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Soliton and wave solutions to the extended Gerdjikov-Ivanov equation in DWDM system with auxiliary equation method

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Abstract: This article investigates soliton and other new solutions for both kerr law and parabolic law nonlinearities, the extended Gerdjikov-Ivanov equation in dense wavelength division multiplexed (DWDM) system was resolved by using the auxiliary equation method. These solutions are bright soliton, singular soliton, Bell-type wave, traveling, and trigonometric waves.

Keywords: Auxiliary equation method, kerr law, dense wavelength division multiplexed, parabolic law nonlinearities, soliton solutions.

1 Introduction

The extended Gerdjikov-Ivanov in DWDM is a model used to characterize a range of physical phenomena, including transmission infrastructure, transatlantic and Transboundary distances, data processing and telecoms. This model was studied for the first time in [1] together with the extended simple equation method. DWDM technology is an integral function which must be included in the fiber-optic Communication process [2,3,4,5,6].

It greatly multiplies the ability to bring information across these fibers. Thus, data can be transmitted in parallel over transcontinental and trans- distances in only a few femto-seconds. Only perfecting DWDM technology can accomplish such an engineering marvel. Recently, researchers have become accustomed to using methods such as Symmetry Method, Modified Kudryashov Method and Spectral Legendre-Chebyshev collocation method to address some nonlinear differential equations such as b -Family Equations, Witham-Broer-Kaup Equation, two dimensional Linear and Nonlinear Mixed Volterra-Fredholm Integral Equation [7,8,9].

In this article, the extended auxiliary equation method has been applied to the extended GI model in DWDM system for both kerr law and parabolic law nonlinearities which causes a further improvement of the model. Strategic solutions bright soliton, singular soliton, Bell-type, traveling and trigonometric waves are retrieved.

2 Governing model

The (GI) equation [10] is represented as

$$i\,\psi_t + a\,\psi_{xx} + b\,|\psi|^4\,\psi + i\,e\,\psi^2\,\psi_x^* = 0. \qquad (1)$$

Under this very significant governing model, the first term is correlated with the temporal evolution of pulses while the coefficient of a provides the existence of group velocity dispersion. The $\psi(x,t)$ complex and valued function is called the wave profile. The coefficient of b is known as the nonlinear notion, which implies nonlinear quinticity. The existence of a e coefficient dispersive phenomenon is assumed.

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2.1 Kerr law nonlinearity

The extended (GI) reads [1]

$$i\psi_t^{(l)} + a_l\psi_{xx}^{(l)} + b_l|\psi^{(l)}|^4\psi^{(l)} + ie_l(\psi^{(l)})^2(\psi_x^{(l)})^* + \left\{c_l\psi_{xt}^{(l)} + d_l|\psi^{(l)}|^2\psi^{(l)} + \sum_{n\neq l}^{N}\alpha_{ln}|\psi^{(n)}|^2\psi^{(l)}\right\} = 0. \quad (2)$$

The coefficients *al* and *cl* correspond to the group velocity dispersion and spatio-temporal dispersion, respectively. In addition, the coefficients of *dl* represent self-phase modulation, while the coefficients of αln indicate the effect of cross-phase modulation. The $\psi^{(l)}(x,t)$ dependent variable describes a solution profile for $1 \leq l \leq N$ in every single path.

In this subsection, by using the extended auxiliary equation method [11,12,13], we shall obtain bright soliton solutions, singular soliton solutions, Bell-type wave solutions, traveling wave solutions and trigonometric wave solutions to Eq.2. For this purpose, we use

$$\psi^{(l)}(x,t) = w_l(\zeta(x,t))e^{i\theta(x,t)}, \quad (3)$$

where

$$\zeta(x,t) = k_1x - \nu t, \quad (4)$$

$$\theta(x,t) = -k_2x + \mu t + k_3. \quad (5)$$

Putting Eq.3 along with Eq.4 and Eq.5 into Eq.2 we get

$$-\mu w_l - i\nu w_l' - k_2^2a_lw_l - 2ia_lk_1k_2w_l' + k_1^2a_lw_l'' + b_lw_l^5 + k_2c_l\mu w_l + ic_lk_1\mu w_l' + ic_lk_2\nu w_l' - k_1c_l\nu w_l'' + d_lw_l^3 - k_2e_lw_l^3 + ik_1e_lw_l^2w_l' + \left(\sum_{n\neq l}^{N}\alpha_{ln}w_n^2\right)w_l = 0, \quad (6)$$

