Journal of Statistics Applications & Probability Letters
An International Journal

http://dx.doi.org/10.18576/jsapl/110303

A Simulation Study on Some Confidence Intervals for Estimating the Population Mean Under Asymmetric and Symmetric Distribution Conditions

H M Nayem and B. M. Golam Kibria*

Department of Mathematics and Statistics, Florida International University, Miami, FL 33199, USA

Received: 22 May 2024, Revised: 2 Jun. 2024, Accepted: 21 Jul. 2024

Published online: 1 Sep. 2024

Abstract: This study presents a comprehensive review and comparison of several methods for estimating the population mean using confidence intervals. The analysis considers both symmetric and asymmetric distributions while accounting for outliers. It evaluates 23 different estimators within classical and modified-t approaches by conducting a simulation study, covering symmetric and skewed distributions. The simulation results reveal that the proposed Q1-t, Q3-t, Q1Q3-t, Wizard-t, and Wizard-t from the median are particularly robust for moderate sample sizes and asymmetric populations. Conversely, Student-t emerges as the top performer for small sample sizes for symmetric distribution. Additionally, Chen-t, Median-t, T1, AADM-t, and Median-t estimators show promise for skewed distributions. Findings indicate that the ordinary t estimator performs optimally for symmetric distributions and small sample sizes, exhibiting a superior coverage rate and minimum width compared to other estimators. For skewed distributions, the Median-t, AADM-t, Median T1, Chen-t, and YY-t statistics are proposed as effective options for mean estimation. Notably, for moderate sample sizes (>50), the newly proposed Wizard-t and Wizard-t from median methods consistently demonstrate higher coverage rates and smaller confidence interval widths, surpassing other test statistics. Real-life data analysis further supports these findings. This study contributes valuable insights for practitioners by offering a comprehensive overview of available estimators for estimating the population mean across various distributional scenarios.

Keywords: Coverage probability, Simulation study, Skewed distributions, Symmetric distributions, Wizart-t

1 Introduction

The foundation of many sophisticated statistical theories is the normality assumption. Neyman's estimate theory for building the confidence interval (CI) is one of these theories [1]. But in reality, many data are skewed instead of being bell-shaped, meaning the distribution is not symmetrical about the mean. Positively skewed data, commonly observed across many fields such as psychology [2], health science [3–5], environmental science [6], and engineering, are prevalent. Real-life data often conform to right-skewed distributions, particularly noticeable when the sample size is small [7–9]. An example of a left-skewed distribution in real-life scenarios is the distribution of household income in many impoverished and developing countries. Other instances include student scores, where the majority score below average, waiting times at doctor's offices, characterized by mostly short waits but occasional long ones, commute times in congested cities, and the sizes of natural disasters. A CI functions as an interval estimator specifically created to encapsulate the true parameter value across multiple samples [10]. It provides a range of values to indicate the precision of parameter estimates. When constructing confidence intervals for the population mean (μ), normal theory is often relied upon in practice, but this approach becomes problematic when dealing with skewed or non-normal populations [8].

* Corresponding author e-mail: hnaye001@fiu.edu

Consequently, there's a necessity to develop confidence intervals for a population mean (μ) that aren't constrained by normality assumption. A study emphasized the need for robust estimators capable of handling deviations from normality, considering its common occurrence in applied research [11]. Recognizing the limitations, Johnson modified the student-t CI in 1978 for asymmetric populations [12], which has been further explored by multiple researchers [2,3,8,10,12–16]. In such instances, various methods like nonparametric, transformation-based, Bayesian, or bootstrap CIs can be employed.

In both left-skewed and right-skewed distributions, skewness can offer valuable insights into rare events, extreme behaviors, or measurement errors. However, it is crucial to identify and manage outliers appropriately during data analysis to prevent undue influence on statistical outcomes or conclusions. Additionally, CIs provide clinically relevant information beyond p-values and conventional significance testing. Therefore, in this study, 23 existing statistics for estimating population mean via CI methods are reviewed, and five alternative CIs are proposed for asymmetric populations with moderately large samples, building upon the entstudent-t. The Bootstrap method was excluded due to its inferior performance compared to transformed T's [18] and its complexity in execution. The evaluated CI methods include Student-t, Johnson-t, Median-t, Mad-t, AADM-t, and the proposed Wizard-t, Wizard-t from median, T1, T2, T3, Median T1, Median T2, Median T3, Mad T1, Mad T2, Mad T3, Chen-t, Yanagihara and Yuan-t, DMSD-t, and Downton-t. Evaluation criteria include coverage probability, which indicates the likelihood of encompassing the actual parameter, and CI width, where a smaller width signifies a better interval.

The paper is organized as follows: Section 2 contains the existing and proposed interval estimators, as well as the methodology of the simulation study. Section 3 discusses the results of the simulation study to compare the performance of the interval estimators and also includes two real-life examples. The acknowledgments can be found in section 4, with the references presented in section 5.

2 Methodology

Let us have independent and identically distributed (iid) random variables X_1, X_2, … X_n come from a skewed and symmetric distribution with unknown mean μ and standard deviation σ. Under the simulation study, we calculated 100(1-α)% CI for estimating the mean μ. We used the R programming language for the analysis. Some existing interval estimators will be reviewed in this section along with some proposed ones.

2.1. Classical parametric approach

2.1.1 Student-t: The student-t CI is used when sample sizes are below 30 and/or when the standard deviation is unknown [17]. The 100(1-α)% CI for estimating the population mean, μ is

$$\bar{x} \pm t_{\frac{\alpha}{2},\ n-1} \frac{s}{\sqrt{n}} ,$$

where $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$, the sample mean, and $s = \sqrt{\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n-1}}$, the sample standard deviation . And, $t_{\frac{\alpha}{2},\ n-1}$ is the upper $(\frac{\alpha}{2})$th percentile of the student-t distribution with (n-1) degrees of freedom.

2.1.2 Johnson-t: Johnson (1978) gave the following 100(1-α)% CI for estimating μ [12]

$$\bar{x} + \left(\frac{\widehat{\mu}_3}{6s^2 n}\right) \pm t_{\frac{\alpha}{2},\ n-1} \frac{s}{\sqrt{n}} ,$$

where $\widehat{\mu}_3 = \frac{n\sum_{i=1}^{n}(x_i-\bar{x})^3}{(n-1)(n-2)}$ is the estimator of the third central moment.

2.1.3 Median-t: Median-t is based on the standard deviation calculated using the median instead of the mean [8]. The 100(1-α)% CI for μ is

$$\bar{x} \pm t_{\frac{\alpha}{2},\ n-1} \frac{\widetilde{s}}{\sqrt{n}} ,$$

where the sample standard deviation, $\widetilde{s} = \sqrt{\frac{\sum_{i=1}^{n}(x_i-\widetilde{x})^2}{n-1}}$ where $\widetilde{x}$ is the sample median.

2.1.4 Mad-t: Mad-t proposed by Shi and Kibria in 2007 which is on the sample mean absolute deviation (MAD) [8]. The 100(1-α)% CI for µ is

$$\overline{x} \pm t_{\frac{\alpha}{2},\ n-1} \frac{\widehat{s}}{\sqrt{n}},$$

where the sample MAD, $\widehat{s} = \frac{\sum_{i=1}^{n} |x_i - \overline{x}|}{n}$.

2.1.5 AADT-t: Abu-Shawiesh et al. modified the student-t for asymmetric distribution [15]. The 100(1-α)% CI for µ is

$$\overline{x} \pm t_{\frac{\alpha}{2},\ n-1} \frac{\text{AADM}}{\sqrt{n}},$$

where $AADM = \frac{\sqrt{\pi/2}}{n} \sum_{i=1}^{n} |x_i - Median|$ is the average absolute deviation from the sample median [19].

2.1.6 Chen-t: Chen used the Edgeworth expansion to modify the central limit theory approach and proposed the following CI for the population mean (μ) [2]

$$\overline{x} \pm \left[t_{\frac{\alpha}{2},n-1} + \frac{\widehat{\gamma}\left(1 + 2t^2_{\frac{\alpha}{2},n-1}\right)}{6\sqrt{n}} + \frac{\widehat{\gamma}^2\left(t_{\frac{\alpha}{2},n-1} + 2t^2_{\frac{\alpha}{2},n-1}\right)}{9n} \right] \frac{s}{\sqrt{n}},$$

where $\widehat{\gamma} = \frac{\widehat{\mu}_3}{S^3}$ is the estimate of the population coefficient of skewness.

2.1.7 Yanagihara and Yuan-t (YY-t): Yanagihara and Yuan proposed the following CI for the population mean to reduce the effects of mean bias and population skewness [20]

$$\overline{x} + \left(\frac{\text{S}\ \widehat{k}_3}{(4n)(2 + \frac{15}{n})} \right) \pm t_{\frac{\alpha}{2},\ n-1} \frac{s}{\sqrt{n}},$$

where

$$\widehat{k}_3 = \frac{\sum_{i=1}^{n} \left(\frac{(x_i - \overline{x})^3}{n}\right)}{\left(\sum_{i=1}^{n} \left(\frac{(x_i - \overline{x})^3}{n}\right)\right)^{\frac{3}{2}}}$$

2.1.8 $\text{DMSD}_{\text{DM}} - t$*:* It is the modification of Student-t based on Decile Mean (DM). The 100(1-α)% CI for the population mean µ is

$$DM \pm \text{t}_{\frac{\alpha}{2},\ n-1} \frac{\text{SD}_{\text{DM}}}{\sqrt{n}},$$

Decile mean standard deviation, $\text{SD}_{\text{DM}} = \sqrt{\frac{1}{8} \sum_{i=1}^{9} (D_i - D_m)^2}$ which is an alternative to the sample standard deviation (s) to measure of dispersion.

2.1.9 Downton-t: Downtown-t (σ^*) based on the Gini's mean difference (G), as an estimator of standard deviation σ. Sample median, MD is an estimator of μ. The 100(1-α)% CI for µ is

$$MD \pm 1.253\, t_{\frac{\alpha}{2},\ n-1} \frac{\sigma^*}{\sqrt{n}},$$

where $\sigma^* = \frac{1}{2}\sqrt{\pi}$ G and Gini Mean Difference, $G = \frac{2}{n(n-1)} \sum_{k=1}^{n} \sum_{l=k+1}^{n} |X_k - X_l|$.

