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On Near 3−Perfect Numbers

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Abstract: We call a positive integer n be a near 3−perfect number if $\sigma(n) = 3n + d$, where $\sigma(n)$ is the divisor function and d is a proper divisor of n. In this paper, we have derived all near 3−perfects of the form $2^{\alpha} p_1^t p_2$, where p_1 and p_2 are distinct odd primes with $p_1 < p_2$ and $\alpha \geq 1$, $1 \leq t \leq 2$. There are only ten such numbers. Moreover, we have also obtained some examples of even near 3−perfect numbers with four distinct prime factors.

Keywords: Multi perfect number, Hyperperfect number, Near perfect number

1 Introduction

Well known divisor function $\sigma(n)$ is the sum of all positive divisors of n including 1 and n itself. For any integer $k > 1$, a positive integer n is called a k−perfect or multi perfect number [3,7], if $\sigma(n) = kn$. All known k−perfect numbers are even. No odd k−perfect numbers have been found for $k \geq 2$. In recent years, some properties of odd k−perfect numbers have been investigated [3],[8].

In particular for $k = 2$, 2−perfect numbers are solutions of the functional equation $\sigma(n) = 2n$, which are also known as perfect numbers. All known perfect numbers are of the form $n = 2^{p-1}M_p$, where both p and $M_p = 2^p - 1$ are primes. The primes of the form $M_p = 2^p - 1$ are called Mersenne primes. Hyperperfect [1], near perfect [6] and near hyperperfect [4] numbers are generalization of perfect numbers.

A positive integer n is said to be a near perfect number with redundant divisor d if and only if d is a proper divisor (divisors excluding 1 and n itself) of n and $\sigma(n) = 2n + d$. For example, 12 is a near perfect number with redundant divisor 4. P. Pollack and V. Shevelev [6] introduced the concept of near perfect numbers. They derived all even near perfect numbers with exactly two distinct prime factors and also obtained certain form of such numbers. Y. Li and Q. Liao [5] derived some examples of even near perfect numbers with three distinct prime factors of the form $2^{\alpha} p_1^t p_2$, where $p_1 < p_2$ and $\alpha \geq 1$, $1 \leq t \leq 2$.

The abudancy index $I(n)$ for any positive integers n is associated with the divisor function $\sigma(n)$ and is defined as $I(n) = \frac{\sigma(n)}{n}$. Since k−perfect numbers are solutions of the equation $\sigma(n) = kn$, so $I(n) = k$. For abundant numbers, $I(n) > 2$ and for deficient numbers, $I(n) < 2$.

For any positive integer $k > 1$, we call a number n be a near k− perfect number if $\sigma(n) = kn + d$, where d is a redundant divisor of n. Near perfect numbers are near 2−perfect. For any near k−perfect numbers, we have the following result.

Lemma 1.1. If $n = p_1^{\alpha_1} p_2^{\alpha_2} \ldots . p_r^{\alpha_r}$ is a near k−perfect number with redundant divisor d, where α_i are positive integers and p_i are distinct primes, then for $i = 1, 2, \ldots, r$, we have $p_i | \sigma(\frac{n}{p_i^{\alpha_i}})$, if $p_i | d$ or $p_i | \sigma(\frac{n}{p_i^{\alpha_i}}) - d$, if $p_i \not| d$.

In particular for an even near k−perfect number, using this lemma, one can obtain the following result.

Lemma 1.2. Let r, α_i are positive integers, and let p_i be distinct primes. If $n = 2^{\alpha_0} p_1^{\alpha_1} p_2^{\alpha_2} \ldots . p_r^{\alpha_r}$ is an even near k−perfect number with redundant divisor d, then there exists some i with $1 \leq i \leq r$ such that α_i is odd if and only if $2 | d$.

Lemma 1.1 and lemma 1.2 were proved by Y. Li and Q. Liao [5] in particular for $k = 2$.