break down into real and imaginary parts we get

$$(-\mu - k_2^2a_l + k_2c_l\mu)w_l + b_lw_l^5 + (k_1^2a_l - k_1c_l\nu)w_l'' + \left(d_l - k_2e_l + \sum_{n\neq l}^{N}\alpha_{ln}\right)w_l^3 = 0, \quad (7)$$

$$(-\nu - 2a_lk_1k_2 + k_1c_l\mu + k_2c_l\nu + k_1e_lw_l^2)w_l^2 = 0, \quad (8)$$

from Eq.8 we have the velocity

$$\nu = \frac{2a_lk_1k_2 - k_1c_l\mu}{k_2c_l - 1}, \quad (9)$$

and we obtain the conditions $e_l = 0$, and $k_2c_l \neq 1$. Substituting with the value of ν into Eq. 7 gives

$$4(k_2c_l - 1)(-\mu - k_2^2a_l + k_2c_l\mu)w_l + 4b_l(k_2c_l - 1)w_l^5 + (k_1^2c_l^2\mu - k_1^2a_l - k_1^2k_2a_lc_l)w_l'' + 4(k_2c_l - 1)\left(d_l - k_2e_l + \sum_{n\neq l}^{N}\alpha_{ln}\right)w_l^3 = 0, \quad (10)$$

Multiply Eq. 10 by w_l' and integrating with respect to ζ yield

$$4(k_2c_l - 1)(-\mu - k_2^2a_l + k_2c_l\mu)w_l^2 + \frac{4}{3}b_l(k_2c_l - 1)w_l^6 + (k_1^2c_l^2\mu - k_1^2a_l - k_1^2k_2a_lc_l)[w_l']^2 + 2(k_2c_l - 1)\left(d_l - k_2e_l + \sum_{n\neq l}^{N}\alpha_{ln}\right)w_l^4 + 2C = 0, \quad (11)$$

where C is the integration constant.
Balancing $[w_l']^2$ with w_l^6 in Eq. 11 gives $N = \frac{1}{2}$, since N is not integer we set $w_l = \sqrt{\varphi_l}$. Substituting into Eq. 11 and multiplying by $4\varphi_l\sqrt{\varphi_l}$ we get

$$16(k_2c_l - 1)(-\mu - k_2^2a_l + k_2c_l\mu)\varphi_l^2 + \frac{16}{3}b_l(k_2c_l - 1)\varphi_l^4 + (k_1^2c_l^2\mu - k_1^2a_l - k_1^2k_2a_lc_l)[\varphi_l']^2 + 8(k_2c_l - 1)\left(d_l - k_2e_l + \sum_{n\neq l}^{N}\alpha_{ln}\right)\varphi_l^3 + 8C\varphi_l = 0, \quad (12)$$

Balancing $[\varphi_l']^2$ with φ^4 in Eq. 12 we get $N = 1$

The following assumption is made to retrieve bell-type solitary wave and trigonometric function solutions to Eq. 10 using the extended auxiliary equation method.

$$\varphi_l = \sum_{j=0}^{N}B_j\phi(\zeta)^j = B_0 + B_1\phi(\zeta), \quad (13)$$

where $(B_N \neq 0)$ are constants, $\phi(\zeta)$ satisfies the following equation

$$[\phi(\zeta)']^2 = m_1\phi^2(\zeta) + m_2\phi^4(\zeta) + m_3\phi^6(\zeta), \quad (14)$$

where m_1, m_2, m_3 are constants to be determinant later.

When $\Delta = m_2^2 - 4m_1m_3$, and $\varepsilon = \pm 1$. Eq. 14 admit the following solutions [11] when $m_1 > 0$ then

$$\phi(\zeta) = \sqrt{\frac{-m_1 m_2 sech^2(\sqrt{m_1}\zeta)}{m_2^2 - m_1 m_3(1-\varepsilon\tanh(\sqrt{m_1}\zeta))^2}}$$

when $m_1 > 0$ then

$$\phi(\zeta) = \sqrt{\frac{m_1 m_2 csch^2(\sqrt{m_1}\zeta)}{m_2^2 - m_1 m_3(1-\varepsilon\coth(\sqrt{m_1}\zeta))^2}}$$