2.2 Transformed Approach

2.2.1 T_1& T_2 Transformation: Hall introduced two transformations from Edgeworth expansion which corrects both bias and skewness [21]. By doing some implications, Zhou and Dinh modified these transformations by their inverses [9].

$$T_1{}^{-1}(t) = \left(\frac{3}{\widehat{\gamma}}\right)[1+\widehat{\gamma}\ (t-(\widehat{\gamma}/6n))(1/3)] - \left(\frac{3}{\widehat{\gamma}}\right)$$

$$T_2{}^{-1}(t) = \left(\frac{3\sqrt{n}}{2\widehat{\gamma}}\right)\log\left[\frac{2\widehat{\gamma}}{3\sqrt{n}}\left(t-\frac{\widehat{\gamma}}{6n}\right)+1\right]$$

where $\widehat{\gamma}$, the population skewness, $\widehat{\gamma} = \frac{\sum_{i=1}^{n}(x_i-\overline{x})^3}{s^3}$. The $100(1-\alpha)\%$ confidence interval (CI) for μ is given by

$$\text{Lower Limit} = \overline{x} - T_i^{-1}\left(\frac{\varnothing\left(1-\frac{\alpha}{2}\right)}{\sqrt{n}}\right)s$$

$$\text{Upper Limit} = \overline{x} + T_i^{-1}\left(\frac{\varnothing\left(\frac{\alpha}{2}\right)}{\sqrt{n}}\right)s$$

where $\varnothing$ is the quantile of the standard normal.

2.2.2 T_3 and Modified T: Zhou and Dinh also proposed a new transformation, called the T_3 transformation [9].

$$T_3^{-1}(t) = \left[1+3\left(t-\frac{\widehat{\gamma}}{6n}\right)\right]^{1/3} - 1$$

The $100(1-\alpha)\%$ confidence interval (CI) for μ is given by the following formulas

$$\text{Lower Limit} = \left(\overline{x} - T_i^{-1}\left(\frac{\varnothing\left(1-\frac{\alpha}{2}\right)}{\sqrt{n}}\right)\right)s$$

$$\text{Upper Limit} = \left(\overline{x} + T_i^{-1}\left(\frac{\varnothing\left(\frac{\alpha}{2}\right)}{\sqrt{n}}\right)\right)s$$

where $\varnothing$ is the quantile of the standard normal.

Almonte and Kibria modified the CI of T_1 , T_2 , and T_3 as Median T_i and Mad T_i (i=1,2,3) so that the sample standard deviation is calculated from the sample median instead of the mean as in the Median-t and the sample mean absolute deviation as in the Mad-t [18].

2.3 Proposed Approach

2.3.1 Wizard-t: For asymmetric population, Wizard-t is the modification of student-t. The following is a 100(1-α)% CI for estimating μ

$$\overline{x} \pm t_{\frac{\alpha}{2},\ n-1}\frac{s^*}{\sqrt{n}},$$

where s^* is the sample standard deviation from the wizard mean, $s^* = \sqrt{\frac{\sum_{i=1}^{n}(x_i-\widetilde{x})^2}{n-1}}$ where $\widetilde{x}$ is the sample wizard mean. To calculate the wizard mean, 20% of observations are chopped. For a symmetric distribution, 10% from each tail. For asymmetric distribution, 20% from the right or left tail is based on the data structure.

2.3.2 Wizard-t from Median: For asymmetric population, Wizard-t from the median is also a modified version of student-t. The following is a 100(1-α)% CI for estimating μ

$$\overline{x} \pm t_{\frac{\alpha}{2},\ n-1}\frac{s^*}{\sqrt{n}},$$

where s^* is the sample standard deviation from the wizard median, $s^* = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \widetilde{x}^*)^2}{n-1}}$ and $\widetilde{x}^*$ is the sample wizard median. To calculate the wizard median, 20% of the observations are removed based on the data structure.

2.3.3 1st-quartile t (Q1-t): The following is a 100(1-α)% CI for estimating µ

$$\overline{x} \pm t_{\frac{\alpha}{2},\; n-1} \frac{s^*}{\sqrt{n}},$$

where $s^* = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \widetilde{x}^{**})^2}{n-1}}$ is the sample standard deviation from the first quartile (Q1 and $\widetilde{x}^{**}$ is the first quartile (Q1) from the sample.

2.3.4 3rd - quartile t (Q3-t): The following is a 100(1-α)% CI for estimating µ

$$\overline{x} \pm t_{\frac{\alpha}{2},\; n-1} \frac{s^*}{\sqrt{n}},$$

where $s^* = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \widetilde{x}^{***})^2}{n-1}}$ is the sample standard deviation from the third quartile (Q3) and $\widetilde{x}^{***}$ is the third quartile (Q3) from the sample.

2.3.5 Q1Q3-t: The following is a 100(1-α)% CI for estimating µ

$$\overline{x} \pm t_{\frac{\alpha}{2},\; n-1} \frac{s^*}{\sqrt{n}},$$

where $s^* = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \widetilde{x}^{****})^2}{n-1}}$ is the sample standard deviation from the average of the first and third quartile and $\widetilde{x}^{****}$ is the average of the first and third quartile from the sample.

2.4. Simulation Study

The paper aims to explore statistical methods for estimating the population mean using the CI for symmetric and asymmetric distributions with varying skewness. It also introduces five new estimators for asymmetric populations. As a theoretical comparison is challenging, a simulation study is conducted to compare the statistics based on sample sizes, coverage probability, and mean width of confidence intervals for estimating the mean.

2.4.1. Simulation Technique

For simulation purposes, we consider n =10, 20, 30, 50, 70, and 100 random samples generated from the Normal, Beta, Gamma, and Log-normal distributions with various various parametric conditions (Table 2.1). The steps of the simulation study:

I. Selecting the sample size (n), the number of simulations, M = 2500, and the level of significance, α=0.05.

II. Generating random samples from the below-mentioned distributions (Table 2.1).

III. Constructing the CIs for all the estimators at 100(1-α)% confidence level.

IV. if the CI includes the population mean µ, then for those that contain the mean, record the width and simulated coverage probability.

V. Repeat (I – IV) M times. Simulation results (based on different considered distributions) are presented in Tables 3.1 to 3.4 for selected n.

Table 2.1: Probability distributions, their parameters and skewness

Distribution	Parameter	Skewness
Normal (μ, σ^2)	$\mu = 10, \sigma^2 = 7$	0
Gamma (α, β)	$\alpha = 4,\ \beta = 1$	1
	$\alpha = 1,\ \beta = 1$	2
	$\alpha = 0.5,\ \beta = 1$	4

Beta (α,β)	$\alpha=10,\ \beta=0.3$	-3
	$\alpha=6,\ \beta=0.6$	-2
	$\alpha=6,\ \beta=0.1$	-5
Lognormal (μ,σ^2)	$\mu=1,\sigma^2=0.8$	3.6
	$\mu=1,\sigma^2=0.6$	2.2
	$\mu=1,\sigma^2=1$	6

2.5. Probability Distributions for Simulation

To study the effect of skewness, we consider two cases for simulation observations: symmetric and asymmetric distributions.

2.5.1. Symmetric distribution: The probability density function (pdf) of a normal random variable X with a mean μ and standard deviation σ, $N\left(\mu,\sigma^2\right)$, is given as follows:

$$f(x;\mu,\sigma)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2};-\infty<x<\infty,-\infty<\mu<\infty,\sigma>0$$

2.5.2. Asymmetric distributions: The skewness of a probability distribution measures the degree to which the distribution deviates from being symmetrical. A distribution with a longer tail on the left side is considered negatively skewed, while one with a longer tail on the right side is positively skewed.

2.5.2.1. Gamma distribution: The gamma distribution, Gamma(α,β), where shape parameter $\alpha=k$ and an inverse scale parameter $\beta=1/\theta$, called a rate parameter. The corresponding probability density function in the shape-rate parameterization is

$$f(x;\alpha,\beta)=\frac{x^{\alpha-1}e^{-\beta x}\beta^{\alpha}}{\Gamma(\alpha)}\quad \text{for } x>0 \text{ and } \alpha,\beta>0$$

where $\Gamma(\alpha)$ is the gamma function and for all positive integers, $\Gamma(\alpha)=(\alpha-1)!$, mean, $\mu=\frac{\alpha}{\beta}$, variance, $\sigma^2=\frac{\alpha}{\beta^2}$, and coefficient of skewness, $\sqrt{2/\alpha}$.

2.5.2.2. Log-normal distribution: Let the random varibale X follow the log-normal distribution with mean μ and standard deviation σ, then the probability density function of X is defined as $X\sim Lognormal\ \left(\mu,\sigma^2\right)$. The corresponding probability density function is

$$f(x;\alpha,\beta)=\frac{1}{x\sigma\sqrt{2\pi}}\exp\left(-\frac{(\ln x-\mu)^2}{2\sigma^2}\right)\quad -\infty<x<\infty,\ -\infty<\mu<\infty,\ \sigma>0$$

where mean, $\mu=\exp\left(\mu+\frac{\sigma^2}{2}\right)$, variance, $\sigma^2=\left[\exp\left(\sigma^2\right)-1\right]\exp\left(2\mu+\sigma^2\right)$ and the coefficient of skewness, $\left[\exp\left(\sigma^2\right)+2\right]\sqrt{\exp\left(\sigma^2\right)-1}$.

2.5.2.3. Beta distribution: The beta distribution, Beta(α,β). The corresponding probability density function in the shape parameterization is

$$f(x;\alpha,\beta)=\frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha,\beta)}\ \text{for } x>0\ \alpha,\beta>0$$

where $B(\alpha,\beta)=\frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$ and Γ is the Gamma function. The mean, $\mu=\frac{\alpha}{\alpha+\beta}$ and variance, $\sigma^2=\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$, coefficient of skewness, $\frac{2(\beta-\alpha)\sqrt{\alpha+\beta+1}}{(\alpha+\beta+2)\sqrt{\alpha\beta}}$.

3 Results and Discussion

We conducted comparisons of interval estimators under symmetric and asymmetric distributions by generating random samples from different distributions with varying skewness, as outlined in sections 3.3.1 to 3.3.4.

3.1. Normal Distribution

The simulation results for 23 interval estimators from Normal distribution with mean=10 and variance=7 are given in Table 3.1. We can see from Table 3.1 that most approaches maintain a high coverage probability (>0.94) across different sample sizes. The mean width of CIs decreases as the sample size increases for all approaches, which is expected. Among the proposed approaches, Wizard-t and Wizard-t from median consistently offer slightly narrower intervals compared to traditional methods like Student-t and Johnson-t. Approaches like Mad-t, Median-t, and AADM-t show variations in coverage probability and mean width across sample sizes. Q1-t, Q3-t, and Q1Q3-t offer high coverage probability with higher mean with compared to all other methods.

