^*, T^*)$ from (67), this gives $AP_{(d)1.2}(\mu^*, t_1^*, T^*)$. Otherwise go to Step 6.

Step 6:For event E_6, determine μ^*, t_1^* and T^* from equation (68), (69) and (70). If μ^*, t_1^* and T^* are in E_6 then calculate $AP_d(\mu^*, t_1^*, T^*)$ from (67), this gives $AP_{(d)1.3}(\mu^*, t_1^*, T^*)$. Otherwise go to Step 7.

Similarly for $I_e \le I_P < I_E$ and for $I_p < I_e < I_E$, we get the optimal total profit $AP_{d(2)}(\mu^*, t_1^*, T^*)$ and $AP_{d(3)}(\mu^*, t_1^*, T^*)$ per unit time. The optimal solution of the inventory system can be found by comparing the average profits of all the sections. Hence the optimal total profit of the system is given by

$$AP_d(\mu^*, t_1^*, T^*)$$
$$= \max[AP_{d(1)}(\mu^*, t_1^*, T^*), AP_{d(2)}(\mu^*, t_1^*, T^*), AP_{d(3)}(\mu^*, t_1^*, T^*)]$$

8 Numerical Examples

The proposed model of the inventory system has been illustrated with the help of two hypothetical numerical examples. The values of different parameters have been displayed in Table 1. Both the examples have been solved by using MS-office Excel's solver to determine the optimal values of mark up rate (μ), selling price (p), time period with positive stock of the items (t_1), breakeven point (B_i), cycle length (T), ordering quantity (Q) along with the maximum profit of the system. The results have been shown in Table 2.

Using the proposed algorithm the optimal solution for Example 1, 2 and 3 are as shown by bold values in Table 2. This is also given as:

For $I_e < I_E \le I_p$:
$\mu^* = 1.48$, $t_1^* = 0.77$ year, $T^* = 1.18$ year, $B^* = 0.42$ year, $W^* = 2391.96$, $Q^* = 37$ units and Total profit $=$ \$1264.25 (Scenario 1.1.1.a)

For $I_e \le I_P < I_E$:
$\mu^* = 1.53$, $t_1^* = 0.99$ year, $T^* = 1.64$ year, $B^* = 0.49$ year, $W^* = 3375.21$, $Q^* = 49$ units and Total profit $=$ \$1317.60 (Scenario 2.1.1.a)

For $I_p < I_e < I_E$:
$\mu^* = 1.46$, $t_1^* = 0.082$ year $= M$, $T^* = 0.53$ year, $B^* =$ 0.8 year, $Q^* = 16$ units and Total profit $=$ \$1130.89 (Scenario 3.1)

9 Sensitivity Analysis

To study the effects of changes of different parameters like, A (ordering cost), a, b (location parameter of demand), h (holding cost), c (unit purchase cost of retailer), M (Permissible delay in payment) on the optimal policies, sensitivity analyses have been performed numerically. These analyses have been carried out by changing -20% to $+20\%$ for one parameter keeping other parameters as same. The results of these analyses have been displayed in Table 3.

From Table 3, the following inferences can be made:

1. One can easily observe that with the increase in value of ordering cost (A), the optimal cycle length (T), optimal order quantity (Q) increases; but the total profit (AP) decreases.

Table 1: Values of parameters of different examples

Example	A (\$)	c (\$)	h (\$)	π (\$)	a	b	I_e (per\$/year)	I_E (per\$/year)	I_p (per\$/year)	M (years)
1. $I_e < I_E \leq I_p$	200	100	10	50	(140,150,160)	(0.78,0.80,0.82)	0.12	0.14	0.15	30/365 = 0.082
2. $I_e \leq I_p < I_E$	200	100	10	50	(140,150,160)	(0.78,0.80,0.82)	0.12	0.18	0.15	30/365 = 0.082
3. $I_p < I_e < I_E$	200	100	10	50	(140,150,160)	(0.78,0.80,0.82)	0.18	0.2	0.15	30/365 = 0.082

Table 2: Result of Example 1, 2 and 3 for different cases, sub cases and scenarios