when $m_1 > 0$ then

$$\phi(\zeta) = 4\left(\sqrt{\frac{m_1 e^{2\varepsilon\sqrt{m_1}\zeta}}{(e^{2\varepsilon\sqrt{m_1}\zeta} - 4m_2)^2 - 64 m_1 m_3}}\right)$$

when $m_1 > 0, \Delta > 0$ then

$$\phi(\zeta) = \sqrt{\frac{2m_1}{\varepsilon\sqrt{\Delta}\cosh(2\sqrt{m_1}\zeta) - m_2}}$$

when $m_1 < 0, \Delta > 0$ then

$$\phi(\zeta) = \sqrt{\frac{2m_1}{\varepsilon\sqrt{\Delta}\cos(2\sqrt{m_1}\zeta) - m_2}}$$

when $m_1 > 0, \Delta < 0$ then

$$\phi(\zeta) = \sqrt{\frac{2m_1}{\varepsilon\sqrt{-\Delta}\sinh(2\sqrt{m_1}\zeta) - m_2}}$$

when $m_1 < 0, \Delta > 0$ then

$$\phi(\zeta) = \sqrt{\frac{2m_1}{\varepsilon\sqrt{\Delta}\sin(2\sqrt{-m_1}\zeta) - m_2}}$$

when $m_1 > 0, m_3 > 0$ then

$$\phi(\zeta) = \sqrt{\frac{-m_1 sech^2(\sqrt{m_1}\zeta)}{m_2 + 2\varepsilon\sqrt{m_1 m_3}\tanh(\sqrt{m_1}\zeta)}}$$

when $m_1 < 0, m_3 > 0$ then

$$\phi(\zeta) = \sqrt{\frac{-m_1 sec^2(\sqrt{m_1}\zeta)}{m_2 + 2\varepsilon\sqrt{-m_1 m_3}\tan(\sqrt{-m_1}\zeta)}}$$

when $m_1 > 0, m_3 > 0$ then

$$\phi(\zeta) = \sqrt{\frac{m_1 csch^2(\sqrt{m_1}\zeta)}{m_2 + 2\varepsilon\sqrt{m_1 m_3}\coth(\sqrt{m_1}\zeta)}}$$

when $m_1 < 0, m_3 > 0$ then

$$\phi(\zeta) = \sqrt{\frac{-m_1 csc^2(\sqrt{-m_1}\zeta)}{m_2 + 2\varepsilon\sqrt{-m_1 m_3}\cot(\sqrt{-m_1}\zeta)}}$$

when $m_1 > 0, \Delta = 0$ then

$$\phi(\zeta) = \sqrt{-\frac{m_1}{m_2}(1+\varepsilon\tanh(\frac{\sqrt{m_1}}{2}\zeta))}$$

when $m_1 > 0, \Delta = 0$ then

$$\phi(\zeta) = \sqrt{-\frac{m_1}{m_2}(1+\varepsilon\coth(\frac{\sqrt{m_1}}{2}\zeta))}$$

when $m_1 > 0, m_2 = 0$ then

$$\phi(\zeta) = 4\left(\sqrt{\frac{\pm m_1 e^{2\varepsilon\sqrt{m_1}\zeta}}{1 - 64 m_1 m_3 e^{4\varepsilon\sqrt{m_1}\zeta}}}\right)$$

Substituting Eq. 11 and Eq. 12 into Eq. 10 and equating the coefficients of $\phi^j(\zeta)$ for $(j = 0, 1, 2, ..., N)$ to zero We have the following algebraic equations.:

$\phi^0(\zeta)$:
$16(k_2c_l - 1)(k_2\mu c_l - \mu - k_2^2 a_l)B_0 + \frac{16b_l}{3}(k_2c_l - 1)B_0^3 + 8(k_2c_l - 1)(d_l + k_2e_l + \alpha_{l1})B_0^2 + 8C = 0$,

$\phi^1(\zeta)$:
$32(k_2c_l - 1)(k_2\mu c_l - \mu - k_2^2 a_l)B_0 + \frac{64b_l}{3}(k_2c_l - 1)B_0^3 + 24(k_2c_l - 1)(d_l + k_2e_l + \alpha_{l1})B_0^2 + 8C = 0$,

