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Estimation of Finite Population Mean under Stratified Adaptive Cluster Sampling using Calibration Approach

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Abstract: The paper puts forward the calibrated estimators of finite population mean under stratified adaptive cluster sampling technique. The paper proceeds with application the mean and logarithmic mean of the available auxiliary information in defining the calibration constraints to obtain the proposed calibration estimators in the scenario of stratified adaptive cluster sampling. The expressions of mean squared errors of the suggested estimators have been derived and compared with the traditionally combined ratio estimator as well as other existing estimators. The obtained results have been compared with the help of a simulation study conducted on a data which comprise of simulated values of X and Y variables using R-software.

Keywords: Adaptive cluster sample, Auxiliary variable, Calibration estimation, Mean, Stratified sampling

1 Introduction

In case of rare, hidden and clustered populations such as rare and endangered species of animals and plants, study of disease that occur occasionally and unpredictably, etc., the most suitable and applicable sampling scheme is Adaptive cluster sampling (ACS). The concept of the adaptive cluster sampling was first suggested in [1] which modified the Horvitz-Thompson's and Hansen-Hurwitz estimators in case of ACS. Several literatures suggested different estimators in the field of adaptive cluster sampling, such as [2] recommended the ratio estimator in ACS. Dryver and Chao [3] discussed a classical ratio estimator in ACS and developed two new types of ratio estimators. Chutiman [4,5] considered ratio estimator suggested in [6] by making use of available auxiliary variable in case of stratified sampling. A ratio estimator without replacement of the networks is proposed in [4], whereas [7] suggested a regression cum exponential type ratio estimator to estimate the population mean with the help of two auxiliary variables under ACS. Younis and Shabbir [8] gave modified ratio estimators and Qureshi et al. [9] have developed the generalized ratio estimator for one auxiliary variable in case of stratified ACS.

In ACS, the first sample can be chosen using any conventional sample design. Here we define an explicit condition C and if the value of the i^{th} selected unit satisfies the pre-defined condition, i.e., $y_i > c$, those neighborhood units of the ith selected unit which satisfy the pre-defined condition, will also be included in the sample. This procedure will be continuing until no further unit left which satisfies the pre-defined condition. In this way, the resultant sample will consist of all units including the first selected sample. All units that fulfil the pre-defined condition are called the network while the units which do not satisfy the pre-defined condition are labelled as edge units. The combination of networks and edge units are termed as clusters.

The calibration approach was first introduced by [12] which modified the design weights to obtain more precise estimators of the population parameters. This paper introduces the calibrated estimators for finite population mean in case of stratified adaptive cluster sampling. Section 2 includes the suggested estimators using (i) mean and (ii) logarithmic mean of the available auxiliary information in defining the calibration constraints. These proposed estimators have been compared with traditional combined ratio estimator defined in case of stratified random sampling and the estimator suggested in [1] . The simulation study has also been conducted on the dataset given in [10] which contains the simulated values of X and Y variable taken from [2].

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1.1 Notations

The following notations have been used in stratified adaptive cluster sampling (ACS) in the subsequent sections:

L : Number of strata

N_h : Size of the h^{th} stratum, such a way that $N = \sum_{h=1}^{L} N_h$

n_h : Size of the sampled units in the h^{th} stratum, such a way that $n = \sum_{h=1}^{L} n_h$

$W_h = \frac{N_h}{N}$: h^{th} stratum weight

y_{hj} : i^{th} value of primary variable in the h^{th} stratum.

x_{hj} : i^{th} value of the available auxiliary variable in the h^{th} stratum.

$\bar{Y}_h = \frac{1}{N_h}\sum_{i=1}^{N_h} y_{hj}$: Population mean of Y-values in the h^{th} stratum.

$\bar{X}_h = \frac{1}{N_h}\sum_{i=1}^{N_h} x_{hj}$: Population mean of X-values in the h^{th} stratum.

A_{hi} : i^{th} network which includes i^{th} unit of the initial sample in the h^{th} stratum

m_{hi} : Total number of units in the i^{th} selected network for the h^{th} stratum

$\omega_{yhi} = \frac{1}{m_{hi}}\sum_{j\in A_{hi}} y_{hj}$: i^{th} network's mean of the study variable in the h^{th} stratum

$\omega_{xhi} = \frac{1}{m_{hi}}\sum_{j\in A_{hi}} x_{hj}$: i^{th} network's mean of the auxiliary variable in the h^{th} stratam.