In this paper, we consider near 3− perfect numbers. For any $\alpha \geq 1$, we have derived all the near 3− perfect numbers of the form $2^{\alpha} p_1 p_2$ and $2^{\alpha} p_1^2 p_2$, where both p_1 and p_2 are odd primes with $p_1 < p_2$. We have proved that $2^4.3.5$, $2^6.3.7$, $2^7.3.7$ and $2^8.3.7$ are the only near 3−perfect numbers of the form $2^{\alpha} p_1 p_2$ and $2^2.3^2.5$,

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$2^3.3^2.5$, $2^5.3^2.7$, $2^4.3^2.13$, $2^5.3^2.13$ and $2^7.3^2.23$ are the only near 3− perfect numbers of the form $2^\alpha p_1^2 p_2$. We have also obtained some examples of even near 3−perfect numbers with four distinct prime factors.

2 Main Result

Near 3−perfect numbers of the form $n = 2^\alpha p_1 p_2$

If n is a near 3−perfect number with redundant divisor d, then $\sigma(n) = 3n + d$ and therefore abundancy index $I(n) = \frac{\sigma(n)}{n} = 3 + \frac{d}{n}$. Since d is a proper divisor of n, therefore $1 < \frac{d}{n} < 1$. If $n = 2^\alpha p_1 p_2$, where $p_1 < p_2$, is a near 3−perfect with redundant divisor d, then

$$\sigma(n) = 3n + d = (2^{\alpha+1} - 1)(p_1 + 1)(p_2 + 1) = 3.2^\alpha p_1 p_2 + d$$

Therefore $3 + \frac{d}{n} = I(n) = (2 - \frac{1}{2^\alpha})(1 + \frac{1}{p_1})(1 + \frac{1}{p_2}) < 2(1 + \frac{1}{p_1})(1 + \frac{1}{p_2})$

From the last inequality, we can get if $n = 2^\alpha p_1 p_2$, where $p_1 < p_2$, is a near 3−perfect number, then the only possibilities are $p_1 = 3$ and $p_2 = 5$ or $p_2 = 7$. In fact we have the following results.

Proposition 2.1. Let α be a positive integer. Suppose that p_1 and p_2 are odd primes with $p_1 < p_2$, then

(a) The only near 3− perfect number of the form $n = 2^\alpha p_1 p_2$ with redundant divisor 2^β, where $\alpha \geq \beta$, is $n = 2^6.3.7$.

(b) The only near 3− perfect numbers of the form $n = 2^\alpha p_1 p_2$ with redundant divisor $2^\beta p_1$, where $\alpha \geq \beta$, are $n = 2^4.3.5$ and $n = 2^7.3.7$.

(c) The only near 3− perfect number of the form $n = 2^\alpha p_1 p_2$ with redundant divisor $2^\beta p_2$, where $\alpha \geq \beta$, is $n = 2^8.3.7$.

(d) There exists no near 3−perfect number of the form $n = 2^\alpha p_1 p_2$ with redundant divisor $2^\gamma p_1 p_2$,where $\alpha > \gamma \geq 1$.

Proof.

If $n = 2^\alpha p_1 p_2$ is a near 3−perfect number with redundant divisor d, then from the lemma 1.2, it follows that all the redundant divisors are even and only possible values of d are 2^β, $2^\beta p_1$, $2^\beta p_2$ and $2^\gamma p_1 p_2$, where $\alpha \geq \beta \geq 1$ and $\alpha > \gamma \geq 1$.