$\phi^2(\zeta)$: $16(k_2c_l - 1)(k_2\mu c_l - \mu - k_2^2 a_l)B_1^2 + 32b_l(k_2c_l - 1)B_0^2B_1^2 + 24(k_2c_l - 1)(d_l + k_2e_l + \alpha_{l1})B_0B_1^2 + (k_1^2c_l^2\mu - k_1^2 a_l - k_1^2k_2a_lc_l)m_1 = 0$,

$\phi^3(\zeta)$:
$\frac{64b_l}{3}(k_2c_l - 1)B_0B_1^3 + 8(k_2c_l - 1)(d_l + k_2e_l + \alpha_{l1})B_1^3 = 0$,

$\phi^4(\zeta))$:
$\frac{16b_l}{3}(k_2c_l - 1)B_1^4 + (k_1^2c_l^2\mu - k_1^2a_l - k_1^2k_2a_lc_l)m_2 = 0$.

solving this system using Matlab, we get:

$$B_0 = -\frac{3(d_l + k_2e_l + \alpha_{l1})}{8b_l}, \; B_1 = B_1,$$

$$m_1 = \frac{48B_1^2(k_2c_l - 1)^2(d_l + k_2e_l + \alpha_{l1})^2}{k_1^2(64a_lb_l + 15c_l^2(d_l + k_2e_l + \alpha_{l1})^2)},$$

$$m_2 = -\frac{1024 b_l^2 B_1^4 (k_2c_l - 1)^2}{3k_1^2(64a_lb_l + 15c_l^2(d_l + k_2e_l + \alpha_{l1})^2)},$$

$$m_3 = 0,$$

$$C = \frac{9(k_2c_l - 1)(d_l + k_2e_l + \alpha_{l1})^3}{128 b_l^2},$$

$$\mu = \frac{64a_lb_lk_2^2 + 15(d_l + k_2e_l + \alpha_{l1})^2}{64 b_l(k_2c_l - 1)}.$$

Remark: We notes that $m_2 = -\frac{64 b_l^2 B_1^2}{9(d_l + k_2e_l + \alpha_{l1})^2} m_1$.

Since $m_3 = 0$ then $\Delta = m_2^2$ and its clear that $m_2 \neq 0$ with this information the available solutions of Eq. 2 are as follows:

When $m_1 > 0$, a bright soliton solution as

$$\psi^{(l)}(x,t) = \sqrt{Y_1\left(sech(\sqrt{m_1}(k_1x - vt)) - 1\right)} \times e^{i(k_2x + (\frac{64a_lb_lk_2^2 + 15(d_l + k_2e_l + \alpha_{l1})^2}{64b_l(c_lk_2 - 1)})t + k_3)}, \tag{15}$$

a singular soliton solution as

$$\psi^{(l)}(x,t) = \sqrt{Y_1\left(i\,csch(\sqrt{m_1}(k_1x - vt)) - 1\right)} \times e^{i(k_2x + (\frac{64a_lb_lk_2^2 + 15(d_l + k_2e_l + \alpha_{l1})^2}{64b_l(c_lk_2 - 1)})t + k_3)}, \tag{16}$$

$$\psi^{(l)}(x,t)=\sqrt{Y_1+4B_1\sqrt{\frac{m_1e^{2\varepsilon\sqrt{m_1}(k_1x-vt)}}{\left(e^{2\varepsilon\sqrt{m_1}(k_1x-vt)}-4m_2\right)^2}}}\times e^{i(k_2x+(\frac{64a_lb_lk_2^2+15(d_l+k_2e_l+\alpha_{l1})^2}{64b_l(c_lk_2-1)})t+k_3)}, \tag{17}$$

When $m_1>0$, $\Delta>0$, Bell-type solitary wave solution as

$$\psi^{(l)}(x,t)=\sqrt{Y_1\left(\sqrt{\frac{2}{\varepsilon\cosh(2\sqrt{m_1}(k_1x-vt))-1}}-1\right)}\times e^{i(k_2x+(\frac{64a_lb_lk_2^2+15(d_l+k_2e_l+\alpha_{l1})^2}{64b_l(c_lk_2-1)})t+k_3)}, \tag{18}$$