Table 3.1: Coverage Probability and Mean width for different sample sizes from Normal (10,7)

Statistics	Measuring Criteria	Sample Size					
		10	20	30	50	70	100
Student t	Coverage probability	0.950	0.950	0.946	0.943	0.947	0.952
	Mean width	9.740	6.477	5.172	3.954	3.317	2.768
Johnson t	Coverage probability	0.950	0.950	0.946	0.943	0.947	0.952
	Mean width	9.740	6.477	5.172	3.954	3.317	2.768
Median t	Coverage probability	0.953	0.952	0.948	0.944	0.948	0.953
	Mean width	9.937	6.556	5.215	3.975	3.329	2.776
Mad t	Coverage probability	0.879	0.890	0.880	0.871	0.881	0.887
	Mean width	7.573	5.107	4.089	3.138	2.635	2.203
AADM t	Coverage probability	0.940	0.946	0.940	0.937	0.946	0.953
	Mean width	9.265	6.317	5.079	3.911	3.289	2.753
Wizard t	Coverage probability	0.904	0.930	0.933	0.933	0.942	0.949
	Mean width	8.164	5.908	4.860	3.807	3.228	2.716
Wizard t from median	Coverage probability	0.910	0.933	0.936	0.934	0.942	0.949
	Mean width	8.268	5.959	4.889	3.822	3.237	2.722
T1	Coverage probability	0.914	0.938	0.938	0.936	0.944	0.950
	Mean width	8.441	6.065	4.956	3.856	3.259	2.735
T2	Coverage probability	0.573	0.607	0.597	0.602	0.614	0.600
	Mean width	3.665	2.634	2.152	1.675	1.415	1.188
T3	Coverage probability	0.777	0.845	0.856	0.870	0.890	0.908
	Mean width	5.710	4.468	3.811	3.110	2.701	2.325
Median T1	Coverage probability	0.918	0.941	0.940	0.936	0.946	0.950
	Mean width	8.611	6.140	4.997	3.876	3.271	2.742
Median T2	Coverage probability	0.580	0.614	0.601	0.606	0.616	0.601
	Mean width	3.739	2.666	2.170	1.684	1.421	1.191
Median T3	Coverage probability	0.786	0.850	0.862	0.871	0.891	0.908
	Mean width	5.825	4.522	3.843	3.126	2.711	2.332
Mad T1	Coverage probability	0.825	0.870	0.868	0.861	0.872	0.880
	Mean width	6.562	4.783	3.918	3.060	2.589	2.176
Mad T2	Coverage probability	0.461	0.502	0.489	0.504	0.509	0.488
	Mean width	2.849	2.077	1.702	1.329	1.124	0.945
Mad T3	Coverage probability	0.658	0.749	0.751	0.776	0.809	0.810
	Mean width	4.439	3.523	3.013	2.468	2.145	1.851
Chen t	Coverage probability	0.950	0.950	0.946	0.943	0.947	0.952
	Mean width	9.740	6.477	5.172	3.954	3.317	2.768
YY t	Coverage probability	0.949	0.950	0.946	0.943	0.947	0.952
	Mean width	9.740	6.477	5.172	3.954	3.317	2.768
DMSD t	Coverage probability	0.658	0.749	0.751	0.776	0.809	0.810
	Mean width	10.668	8.014	6.778	5.529	4.810	4.157
	Mean width	10.856	7.689	6.227	4.828	4.082	3.426
Downton t	Coverage probability	1.000	1.000	1.000	1.000	1.000	1.000
	Mean width	25.101	16.448	13.075	9.960	8.341	6.957

Statistics	Measuring Criteria	Sample Size					
		10	20	30	50	70	100
Q1 t	Coverage probability	0.949	0.964	0.967	0.969	0.978	0.983
	Mean width	9.698	7.080	5.842	4.581	3.882	3.272
Q1Q3 t	Coverage probability	0.948	0.966	0.965	0.969	0.978	0.980
	Mean width	9.695	7.085	5.823	4.571	3.888	3.270
Q3 t	Coverage probability	0.949	0.965	0.967	0.968	0.978	0.981
	Mean width	9.696	7.083	5.833	4.576	3.885	3.271

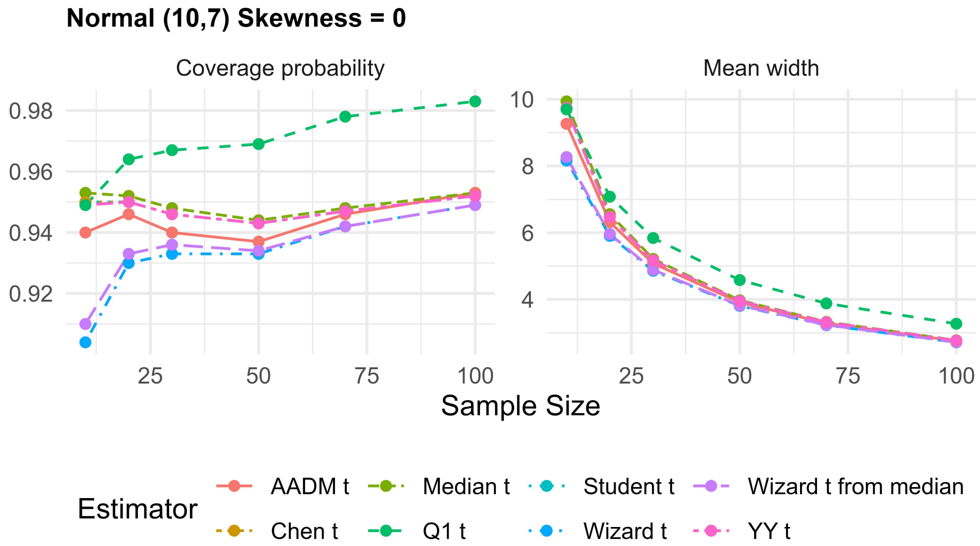

Fig. 1: Coverage probability and Mean width of Normal (10,7)

3.2. Beta Distribution

Table 3.2 demonstrates the coverage probability and mean width comparisons for different sample sizes for beta distribution with different skewness.

Table 3.2: Coverage Probability and Mean Width for different sample sizes from Beta Distribution with different skewness

Distributions	Approaches	Measuring Criteria	Sample Size					
			10	20	30	50	70	100
Beta (Skewness = -3)	Student t	Coverage probability	0.836	0.880	0.895	0.917	0.920	0.935
		Mean width	0.061	0.043	0.036	0.028	0.023	0.020
	Johnson t	Coverage probability	0.843	0.887	0.900	0.921	0.922	0.940
		Mean width	0.061	0.043	0.036	0.028	0.023	0.020
	Median t	Coverage probability	0.854	0.896	0.912	0.935	0.939	0.952
		Mean width	0.067	0.047	0.039	0.030	0.025	0.021
	Mad t	Coverage probability	0.764	0.795	0.793	0.805	0.795	0.804
		Mean width	0.044	0.030	0.024	0.018	0.016	0.013
	AADM t	Coverage probability	0.781	0.814	0.810	0.818	0.818	0.822
		Mean width	0.047	0.031	0.025	0.019	0.016	0.014
	Wizard t	Coverage probability	0.812	0.873	0.898	0.924	0.930	0.947
		Mean width	0.056	0.042	0.036	0.029	0.024	0.021
	Wizard t from median	Coverage probability	0.825	0.886	0.909	0.934	0.940	0.955
		Mean width	0.059	0.044	0.038	0.030	0.026	0.022
	T1	Coverage probability	0.813	0.866	0.889	0.910	0.914	0.931
		Mean width	0.056	0.041	0.034	0.027	0.023	0.019
	T2	Coverage probability	0.534	0.577	0.575	0.581	0.589	0.585
		Mean width	0.024	0.017	0.015	0.012	0.010	0.008
	T3	Coverage probability	0.699	0.774	0.811	0.854	0.869	0.878

Distributions	Approaches	Measuring Criteria	Sample Size					
			10	**20**	**30**	**50**	**70**	**100**
		Mean width	0.036	0.029	0.026	0.022	0.019	0.016
	Median T1	Coverage probability	0.831	0.886	0.907	0.930	0.938	0.950
		Mean width	0.061	0.045	0.038	0.029	0.025	0.021
	Median T2	Coverage probability	0.580	0.615	0.622	0.628	0.630	0.624
		Mean width	0.026	0.019	0.016	0.013	0.011	0.009
	Median T3	Coverage probability	0.720	0.807	0.840	0.877	0.894	0.910
		Mean width	0.040	0.032	0.029	0.024	0.021	0.018
	Mad T1	Coverage probability	0.736	0.777	0.781	0.798	0.788	0.798
		Mean width	0.040	0.028	0.024	0.018	0.015	0.013
	Mad T2	Coverage probability	0.400	0.428	0.413	0.420	0.416	0.419
		Mean width	0.017	0.012	0.010	0.008	0.007	0.006
	Mad T3	Coverage probability	0.588	0.650	0.670	0.700	0.706	0.718
		Mean width	0.026	0.021	0.018	0.015	0.013	0.011
	Chen t	Coverage probability	0.772	0.818	0.832	0.860	0.869	0.880
		Mean width	0.046	0.033	0.028	0.022	0.019	0.016
	YY t	Coverage probability	0.838	0.882	0.899	0.920	0.922	0.938
		Mean width	0.061	0.043	0.036	0.028	0.023	0.020
		Coverage probability	0.588	0.650	0.670	0.700	0.706	0.718
	DMSD t	Mean width	0.067	0.055	0.051	0.044	0.040	0.036
		Mean width	0.045	0.030	0.024	0.018	0.015	0.013
	Downton t	Coverage probability	0.913	0.901	0.861	0.731	0.531	0.280
		Mean width	0.133	0.086	0.070	0.053	0.044	0.037
	Q1 t	Coverage probability	0.812	0.866	0.889	0.914	0.916	0.932
		Mean width	0.055	0.040	0.034	0.027	0.023	0.019
	Q1Q3 t	Coverage probability	0.838	0.896	0.916	0.936	0.944	0.959
		Mean width	0.061	0.046	0.039	0.031	0.026	0.022
	Q3 t	Coverage probability	0.825	0.880	0.904	0.929	0.935	0.948
		Mean width	0.058	0.043	0.037	0.029	0.025	0.021
	Student t	Coverage probability	0.896	0.922	0.929	0.941	0.934	0.942
		Mean width	0.137	0.095	0.076	0.058	0.049	0.041
	Johnson t	Coverage probability	0.901	0.924	0.930	0.944	0.936	0.944
		Mean width	0.137	0.095	0.076	0.058	0.049	0.041
	Median t	Coverage probability	0.907	0.931	0.937	0.952	0.946	0.957
		Mean width	0.146	0.101	0.081	0.062	0.052	0.044
	Mad t	Coverage probability	0.826	0.852	0.848	0.863	0.856	0.859
		Mean width	0.104	0.071	0.057	0.044	0.037	0.031
	AADM t	Coverage probability	0.868	0.898	0.893	0.906	0.902	0.914
Beta (Skewness = -2)		Mean width	0.120	0.082	0.065	0.051	0.042	0.036
	Wizard t	Coverage probability	0.874	0.919	0.930	0.947	0.944	0.957
		Mean width	0.123	0.093	0.077	0.061	0.051	0.044
	Wizard t from median	Coverage probability	0.884	0.927	0.938	0.957	0.956	0.967
		Mean width	0.130	0.098	0.081	0.064	0.054	0.046
	T1	Coverage probability	0.873	0.912	0.922	0.934	0.930	0.937
		Mean width	0.123	0.090	0.073	0.057	0.048	0.041
	T2	Coverage probability	0.558	0.587	0.586	0.604	0.608	0.606
		Mean width	0.052	0.039	0.032	0.025	0.021	0.018
	T3	Coverage probability	0.748	0.817	0.841	0.872	0.878	0.902
		Mean width	0.081	0.065	0.056	0.046	0.040	0.035
	Median T1	Coverage probability	0.884	0.922	0.932	0.948	0.943	0.954
		Mean width	0.131	0.096	0.078	0.061	0.051	0.043
	Median T2	Coverage probability	0.582	0.616	0.613	0.636	0.639	0.631
		Mean width	0.056	0.041	0.034	0.026	0.022	0.019
	Median T3	Coverage probability	0.763	0.837	0.861	0.890	0.896	0.920
		Mean width	0.086	0.070	0.059	0.049	0.042	0.037
	Mad T1	Coverage probability	0.799	0.832	0.837	0.855	0.851	0.853
		Mean width	0.093	0.068	0.055	0.043	0.036	0.031
	Mad T2	Coverage probability	0.446	0.460	0.471	0.484	0.476	0.480
		Mean width	0.040	0.029	0.024	0.019	0.016	0.013

