Journal of Statistics Applications & Probability Letters
An International Journal

http://dx.doi.org/10.18576/jsapl/080206

Bayesian Prediction Based on Dual Generalized Order Statistics Using Multiply Type-II Censoring

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Received: 1 Aug. 2020, Revised: 12 Dec. 2020, Accepted: 3 Jan. 2021
Published online: 1 May 2021

Abstract: This paper provides Bayesian prediction for a past ordered observations based on dual generalized order statistics (dgos) by using multiply type-II censoring. A general class of distribution and a general class of prior distribution are used to illustrate the main idea. The predictive reliability function can be obtained in one sample case. A simulation data for Power distribution is used to clear the purposed results.

Keywords: Bayesian prediction, Dual generalized order statistics, Order statistics, one sample scheme, A general class of distribution.

1 Introduction

Generalized order statistics have been studied by many authors, one of the earliest papers on gos is that by Kamps [1], in which he considered a new model of gos as a unification for several models of ordered random variables (e.g., ordinary order statistics, k-record values and progressively type-II censoring). There are many authors discussed about gos, dgos and Bayesian prediction such as Abushal [2], Al-Hussaini and Ahmad [3], Burkschat et al. [4], Cramer and Kamps [5], Ghafoori et al. [6], Jaheen and Harbi [7], Kamps and Gather [8], Kamps and Cramer [9], Khan et al. [10], Pandey and Rao. [11] and Raqab [12] and among others. Suppose that F be an absolutely continuous cumulative distribution function (cdf) with density function (pdf) f with parameters $n \in N$, $\widetilde{m} = (m_1, ..., m_{n-1}) \in R_{n-1}$, $k \geq 1$, be given constants such that for all $1 \leq i \leq n-1, \gamma_i = k+n-i+M_i > 0$, where $M_i = \sum_{j=i}^{n-1} m_j$. The random variables $X_i \equiv X_{i,n,m,k}$, $i = 1, \ldots, n$ are said to be dgos from an absolutely continuous distribution function if their joint density function in the form (see Burkschat et al. [4])

$$f_{1,2,\ldots,n}(x_1,x_2,\ldots,x_n) = k\left(\prod_{j=1}^{n-1}\gamma_j\right)\left(\prod_{j=1}^{n-1}(F(x_j))^{m_j} f(x_j)\right)(F(x_n))^{k-1} f(x_n),$$
$$F^{-1}(0) < x_n < x_{n-1} < \ldots < x_2 < x_1 < F^{-1}(1). \tag{1}$$

Multiply type-II censoring can be considered as a more general form of censoring scheme where the j_1th,...,j_rth failure times are available from n, $1 \leq j_1 < \ldots < j_r \leq n$. It has been discussed by Abdel-aty et al. [13], Balakrishnan [14] and Mohie El-Din [15].

The joint density function of multiply type-II censoring $X_{j_1}, \ldots, X_{j_r}$ dgos is given by

$$f_{j_1,\ldots,j_r}(\mathbf{x}) = c\prod_{i=1}^{r}\sum_{\omega=j_{i-1}+1}^{j_i} c_{\omega}^{(j_{i-1})}(j_i)\left(\frac{F(x_{j_i})}{F(x_{j_{i-1}})}\right)^{\gamma_\omega}\frac{f(x_{j_i})}{F(x_{j_i})},$$
$$F^{-1}(0) < x_n < x_{n-1} < \ldots < x_2 < x_1 < F^{-1}(1). \tag{2}$$

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where $\mathbf{x} = (x_{j_1}, x_{j_2}, \ldots, x_{j_r})$, $j_0 = 0$, $c = \prod_{i=1}^{r} \prod_{\omega=j_{i-1}+1}^{j_i} \gamma_\omega$, $c_\omega^{(j_{i-1})}(j_i) = \prod_{q=j_{i-1}+1, q\neq\omega}^{j_i} \frac{1}{\gamma_q - \gamma_\omega}$.