When $m_1<0$, $\Delta>0$, trigonometric function solution as

$$\psi^{(l)}(x,t)=\sqrt{Y_1\left(\sqrt{\frac{2}{\varepsilon\cos(2\sqrt{m_1}(k_1x-vt))-1}}-1\right)}\times e^{i(k_2x+(\frac{64a_lb_lk_2^2+15(d_l+k_2e_l+\alpha_{l1})^2}{64b_l(c_lk_2-1)})t+k_3)}, \tag{19}$$

a traveling wave solution as

$$\psi^{(l)}(x,t)=\sqrt{Y_1\left(\sqrt{\frac{2}{\varepsilon\sin(2\sqrt{m_1}(k_1x-vt))-1}}-1\right)}\times e^{i(k_2x+(\frac{64a_lb_lk_2^2+15(d_l+k_2e_l+\alpha_{l1})^2}{64b_l(c_lk_2-1)})t+k_3)}, \tag{20}$$

where $Y_1=\frac{3(d_l+k_2e_l+\alpha_{l1})}{8b_l}$.

2.2 Parabolic law nonlinearity

The extended (GI) reads [1]

$$i\psi_t^{(l)}+a_l\psi_{xx}^{(l)}+b_l|\psi^{(l)}|^4\psi^{(l)}+ie_l(\psi^{(l)})^2(\psi_x^{(l)})^*+\left\{c_l\psi_{xt}^{(l)}+\delta_l|\psi^{(l)}|^4\psi^{(l)}+(d_l+\sum_{n\neq l}^{N}\gamma_{ln}|\psi^{(n)}|^2)|\psi^{(l)}|^2\psi^{(l)}+\sum_{n\neq l}^{N}(\alpha_{ln}+\beta_{ln}|\psi^{(n)}|^2)|\psi^{(n)}|^2\psi^{(l)}\right\}=0. \tag{21}$$

The coefficients al and cl correspond to the group velocity dispersion and spatio-dispersion respectively. In addition, the self-phase modulation coefficients dl and δln are represented, while the cross-phase modulation coefficients γln, αln and βln are represented. A solitary profile is the dependent variable $\psi^{(l)}(x,t)$ for $1\le l\le N$ in every single channel.

Substituting, Eq.3 along with Eq.4 and Eq.5 into Eq.21, Gets the same imaginary component as the one given by Eq.7 and so the velocity is same as Eq.9. However, the real component of Eq. 13 is

$$(-\mu-k_2^2a_l+k_2c_l\mu)w_l+(b_l+\delta_l+\sum_{n\neq l}^{N}(\beta_{ln}+\gamma_{ln}))w_l^5+(k_1^2a_l-k_1c_lv)w_l''+\left(d_l-k_2e_l+\sum_{n\neq l}^{N}\alpha_{ln}\right)w_l^3=0, \tag{22}$$

We can see the difference between Eq. 7 and Eq. 22 is the adding term $(\delta_l+\sum_{n\neq l}^{N}(\beta_{ln})$ and so we have

$$B_0=-\frac{3(d_l+k_2e_l+\alpha_{l1})}{8(b_l+\delta_l+\beta_{l1}+\gamma_{l1})},\quad B_1=B_1,$$

$$m_1=\frac{48B_1^2(k_2c_l-1)^2(d_l+k_2e_l+\alpha_{l1})^2}{k_1^2(64a_l(b_l+\delta_l+\beta_{l1}+\gamma_{l1})+15c_l^2(d_l+k_2e_l+\alpha_{l1})^2)},$$

$$m_2=-\frac{1024b_l^2B_1^4(k_2c_l-1)^2}{3k_1^2(64a_l(b_l+\delta_l+\beta_{l1}+\gamma_{l1})+15c_l^2(d_l+k_2e_l+\alpha_{l1})^2)},$$

$$m_3=0,$$

$$C=\frac{9(k_2c_l-1)(d_l+k_2e_l+\alpha_{l1})^3}{128(b_l+\delta_l+\beta_{l1}+\gamma_{l1})^2},$$

$$\mu=\frac{64a_lk_2^2(b_l+\delta_l+\beta_{ln}+\gamma_{l1})+15(d_l+k_2e_l+\alpha_{l1})^2}{64(k_2c_l-1)(b_l+\delta_l+\beta_{l1}+\gamma_{l1})}.$$