Section	**Case**	**Sub-case**	**Scenarios**	μ	t_1	T	B	W_i	$p = \mu c$	Q	**Profit**	**Remark**
$I_e < I_E \leq I_p$	1.1	1.1.1	**1.1.1.1.a**	**1.48**	**0.77**	**1.18**	**0.42**	**2391.96**	**148**	**37**	**1264.25**	**1.1.1.1.a**
			1.1.1.1.b	1.46	0.62	0.97	0.53	2067.37	146	32	1231.03	
			1.1.1.2	1.31	0.53	0.94	0.52	2123.47	131	28	1187.63	
		1.1.2	-	1.50	0.36	0.83	-	2502.01	150	25	1097.56	
	1.2	-	-	1.49	M	0.53	-	2407.25	149	16	948.35	
	1.3	-	-	1.46	0.05	M	-	2463.37	146	12	787.60	
$I_e \leq I_p < I_E$	2.1	2.1.1	**2.1.1.1.a**	**1.53**	**0.99**	**1.64**	**0.49**	**3375.21**	**153**	**49**	**1317.60**	**2.1.1.1.a**
			2.1.1.1.b	1.49	0.87	1.43	0.54	3068.73	149	37	1292.57	
			2.1.1.2	1.49	0.82	1.28	0.64	2734.89	149	35	1227.65	
		2.1.2	-	1.47	0.61	1.04	-	2376.93	147	32	1175.28	
	2.2	-	-	1.48	M	0.47	-	2226.07	148	14	825.38	
	2.3	-	-	1.44	0.06	M	-	1489.25	144	7	524.72	
$I_p < I_e < I_E$	3.1	-	-	**1.46**	$\boldsymbol{M}$	**0.53**	**-**	**-**	**146**	**16**	**1130.89**	**3.1**
	3.2	-	-	1.45	0.06	M	-	-	145	8	975.47	

Table 3: Effect of changes in the system parameters

Parameters	**% changes**	π	t_1	T	B	Q	**Profit**
A	-20%	-1.08	-25.81	-25.49	-25.31	-22.26	2.15
	-10%	-0.56	-13.60	-13.51	-13.18	-11.57	1.03
	10%	0.72	18.18	18.18	16.95	14.81	-0.86
	20%	1.43	36.42	36.42	32.81	28.31	-1.86
a	-20%	-13.04	23.45	28.64	47.65	-35.08	-75.34
	-10%	-6.23	10.57	11.75	23.05	-15.69	-42.81
	10%	6.13	-7.34	-9.23	-15.86	15.02	49.83
	20%	12.53	-11.46	-12.45	-24.63	29.65	93.78
b	-20%	4.04	38.67	58.53	15.07	53.91	17.89
	-10%	3.09	32.45	34.04	-23.09	32.76	13.57
	10%	-3.18	-31.83	-34.98	-33.11	-33.05	-13.90
	20%	-3.92	-37.97	-57.31	9.87	-50.61	-17.07
h	-20%	-0.14	4.76	6.48	7.47	7.23	3.72
	-10%	-0.08	2.48	3.54	3.58	3.47	1.53
	10%	0.49	-2.36	-2.96	-3.08	-3.26	-1.71
	20%	0.73	-4.73	-6.53	-5.87	-6.47	-3.21
c	-20%	16.47	3.45	5.23	-28.51	32.72	51.87
	-10%	7.42	2.45	2.57	-13.67	15.33	29.25
	10%	-5.83	-4.73	-2.34	13.26	-14.86	-32.84
	20%	-12.55	-6.39	-5.69	27.14	-35.56	-68.15
π	-20%	13.46	12.36	19.64	9.71	31.38	2.89
	-10%	9.25	5.46	9.58	6.83	16.22	1.47
	10%	-0.45	-9.31	-12.68	-5.14	-11.12	-1.28
	20%	-0.69	-13.68	-19.55	-6.60	-17.34	-2.31
M	-20%	0.03	0.96	1.02	1.34	-1.32	-1.07
	-10%	0.01	0.53	0.58	0.67	-0.65	-0.63
	10%	-0.01	-0.56	-0.48	-0.71	0.57	0.51
	20%	-0.02	-1.03	-0.94	-1.47	0.99	1.05

2. With the increase in the holding cost (h), the total profit (AP) decreases as there is an increase in carrying cost.
3. With the increase in the value of (a), the total profit (AP) increases whereas as the value of (b) increases, the total profit (AP) decreases.
4. As the cost per unit (c) increases, there is a decrement in the value of total profit (AP). This reveals the natural trend of cost-profit analysis.
5. It can be clearly observe from Table 2 as the shortage cost per unit (π) increases, optimal cycle length (T), optimal order quantity (Q) and the total profit (AP) decreases.
6. It is clearly observe that as the credit period (M) increases, both optimal order quantity (Q) and total profit (AP) increases.

10 Conclusion

In this study, a fuzzy economic order quantity model with price-dependent demand under conditions of permissible delay in payments by considering higher interest earn rate on fixed sales revenue has been proposed. Shortages are allowed and fully backlogged. The present is a generalized one under permissible delay in payment as it considers various financial scenarios. The proper mathematical models are developed to determine the optimal order quantity, the time period in which the inventory of the positive stock is finished and the total cycle length by maximizing the total fuzzy profit function for the different cases. The arithmetic operations are defined under the function principle and for defuzzification, signed distance method is employed to evaluate the optimal cycle length T, mark rate and payoff time which maximize the total profit. The numerical examples present the validity of the model. A sensitivity analysis is also conducted to explore the effects of the parameters A, a, c, h and M on the optimal result. Findings suggest that the presence of A, a, c, h and M have got an affirmative effect on retailer's ordering policy (Table 3).

The proposed model can be extended for stock-dependent demand, two-level trade credit, cash discount and many other realistic situations.

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