The general form of distributions which includes distributions such as doubly truncated inverted Weibull, inverted Gompertz, generalized logistic, Burr type X, Burr type XII, logistic, inverted Pareto, inverted compound Weibull, Gumbel and compound Gompertz. A random variable (r.v) X is said to have a general form of absolutely continuous distributions if its cdf and pdf are given, respectively, by

$$f(x;\theta) = -\grave{\psi}(x) \exp\{-\psi(x)\}, \quad x \geq 0, \quad \theta \in \Theta, \tag{3}$$

where $\grave{\psi}(x) = \frac{d}{dx}\psi(x)$,

and cdf is

$$F(x;\theta) = \exp\{-\psi(x)\}. \tag{4}$$

In the current investigation, Bayesian prediction in one sample scheme by using multiply type-II censoring are obtained by using general class of distribution and a general class of prior distribution in Section 3. Power distribution is used as application example to obtain Bayesian prediction in one sample scheme in Section 4. Simulation result is presented in Sections 5. Finally, we make some concluding remarks in Section 6.

2 Bayes Prediction

We will present Bayesian prediction in one sample scheme by using multiply type-II censoring based on dgos, when the parameter θ is unknown.

2.1 Prior and posterior distributions

Accordingly to Eq.(2), we find the likelihood function of the observed data from general class distribution is

$$L(\theta|\mathbf{x}) \propto \prod_{i=1}^{r} \sum_{\omega=j_{i-1}+1}^{j_i} c_\omega^{(j_{i-1})}(j_i) \grave{\psi}(x_{j_i}) \exp\{-\gamma_\omega(\psi(x_{j_i}) - \psi(x_{j_{i-1}}))\}, \tag{5}$$

where $x_{j_0} = \infty$ and $j_0 = 0$.

Under the assumption that parameter θ is unknown, a class prior which was introduced by Abdel-aty et al. [13] is given by

$$\pi(\theta|\delta) \propto C(\theta|\delta) \exp\{-D(\theta|\delta)\}, \tag{6}$$

where δ be a vector of hyper-parameters.

Using (5) and (6), the posterior density of θ is

$$\pi^*(\theta|\mathbf{x}) = R^{-1} \prod_{i=1}^{r} \sum_{\omega=j_{i-1}+1}^{j_i} c_\omega^{(j_{i-1})}(j_i) \eta_i(\theta|\mathbf{x}) \exp\{-z_{i\omega}(\theta|\mathbf{x})\}, \tag{7}$$

where

$$R = \int_0^\infty \prod_{i=1}^{r} \sum_{\omega=j_{i-1}+1}^{j_i} c_\omega^{(j_{i-1})}(j_i) \eta_i(\theta|\mathbf{x}) \exp\{-z_{i\omega}(\theta|\mathbf{x})\} d\theta, \tag{8}$$

$$z_{i\omega}(\theta|\mathbf{x}) = \gamma_\omega \left(\psi(x_{j_i}|\theta) - \psi(x_{j_{i-1}}|\theta) + \frac{D(\theta|\delta)}{\gamma_\omega r} \right), \tag{9}$$

and

$$\eta_i(\theta|\mathbf{x}) = \grave{\psi}(x_{j_i}|\theta)(C(\theta|\delta))^{\frac{1}{r}}. \tag{10}$$

2.2 Bayesian prediction intervals

2.2.1 One sample Bayesian prediction

Let $Y_s = X_{j_r+s}$, $s = 1,2,...,n-j_r$ denote the past ordered observations. If we want to find prediction bounds for Y_s, the conditional density function of Y_{j_r+s} given X_{j_r} of dgos is given by (see Burkschat et al. [4])

$$f_{s|j_r,n,m,k}(y_s|x_{j_r}) = \frac{c_{j_r+s-1}}{(s-1)!c_{j_r-1}}(F(x_{j_r}))^{m-\gamma_{j_r}+1}(F(y_s))^{\gamma_{j_r+s}-1} \times (h_m(F(y_s)) - h_m(F(x_{j_r})))^{s-1} f(y_s), \quad y_s < x_{j_r}, \tag{11}$$

where $0 < z < 1$ and

$$h_m(z) = \begin{cases} -z^{m+1}/(m+1), & m \neq -1, \\ -\ln z, & m = -1. \end{cases} \tag{12}$$