Remark: We notes that $m_2=-\frac{64B_1^2(b_l+\delta_l+\beta_{l1}+\gamma_{l1})^2}{9(d_l+k_2e_l+\alpha_{l1})^2}m_1$.
And the solutions will be as follows:
When $m_1>0$, bright soliton solution as

$$\psi^{(l)}(x,t)=\sqrt{Y_2\,(sech(\sqrt{m_1}(k_1x-vt))-1)}\times e^{i(k_2x+(\frac{64a_lk_2^2(b_l+\delta_l+\beta_{l1}+\gamma_{l1})+15(d_l+k_2e_l+\alpha_{l1})^2}{64(b_l+\delta_l+\beta_{l1}+\gamma_{l1})(c_lk_2-1)})t+k_3)}, \tag{23}$$

singular soliton solution as

$$\psi^{(l)}(x,t)=\sqrt{Y_2\,(icsch(\sqrt{m_1}(k_1x-vt))-1)}\times e^{i(k_2x+(\frac{64a_lk_2^2(b_l+\delta_l+\beta_{l1}+\gamma_{l1})+15(d_l+k_2e_l+\alpha_{l1})^2}{64(b_l+\delta_l+\beta_{l1}+\gamma_{l1})(c_lk_2-1)})t+k_3)}, \tag{24}$$

$$\psi^{(l)}(x,t)=\sqrt{Y_2+4B_1\sqrt{\frac{m_1e^{2\varepsilon\sqrt{m_1}(k_1x-\nu t)}}{\left(e^{2\varepsilon\sqrt{m_1}(k_1x-\nu t)}-4m_2\right)^2}}}$$
$$\times e^{i\left(k_2x+\left(\frac{64a_lk_2^2(b_l+\delta_l+\beta_{l1}+\gamma_{l1})+15(d_l+k_2e_l+\alpha_{l1})^2}{64(b_l+\delta_l+\beta_{l1}+\gamma_{l1})(c_lk_2-1)}\right)t+k_3\right)}, \tag{25}$$

When $m_1>0$, $\Delta>0$, Bell-type solitary wave solution as

$$\psi^{(l)}(x,t)=\sqrt{Y_2\left(\sqrt{\frac{2}{\varepsilon\cosh(2\sqrt{m_1}(k_1x-\nu t))-1}}-1\right)}$$
$$\times e^{i\left(k_2x+\left(\frac{64a_lk_2^2(b_l+\delta_l+\beta_{l1}+\gamma_{l1})+15(d_l+k_2e_l+\alpha_{l1})^2}{64(b_l+\delta_l+\beta_{l1}+\gamma_{l1})(c_lk_2-1)}\right)t+k_3\right)}, \tag{26}$$

When $m_1<0$, $\Delta>0$, trigonometric function solution as

$$\psi^{(l)}(x,t)=\sqrt{Y_2\sqrt{\frac{2}{\varepsilon\cos(2\sqrt{m_1}(k_1x-\nu t))-1}}-1}$$
$$\times e^{i\left(k_2x+\left(\frac{64a_lk_2^2(b_l+\delta_l+\beta_{l1}+\gamma_{l1})+15(d_l+k_2e_l+\alpha_{l1})^2}{64(b_l+\delta_l+\beta_{l1}+\gamma_{l1})(c_lk_2-1)}\right)t+k_3\right)}, \tag{27}$$

traveling wave solution as

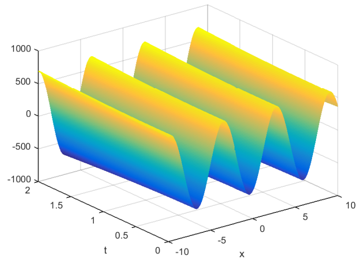
$$\psi^{(l)}(x,t)=\sqrt{Y_2\sqrt{\frac{2}{\varepsilon\sin(2\sqrt{m_1}(k_1x-\nu t))-1}}-1}$$
$$\times e^{i\left(k_2x+\left(\frac{64a_lk_2^2(b_l+\delta_l+\beta_{l1}+\gamma_{l1})+15(d_l+k_2e_l+\alpha_{l1})^2}{64(b_l+\delta_l+\beta_{l1}+\gamma_{l1})(c_lk_2-1)}\right)t+k_3\right)}, \tag{28}$$

where $Y_2=\frac{3(d_l+k_2e_l+\alpha_{l1})}{8(b_l+\delta_l+\beta_{l1}+\gamma_{l1})}$

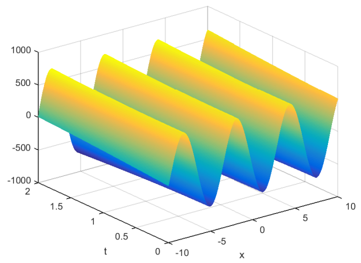
3 Physical illustrations

In this section, we present numerical simulation of the extended GI model.