$\bar{\omega}_{yh} = \frac{\sum_{i=1}^{n_h}\omega_{yhi}}{n_h}$: h^{th} stratum sample mean of study variable.

$\bar{\omega}_{xh} = \frac{\sum_{i=1}^{n_h}\omega_{xhi}}{n_h}$: h^{th} stratum sample mean of the auxiliary variable.

1.2 Existing Estimators

(i) A combined ratio estimator in the case of usual stratified sampling technique is given as:

$$\bar{y}_{R.st} = \frac{\bar{y}_{st}}{\bar{x}_{st}}\bar{X} = \hat{R}\bar{X} \tag{1}$$

where $\bar{y}_{st} = \sum_{h=1}^{L}\frac{N_h}{N}\bar{y}_h$, $\bar{x}_{st} = \sum_{h=1}^{L}\frac{N_h}{N}\bar{x}_h$ and $\hat{R} = \frac{\bar{y}_{st}}{\bar{x}_{st}}$ is the sample ratio of the stratified means of the study and auxiliary variables.

(ii) The mean estimator given by Thompson [11] in the case of stratified adaptive cluster sampling (SACS) technique is given as:

$$\bar{y}_{th.sac} = \sum_{h=1}^{L}\frac{N_h}{N}\bar{\omega}_{yh} \tag{2}$$

where $\bar{\omega}_{yh} = \frac{\sum_{i=1}^{n_h}\omega_{yhi}}{n_h}$ is the h^{th} stratum sample mean of the study variable under stratified adaptive cluster sampling.

2 Proposed Calibration Estimators in SACS

In SACS, the first sample of size $n_h$in the h^{th} stratum is drawn using SRSWOR. The units which fulfil the pre-specified condition of interest along with their neighborhood units which are also fulfilling the predefined condition are included in the sample. We recommend two calibration estimators of population mean assuming (i) usual mean and (ii) logarithmic mean of the available auxiliary variable in the following subsections:

2.1 Calibration Estimator using Mean of the Auxiliary Variable

The calibration estimator of population mean using known population mean $\bar{X}_h$ in defining a calibration constraint for stratified adaptive cluster sampling is suggested as:

$$\bar{y}_{c1.sac} = \sum_{h=1}^{L} \psi_h \bar{\omega}_{yh} \tag{3}$$

Here, ψ_h; h=1, 2, ..., L are carefully chosen calibration weights for minimizing the Chi-square type distance function specified as:

$$\sum_{h=1}^{L} \frac{(\psi_h - W_h)^2}{W_h Q_h} \tag{4}$$

depending on the subsequent calibration constraints:

$$\sum_{h=1}^{L} \psi_h = \sum_{h=1}^{L} W_h \tag{5}$$

$$\sum_{h=1}^{L} \psi_h \bar{\omega}_{xh} = \sum_{h=1}^{L} W_h \bar{X}_h \tag{6}$$

The Lagrange equation in this situation can be written as follows:

$$\Phi = \sum_{h=1}^{L} \frac{(\psi_h - W_h)^2}{Q_h W_h} - 2\eta_1 (\sum_{h=1}^{L} \psi_h - \sum_{h=1}^{L} W_h) - 2\eta_2 (\sum_{h=1}^{L} \psi_h \bar{\omega}_{xh} - \sum_{h=1}^{L} W_h \bar{X}_h) \tag{7}$$

where, η_1 *and* η_2 are constant, known as Lagrange's multipliers. In order to obtain the optimal values of calibrated weights ψ_h $(h = 1, 2, ..., L)$, we differentiate the Lagrange function given in equation (7) with respect to ψ_h and equate it to zero. Hence, the calibration weights are found as given below:

$$\Omega_h = W_h + W_h Q_h (\eta_1 + \eta_2 \bar{\omega}_{xh}) \tag{8}$$

Here η_1 *and* η_2 are obtained after substituting ψ_h from equation (8) to equation (5) and (6), thus, the calibrated weights become:

$$\psi_h = W_h + W_h Q_h \left[\frac{-(\sum_{h=1}^{L} W_h (\bar{X}_h - \bar{\omega}_{xh}))(\sum_{h=1}^{L} W_h Q_h \bar{\omega}_{xh})}{(\sum_{h=1}^{L} W_h Q_h \bar{\omega}_{xh}^2)(\sum_{h=1}^{L} W_h Q_h) - (\sum_{h=1}^{L} W_h Q_h \bar{\omega}_{xh})^2} \right] + W_h Q_h \bar{\omega}_{xh} \left[\frac{(\sum_{h=1}^{L} W_h Q_h)(\sum_{h=1}^{L} W_h (\bar{X}_h - \bar{\omega}_{xh}))}{(\sum_{h=1}^{L} W_h Q_h \bar{\omega}_{xh}^2)(\sum_{h=1}^{L} W_h Q_h) - (\sum_{h=1}^{L} W_h Q_h \bar{\omega}_{xh})^2} \right] \tag{9}$$

Putting the value of ψ_h from equation (9) to equation (3), the recommended calibrated estimator for stratified adaptive cluster sampling gets the form given as:

$$\bar{y}_{c1.sac} = \sum_{h=1}^{L} W_h \bar{\omega}_{yh} + \hat{\beta}_{c1.sac} \left[\sum_{h=1}^{L} W_h (\bar{X}_h - \bar{\omega}_{xh}) \right] \tag{10}$$

where $\hat{\beta}_{c1.sac} = \left[\frac{(\sum_{h=1}^{L} W_h Q_h)(\sum_{h=1}^{L} W_h Q_h \bar{\omega}_{yh} \bar{\omega}_{xh}) - (\sum_{h=1}^{L} W_h Q_h \bar{\omega}_{xh})(\sum_{h=1}^{L} W_h Q_h \bar{\omega}_{yh})}{(\sum_{h=1}^{L} W_h Q_h \bar{\omega}_{xh}^2)(\sum_{h=1}^{L} W_h Q_h) - (\sum_{h=1}^{L} W_h Q_h \bar{\omega}_{xh})^2} \right]$

The proposed estimator given in equation (10) can be expressed as:

$$\bar{y}_{c1.sac} = \sum_{h=1}^{L} W_h \phi \left(\bar{\omega}_{yh}, \bar{\omega}_{xh}, \hat{\beta}_{c1.sac}, \bar{X}_h \right) \tag{11}$$

Hence, we obtain the approximate mean squared error (MSE) by applying Taylor linearization method as:

$$MSE\left(\bar{y}_{c1.sac}\right) = E[\bar{y}_{c1.sac} - \bar{Y}_h]^2 = \sum_{h=1}^{L} W_h^2 E\left[\phi\left(\bar{\omega}_{yh},\, \bar{\omega}_{xh},\, \hat{\beta}_{c1.sac},\, \bar{X}_h\right) - \bar{Y}_h\right]^2$$

$$E\left[\phi\left(\bar{\omega}_{yh},\, \bar{\omega}_{xh},\, \hat{\beta}_{c1.sac},\, \bar{X}_h\right) - \bar{Y}_h\right]^2 = \left[\frac{\partial \phi}{\partial \bar{\omega}_{yh}}\bigg|_P\right]^2 \times E\left(\bar{\omega}_{yh} - \bar{Y}_h\right)^2 + \left[\frac{\partial \phi}{\partial \bar{\omega}_{xh}}\bigg|_P\right]^2 \times E\left(\bar{\omega}_{xh} - \bar{X}_h\right)^2$$

$$+2 \times \left[\frac{\partial \phi}{\partial \bar{\omega}_{yh}}\bigg|_P\right]\left[\frac{\partial \phi}{\partial \bar{\omega}_{xh}}\bigg|_P\right] \times E\left[\left(\bar{\omega}_{yh} - \bar{Y}_h\right)\left(\bar{\omega}_{xh} - \bar{X}_h\right)\right]$$

$$+ \; higher\, order\, terms$$

Theorem 2.1. The mean squared error (MSE) of the proposed calibration estimator $\bar{y}_{c1.sac}$ for $Q_h = 1$ followed by first order Taylor expansion up to second order of approximation can be derived as:

$$MSE\left(\bar{y}_{cl.sac}\right) = \sum_{h=1}^{L} W_h^2 f'_h \left[\bar{Y}_h^2 C_{yh.sac}^2 + \beta_{c1.sac}^2 \bar{X}_h^2 C_{xh.sac}^2 - 2\beta_{cl.sac}\rho_{yxh.sac}\bar{Y}_h\bar{X}_h C_{yh.sac}C_{xh.sac}\right] \tag{12}$$

where $\beta_{c1.sac} = \left[\frac{(\sum_{h=1}^{L} W_h\bar{Y}_h\bar{X}_h) - (\sum_{h=1}^{L} W_h\bar{X}_h)(\sum_{h=1}^{L} W_h\bar{Y}_h)}{(\sum_{h=1}^{L} W_h\bar{X}_h^2) - (\sum_{h=1}^{L} W_h\bar{X}_h)^2}\right]$