When $m \neq -1$, using (3) and (4) in (11), we obtain

$$\begin{aligned} f_{s|j_r,n,m,k}(y_s|\theta,x_{j_r}) &= \frac{-c_{j_r+s-1}}{(s-1)!c_{j_r-1}}\grave{\psi}(y_s|\theta)\exp\left(-[\gamma_{j_r+s}\psi(y_s|\theta) + (m-\gamma_{j_r}+1)\psi(x_{j_r}|\theta)]\right) \\ &\times \left(\frac{e^{-(m+1)\psi(x_{j_r}|\theta)} - e^{-(m+1)\psi(y_s|\theta)}}{m+1}\right)^{s-1} \\ &= c_{j_r,s}\sum_{\kappa=0}^{s-1}\frac{(-1)^{\kappa+1}}{(s-\kappa-1)!\kappa!}\grave{\psi}(y_s|\theta)\exp\{-\gamma_{j_r+s-\kappa}(\psi(y_s|\theta) \\ &- \psi(x_{j_r}|\theta))\}, \end{aligned} \tag{13}$$

where $c_{j_r,s} = \left(\prod_{i=j_r+1}^{j_r+s}\gamma_i\right)/(m+1)^{s-1}$.

Using (7) and (13), the predictive density function of Y_s given X_{j_r} dgos is given by

$$\begin{aligned} f_{s|j_r,n,m,k}(y_s|x_{j_r}) &= \int_0^\infty f_{s|j_r,n,m,k}(y_s|\theta,x_{j_r})\pi^*(\theta|\mathbf{x})\,\mathrm{d}\theta \\ &= R^{-1}c_{j_r,s}\sum_{\kappa=0}^{s-1}\frac{(-1)^{\kappa+1}}{(s-\kappa-1)!\kappa!}\prod_{i=1}^{r}\sum_{\omega=j_{i-1}+1}^{j_i}c_\omega^{(j_{i-1})}(j_i)\int_0^\infty\grave{\psi}(y_s|\theta)\eta_i(\theta|\mathbf{x}) \\ &\times \exp\{-[z_{i\omega}(\theta|\mathbf{x}) + \gamma_{j_r+s-\kappa}(\psi(y_s|\theta) - \psi(x_{j_r}|\theta))]\}\,\mathrm{d}\theta. \end{aligned} \tag{14}$$

Hence, the predictive survival function for the (j_r+s)th past dgos is given by

$$\begin{aligned} P[Y_s > \nu|x_{j_r}] &= \int_\nu^{x_{j_r}} f_{s|j_r,n,m,k}(y_s|x_{j_r})\,\mathrm{d}y_s \\ &= R^{-1}c_{j_r,s}\sum_{\kappa=0}^{s-1}\frac{(-1)^{\kappa+1}}{(s-\kappa-1)!\kappa!}\prod_{i=1}^{r}\sum_{\omega=j_{i-1}+1}^{j_i}c_\omega^{(j_{i-1})}(j_i)\int_\nu^{x_{j_r}}\int_0^\infty\grave{\psi}(y_s|\theta)\eta_i(\theta|\mathbf{x}) \\ &\times \exp\{-[z_{i\omega}(\theta|\mathbf{x}) + \gamma_{j_r+s-\kappa}(\psi(y_s|\theta) - \psi(x_{j_r}|\theta))]\}\,\mathrm{d}\theta\mathrm{d}y_s. \end{aligned} \tag{15}$$

When $m_1 = \ldots = m_{n-1} = m = -1$ and $k = 1$, Eq.(11) can be written as

$$\begin{aligned} f_{s|j_r,n,m,k}(y_s|\theta,x_{j_r}) &= \sum_{\kappa=0}^{s-1}\frac{(-1)^{\kappa+1}}{(s-\kappa-1)!\kappa!}\grave{\psi}(y_s|\theta)(\psi(y_s|\theta))^{s-\kappa-1}(\psi(x_{j_r}|\theta))^\kappa \\ &\times \exp\{-(\psi(y_s|\theta) - \psi(x_{j_r}|\theta))\}, \end{aligned} \tag{16}$$