Distributions	Approaches	Measuring Criteria	Sample Size					
			10	20	30	50	70	100
	Mad T3	Coverage probability	0.635	0.702	0.721	0.773	0.772	0.776
		Mean width	0.061	0.049	0.042	0.035	0.030	0.026
	Chen t	Coverage probability	0.844	0.879	0.885	0.898	0.901	0.919
		Mean width	0.108	0.077	0.063	0.050	0.043	0.036
	YY t	Coverage probability	0.896	0.923	0.929	0.943	0.934	0.943
		Mean width	0.137	0.095	0.076	0.058	0.049	0.041
	DMSD t	Coverage probability	0.635	0.702	0.721	0.773	0.772	0.776
		Mean width	0.150	0.119	0.102	0.085	0.075	0.066
		Mean width	0.136	0.097	0.076	0.060	0.050	0.042
	Downton t	Coverage probability	0.977	0.975	0.960	0.931	0.854	0.732
		Mean width	0.326	0.217	0.171	0.131	0.109	0.092
	Q1 t	Coverage probability	0.882	0.922	0.930	0.951	0.946	0.956
		Mean width	0.127	0.094	0.077	0.061	0.051	0.043
	Q1Q3 t	Coverage probability	0.902	0.947	0.955	0.969	0.970	0.975
		Mean width	0.142	0.107	0.088	0.070	0.059	0.050
	Q3 t	Coverage probability	0.893	0.938	0.943	0.962	0.959	0.967
		Mean width	0.135	0.101	0.082	0.065	0.055	0.047
Beta (Skewness = -5)	Student t	Coverage probability	0.704	0.790	0.818	0.854	0.880	0.916
		Mean width	0.051	0.037	0.032	0.025	0.021	0.018
	Johnson t	Coverage probability	0.714	0.796	0.828	0.858	0.883	0.920
		Mean width	0.051	0.037	0.032	0.025	0.021	0.018
	Median t	Coverage probability	0.723	0.802	0.834	0.864	0.889	0.925
		Mean width	0.056	0.040	0.034	0.026	0.023	0.019
	Mad t	Coverage probability	0.637	0.686	0.678	0.682	0.687	0.696
		Mean width	0.033	0.022	0.018	0.014	0.011	0.010
	AADM t	Coverage probability	0.620	0.635	0.612	0.608	0.613	0.621
		Mean width	0.030	0.019	0.016	0.011	0.010	0.008
	Wizard t	Coverage probability	0.687	0.784	0.815	0.858	0.884	0.922
		Mean width	0.046	0.036	0.032	0.025	0.022	0.019
	Wizard t from median	Coverage probability	0.693	0.789	0.818	0.859	0.886	0.922
		Mean width	0.047	0.037	0.032	0.025	0.022	0.019
	T1	Coverage probability	0.691	0.783	0.810	0.852	0.877	0.914
		Mean width	0.047	0.036	0.031	0.024	0.021	0.018
	T2	Coverage probability	0.472	0.516	0.526	0.566	0.578	0.589
		Mean width	0.019	0.015	0.013	0.010	0.009	0.008
	T3	Coverage probability	0.598	0.716	0.753	0.805	0.831	0.876
		Mean width	0.030	0.026	0.023	0.019	0.017	0.015
	Median T1	Coverage probability	0.707	0.796	0.829	0.861	0.888	0.923
		Mean width	0.051	0.039	0.033	0.026	0.022	0.019
	Median T2	Coverage probability	0.509	0.554	0.559	0.596	0.606	0.616
		Mean width	0.021	0.016	0.014	0.011	0.010	0.008
	Median T3	Coverage probability	0.623	0.736	0.772	0.822	0.846	0.890
		Mean width	0.033	0.028	0.025	0.021	0.018	0.016
	Mad T1	Coverage probability	0.618	0.673	0.666	0.671	0.680	0.692
		Mean width	0.031	0.021	0.018	0.013	0.011	0.010
	Mad T2	Coverage probability	0.327	0.342	0.312	0.333	0.342	0.353
		Mean width	0.013	0.009	0.008	0.006	0.005	0.004
	Mad T3	Coverage probability	0.484	0.544	0.546	0.576	0.595	0.622
		Mean width	0.020	0.015	0.013	0.011	0.009	0.008
	Chen t	Coverage probability	0.653	0.736	0.760	0.801	0.816	0.857
		Mean width	0.038	0.028	0.024	0.019	0.016	0.014
	YY t	Coverage probability	0.708	0.792	0.824	0.858	0.883	0.920
		Mean width	0.051	0.037	0.032	0.025	0.021	0.018
	DMSD t	Coverage probability	0.484	0.544	0.546	0.576	0.595	0.622
		Mean width	0.055	0.050	0.049	0.045	0.042	0.040
		Mean width	0.016	0.008	0.006	0.004	0.003	0.003
	Downton t	Coverage probability	0.738	0.742	0.693	0.540	0.375	0.172
		Mean width	0.093	0.059	0.048	0.036	0.030	0.025

Distributions	**Approaches**	**Measuring Criteria**	**Sample Size**					
			10	**20**	**30**	**50**	**70**	**100**
	Q1 t	Coverage probability	0.682	0.777	0.809	0.852	0.879	0.916
		Mean width	0.044	0.035	0.031	0.024	0.021	0.018
	Q1Q3 t	Coverage probability	0.694	0.789	0.819	0.859	0.886	0.923
		Mean width	0.047	0.037	0.032	0.025	0.022	0.019
	Q3 t	Coverage probability	0.689	0.785	0.814	0.856	0.882	0.920
		Mean width	0.046	0.036	0.031	0.025	0.022	0.019

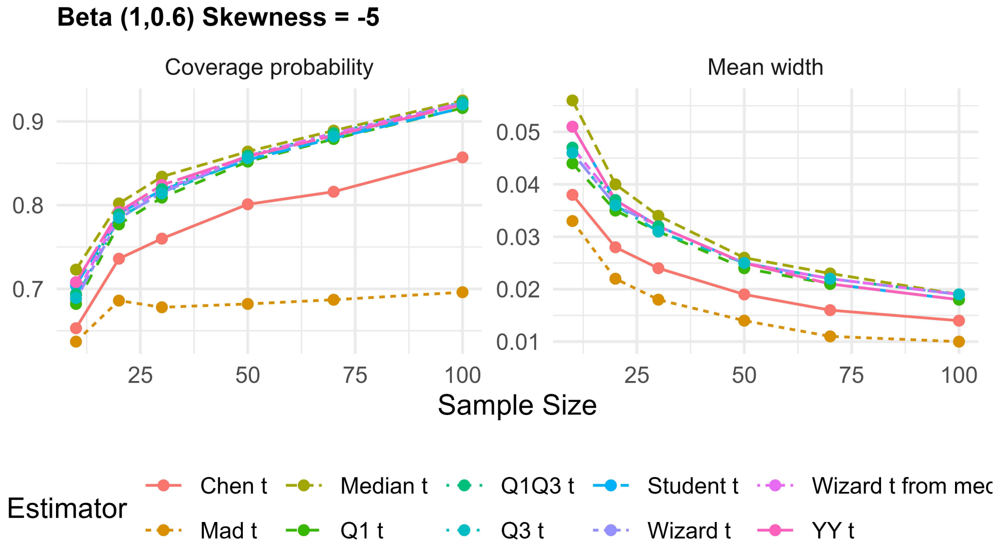

Fig. 2: Coverage probability and Mean width of Beta (1,0.6)

3.2.1. Beta with Skewness = -2

Similar trends are observed compared to skewness = -3, with the proposed Q1-t, Q3-t, Q1Q3-t, and Wizard-t approach continuing to outperform traditional methods. Median-t performs consistently better in terms of coverage probability but with a slightly wider width compared to the proposed Wizard-t approaches. The Mad-t still demonstrates relatively lower coverage probability, but narrower width compared to traditional methods.

3.2.2. Beta with Skewness = -3

The proposed Q1-t, Q3-t, Q1Q3-t, and Wizard-t approach consistently outperform traditional methods like Student-t, Johnson-t, and AADM-t in terms of maintaining higher coverage probability and providing narrower width. Median-t offers higher coverage probability compared to other traditional methods but with slightly wider width. Mad-t demonstrates relatively lower coverage probability but provides narrower width. Other methods show varying degrees of performance, with coverage probability and mean width of CIs differing across methods and sample sizes.

3.2.3. Beta with Skewness = -5

Similar trends are observed compared to skewness = -3, with the proposed Q1-t, Q3-t, Q1Q3-t, and Wizard-t approach continuing to outperform traditional methods. Median-t performs consistently well in terms of coverage probability but with slightly wider width compared to the proposed Wizard-t approaches. The Mad-t still demonstrates relatively lower Coverage probability, but narrower CIs compared to traditional methods.

All methods exhibit decreased performance compared to skewness = -3 and -2. Coverage probabilities are generally lower across all methods. The proposed Q1-t, Q3-t, Q1Q3-t, and Wizard-t approach maintain relatively higher coverage probability and narrower widths compared to traditional methods, albeit with reduced effectiveness compared to skewness = -3 and -2. Median-t still offers higher coverage probability but with wider CIs compared to the proposed Wizard-t approach.

3.3. Gamma Distribution

Table 3.3 demonstrates the coverage probability and mean width comparisons for different sample sizes for the gamma distribution with different skewness.

Table 3.3: Coverage Probability and Mean Width for different sample sizes from Gamma Distribution with different skewness.