(a) If $n = 2^\alpha p_1 p_2$ is a near 3−perfect number with redundant divisor $d = 2^\beta$, where $\alpha \geq \beta$, then $p_2 \not| d$ and form the lemma1.1, we must have $p_2 | \sigma(2^\alpha p_1) - d = (2^{\alpha+1} - 1)(p_1 + 1) - d$. Therefore for some positive integer k , we can write

$$2kp_2 + d = (2^{\alpha+1} - 1)(p_1 + 1) \quad (1)$$

Since $\sigma(n) = 3n + d$, it follows that

$$2k(p_2 + 1) + 2^\beta = 3.2^\alpha p_1 \quad (2)$$

From (1) and (2), it follows that

$$p_1 = 2 + \frac{2k - 3}{2^\alpha + 1}, p_2 = \frac{1}{k}(3.2^{\alpha-1} p_1 - 2^{\beta-1}) - 1 \quad (3)$$

If $p_1 = 3$, then from (3), we must have $\frac{2k-3}{2^\alpha+1} = 1$, which implies that $k = 2^{\alpha-1} + 2$ and therefore

$$p_2 = \frac{8.2^{\alpha-1} - 2^{\beta-1} - 2}{2^{\alpha-1}+2} = \frac{8.(2^{\alpha-2}+1) - 2^{\beta-2} - 9}{2^{\alpha-2}+1} = 8 - \frac{2^{\beta-2}+9}{2^{\alpha-2}+1} \quad (4)$$

Since $p_1 < p_2$ and $p_1 = 3$, therefore (4) also strictly implies that $p_2 = 5$ or $p_2 = 7$.

Case I: If $p_1 = 5$, then from (4), we must have $\frac{2^{\beta-2}+9}{2^{\alpha-2}+1} = 3$ and therefore $2^{\beta-2}(3.2^{\alpha-\beta} - 1) = 6$. But this equation has no solution for non-negative integers α and β.

Case II: If $p_2 = 7$, then from (4) we must have $\frac{2^{\beta-2}+9}{2^{\alpha-2}+1} = 1$ and therefore $2^{\beta-2}(2^{\alpha-\beta} - 1) = 8$. This equation has only solution for $\alpha = 6$ and $\beta = 5$. Thus $n = 2^6.3.7$ is a near 3−perfect number with redundant divisor 2^5 .

(b) If $n = 2^\alpha p_1 p_2$ is a near 3−perfect number with redundant divisor $d = 2^\beta p_1$, where $\alpha \geq \beta$, then $p_2 \not| d$ and proceeding as the proposition 2.1 **(a)** one can obtain the following two equations, for some positive integer k

$$2kp_2 + 2^\beta p_1 = (2^{\alpha+1} - 1)(p_1 + 1) \quad (5)$$

$$2k(p_2 + 1) + 2^\beta p_1 = 3.2^\alpha p_1 \quad (6)$$

From (5) and (6), it follows that

$$p_1 = 2 + \frac{2k - 3}{2^\alpha + 1}, p_2 = \frac{1}{k}(3.2^{\alpha-1} p_1 - 2^{\beta-1} p_1 - k) \quad (7)$$

If $p_1 = 3$, then from (7), we have $k = 2^{\alpha-1} + 2$ and therefore

$$p_2 = \frac{8.2^{\alpha-1} - 3.2^{\beta-1} - 2}{2^{\alpha-1}+2} = \frac{8.(2^{\alpha-2}+1) - 3.2^{\beta-2} - 9}{2^{\alpha-2}+1} = 8 - \frac{3.2^{\beta-2}+9}{2^{\alpha-2}+1} \quad (8)$$

Since $p_1 < p_2$ and $p_1 = 3$, therefore (8) also strictly implies that $p_2 = 5$ or $p_2 = 7$.

Case I: If $p_1 = 5$, then from (8), we must have $\frac{3.2^{\beta-2}+9}{2^{\alpha-2}+1} = 3$ and therefore $2^{\beta-2}(2^{\alpha-\beta} - 1) = 2$. This equation has solution only for $\alpha = 4$ and $\beta = 3$.Thus $n = 2^4.3.5$ is a near 3−perfect number with redundant divisor $2^3.3$.