The predictive density function of Y_s is given by

$$\begin{aligned} f_{s|j_r,n,m,k}(y_s|x_{j_r}) &= R^{-1}\sum_{\kappa=0}^{s-1}\frac{(-1)^{\kappa+1}}{(s-\kappa-1)!\kappa!}\prod_{i=1}^{r}\sum_{\omega=j_{i-1}+1}^{j_i}\int_0^\infty\grave{\psi}(y_s|\theta)\eta_i(\theta|\mathbf{x}) \\ &\times (\psi(y_s|\theta))^{s-\kappa-1}(\psi(x_{j_r}|\theta))^\kappa\exp\{-[z_{i\omega}(\theta|\mathbf{x}) + \psi(y_s|\theta) \\ &- \psi(x_{j_r}|\theta)]\}\,\mathrm{d}\theta. \end{aligned} \tag{17}$$

Hence, the predictive survival function for the (j_r+s)th past dgos is given by

$$P[Y_s > \nu | x_{j_r}] = R^{-1}\sum_{\kappa=0}^{s-1}\frac{(-1)^{\kappa+1}}{(s-\kappa-1)!\kappa!}\prod_{i=1}^{r}\sum_{\omega=j_{i-1}+1}^{j_i}\int_{\nu}^{x_{jr}}\int_0^\infty \grave{\psi}(y_s|\theta)\eta_i(\theta|\mathbf{x}) \times (\psi(y_s|\theta))^{s-\kappa-1}(\psi(x_{j_r}|\theta))^{\kappa}\exp\{-[z_{i\omega}(\theta|\mathbf{x})+\psi(y_s|\theta) - \psi(x_{j_r}|\theta)]\}\,d\theta dy_s. \quad (18)$$

2.2.2 Two sample Bayesian prediction

Let Y_s, $s=1,2,\ldots,n_1$ denote the past ordered observations and the goal is to predict a past sample of size n_1 by using the informative sample. So the density function of Y_s of dgos is given by (see Burkschat et al. [4])

$$f_{s,n,m,k}(y_s) = \frac{c_{s-1}}{(s-1)!}(F(y_s))^{\gamma_s-1}f(y_s)g_m^{s-1}(F(y_s)), \quad (19)$$

where $c_{s-1}=\prod_{j=1}^{s}\gamma_j$, $g_m(z)=h_m(z)-h_m(1)$, $0\le z<1$,

$$h_m(z) = \begin{cases} \frac{-1}{m+1}z^{m+1}, & m\neq -1,\\ -\ln z, & m=-1. \end{cases} \quad (20)$$

When $m\neq -1$, using (3) and (4) in (19), we obtain

$$f_{s,n,m,k}(y_s) = \sum_{\kappa=0}^{s-1}\frac{(-1)^s c_{s-1}}{(s-\kappa-1)!\kappa!(m+1)^{s-1}}\grave{\psi}(y_s|\theta)\exp\{-\gamma_{s-\kappa}(\psi(y_s|\theta))\}. \quad (21)$$

Using (7) and (21), the predictive density function of Y_s dgos is given by

$$f_{s,n,m,k}(y_s|x_{j_r}) = \int_0^\infty f_{s,n,m,k}(y_s)\pi^*(\theta|\mathbf{x})\,d\theta = R^{-1}\sum_{\kappa=0}^{s-1}\frac{(-1)^s c_{s-1}}{(s-\kappa-1)!\kappa!(m+1)^{s-1}}\prod_{i=1}^{r}\sum_{\omega=j_{i-1}+1}^{j_i}c_\omega^{(j_{i-1})}(j_i)\int_0^\infty \grave{\psi}(y_s|\theta)\eta_i(\theta|\mathbf{x}) \times \exp\{-[z_{i\omega}(\theta|\mathbf{x})+\gamma_{s-\kappa}(\psi(y_s|\theta))]\}\,d\theta. \quad (22)$$