Distributions	**Approaches**	**Measuring Criteria**	**Sample Size**					
			10	**20**	**30**	**50**	**70**	**100**
	Student t	Coverage probability	0.922	0.930	0.942	0.945	0.947	0.948
		Mean width	0.226	0.150	0.120	0.092	0.077	0.064
	Johnson t	Coverage probability	0.925	0.930	0.944	0.946	0.948	0.949
		Mean width	0.226	0.150	0.120	0.092	0.077	0.064
	Median t	Coverage probability	0.930	0.932	0.947	0.948	0.952	0.953
		Mean width	0.236	0.156	0.124	0.095	0.080	0.067
	Mad t	Coverage probability	0.867	0.882	0.870	0.883	0.894	0.872
		Mean width	0.177	0.119	0.095	0.073	0.062	0.052
	AADM t	Coverage probability	0.904	0.924	0.934	0.939	0.941	0.938
		Mean width	0.211	0.143	0.115	0.089	0.075	0.063
	Wizard t	Coverage probability	0.898	0.924	0.943	0.950	0.958	0.961
		Mean width	0.202	0.147	0.121	0.095	0.081	0.068
Gamma (Skewness =1)	Wizard t from median	Coverage probability	0.903	0.929	0.950	0.954	0.962	0.966
		Mean width	0.210	0.152	0.125	0.098	0.083	0.071
	T1	Coverage probability	0.894	0.920	0.932	0.942	0.945	0.944
		Mean width	0.200	0.142	0.115	0.090	0.076	0.064
	T2	Coverage probability	0.582	0.616	0.585	0.609	0.599	0.601
		Mean width	0.086	0.061	0.050	0.039	0.033	0.028
	T3	Coverage probability	0.772	0.838	0.849	0.876	0.899	0.890
		Mean width	0.133	0.104	0.088	0.072	0.063	0.054
	Median T1	Coverage probability	0.903	0.924	0.939	0.946	0.950	0.952
		Mean width	0.209	0.147	0.120	0.093	0.078	0.066
	Median T2	Coverage probability	0.604	0.628	0.604	0.623	0.618	0.616
		Mean width	0.089	0.063	0.052	0.040	0.034	0.029
	Median T3	Coverage probability	0.786	0.848	0.859	0.889	0.908	0.904
		Mean width	0.138	0.107	0.092	0.075	0.065	0.056
	Mad T1	Coverage probability	0.831	0.866	0.858	0.876	0.889	0.870
		Mean width	0.156	0.112	0.092	0.072	0.061	0.051
	Mad T2	Coverage probability	0.472	0.508	0.480	0.498	0.501	0.506
		Mean width	0.067	0.048	0.040	0.031	0.026	0.022
	Mad T3	Coverage probability	0.670	0.743	0.743	0.789	0.812	0.800
		Mean width	0.104	0.082	0.070	0.058	0.050	0.043
	Chen t	Coverage probability	0.889	0.904	0.907	0.927	0.930	0.926
		Mean width	0.190	0.130	0.106	0.083	0.071	0.060
	YY t	Coverage probability	0.923	0.929	0.944	0.946	0.948	0.949
		Mean width	0.226	0.150	0.120	0.092	0.077	0.064
		Coverage probability	0.670	0.743	0.743	0.789	0.812	0.800
	DMSD t	Mean width	0.247	0.184	0.155	0.125	0.109	0.094
		Mean width	0.254	0.180	0.146	0.115	0.096	0.081
	Downton t	Coverage probability	0.994	0.995	0.991	0.985	0.980	0.951
		Mean width	0.566	0.370	0.294	0.224	0.187	0.156
	Q1 t	Coverage probability	0.910	0.938	0.954	0.958	0.969	0.972
		Mean width	0.218	0.158	0.130	0.102	0.086	0.073
	Q1Q3 t	Coverage probability	0.926	0.952	0.968	0.974	0.978	0.982
		Mean width	0.236	0.173	0.143	0.113	0.096	0.081
	Q3 t	Coverage probability	0.920	0.946	0.964	0.966	0.974	0.979
		Mean width	0.227	0.165	0.136	0.107	0.091	0.077
	Student t	Coverage probability	0.886	0.917	0.918	0.936	0.934	0.945
		Mean width	0.089	0.061	0.049	0.038	0.032	0.027
Gamma (Skewness =2)	Johnson t	Coverage probability	0.890	0.920	0.922	0.936	0.938	0.946
		Mean width	0.089	0.061	0.049	0.038	0.032	0.027
	Median t	Coverage probability	0.893	0.928	0.931	0.946	0.948	0.954

Distributions	**Approaches**	**Measuring Criteria**	**Sample Size**					
			10	**20**	**30**	**50**	**70**	**100**
		Mean width	0.095	0.065	0.052	0.040	0.034	0.028
	Mad t	Coverage probability	0.819	0.846	0.840	0.844	0.833	0.851
		Mean width	0.067	0.045	0.036	0.028	0.023	0.020
	AADM t	Coverage probability	0.859	0.887	0.881	0.897	0.889	0.900
		Mean width	0.076	0.052	0.041	0.032	0.027	0.022
	Wizard t	Coverage probability	0.862	0.915	0.922	0.944	0.945	0.953
		Mean width	0.080	0.060	0.049	0.039	0.033	0.028
	Wizard t from median	Coverage probability	0.873	0.925	0.932	0.952	0.954	0.961
		Mean width	0.084	0.063	0.052	0.041	0.035	0.030
	T1	Coverage probability	0.861	0.908	0.910	0.930	0.931	0.943
		Mean width	0.080	0.058	0.047	0.037	0.031	0.026
	T2	Coverage probability	0.552	0.596	0.586	0.592	0.596	0.601
		Mean width	0.034	0.025	0.020	0.016	0.014	0.011
	T3	Coverage probability	0.736	0.815	0.833	0.863	0.871	0.895
		Mean width	0.052	0.042	0.036	0.030	0.026	0.022
	Median T1	Coverage probability	0.874	0.917	0.922	0.942	0.944	0.952
		Mean width	0.085	0.062	0.050	0.039	0.033	0.028
	Median T2	Coverage probability	0.580	0.623	0.613	0.627	0.618	0.639
		Mean width	0.036	0.027	0.022	0.017	0.014	0.012
	Median T3	Coverage probability	0.752	0.830	0.850	0.884	0.895	0.917
		Mean width	0.056	0.045	0.038	0.032	0.028	0.024
	Mad T1	Coverage probability	0.788	0.828	0.830	0.834	0.828	0.846
		Mean width	0.060	0.043	0.035	0.027	0.023	0.019
	Mad T2	Coverage probability	0.444	0.463	0.462	0.451	0.462	0.471
		Mean width	0.025	0.019	0.015	0.012	0.010	0.008
	Mad T3	Coverage probability	0.620	0.700	0.714	0.744	0.749	0.779
		Mean width	0.039	0.031	0.027	0.022	0.019	0.016
	Chen t	Coverage probability	0.837	0.867	0.866	0.890	0.895	0.912
		Mean width	0.069	0.049	0.040	0.032	0.027	0.023
	YY t	Coverage probability	0.888	0.918	0.921	0.936	0.935	0.945
		Mean width	0.089	0.061	0.049	0.038	0.032	0.027
		Coverage probability	0.620	0.700	0.714	0.744	0.749	0.779
	DMSD t	Mean width	0.097	0.077	0.067	0.057	0.050	0.045
		Mean width	0.083	0.059	0.047	0.037	0.031	0.026
	Downton t	Coverage probability	0.969	0.971	0.958	0.918	0.832	0.702
		Mean width	0.208	0.138	0.109	0.083	0.070	0.058
	Q1 t	Coverage probability	0.866	0.917	0.921	0.944	0.942	0.951
		Mean width	0.081	0.060	0.049	0.039	0.033	0.028
	Q1Q3 t	Coverage probability	0.891	0.939	0.945	0.965	0.964	0.969
		Mean width	0.091	0.068	0.056	0.045	0.038	0.032
	Q3 t	Coverage probability	0.882	0.930	0.936	0.955	0.957	0.962
		Mean width	0.086	0.064	0.053	0.042	0.035	0.030
Gamma (Skewness 4)	Student t	Coverage probability	0.836	0.880	0.895	0.917	0.920	0.935
		Mean width	0.061	0.043	0.036	0.028	0.023	0.020
	Johnson t	Coverage probability	0.843	0.887	0.900	0.921	0.922	0.940
		Mean width	0.061	0.043	0.036	0.028	0.023	0.020
	Median t	Coverage probability	0.854	0.896	0.912	0.935	0.939	0.952
		Mean width	0.067	0.047	0.039	0.030	0.025	0.021
	Mad t	Coverage probability	0.764	0.795	0.793	0.805	0.795	0.804
		Mean width	0.044	0.030	0.024	0.018	0.016	0.013
	AADM t	Coverage probability	0.781	0.814	0.810	0.818	0.818	0.822
		Mean width	0.047	0.031	0.025	0.019	0.016	0.014
	Wizard t	Coverage probability	0.812	0.873	0.898	0.924	0.930	0.947
		Mean width	0.056	0.042	0.036	0.029	0.024	0.021
	Wizard t from median	Coverage probability	0.825	0.886	0.909	0.934	0.940	0.955
		Mean width	0.059	0.044	0.038	0.030	0.026	0.022
	T1	Coverage probability	0.813	0.866	0.889	0.910	0.914	0.931
		Mean width	0.056	0.041	0.034	0.027	0.023	0.019

Distributions	Approaches	Measuring Criteria	Sample Size					
			10	20	30	50	70	100
	T2	Coverage probability	0.534	0.577	0.575	0.581	0.589	0.585
		Mean width	0.024	0.017	0.015	0.012	0.010	0.008
	T3	Coverage probability	0.699	0.774	0.811	0.854	0.869	0.878
		Mean width	0.036	0.029	0.026	0.022	0.019	0.016
	Median T1	Coverage probability	0.831	0.886	0.907	0.930	0.938	0.950
		Mean width	0.061	0.045	0.038	0.029	0.025	0.021
	Median T2	Coverage probability	0.580	0.615	0.622	0.628	0.630	0.624
		Mean width	0.026	0.019	0.016	0.013	0.011	0.009
	Median T3	Coverage probability	0.720	0.807	0.840	0.877	0.894	0.910
		Mean width	0.040	0.032	0.029	0.024	0.021	0.018
	Mad T1	Coverage probability	0.736	0.777	0.781	0.798	0.788	0.798
		Mean width	0.040	0.028	0.024	0.018	0.015	0.013
	Mad T2	Coverage probability	0.400	0.428	0.413	0.420	0.416	0.419
		Mean width	0.017	0.012	0.010	0.008	0.007	0.006
	Mad T3	Coverage probability	0.588	0.650	0.670	0.700	0.706	0.718
		Mean width	0.026	0.021	0.018	0.015	0.013	0.011
	Chen t	Coverage probability	0.772	0.818	0.832	0.860	0.869	0.880
		Mean width	0.046	0.033	0.028	0.022	0.019	0.016
	YY t	Coverage probability	0.838	0.882	0.899	0.920	0.922	0.938
		Mean width	0.061	0.043	0.036	0.028	0.023	0.020
	DMSD t	Coverage probability	0.588	0.650	0.670	0.700	0.706	0.718
		Mean width	0.067	0.055	0.051	0.044	0.040	0.036
		Mean width	0.045	0.030	0.024	0.018	0.015	0.013
	Downton t	Coverage probability	0.913	0.901	0.861	0.731	0.531	0.280
		Mean width	0.133	0.086	0.070	0.053	0.044	0.037
	Q1 t	Coverage probability	0.812	0.866	0.889	0.914	0.916	0.932
		Mean width	0.055	0.040	0.034	0.027	0.023	0.019
	Q1Q3 t	Coverage probability	0.838	0.896	0.916	0.936	0.944	0.959
		Mean width	0.061	0.046	0.039	0.031	0.026	0.022
	Q3 t	Coverage probability	0.825	0.880	0.904	0.929	0.935	0.948
		Mean width	0.058	0.043	0.037	0.029	0.025	0.021