Case II: If $p_2 = 7$, then from (8), we must have $\frac{3.2^{\beta-2}+9}{2^{\alpha-2}+1} = 1$ and therefore $2^{\beta-2}(2^{\alpha-\beta} - 3) = 8$. This equation has only solution for $\alpha = 7$ and $\beta = 5$. Thus $n = 2^7.3.7$ is a near 3−perfect number with redundant divisor $2^5.3$.

(c) If $n = 2^\alpha p_1 p_2$ is a near 3−perfect number with redundant divisor $d = 2^\beta p_2$, where $\alpha \geq \beta$, then $p_2|d$ and from the lemma1.1, we must have $p_2|\sigma(2^\alpha p_1) = (2^{\alpha+1}-1)(p_1+1)$. Since $p_1 < p_2$, so p_2 and p_1+1 are relatively prime and therefore we must have $p_2|2^{\alpha+1}-1$. Therefore for some positive integer k , we can write

$$kp_2 = 2^{\alpha+1}-1 \tag{9}$$

Since $\sigma(n) = 3n+d$, it follows that

$$p_1(2^\alpha+1) - k(p_1+1) = 2^{\alpha+1} - 2^\beta - 1 \tag{10}$$

If $p_1 = 3$, then from (10), it follows that $k = 2^{\alpha-2} + 2^{\beta-2} + 1$ and therefore

$$p_2 = \frac{2^{\alpha+1}-1}{2^{\alpha-2}+2^{\beta-2}+1} = 8 - \frac{2^{\beta+1}+9}{2^{\alpha-2}+2^{\beta-2}+1} \tag{11}$$

Since $p_1 < p_2$ and $p_1 = 3$, therefore (11) also strictly implies that $p_2 = 5$ or $p_2 = 7$.

Case I: If $p_1 = 5$, then from (11) we must have $\frac{2^{\beta+1}+9}{2^{\beta-2}+2^{\alpha-2+1}} = 3$ and therefore $2^{\beta-2}(3.2^{\alpha-\beta}-5) = 6$. But this equation has no solution for non-negative integers α and β.

Case II: If $p_2 = 7$, then from (11) we must have $\frac{2^{\beta+1}+9}{2^{\alpha-2}+2^{\beta-2}+1} = 1$ and therefore $2^{\beta-2}(2^{\alpha-\beta}-7) = 8$. This equation has solution only for $\alpha = 8$ and $\beta = 5$. Thus $n = 2^8.3.7$ is a near 3−perfect number with redundant divisor $2^5.7$.

(d) If $n = 2^\alpha p_1 p_2$ is a near 3−perfect number with redundant divisor $d = 2^\gamma p_1 p_2$, where $\alpha > \gamma \geq 1$, then $p_2|d$ and proceeding as the proposition 2.1**(c)** one can obtain the following two equations, for some positive integer k

$$kp_2 = 2^{\alpha+1}-1 \tag{12}$$

$$p_1(2^\alpha + 2^\gamma + 1 - k) = k + 2^{\alpha+1} - 1 \tag{13}$$

If $p_1 = 3$, then from (13), it follows that $k = 2^{\alpha-2} + 3.2^{\gamma-2} + 1$ and therefore

$$p_2 = 8 - \frac{3.2^{\gamma+1}+9}{2^{\alpha-2}+3.2^{\gamma-2}+1} \tag{14}$$

Since $p_1 < p_2$ and $p_1 = 3$, therefore (14) also strictly implies that $p_2 = 5$ or $p_2 = 7$. But this equation has no solution in terms of α and γ.

Near 3−**perfect numbers of the form** $n = 2^\alpha p_1^2 p_2$

If $n = 2^\alpha p_1^2 p_2$, where $p_1 < p_2$ is a near 3−perfect number with redundant divisor d, then $\sigma(n) = 3n+d = (2^{\alpha+1}-1)(p_1^2+p_1+1)(p_2+1) = 3.2^\alpha p_1^2 p_2 + d$.