The real components (a) and the imaginary components (b) are represents traveling wave solutions $\psi^l(x,t)$ as they are depicted in Figures $1-4$. We plot 3D graphics of bright soliton solution Eq.15 and trigonometric function solution Eq.19 for the Kerr law nonlinearity, and bright soliton solution Eq.23 and trigonometric function solution Eq.27 for the parabolic law nonlinearity respectively, which signified the dynamic of the solutions under the following selected parameters, when $b_l=1$, $k_1=1$, $k_2=1$, $k_3=0$, $c_l=2$; $B_1=1$, $d_l=1$, $\nu=-2$, $\alpha_{l1}=1$, $\delta_l=1$, $\beta_{l1}=1$, $\gamma_{l1}=1$ and $t\in[0.2], x\in[-10,10]$.

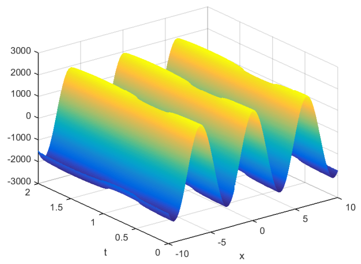


(a) The real part.

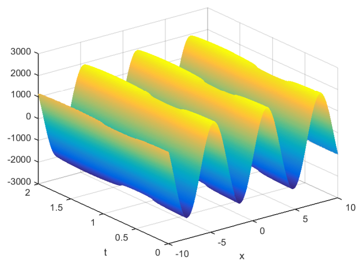


(b) The imaginary part.

Fig. 1: Bright soliton solution $\psi^{(l)}(x,t)$ of Eq.15 with $a=1$.

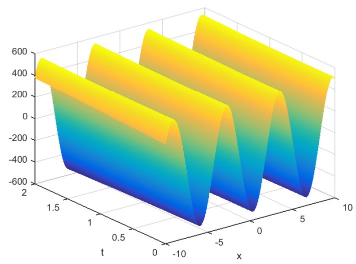


(a) The real part.

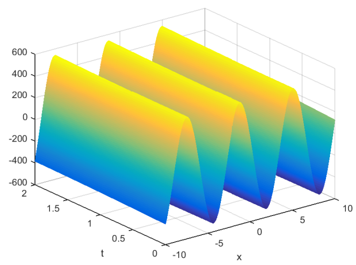


(b) The imaginary part.

Fig. 2: Trigonometric function solution $\psi^{(l)}(x,t)$ of Eq.19 with $a=-1$.

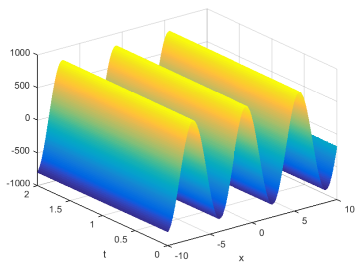


(a) The real part.

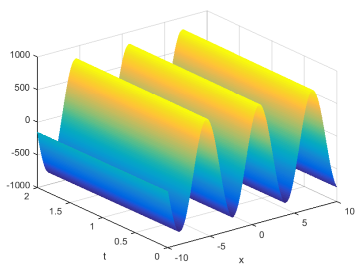


(b) The imaginary part.

Fig. 3: Bright soliton solution $\psi^{(l)}(x,t)$ of Eq.23 with $a = 1$.



(a) The real part.



(b) The imaginary part.

Fig. 4: Trigonometric function solution $\psi_1^{(l)}(x,t)$ of Eq.27 with $a = -1$.

4 Conclusion

The extended Gerdjikov-Ivanov equation in DWDM for kerr law and parabolic law nonlinearities was investigated Based on gaining bright, singular soliton solutions, Bell-type solitary wave solutions, traveling and trigonometric wave solutions were presented by the extended auxiliary equation method.

Conflict of Interest

The authors declare that they have no conflict of interest.

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