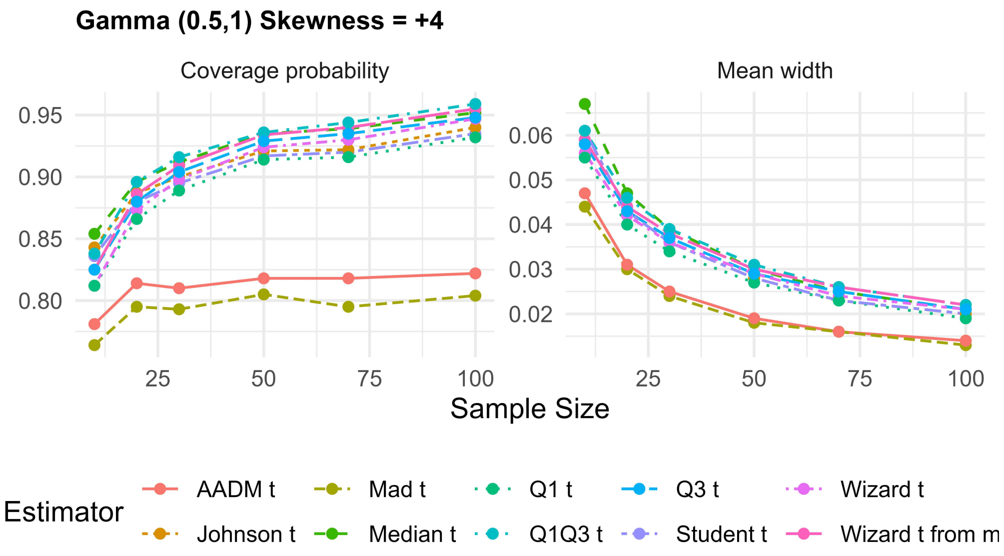

Fig. 3: Coverage probability and Mean width of Gamma (0.5,1)

3.3.1. Gamma with Skewness = 1

This distribution exhibits positive skewness, and coverage probability varies across approaches and sample sizes. The proposed Q1-t, Q3-t, Q1Q3-t, and Wizard-t approaches generally maintain higher coverage probability compared to traditional methods, especially as the sample size increases. As expected, the mean width of intervals decreases with increasing sample size for most approaches. Median-t and Johnson-t also show competitive performance in terms of coverage probability and mean widths.

3.3.2. Gamma with Skewness = 2

With higher skewness compared to the previous gamma distribution, coverage probability tends to decrease for all approaches. The proposed Q1-t, Q3-t, Q1Q3-t, and Wizard-t approaches consistently exhibit higher coverage probability compared to other methods across different sample sizes. Median t and Johnson-t also maintain relatively high coverage probability, especially at larger sample sizes. As the sample size increases, the mean width of intervals decreases for most approaches, but the difference in mean widths among different methods remains consistent.

3.3.3. Gamma with Skewness = 4

This distribution represents significant positive skewness, leading to lower coverage probability for most approaches. Despite the challenging skewness, the proposed "Wizard-t" approaches generally maintain relatively higher coverage probability compared to traditional methods. As the sample size increases, the coverage probability improves slightly for some approaches, but the difference in performance between methods persists. Median-t and Johnson-t also show reasonable performance but with slightly lower coverage probability compared to the "Wizard-t" approaches. Also, proposed Q1-t, Q3-t, and Q1Q3-t consistently maintain relatively higher coverage probability and lower mean width compared to traditional methods.

3.4. Log-normal Distribution

Table 3.4 demonstrates the coverage probability and mean width for different sample sizes for the log-normal distribution with different skewness.

3.4.1. Log-normal with Skewness 2.2

Overall, the interval estimators tend to perform relatively well at this skewness level. Most methods achieve high coverage probabilities, indicating that their CIs capture the true parameter value with high probability. As the sample size increases, the mean width of the CIs generally decreases, indicating increased precision. Proposed Q1-t, Q3-t, Q1Q3-t, Wizard-t, and Wizard-t from median consistently demonstrate good performance across different sample sizes, with high coverage probability and relatively narrow mean width.

3.4.2. Log-normal with Skewness 3.6

At higher skewness levels, the performance of statistical methods becomes more variable. While some methods still maintain high coverage probability, others experience a slight decrease, especially at smaller sample sizes. The mean width of CIs tends to increase compared to skewness 2.2, indicating decreased precision in estimation. Proposed Q1-t, Q3-t, Q1Q3-t, Median-t, and Wizard-t from median remain performers, but their performance may degrade slightly compared to skewness 2.2.

Table 3.4: Coverage Probability and Mean Width for different sample sizes from Lognormal Distribution with different skewness.

Distributions	**Approaches**	**Measuring Criteria**	**Sample Size**					
			10	**20**	**30**	**50**	**70**	**100**
Lognormal (Skewness 2.2)	Student t	Coverage probability	0.906	0.921	0.927	0.936	0.942	0.941
		Mean width	2.801	1.902	1.543	1.191	0.999	0.839
	Johnson t	Coverage probability	0.910	0.926	0.930	0.938	0.943	0.943
		Mean width	2.801	1.902	1.543	1.191	0.999	0.839
	Median t	Coverage probability	0.911	0.929	0.934	0.942	0.948	0.950
		Mean width	2.923	1.977	1.599	1.231	1.032	0.866
	Mad t	Coverage probability	0.834	0.848	0.835	0.828	0.852	0.841
		Mean width	2.087	1.407	1.129	0.865	0.726	0.606
	AADM t	Coverage probability	0.889	0.898	0.899	0.900	0.917	0.908
		Mean width	2.448	1.665	1.342	1.032	0.868	0.726
	Wizard t	Coverage probability	0.881	0.916	0.928	0.943	0.951	0.954
		Mean width	2.512	1.860	1.556	1.231	1.044	0.884
	Wizard t from median	Coverage probability	0.887	0.920	0.928	0.947	0.954	0.956
		Mean width	2.572	1.900	1.584	1.251	1.061	0.897
	T1	Coverage probability	0.866	0.904	0.918	0.928	0.938	0.938
		Mean width	2.362	1.760	1.468	1.157	0.979	0.828
	T2	Coverage probability	0.550	0.575	0.569	0.592	0.597	0.594
		Mean width	1.042	0.770	0.641	0.504	0.426	0.360
	T3	Coverage probability	0.737	0.819	0.833	0.856	0.895	0.896
		Mean width	1.638	1.310	1.136	0.936	0.813	0.705
	Median T1	Coverage probability	0.878	0.911	0.924	0.934	0.944	0.947

Distributions	**Approaches**	**Measuring Criteria**	**Sample Size**					
			10	**20**	**30**	**50**	**70**	**100**
		Mean width	2.464	1.829	1.521	1.196	1.012	0.854
	Median T2	Coverage probability	0.568	0.593	0.591	0.605	0.612	0.603
		Mean width	1.088	0.801	0.664	0.521	0.440	0.372
	Median T3	Coverage probability	0.751	0.830	0.844	0.866	0.903	0.908
		Mean width	1.709	1.362	1.178	0.968	0.840	0.728
	Mad T1	Coverage probability	0.777	0.825	0.815	0.816	0.844	0.834
		Mean width	1.762	1.303	1.075	0.840	0.712	0.598
	Mad T2	Coverage probability	0.435	0.452	0.438	0.460	0.451	0.448
		Mean width	0.777	0.570	0.469	0.366	0.310	0.260
	Mad T3	Coverage probability	0.626	0.692	0.705	0.740	0.765	0.763
		Mean width	1.221	0.970	0.832	0.680	0.591	0.509
	Chen t	Coverage probability	0.932	0.952	0.960	0.963	0.966	0.964
		Mean width	4.071	2.540	1.986	1.470	1.195	0.983
	YY t	Coverage probability	0.907	0.924	0.928	0.938	0.943	0.942
		Mean width	2.801	1.902	1.543	1.191	0.999	0.839
		Coverage probability	0.626	0.692	0.705	0.740	0.765	0.763
	DMSD t	Mean width	3.058	2.424	2.143	1.847	1.646	1.496
		Mean width	2.652	1.861	1.501	1.160	0.985	0.823
	Downton t	Coverage probability	0.993	0.992	0.992	0.986	0.976	0.960
		Mean width	6.777	4.444	3.549	2.702	2.261	1.885
	Q1 t	Coverage probability	0.911	0.945	0.956	0.962	0.972	0.974
		Mean width	2.836	2.103	1.756	1.387	1.176	0.995
	Q1Q3 t	Coverage probability	0.896	0.927	0.936	0.950	0.954	0.957
		Mean width	2.593	1.898	1.572	1.238	1.050	0.887
	Q3 t	Coverage probability	0.903	0.939	0.948	0.957	0.963	0.967
		Mean width	2.715	2.000	1.664	1.312	1.113	0.941
	Student t	Coverage probability	0.870	0.898	0.907	0.915	0.934	0.930
		Mean width	4.388	3.020	2.473	1.929	1.619	1.370
	Johnson t	Coverage probability	0.875	0.899	0.912	0.918	0.937	0.932
		Mean width	4.388	3.020	2.473	1.929	1.619	1.370
	Median t	Coverage probability	0.880	0.904	0.917	0.926	0.942	0.940
		Mean width	4.631	3.175	2.592	2.016	1.691	1.430
	Mad t	Coverage probability	0.801	0.821	0.803	0.799	0.820	0.799
		Mean width	3.182	2.138	1.717	1.313	1.102	0.920
Lognormal (Skewness 3.6)	AADM t	Coverage probability	0.847	0.866	0.862	0.857	0.881	0.866
		Mean width	3.616	2.451	1.976	1.518	1.275	1.066
	Wizard t	Coverage probability	0.846	0.892	0.910	0.926	0.942	0.943
		Mean width	3.945	2.955	2.493	1.990	1.688	1.439
	Wizard t from median	Coverage probability	0.852	0.897	0.915	0.932	0.947	0.947
		Mean width	4.063	3.037	2.552	2.034	1.725	1.469
	T1	Coverage probability	0.834	0.882	0.894	0.909	0.931	0.927
		Mean width	3.673	2.785	2.348	1.872	1.586	1.351
	T2	Coverage probability	0.530	0.556	0.562	0.580	0.592	0.587
		Mean width	1.628	1.222	1.027	0.816	0.690	0.588
	T3	Coverage probability	0.718	0.795	0.818	0.850	0.884	0.885
		Mean width	2.564	2.080	1.821	1.516	1.318	1.151
	Median T1	Coverage probability	0.842	0.889	0.903	0.919	0.938	0.938
		Mean width	3.875	2.927	2.460	1.956	1.657	1.409
	Median T2	Coverage probability	0.552	0.579	0.586	0.599	0.616	0.608
		Mean width	1.718	1.285	1.076	0.853	0.721	0.613
	Median T3	Coverage probability	0.735	0.812	0.832	0.865	0.896	0.895
		Mean width	2.706	2.187	1.908	1.584	1.377	1.201
	Mad T1	Coverage probability	0.743	0.794	0.785	0.791	0.814	0.794
		Mean width	2.669	1.973	1.630	1.275	1.079	0.907
	Mad T2	Coverage probability	0.412	0.419	0.408	0.420	0.418	0.418
		Mean width	1.182	0.865	0.713	0.556	0.470	0.395
	Mad T3	Coverage probability	0.597	0.646	0.668	0.704	0.728	0.722
		Mean width	1.860	1.473	1.264	1.032	0.897	0.773