2.2 Calibration Estimator using Logarithmic Mean

The second proposed calibration estimator using logarithmic mean of $\bar{X}_h$ in calibration constraint under SACS is defined as:

$$\bar{y}_{c2.sac} = \sum_{h=1}^{L} \psi_h \bar{y}_{h.sac} \tag{13}$$

The calibration weights $\psi_h\, for\, (h = 1, 2, ..., L)$, are selected in such a way so as for minimizing the Chi-square distance function under the subsequent calibration constraints:

$$\sum_{h=1}^{L} \psi_h = \sum_{h=1}^{L} W_h \tag{14}$$

$$\sum_{h=1}^{L} \psi_h \log(\bar{\omega}_{xh}) = \sum_{h=1}^{L} W_h \log(\bar{X}_h) \tag{15}$$

The Lagrange equation for the aforesaid calibration estimator is specified as:

$$\Phi = \sum_{h=1}^{L} \frac{(\psi_h - W_h)^2}{Q_h W_h} - 2\eta_1\left(\sum_{h=1}^{L} \psi_h - \sum_{h=1}^{L} W_h\right) - 2\eta_2\left(\sum_{h=1}^{L} \psi_h \log(\bar{\omega}_{xh}) - \sum_{h=1}^{L} W_h \log(\bar{X}_h)\right) \tag{16}$$

where $\eta_1\, and\, \eta_2$ are the Lagrange's multiplier. For finding the optimum $\psi_h\, for\, (h = 1, 2, ..., L)$, the Lagrange equation mentioned in equation (16) is differentiated with respect to ψ_h and equate to zero.

Henceforth, the calibrated weights are derived as:

$$\Omega_h = W_h + W_h Q_h(\eta_1 + \eta_2 \log(\bar{\omega}_{xh})) \tag{17}$$

Here $\eta_1\, and\, \eta_2$ are attained by placing the value of ψ_h from equation (17) to equations (14) and (15), thus, the resultant calibrated weights are:

$$\psi_h = W_h + \left[\frac{W_h Q_h \bar{\omega}_{xh} \sum_{h=1}^{L} W_h Q_h \left(\sum_{h=1}^{L} W_h\left(\log(\bar{X}_h) - \log(\bar{\omega}_{xh})\right)\right) - W_h Q_h \left(\sum_{h=1}^{L} W_h\left(\log(\bar{X}_h) - \log(\bar{\omega}_{xh})\right)\right) \sum_{h=1}^{L} W_h Q_h \log(\bar{\omega}_{xh})}{\left(\sum_{h=1}^{L} W_h Q_h \log(\bar{\omega}_{xh}^2)\right)\left(\sum_{h=1}^{L} W_h Q_h\right) - \left(\sum_{h=1}^{L} W_h Q_h \log(\bar{\omega}_{xh})\right)^2}\right] \tag{18}$$

Therefore, after putting ψ_h from equation (18) to equation (13), the proposed calibrated estimator for SACS attains the final form given as:

$$\bar{y}_{c2.sac} = \sum_{h=1}^{L} W_h \bar{\omega}_{yh} + \hat{\beta}_{c2.sac} \left[\sum_{h=1}^{L} W_h (\log \bar{X}_h - \log \bar{\omega}_{xh}) \right] \tag{19}$$

where $\hat{\beta}_{c2.sac} = \left[\frac{(\sum_{h=1}^{L} W_h Q_h)(\sum_{h=1}^{L} W_h Q_h \bar{\omega}_{yh} \log(\bar{\omega}_{xh})) - (\sum_{h=1}^{L} W_h Q_h \log(\bar{\omega}_{xh}))(\sum_{h=1}^{L} W_h Q_h \bar{\omega}_{yh})}{(\sum_{h=1}^{L} W_h Q_h \log(\bar{\omega}_{xh}^2))(\sum_{h=1}^{L} W_h Q_h) - (\sum_{h=1}^{L} W_h Q_h \log(\bar{\omega}_{xh}))^2} \right]$