Hence, the predictive survival function for the sth past dgos is given by

$$P[Y_s>\varepsilon|x_{j_r}] = \int_\varepsilon^{x_{jr}} f_{s,n,m,k}(y_s|x_{j_r})\,dy_s = R^{-1}\sum_{\kappa=0}^{s-1}\frac{(-1)^s c_{s-1}}{(s-\kappa-1)!\kappa!(m+1)^{s-1}}\prod_{i=1}^{r}\sum_{\omega=j_{i-1}+1}^{j_i}c_\omega^{(j_{i-1})}(j_i)\int_\varepsilon^{x_{jr}}\int_0^\infty \grave{\psi}(y_s|\theta)\eta_i(\theta|\mathbf{x}) \times \exp\{-[z_{i\omega}(\theta|\mathbf{x})+\gamma_{s-\kappa}(\psi(y_s|\theta))]\}\,d\theta dy_s. \quad (23)$$

When $m_1=\ldots=m_{n-1}=m=-1$ and $k=1$, Eq.(19) can be written as

$$f_{s,n,m,k}(y_s) = \frac{-1}{(s-1)!}\grave{\psi}(y_s|\theta)(\psi(y_s|\theta))^{s-1}. \quad (24)$$

The predictive density function of Y_s is given by

$$f_{s,n,m,k}(y_s|x_{j_r}) = \frac{-R^{-1}}{(s-1)!}\prod_{i=1}^{r}\sum_{\omega=j_{i-1}+1}^{j_i}c_\omega^{(j_{i-1})}(j_i)\int_0^\infty \grave{\psi}(y_s|\theta)(\psi(y_s|\theta))^{s-1}\eta_i(\theta|\mathbf{x}) \times \exp\{-z_{i\omega}(\theta|\mathbf{x})\}\,d\theta. \quad (25)$$

Hence, the predictive survival function for the sth past dgos is given by

$$P[Y_s>\varepsilon|x_{j_r}] = \frac{-R^{-1}}{(s-1)!}\prod_{i=1}^{r}\sum_{\omega=j_{i-1}+1}^{j_i}c_\omega^{(j_{i-1})}(j_i)\int_\varepsilon^{x_{jr}}\int_0^\infty \grave{\psi}(y_s|\theta)(\psi(y_s|\theta))^{s-1}\eta_i(\theta|\mathbf{x}) \times \exp\{-z_{i\omega}(\theta|\mathbf{x})\}\,d\theta dy_s. \quad (26)$$

3 Example

3.1 Power distribution

In this subsection, we applied the previous theoretical results by using Power distribution when the two parameters (β,λ) are unknown.
Choosing $\psi(x|\beta,\lambda) = -\beta\ln(\frac{x}{\lambda})$, $\grave{\psi}(x|\beta,\lambda) = \frac{-\beta}{x}$, $\theta = (\lambda,\beta)$. So Eqs.(3) and (4) can be written as

$$f(x|\beta,\lambda) = \frac{\beta}{\lambda}\left(\frac{x}{\lambda}\right)^{\beta-1}, \quad 0\le x\le\lambda, \quad \lambda>0, \quad \beta>0, \tag{27}$$

and

$$F(x|\beta,\lambda) = \left(\frac{x}{\lambda}\right)^{\beta}. \tag{28}$$

If we have β has prior gamma (c,d) and λ has the conditional distribution prior Power distribution $(\beta a,b)$, then Eq.(6) becomes

$$\pi(\beta,\lambda,\delta) \propto \beta^{c}\lambda^{-1}\exp\left(-\beta\left[a(\ln b-\ln\lambda)+d\right]\right), \quad \lambda\le b \tag{29}$$

where $\delta=(a,b,c,d)$, $a,b,c,d>0$, $C(\beta,\lambda|\delta)=\beta^{c}\lambda^{-1}$ and
$D(\beta,\lambda|\delta)=\beta\left[a(\ln b-\ln\lambda)+d\right]$.