Distributions	Approaches	Measuring Criteria	Sample Size					
			10	20	30	50	70	100
	Chen t	Coverage probability	0.921	0.931	0.951	0.953	0.963	0.961
		Mean width	7.027	4.416	3.468	2.584	2.086	1.723
	YY t	Coverage probability	0.870	0.899	0.910	0.918	0.937	0.932
		Mean width	4.388	3.020	2.473	1.929	1.619	1.370
	DMSD t	Coverage probability	0.597	0.646	0.668	0.704	0.728	0.722
		Mean width	4.779	3.920	3.552	3.170	2.868	2.682
		Mean width	3.681	2.559	2.056	1.585	1.347	1.123
	Downton t	Coverage probability	0.982	0.980	0.975	0.954	0.929	0.856
		Mean width	10.167	6.658	5.325	4.054	3.388	2.828
	Q1 t	Coverage probability	0.877	0.921	0.934	0.952	0.961	0.966
		Mean width	4.400	3.289	2.762	2.198	1.864	1.586
	Q1Q3 t	Coverage probability	0.858	0.900	0.909	0.925	0.940	0.940
		Mean width	3.974	2.930	2.445	1.942	1.647	1.400
	Q3 t	Coverage probability	0.868	0.911	0.922	0.943	0.954	0.954
		Mean width	4.187	3.109	2.604	2.070	1.755	1.493
Lognormal (Skewness 6)	Student t	Coverage probability	0.833	0.872	0.879	0.897	0.920	0.911
		Mean width	6.721	4.697	3.888	3.076	2.585	2.214
	Johnson t	Coverage probability	0.838	0.876	0.885	0.902	0.925	0.915
		Mean width	6.721	4.697	3.888	3.076	2.585	2.214
	Median t	Coverage probability	0.842	0.881	0.893	0.908	0.933	0.922
		Mean width	7.161	4.978	4.105	3.236	2.718	2.323
	Mad t	Coverage probability	0.763	0.775	0.764	0.768	0.775	0.751
		Mean width	4.735	3.161	2.539	1.943	1.625	1.360
	AADM t	Coverage probability	0.800	0.822	0.809	0.808	0.828	0.797
		Mean width	5.172	3.485	2.809	2.157	1.809	1.514
	Wizard t	Coverage probability	0.807	0.867	0.882	0.906	0.932	0.922
		Mean width	6.048	4.589	3.907	3.159	2.682	2.312
	Wizard t from median	Coverage probability	0.815	0.874	0.890	0.910	0.937	0.928
		Mean width	6.243	4.724	4.007	3.234	2.744	2.362
	T1	Coverage probability	0.791	0.855	0.864	0.889	0.916	0.908
		Mean width	5.589	4.316	3.682	2.981	2.529	2.182
	T2	Coverage probability	0.513	0.532	0.544	0.568	0.572	0.575
		Mean width	2.487	1.899	1.613	1.301	1.102	0.949
	T3	Coverage probability	0.688	0.762	0.793	0.842	0.870	0.868
		Mean width	3.926	3.234	2.862	2.418	2.104	1.860
	Median T1	Coverage probability	0.805	0.867	0.879	0.902	0.930	0.919
		Mean width	5.955	4.576	3.888	3.136	2.659	2.288
	Median T2	Coverage probability	0.535	0.561	0.571	0.590	0.600	0.599
		Mean width	2.649	2.013	1.703	1.368	1.159	0.996
	Median T3	Coverage probability	0.708	0.781	0.807	0.856	0.888	0.882
		Mean width	4.182	3.428	3.022	2.543	2.212	1.951
	Mad T1	Coverage probability	0.710	0.741	0.742	0.755	0.767	0.744
		Mean width	3.947	2.909	2.407	1.883	1.590	1.340
	Mad T2	Coverage probability	0.377	0.387	0.388	0.388	0.383	0.377
		Mean width	1.754	1.278	1.053	0.822	0.693	0.583
	Mad T3	Coverage probability	0.561	0.598	0.621	0.649	0.676	0.675
		Mean width	2.766	2.177	1.869	1.527	1.322	1.142
	Chen t	Coverage probability	0.893	0.915	0.934	0.940	0.954	0.951
		Mean width	11.692	7.453	5.905	4.470	3.598	3.009
	YY t	Coverage probability	0.835	0.873	0.882	0.902	0.923	0.913
		Mean width	6.721	4.697	3.888	3.076	2.585	2.214
	DMSD t	Coverage probability	0.561	0.598	0.621	0.649	0.676	0.675
		Mean width	7.301	6.213	5.776	5.353	4.916	4.744
		Mean width	4.846	3.328	2.661	2.045	1.737	1.445
	Downton t	Coverage probability	0.962	0.956	0.939	0.902	0.832	0.686
		Mean width	14.821	9.676	7.745	5.901	4.922	4.115
	Q1 t	Coverage probability	0.838	0.892	0.912	0.924	0.947	0.946
		Mean width	6.646	5.013	4.244	3.416	2.897	2.489

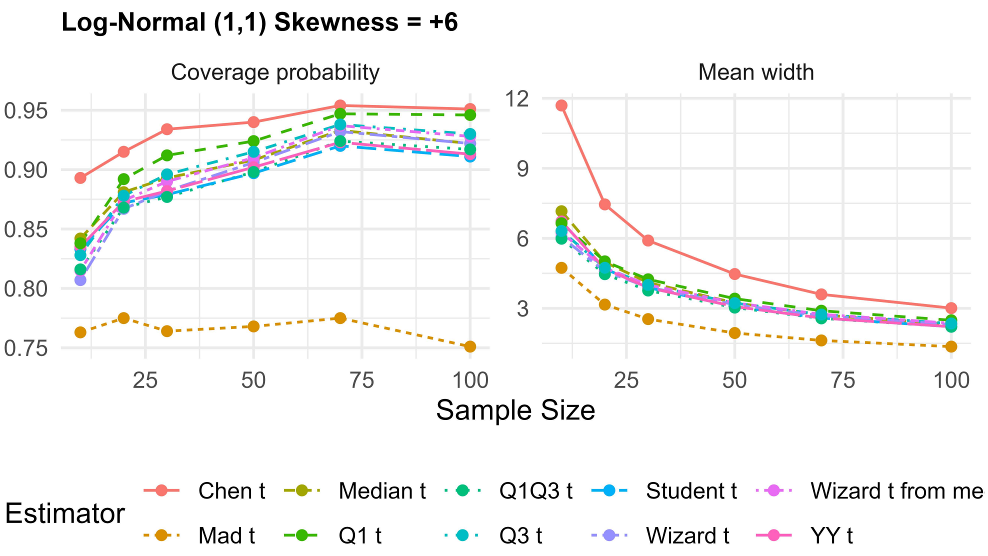

Fig. 4: Coverage probability and Mean width of Lognormal (1,1)

Distributions	Approaches	Measuring Criteria	Sample Size					
			10	20	30	50	70	100
	Q1Q3 t	Coverage probability	0.816	0.868	0.877	0.898	0.924	0.917
		Mean width	5.983	4.463	3.764	3.032	2.571	2.212
	Q3 t	Coverage probability	0.828	0.878	0.896	0.915	0.938	0.930
		Mean width	6.315	4.738	4.004	3.224	2.734	2.351

3.4.3. Log-normal with Skewness 6

This skewness level represents a more extreme distribution where the performance of statistical methods is challenged. Coverage probability tends to decrease across most methods and sample sizes, indicating a higher probability of CIs failing to capture the true parameter. The mean width of CIs increases significantly, suggesting a trade-off between precision and accuracy. Despite these challenges, Median-t and Wizard-t from median provide relatively good performance compared to other methods, but with reduced coverage probability and wider width compared to lower skewness levels. Also, proposed Q1-t, Q3-t, and Q1Q3-t provide high coverage probability with lower mean width compared to the existing methods.

3.5. Application

To illustrate the simulation results, we considered two real-life examples, one for right-skewed and another one for left-skewed data.

3.5.1. Psychotropic drug exposure data

To analyze the average usage of psychotropic drugs among non-antipsychotic drug users, the number of psychotropic drug users was reported for a random sample of n = 20 from various drug categories. The data below shows the number of users. [17]: 43.4, 24, 1.8, 0, 0.1, 170.1, 0.4, 150, 31.5, 5.2, 35.7, 27.3,5, 64.3, 70, 94, 61.9, 9.1, 38.8, and 14.8.

The data were found to be positively skewed (Fig. 5) with skewness = 1.57, kurtosis = 2.06, population mean = 42.37, and population standard deviation = 48.43 [17].

Table 3.5: 95% CIs and widths for psychotropic drug exposure data

Statistics	95% CI		CI Width
	Lower Limit	Upper Limit	
Student_t	19.70	65.04	45.33
Johnson_t	20.34	65.67	45.33
Median_t	18.86	65.88	47.01
Mad_t	25.66	69.08	43.43

Statistics	95% CI		CI Width
	Lower Limit	Upper Limit	
AADM_t	22.66	62.08	44.42
Wizard_t	20.05	64.69	44.63
Wizard_from_Median_t	19.46	65.28	45.12
T1	21.41	63.33	44.91
T2	23.19	61.55	44.36
T3	26.75	57.99	41.23
Median_T1	20.64	64.10	43.47
Median_T2	22.85	61.89	45.04
Median_T3	26.17	68.57	42.39
MAD_T1	26.92	67.82	40.90
MAD_T2	25.60	69.14	43.54
MAD_T3	20.85	63.89	45.03
Chen_t	11.93	72.81	60.88
YY_t	20.02	65.35	45.33
DMSD_t	15.17	69.57	54.39
Downton_t	-23.30	82.10	105.39
Q1_t	16.13	68.61	42.47
Q3_t	19.97	64.77	44.79
Q1Q3_t	18.05	66.69	44.63

The confidence intervals and their respective confidence widths can be found in Table 3.5. From Table 3.5, all the proposed CIs cover the hypothesized true population mean of 42.37. However, Mad-T1 provided the shortest width followed by T2, and Downton-t produced the highest width.