The proposed estimator specified in equation (19) can be rewritten as:

$$\bar{y}_{c2.sac} = \sum_{h=1}^{L} W_h \phi \left(\bar{\omega}_{yh}, \bar{\omega}_{xh}, \hat{\beta}_{c2.sac}, \bar{X}_h \right) \tag{20}$$

Therefore, the approximate mean squared error (MSE) applying Taylor linearization method is obtained as:

$$MSE(\bar{y}_{c2.sac}) = E[\bar{y}_{c2.sac} - \bar{Y}_h]^2 = \sum_{h=1}^{L} W_h^2 E\left[\phi\left(\bar{\omega}_{yh}, \bar{\omega}_{xh}, \hat{\beta}_{c2.sac}, \bar{X}_h\right) - \bar{Y}_h\right]^2$$

$$E\left[\phi\left(\bar{\omega}_{yh}, \bar{\omega}_{xh}, \hat{\beta}_{c2.sac}, \bar{X}_h\right) - \bar{Y}_h\right]^2 = \left[\frac{\partial \phi}{\partial \bar{\omega}_{yh}}\bigg|_P\right]^2 \times E\left(\bar{\omega}_{yh} - \bar{Y}_h\right)^2 + \left[\frac{\partial \phi}{\partial \bar{\omega}_{xh}}\bigg|_P\right]^2 \times E(\bar{\omega}_{xh} - \bar{X}_h)^2$$

$$+2 \times \left[\frac{\partial \phi}{\partial \bar{\omega}_{yh}}\bigg|_P\right]\left[\frac{\partial \phi}{\partial \bar{\omega}_{xh}}\bigg|_P\right] \times E\left[\left(\bar{\omega}_{yh} - \bar{Y}_h\right)(\bar{\omega}_{xh} - \bar{X}_h)\right]$$

$$+ higher\,order\,terms$$

Theorem 2.2. Thus, MSE of the recommended calibration estimator $\bar{y}_{c2.sac}$ for $Q_h = 1$ using first order Taylor expansion up to the 2nd order of approximation can be derived as:

$$MSE(\bar{y}_{c2.sac}) = \sum_{h=1}^{L} W_h^2 f'_h \left[\bar{Y}_h^2 C_{yh.sac}^2 + \beta_{c2.sac}^2 C_{xh.sac}^2 - 2\beta_{c2.sac}\rho_{yxh.sac}\bar{Y}_h C_{yh.sac} C_{xh.sac}\right] \tag{21}$$

where $\beta_{c2.sac} = \left[\frac{(\sum_{h=1}^{L} W_h \bar{Y}_h \log(\bar{X}_h)) - (\sum_{h=1}^{L} W_h \log(\bar{X}_h))(\sum_{h=1}^{L} W_h \bar{Y}_h)}{(\sum_{h=1}^{L} W_h \log(\bar{X}_h^2)) - (\sum_{h=1}^{L} W_h \log(\bar{X}_h))^2} \right]$

$C_{xh.sac}^2$: Population coefficient of variation (CV) of X in the h^{th} stratum. $C_{yh.sac}^2$: Population CV of Y in the h^{th} stratum.

3 Simulation Study

The simulation study has been performed for determining the percentage relative efficiency of the suggested calibrated estimators for SACS. The simulated Y-values and X-values given in [10] are considered for this study as shown in Tables 1 and 2, respectively. The data considered for the simulation study have $20 \times 20 = 400$ observations, divided into four equal size strata, where each stratum consists of $20 \times 5 = 100$ observations arranged into 5 columns and 20 rows.