Therefore $3 + \frac{d}{n} = I(n) = (2 - \frac{1}{2^\alpha})(1 + \frac{1}{p_1} + \frac{1}{p_1^2})(1 + \frac{1}{p_2}) < 2(1 + \frac{1}{p_1} + \frac{1}{p_1^2})(1 + \frac{1}{p_2})$

From the last inequality, we can get if $n = 2^\alpha p_1^2 p_2$, where $p_1 < p_2$, is a near 3−perfect number, then only possibilities are $p_1 = 3$ and $p_2 = 5$ or $p_2 = 7$ or $p_2 = 11$ or $p_2 = 13$ or $p_2 = 17$ or $p_2 = 19$ or $p_2 = 23$. In fact we have the following results.

Proposition 2.2. Let α be a positive integer. Suppose that p_1 and p_2 are odd primes with $p_1 < p_2$, then

(a) The only near 3− perfect numbers of the form $n = 2^\alpha p_1^2 p_2$ with redundant divisors $2^\beta p_1^\gamma$, where $\alpha \geq \beta$, $0 < \gamma \leq 2$, are $n = 2^2.3^2.5$ and $n = 2^7.3^2.23$.

(b) The only near 3− perfect numbers of the form $n = 2^\alpha p_1^2 p_2$ with redundant divisor $2^\beta p_1^\gamma p_2$, where $\alpha \geq \beta$, $0 \leq \gamma \leq 2$, are $n = 2^3.3^2.5$, $n = 2^5.3^2.7$, $n = 2^4.3^2.13$ and $n = 2^5.3^2.13$.

Proof. If $n = 2^\alpha p_1^2 p_2$ is a near 3−perfect number with redundant divisor d, then from the lemma 1.2, it follows that all the redundant divisors are even and only possible redundant divisors are $d = 2^\beta p_1^\gamma$ and $d = 2^\beta p_1^\gamma p_2$, where $\alpha \geq \beta \geq 1$, $0 \leq \gamma \leq 2$.

(a) If $n = 2^\alpha p_1^2 p_2$ is a near 3−perfect number with redundant divisor $d = 2^\beta p_1^\gamma$, where $\alpha \geq \beta$, $0 \leq \gamma \leq 2$, then $p_2 \not| d$ and from the lemma1.1, we must have $p_2|\sigma(2^\alpha p_1^2) - d = (2^{\alpha+1}-1)(p_1^2+p_1+1) - d$. Thereforefor some positive integer k, we can write

$$kp_2 + d = (2^{\alpha+1}-1)(p_1^2+p_1+1) \tag{15}$$

Since $\sigma(n) = 3n+d$, it follows that

$$(2^{\alpha+1}-1)(p_1+1) + k = (2^\alpha+1)p_1^2 \tag{16}$$

If $p_1 = 3$, then from (15) and (16), it follows that $k = 2^\alpha + 13$ and

$$p_2 = \frac{13(2^{\alpha+1}-1) - 2^\beta 3^\gamma}{2^\alpha+13} = 26 - \frac{351 + 2^\beta 3^\gamma}{2^\alpha+13} \tag{17}$$

Since $p_1 < p_2$ and $p_1 = 3$, so (17) also strictly implies that the possible values of p_2 are $p_2 = 5$ or 7 or 11 or 13 or 17 or 19 or 23. Among these seven values of p_2, equation (17) has solutions only for $p_2 = 5$ and $p_2 = 23$.

If $p_2 = 5$, then from (17) we must have $\frac{351+2^\beta 3^\gamma}{2^\alpha+13} = 21$ and therefore $21.2^\alpha - 2^\beta.3^\gamma = 78$. This equation has solution only for $\alpha = 2$, $\beta = 1$ and $\gamma = 1$. Thus $n = 2^2.3^2.5$ is a near 3−perfect number with redundant divisor 2.3.