The posterior density function of λ and β is given by

$$\pi^{*}(\lambda,\beta|\mathbf{x}) = R^{-1}\prod_{i=1}^{r}\sum_{\omega=j_{i-1}+1}^{j_i} c_{\omega}^{(j_{i-1})}(j_i)\frac{-\beta^{\frac{c}{r}+1}}{x_{j_i}}\lambda^{\frac{-1}{r}}\left(\frac{x_{j_{i-1}}}{x_{j_i}}\right)^{-\beta\gamma_{\omega}} \times \exp\left\{\frac{-\beta}{r}\left[a(\ln b-\ln\lambda)+d\right]\right\}, \tag{30}$$

where

$$R = \prod_{i=1}^{r}\sum_{\omega=j_{i-1}+1}^{j_i}\frac{c_{\omega}^{(j_{i-1})}(j_i)}{x_{j_i}}\Gamma(\frac{c}{r}+2)\int_0^{\infty}\lambda^{\frac{-1}{r}}\left[\frac{a}{r}(\ln b-\ln\lambda)+\frac{d}{r} + \gamma_{\omega}\ln\left(\frac{x_{j_{i-1}}}{x_{j_i}}\right)\right]^{-(\frac{c}{r}+2)} d\lambda. \tag{31}$$

So Eqs.(15), (18), (23) and (26) reduce to

$$P[Y_s>\nu|x_{j_r}] = R^{-1}c_{j_r,s}\sum_{\kappa=0}^{s-1}\frac{(-1)^{\kappa+2}}{(s-\kappa-1)!\kappa!}\prod_{i=1}^{r}\sum_{\omega=j_{i-1}+1}^{j_i}\frac{c_{\omega}^{(j_{i-1})}(j_i)}{x_{j_i}}\Gamma(\frac{c}{r}+2) \times \int_{\nu}^{x_{j_r}}\int_0^{\infty}\lambda^{-\frac{1}{r}}[\frac{a}{r}\left(\ln\left(\frac{b}{\lambda}\right)\right)+\frac{d}{r}+\gamma_{\omega}\ln\left(\frac{x_{j_{i-1}}}{x_{j_i}}\right) - \gamma_{j_r+s-\kappa}\left(\ln\left(\frac{x_{j_r}}{\lambda}\right)-\ln\left(\frac{y_s}{\lambda}\right)\right)]^{-(\frac{c}{r}+2)}d\lambda\, dy_s, \tag{32}$$

$$P[Y_s>\nu|x_{j_r}] = R^{-1}\sum_{\kappa=0}^{s-1}\frac{(-1)^{\kappa+s+2}}{(s-\kappa-2)!(\kappa-1)!}\prod_{i=1}^{r}\sum_{\omega=j_{i-1}+1}^{j_i}\frac{c_{\omega}^{(j_{i-1})}(j_i)}{x_{j_i}}\Gamma(\frac{c}{r}+s+2) \times \int_{\nu}^{x_{j_r}}\int_0^{\infty}\lambda^{-\frac{1}{r}}[\frac{a}{r}\left(\ln\left(\frac{b}{\lambda}\right)\right)+\frac{d}{r}+\gamma_{\omega}\ln\left(\frac{x_{j_{i-1}}}{x_{j_i}}\right)+\ln\left(\frac{y_s}{\lambda}\right) - \ln\left(\frac{x_{j_r}}{\lambda}\right)]^{-(\frac{c}{r}+s+2)}y_s^{-1}\ln\left(\frac{y_s}{\lambda}\right)\ln\left(\frac{x_{j_r}}{\lambda}\right)d\lambda\, dy_s, \tag{33}$$