Table 1: Values of the Y-variable

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1
0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	1
2	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	1	0
1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	1	0	1	1	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1
0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	6	0	1
0	0	1	0	0	0	0	0	6	18	3	0	1	0	0	0	9	27	9	0
1	0	0	0	0	0	0	1	14	46	11	0	0	0	0	1	8	25	14	1
0	0	0	0	0	0	0	0	4	7	2	0	0	0	0	0	1	2	1	0

Table 2: Values of the X-variable

0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	1	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	2
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0
0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
0	0	0	1	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1
0	0	0	0	0	0	0	1	2	0	1	0	0	0	0	0	0	0	0	1
0	0	0	0	0	0	0	0	4	6	0	1	0	0	0	4	3	4	2	3
0	0	1	0	0	0	0	4	6	11	4	3	1	0	0	2	6	8	3	7
0	0	0	0	0	0	1	4	4	9	11	4	0	0	2	1	8	10	0	2
0	0	0	0	0	0	1	2	7	4	3	1	1	0	0	0	0	2	2	0

The subsequent table displays summary about the data under consideration:

Table 3: Summary of the data

Stratum	1	2	3	4
$\bar{X}_h$	0.1	0.7	0.37	0.79
$\bar{Y}_h$	0.11	1.05	0.21	1.17
S^2_{xh}	0.091	3.727	1.751	3.481
S^2_{yh}	0.119	26.634	1.359	17.132
S_{xyh}	0.029	7.076	1.376	5.703
$S^2_{xh\_sac}$	0.091	3.259	1.347	2.185
$S^2_{yh\_sac}$	0.119	10.847	0.868	7.305
S_{xyh_sac}	0.029	5.544	0.942	3.165

An initial sample from each stratum is drawn by using SRSWOR. An explicit condition, $C = y; y > 0$ is considered for including preliminary as well as neighborhood units in the sample. We have considered various initial stratified samples of sizes n= 16, 20, 24, 28, 32, 36, 40, 44, 48, 52, 56, 60 and selected samples of equal sizes from each stratum, i.e., nh=4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, respectively, for this study. The simulation study is performed using R-software generating 5000 samples for different values of n, i.e., n= 16, 20, 24, 28, 32, 36, 40, 44, 48, 52, 56 and 60. The performances of the suggested estimators are measured by percentage relative root mean squared error (RRMSE) and percentage relative efficiency (RE) computed as:

$$PRRMSE(\bar{y}_\alpha) = \sqrt{\frac{1}{5000}\sum_{i=1}^{5000}\left(\frac{(\bar{y}_{i\alpha} - \bar{Y})}{\bar{Y}}\right)^2} \times 100\ ;$$

$where\ \alpha = R.st, th.sac, c1.sac, c2.sac.$

$$PRE(\bar{y}_\alpha) = \left(\frac{\bar{y}_{R.st}}{\bar{y}_\alpha}\right) \times 100;\ \ \alpha = R.st,\ th.sac, c1.sac,\ c2.sac$$

The PRRMSE and PRE obtained for ratio estimator $\bar{y}_{R.st}$, estimator given by Thompson [13] $\bar{y}_{th.sac}$ along with the suggested calibrated estimators under stratified adaptive cluster sampling, are presented in Tables 4 and 5, respectively.

Table 4: Percentage Relative Root Mean Squared Error (PRRMSE)

Sample size (n, nh)	$\bar{y}_R$	$\bar{y}_{th}$	$\bar{y}_{c1.sac}$	$\bar{y}_{c2.sac}$
16; 4	108.45	96.24	77.57	66.18
20; 5	98.91	85.03	65.93	61.35
24; 6	90.55	77.35	64.11	54.25
28; 7	79.54	71.78	59.44	49.67
32; 8	77.94	66.56	57.93	47.08
36; 9	65.26	62.99	56.72	43.92
40; 10	62.91	58.22	50.93	40.16
44; 11	57.59	55.63	49.73	38.54
48; 12	53.38	52.49	47.09	37.06
52; 13	51.42	50.08	45.93	35.56
56; 14	50.36	47.73	44.15	32.97
60; 15	46.96	46.4	42.95	32.08

Table 5: Percentage Relative Efficiency (PRE)

Sample size (n, nh)	$\bar{y}_R$	$\bar{y}_{th}$	$\bar{y}_{c1.sac}$	$\bar{y}_{c2.sac}$
16; 4	100	112.68	139.81	163.86
20; 5	100	116.32	150.02	161.22
24; 6	100	117.06	141.24	166.91
28; 7	100	110.81	133.81	160.15
32; 8	100	117.1	134.54	165.53
36; 9	100	103.6	115.04	148.56
40; 10	100	108.05	123.53	156.67
44; 11	100	103.53	115.8	149.41
48; 12	100	101.71	113.35	144.04
52; 13	100	102.68	111.94	144.59
56; 14	100	105.51	114.07	152.75
60; 15	100	101.21	109.35	146.41

4 Conclusion and Discussion

The developed estimators for stratified adaptive cluster sampling under the calibration approach are performing well compared to the classical ratio estimator and the estimator given in [11] for this dataset. The results given in Tables 4 and 5 depict PRRMSE and PRE, respectively.