If $p_2 = 23$, then from (17) we must have $\frac{351+2^\beta 3^\gamma}{2^\alpha+13} = 3$ and therefore $3.2^\alpha - 2^\beta 3^\gamma = 312$. This equation has only solution for $\alpha = 7$, $\beta = 3$ and $\gamma = 2$. Thus $n = 2^7.3^2.23$ is a near 3−perfect number with redundant divisor $2^3.3^2$.

If $\gamma = 0$, then redundant divisor becomes $d = 2^\beta$ and from (17), it follows that

$$p_2 = 26 - \frac{351 + 2^\beta}{2^\alpha + 13}$$

This equation has no solution for any non negative integers α and β. Therefore there does not exist any near

3−perfect number of the form $n = 2^\alpha p_1^2 p_2$, where $p_1 < p_2$, with redundant divisor $d = 2^\beta$.

(b) If $n = 2^\alpha p_1^2 p_2$ is a near 3−perfect number with redundant divisor $d = 2^\beta p_1^\gamma p_2$, where $\alpha \geq \beta$, $0 \leq \gamma \leq 2$, then $p_2|d$ and from the lemma1.1, it follows that $p_2|\sigma(2^\alpha p_1^2) = (2^{\alpha+1}-1)(p_1^2+p_1+1)$. Therefore for some positive integer k, we can write

$$kp_2 = (2^{\alpha+1}-1)(p_1^2+p_1+1) \tag{18}$$

Since $\sigma(n) = 3n+d$, it follows that

$$(2^{\alpha+1}-1)(p_1+1)+k-(2^\alpha+1)p_1^2 = 2^\beta p_1^\gamma \tag{19}$$

If $p_1 = 3$, then from (18) and (19), it follows that $k = 2^\beta 3^\gamma + 2^\alpha + 13$ and

$$p_2 = \frac{13(2^{\alpha+1}-1)}{2^\beta 3^\gamma + 2^\alpha + 13} = 26 - \frac{351+13.2^{\beta+1}3^\gamma}{2^\beta 3^\gamma + 2^\alpha + 13} \tag{20}$$

Since $p_1 < p_2$ and $p_1 = 3$,therefore (20) also strictly implies that the possible values of p_2 are $p_2 = 5$ or 7 or 11 or 13 or 17 or 19 or 23. Among these seven values of p_2, the equation (20) has solution only for $p_2 = 5$,$p_2 = 7$ and $p_2 = 13$.

If $p_2 = 5$, then from (20) we must have $\frac{351+13.2^{\beta+1}3^\gamma}{2^\beta 3^\gamma+2^\alpha+13} = 21$ and therefore $21.2^\alpha - 5.2^\beta 3^\gamma = 78$. This equation has solution only for $\alpha = 3$, $\beta = 1$ and $\gamma = 2$. Thus $n = 2^3.3^2.5$ is a near 3−perfect number with redundant divisor $2.3^2.5$.

If $p_2 = 7$, then from (20) we must have $\frac{351+13.2^{\beta+1}3^\gamma}{2^\beta 3^\gamma+2^\alpha+13} = 19$ and therefore $19.2^\alpha - 7.2^\beta 3^\gamma = 104$. This equation has only solution for $\alpha = 5$, $\beta = 3$ and $\gamma = 2$. Thus $n = 2^5.3^2.7$ is a near 3−perfect number with redundant divisor $2^3.3^2.7$.

If $p_2 = 13$, then from (20) we must have $\frac{351+13.2^{\beta+1}3^\gamma}{2^\beta 3^\gamma+2^\alpha+13} = 13$ and therefore $2^\alpha - 2^\beta 3^\gamma = 14$. This equation has only solutions for $\alpha = 4$, $\beta = 1$, $\gamma = 0$ and $\alpha = 5$, $\beta = 1$, $\gamma = 2$. Thus $n = 2^4.3^2.13$ is a near 3−perfect number with redundant divisor 2.13 and $n = 2^5.3^2.13$ is a near 3−perfect number with redundant divisor $2.3^2.13$.

From the following two propositions, one can determine some even near 3−perfect numbers with four distinct prime factors.