$$P[Y_s>\varepsilon|x_{j_r}] = R^{-1}\sum_{\kappa=0}^{s-1}\frac{(-1)^{s+1}c_{s-1}}{(s-\kappa-1)!\kappa!(m+1)^{s-1}}\Gamma(\frac{c}{r}+2)\prod_{i=1}^{r}\sum_{\omega=j_{i-1}+1}^{j_i}\frac{c_{\omega}^{(j_{i-1})}(j_i)}{x_{j_i}} \times \int_{\varepsilon}^{x_{j_r}}\int_0^{\infty}\lambda^{-\frac{1}{r}}[\frac{a}{r}\left(\ln\left(\frac{b}{\lambda}\right)\right)+\frac{d}{r}+\gamma_{\omega}\ln\left(\frac{x_{j_{i-1}}}{x_{j_i}}\right) - \gamma_{s-\kappa}\left(\ln\left(\frac{y_s}{\lambda}\right)\right)]^{-(\frac{c}{r}+2)}d\lambda\, dy_s, \tag{34}$$

and

$$P[Y_s > \varepsilon | x_{j_r}] = R^{-1}\frac{(-1)^{s+1}}{(s-2)!}\prod_{i=1}^{r}\sum_{\omega=j_{i-1}+1}^{j_i}\frac{c_{\omega}^{(j_{i-1})}(j_i)}{x_{j_i}}\Gamma(\frac{c}{r}+s+2)\int_{\varepsilon}^{x_{jr}}\int_{0}^{\infty}$$
$$\times \lambda^{-\frac{1}{r}}[\frac{a}{r}\left(\ln\left(\frac{b}{\lambda}\right)\right)+\frac{d}{r}+\gamma_{\omega}\ln\left(\frac{x_{j_{i-1}}}{x_{j_i}}\right)+\ln\left(\frac{y_s}{\lambda}\right)$$
$$-\ln\left(\frac{x_{j_r}}{\lambda}\right)]^{-(\frac{c}{r}+s+2)}y_s^{-1}\ln\left(\frac{y_s}{\lambda}\right)d\lambda dy_s. \quad (35)$$

4 Illustrative example

In this section, we present a simulation study to illustrate our previous theoretical results for Power distribution when both parameters are unknown.

4.1 Simulation Study

To illustrate the prediction results for Power distribution, we use the following steps:

1.By choosing the hyper parameters $\delta = (a,b,c,d) = (3,0.7,2,0.8)$, we can generate β from gamma(c,d) and λ from Power $(\beta a,b)$ then we get $\beta = 3.887$ and $\lambda = 1.415$.

2.By using the transformation $X_i = \lambda(U_i)^{\frac{1}{\beta}}$ where U_i from $U(0,1)$, so the generated sample of size $n = 20$ from Power distribution pdf in (27) can be obtained as the following:

3.369	2.243	2.064	1.946	1.741	1.654	1.644	1.640	1.628	1.952
1.562	1.552	1.551	1.547	1.545	1.543	1.521	1.507	1.448	1.427

3.By choosing the parameters $m_i = \delta - 1$, $i = 1,\ldots,n-1$, $k = \delta$, $\gamma_i = (n-i+1)\delta$ and $\delta = 1$, we get ordinary order statistics.

4.If $r = 5$, $(j_1, j_2, j_3, j_4, j_5) = (2,5,8,12,15)$ and $x_0 = \lambda$, Bayesian prediction bounds for X_{16} and X_{20} are displayed in Table 1.

Table 1: The lower, upper and width of the 95% prediction intervals for X_{16} and X_{20}.

order statistics			
s	Lower	Upper	Width
1	1.186	1.560	0.374
5	1.185	1.566	0.381

5.If $r = 7$, $(j_1, j_2, j_3, j_4, j_5, j_6, j_7) = (2,4,6,8,10,12,14)$, Bayesian prediction bounds for X_{16} and X_{20} are displayed in Table 2.

Table 2: The lower, upper and width of the 95% prediction intervals for X_{16} and X_{20}.

order statistics			
s	Lower	Upper	Width
2	1.188	1.557	0.369
6	1.187	1.559	0.372

5 Conclusion

Bayesian prediction intervals for one sample scheme for the general class of distributions based on multiply type-II censoring are obtained under dgos. Power distribution is used as application example to illustrate our results. From Table 1 and 2, we notice that the lengths of the prediction intervals are increasing in s. From Table 2, we obtain better results by using large number of observed sample.

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