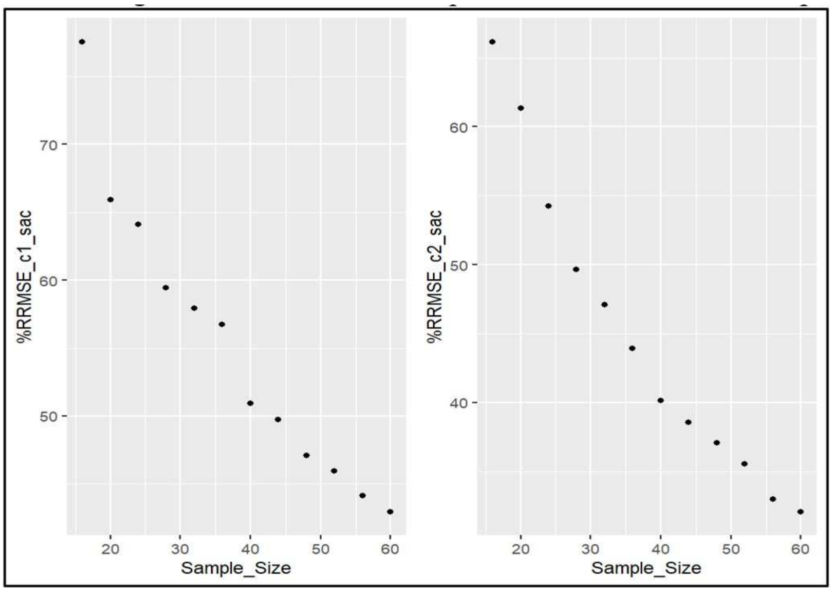

Fig. 1: PRRMSE of the Suggested Estimators

The results are obtained for varying sample sizes. It can be easily observed from Table 5, that as the sample size increases, the PRRMSE decreases, these results are also shown in Figure 1.

Figure 2 depicts that the proposed calibration estimators have less PRRMSE hence, greater percent relative efficiency as compared with the classical ratio estimator as well as the estimator suggested in [11]. Hence, it is concluded that the developed calibration estimators for stratified adaptive cluster sampling using mean and logarithmic mean of available auxiliary variable in the calibration constraints are much more efficient than the other considered estimators for estimating the population mean on the basis of simulation result performed on the data provided in [5].

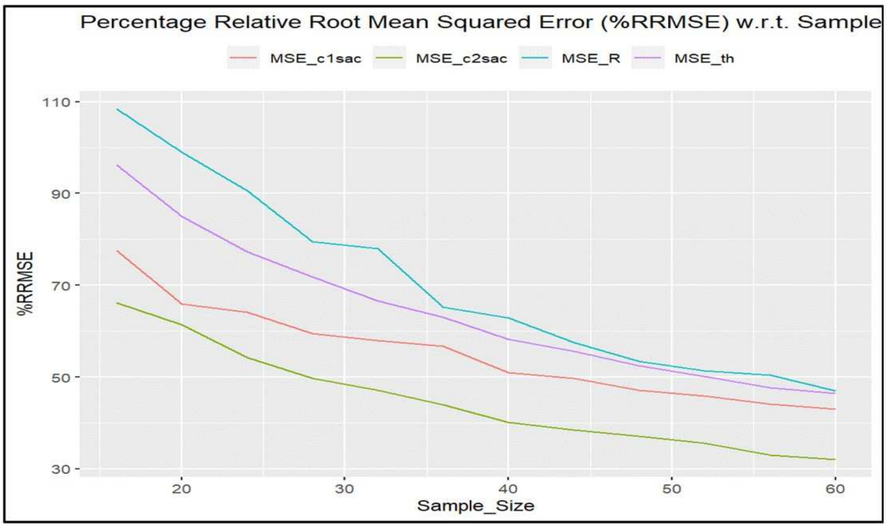

Fig. 2: The PRRMSE with respect to the Sample Size

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