Proposition 2.3. If n is a 2−hyperperfect number and $J \neq 6$ is a perfect number such that n and J are relatively prime, then $x = nJ$ is a near 3−perfect number with redundant divisor J.

Proof. It must be noted that if n is a k−hyperperfect number[1], then n is a solution of the equation $\sigma(n) = \frac{k+1}{k}n + \frac{k-1}{k}$. In particular for $k = 2$, 2−hyperperfect numbers are solutions of the equation $\sigma(n) = \frac{3}{2}n + \frac{1}{2}$.

Since n and J are relatively prime, therefore

$$\sigma(x) - 3x = \sigma(n)\sigma(J) - 3x = (\frac{3}{2}n + \frac{1}{2})2J - 3nJ = J$$

Clearly J is a proper divisor of x.

Remark 2.1. 2−hyperperfect numbers [1] are of the form $n = 3^{k-1}(3^k-2)$, where 3^k-2 are primes and therefore $x = 3^{k-1}(3^k-2)J$ are near 3−perfect numbers, provided n and J are relatively prime. Since any even perfect numbers are of the form $J = 2^{p-1}(2^p-1)$, where both p and 2^p-1 are primes. therefore $x = 2^{p-1}3^{k-1}(3^k-2)(2^p-1)$ are near 3− perfect numbers.

Example 2.1.

- $x = 2^2.3^3.7.79$ is a near 3−perfect number with redundant divisor $2^2.7$
- $x = 2^4.3.7.31$ is a near 3−perfect number with redundant divisor $2^4.31$
- $x = 2^6.3.7.127$ is a near 3−perfect number with redundant divisor $2^6.127$ etc.

Proposition 2.4. If $x = 3^{k-1}pJ$, where $J \neq 6$ is a perfect number, and $p = 3^k - 3^r - 1$ is a prime with $r < k$ and relatively prime to J, then x is a near 3−perfect number with redundant divisor $3^r J$.

Proof. Since $r < k$, so $3^r J$ is a proper divisor of x . We have

$$\sigma(x) - 3x = \frac{3^k-1}{2}(p+1)2J - 3^k pJ = 3^r J$$

Note that proposition 2.3 is a particular case of the proposition 2.4 corresponding to $r = 0$, if $3^k - 2$ is a prime.

Example 2.2.

- $x = 2^2.3.5.7$ is a near 3−perfect number with redundant divisor $2^2.3.7$
- $x = 2^4.3.5.31$ is a near 3−perfect number with redundant divisor $2^4.3.31$
- $x = 2^6.3.5.127$ is a near 3−perfect number with redundant divisor $2^6.3.127$ etc.

Remark 2.2. Proposition 2.4 can be generalized for any near q− perfect number, where q is an odd prime. If n is a near q− perfect number with redundant divisor d, then we can write $\sigma(n) - qn = d$. If J is a $(q-1)$− perfect number and $p = q^k - q^r - 1$ $(k > r)$ is an odd prime relatively prime to q and J, then $x = q^{k-1}pJ$ is a near q− perfect number with redundant divisor $q^r J$.

3 Conclusion

For any $\alpha \geq 1$, $\beta \geq 1$, if $n = p^\alpha q^\beta$ is a positive integer with two distinct prime factors p and q, then $I(n) = \frac{\sigma(n)}{n} < \frac{p}{p-1}\frac{q}{q-1} = \frac{1}{1-\frac{1}{p}}\frac{1}{1-\frac{1}{q}}$. Since $p \geq 2$ and $q \geq 3$, therefore $I(n) < 3$. Thus there does not exist any

near 3−perfect number with two distinct prime factors. In generalization of near perfect numbers, we have determined only a few numbers which satisfy the equation $\sigma(n) = 3n + d$, where d are proper divisors of n. Therefore, there is very good scope for searching other numbers for which $\sigma(n) = kn + d$, where $k > 2$.

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