3.5.2. Long jump distance data

The following data represent the results of the final point scores reported for 40 players in the long jump distance, measured in meters [17]: 8.11, 8.11, 8.09, 8.08, 8.06, 8.03, 8.02, 7.99, 7.99, 7.97, 7.95, 7.92, 7.92, 7.92, 7.89, 7.87, 7.84, 7.79, 7.79, 7.77, 7.76, 7.72, 7.71, 7.66, 7.62, 7.61, 7.59, 7.55, 7.53, 7.5, 7.5, 7.42, 7.38, 7.38, 7.26, 7.25, 7.08, 6.96, 6.84, 6.55.

According to a study, the data is negatively skewed (Fig. 6) with skewness = -1.16, kurtosis = 1.20, population mean = 7.6745, and population standard deviation = 0.37 [17].

Table 3.6: 95% CIs and widths of the final scores for long jump distance data

Statistics	95% CI		CI Width
	Lower Limit	Upper Limit	
Student_t	7.556	7.793	0.237
Johnson_t	7.554	7.791	0.237
Median_t	7.553	7.796	0.244
Mad_t	7.582	7.767	0.185
AADM_t	7.562	7.787	0.225
Wizard_t	7.557	7.792	0.235
Wizard_from_Median_t	7.561	7.788	0.228
T1	7.559	7.790	0.230
T2	7.625	7.724	0.100
T3	7.584	7.765	0.181
Median_T1	7.556	7.793	0.237
Median_T2	7.623	7.726	0.103
Median_T3	7.581	7.768	0.187
MAD_T1	7.585	7.764	0.180
MAD_T2	7.636	7.713	0.078
MAD_T3	7.604	7.745	0.142
Chen_t	7.570	7.779	0.208
YY_t	7.555	7.792	0.237
DMSD_t	7.512	7.837	0.326
Downton_t	7.478	8.052	0.575
Q1_t	7.550	7.799	0.250
Q3_t	7.532	7.817	0.284
Q1Q3_t	7.541	7.808	0.267

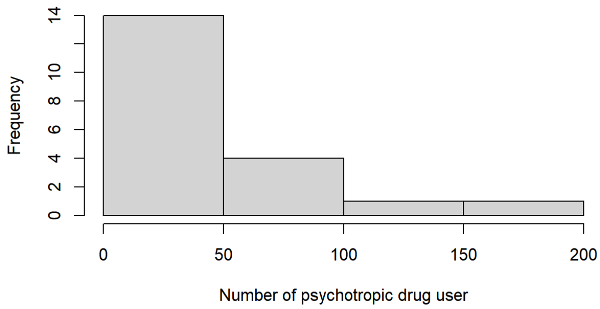

Fig. 5: Histogram of Psychotropic drug exposure data

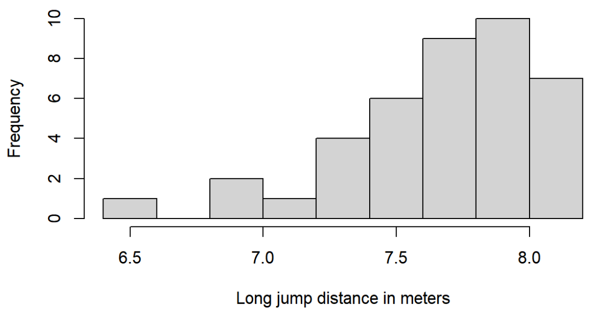

Fig. 6: Histogram of Long jump distance data

The table in 3.6 provides the confidence intervals (CIs) and their respective widths. As this data set is negatively skewed, we determine that the findings in Table 3.6 corroborate the simulation results for the negatively skewed distribution examined in this study. Table 3.6 reveals that all the suggested confidence intervals (CIs) encompass the hypothesized true population mean of 7.6745.

4 Some Conclusion Remarks

This paper considers 23 different interval estimators for estimating the population mean of symmetric and asymmetric distributions within classical and modified-t approaches. As a direct theoretical comparison is unfeasible, a simulation study was conducted to assess the performance of the estimators based on the CI method. The main merit of this paper is to review the existing estimators proposed by several researchers several times under different simulation conditions. Random samples are generated from various left-skewed, right-skewed, and symmetric distributions. Our simulation results indicate that among 23 estimators, for a moderate sample (>50), our proposed Q1-t, Q3-t, Q1Q3-t, Wizard-t, and Wizard-t from median have consistently better coverage probability and average width than the other test statistics, especially for the asymmetric population. We also observed that Student-t performed the best for small sample sizes for symmetric distribution. Overall, our analysis suggests that the Chen-t, Median-t, T1, AADM-t, and Median-t estimators are promising and can also be chosen for estimating the mean for skewed distributions. Two real-life data are analyzed to illustrate the findings of the paper. It is important to consider the trade-offs between precision and accuracy when selecting the best method for a particular application. We are confident that this paper will provide the practitioners with an expanded array of interval estimators, enabling them to make optimal selections from a multitude of options utilized by various researchers across different contexts and timeframes.

Acknowledgement

The authors are thankful to the anonymous referee for meticulously reviewing and providing valuable comments that enhanced this paper.

References

[1] Neyman J. Outline of a Theory of Statistical Estimation Based on the Classical Theory of Probability. Philosophical Transactions of the Royal Society of London. Series A, Mathematical and Physical Sciences 236, 333–380 (1937).
[2] Chen, L. Testing the Mean of Skewed Distributions. J Am Stat Assoc 90, 767 (1995).
[3] Baklizi, A. & Kibria, B. M. G. One and two sample confidence intervals for estimating the mean of skewed populations: an empirical comparative study. J Appl Stat 36, 601–609 (2009).
[4] Banik, S. & Kibria, B. M. G. Comparison of Some Parametric and Nonparametric Type One Sample Confidence Intervals for Estimating the Mean of a Positively Skewed Distribution. Commun Stat Simul Comput 39, 361–389 (2010).
[5] Ghosh, S. & Polansky, A. M. New bootstrap confidence intervals for means of positively skewed distributions. Commun Stat Theory Methods 45, 6915–6927 (2016).
[6] MUDELSEE, M. & ALKIO, M. Quantifying effects in two-sample environmental experiments using bootstrap confidence intervals. Environmental Modelling & Software 22, 84–96 (2007).
[7] Andersson, P. G. ALTERNATIVE CONFIDENCE INTERVALS FOR THE TOTAL OF A SKEWED BIOLOGICAL POPULATION. Ecology 85, 3166–3171 (2004).
[8] Shi, W. & Kibria, B. M. G. On some confidence intervals for estimating the mean of a skewed population. Int J Math Educ Sci Technol 38, 412–421 (2007).
[9] Zhou, X. H. Nonparametric confidence intervals for the one- and two-sample problems. Biostatistics 6, 187–200 (2005).
[10] Abu-Shawiesh, M. O. A. & Saghir, A. Robust Confidence Intervals for the Population Mean Alternatives to the Student-t Confidence Interval. Journal of Modern Applied Statistical Methods 18, 2–20 (2020).
[11] Luh, W. & Guo, J. Transformation works for non-normality? On one-sample transformation trimmed t methods. British Journal of Mathematical and Statistical Psychology 54, 227–236 (2001).
[12] Baklizi, A. Inference about the mean of a skewed population: a comparative study. J Stat Comput Simul 78, 421–435 (2008).
[13] Abu-Shawiesh, M. O. A., Banik, S., Kibria, B. M. G. & Akyüz, H. E. A comparison of some modified confidence intervals based on robust scale estimators for process capability index. Production Engineering 14, 217–229 (2020).
[14] Abu-Shawiesh et. al., M. O. A. Confidence Intervals based on Absolute Deviation for Population Mean of a Positively Skewed Distribution. International Journal of Computational and Theoretical Statistics 5, 1–13 (2018).
[15] Sinsomboonthong, J., Abu-Shawiesh, M. O. A. & Kibria, B. M. G. Performance of Robust Confidence Intervals for Estimating Population Mean Under Both Non-Normality and in Presence of Outliers. Advances in Science, Technology and Engineering Systems Journal 5, 442–449 (2020).
[16] Abu-Shawiesh, M. O. A., Sinsomboonthong, J. & Kibria, B. M. G. A modified robust confidence interval for the population mean of distribution based on deciles. Statistics in Transition New Series 23, 109–128 (2022).
[17] Almonte, C. & Kibria, B. M. G. On some classical, bootstrap and transformation confidence intervals for estimating the mean of an asymmetrical population. Model Assisted Statistics and Applications 4, 91–104 (2009).
[18] Johnson, N. J. Modified t Tests and Confidence Intervals for Asymmetrical Populations. J Am Stat Assoc 73, 536–544 (1978).
[19] Gastwirth, J. L. Statistical properties of a measure of tax assessment uniformity. J Stat Plan Inference 6, 1–12 (1982).
[20] Yanagihara, H. & Yuan, K. Four improved statistics for contrasting means by correcting skewness and kurtosis. British Journal of Mathematical and Statistical Psychology 58, 209–237 (2005).
[21] Hall, P. On the Removal of Skewness by Transformation. J R Stat Soc Series B Stat Methodol 54, 221–228 (1992).

B. M. Golam Kibria is a Professor and Graduate Director (Statistics Division) in the Department of Mathematics and Statistics at Florida International University. Since 1993, he has published about 275 full research articles in various internationally renowned statistical journals, 22 conference proceedings, and co-authored two books. Dr. Kibria has supervised, as a major (or co-major) professor, two Ph.D. and 24 master's students, as well as 28 undergraduate students at FIU. Dr. Kibria is an elected Fellow of the Royal Statistical Society and holds an Honorary Doctorate from Jönköping University, Sweden. Dr. Kibria has been listed four times in a row (2020, 2021, 2022, and 2023) as among the Top 2 percent of scientists in all disciplines for single-year impact.

H. M. Nayem is a PhD student in the Department of Mathematics and Statistics at Florida International University (FIU). He also works as a graduate research assistant at the Department of Civil and Environmental Engineering at FIU, where he contributes to projects that involve statistical modeling in transportation engineering. He received his bachelor's and master's degrees in Statistics from Jahangirnagar University in Dhaka, Bangladesh. His primary research interests are in the field of Data Science, where he explores various statistical methods and their applications in real-world scenarios. Additionally, his research interests are statistical inference, Bayesian statistics, time series analysis, and machine learning.