Applied Mathematics & Information Sciences
An International Journal

http://dx.doi.org/10.18576/amis/110601

The Solution of the Hierarchy of Quantum Kinetic Equations for Correlation Matrices with Generalized Yukawa Potential

Nikolai (Jr.) Bogolubov[1], *Mukhayo Rasulova*[2,*] *and Umarbek Avazov*[2]

[1] Steklov Institute of Mathematics of the Russian Academy of Sciences, Moscow 119991, Russia
[2] Institute of Nuclear Physics Academy of Sciences Republic of Uzbekistan, Tashkent 100214, Republic of Uzbekistan

Received: 2 Sep. 2017,, Revised: 2 Oct. 2017,, Accepted: 4 Oct. 2017
Published online: 1 Nov. 2017

Abstract: In paper the evolution of N identical in mass and charge particles interacting wia generalized Yukawa potential is investigated. The system of particles is considered in a finite area. Using the semi group theory, we prove the existence of a unique solution of the chain of quantum kinetic equations for correlation matrices.

Keywords: generalized Yukawa potential, screened Coulomb potential,the potential of the Debye-Huckel, Yukawa potential, chain of quantum kinetic equations for correlation matrices

1 Introduction

In connection with development of quantum information and quantum calculations, interest on research of correlation matrices and its properties has risen [1]-[2].

For detailed research of its properties, it is necessary to determine their explicit form at first. And for this, it is needed to solve equation, describing behavior of quantum system of many interaction particles both in equilibrium and in non- equilibrium states. The fact that real physical quantum systems of interactive particles are in move attracts interest on determining quantum correlation matrices, solving kinetic equations describing investigated system. As it is known from quantum physics, dynamics of such system is described by equation of Liouville [3]. Unfortunately, solution of equation of Liouville does not give information about real physical process, which is described in Boltzman and Vlasov equations. The most reasonable tool connection Liouville's equation with Boltzman and Vlasov equations is chain of kinetic equations of Bogoliubov-Born-Green-Kirkwood-Yvon (BBGKY) [4].

Quantum analogue of classical BBGKY, describing dynamics of quantum system of particles is chain of quantum kinetic equations of BBGKY [5], [6]. It is complicated system of interconnected integral - differential equations of density matrices of particles, that depends on interaction type of interaction potential between particles. One of universal potential, used in solid state physics, physics of plasma, atomic physics and chemistry is potential that can be determined from generalized Yukawa potential [7]. Namely, this is Debye - Huckel potential [8], screened Coulomb potential [9], Yukawa potential [10].

The present paper solves the Cauchy problem for the BBGKY chain for quantum kinetic equations, describing dynamics of the quantum system of particles interacting with each other by the generalized Yukawa potential. A chain of quantum kinetic equations for correlation matrices is defined on the basis of the BBGKY chain for density matrices. Solution of the chain of equations for correlation matrices using solutions of the Cauchy problem for the chain of quantum kinetic equations BBGKY for density matrices [11].

2 Formulation of the Problem

We consider the hierarchy BBGKY of quantum kinetic equations, which describes the evolution of a system of identical particles with mass m and charge $q = 1$ interacting via generalized Yukawa potential [7]

* Corresponding author e-mail: rasulova@live.com

$\phi(r) = \sum_{k=0}^{N} r^{k-1} \int_{\mu_0}^{\infty} e^{-\mu r} dw_k(\mu)$, which depends on the distance between particles $r \equiv |x_i - x_j| = ((x_i^1 - x_j^1)^2 + (x_i^2 - x_j^2)^2 + (x_i^3 - x_j^3)^2)^{1/2}$ and where $\mu_0 > 0, N$ - integer and $w_0, ..., w_N$ - real (not necessarily positive) measures with finite total variation. These potentials are "superpositions" core Yukawa potential $r^{-1}e^{-\mu r}$ [10]. We assume that the charge is a real constant. In the present paper, the Cauchy problem is formulated for a quantum system of a finite number particles contained in the finite region (vessel) with volume $V = |\Lambda|$ [4],[5],[6]:

$$i\frac{\partial \rho_s^\Lambda(t, x_1, ..., x_s; x_1', ..., x_s')}{\partial t} = [H_s^\Lambda, \rho_s^\Lambda](t, x_1, ..., x_s; x_1', ..., x_s')$$

$$+\frac{N}{V}\left(1 - \frac{s}{N}\right) Tr_{x_{s+1}} \sum_{1 \le i \le s} (\phi_{i,s+1}(|x_i - x_{s+1}|) -$$

$$\phi_{i,s+1}(|x_i' - x_{s+1}|))\rho_{s+1}^\Lambda(t, x_1, ..., x_s, x_{s+1}; x_1', ..., x_s', x_{s+1}), \quad (1)$$

with the initial condition

$$\rho_s^\Lambda(t, x_1, ..., x_s; x_1', ..., x_s')|_{t=0} = \rho_s^\Lambda(0, x_1, ..., x_s; x_1', ..., x_s'). \quad (2)$$

In the problem given by equation (1) and (2) the vector represented by x_i gives the position of ith particle in the 3-dimensional Euclidean space R^3, $x_i = (x_i^1, x_i^2, x_i^3)$, $i = 1, 2, ..., s$, and $x_i^\alpha, \alpha = 1, 2, 3$ are coordinates of a vector x_i. The length of the vector x_i is denoted by

$$|x_i| = ((x_i^1)^2 + (x_i^2)^2 + (x_i^3)^2)^{\frac{1}{2}}.$$

In (1) $\hbar = 1$ is the Planck constant and $[,]$ denotes the Poisson bracket.

The reduced statistical operator of s particles is $\rho_s^\Lambda(x_1, .., x_s; x_1', .., x_s')$ related to the positive symmetric density matrix D of N particles by [5],[6]

$$\rho_s^\Lambda(x_1, ., x_s; x_1', ., x_s') =$$

$$V^s Tr_{x_{s+1},..,x_N} D_N^\Lambda(x_1, ., x_s, x_{s+1}, ., x_N; x_1', ., x_s', x_{s+1}, ., x_N),$$

where $s \in N$, N is the number of particles, and V the volume of the system of particles. The trace is defined in terms of the kernel $\rho^\Lambda(x, x')$ by the formula

$$Tr_x \rho^\Lambda = \int_\Lambda \rho^\Lambda(x, x) dx.$$

The Hamiltonian of system is defined as

$$H_s^\Lambda(x_1, ..., x_s) = \sum_{1 \le i \le s} \left(-\frac{1}{2m} \triangle_{x_i} + u^\Lambda(x_i) \right) +$$

$$+ \sum_{1 \le i < j \le s} \phi_{i,j}(|x_i - x_j|),$$

where $\triangle_i$ is the Laplacian

$$\triangle_i = \frac{\partial^2}{\partial (x_i^1)^2} + \frac{\partial^2}{\partial (x_i^2)^2} + \frac{\partial^2}{\partial (x_i^3)^2},$$

$$\phi(r) = \sum_{k=0}^{N} r^{k-1} \int_{\mu_0}^{\infty} e^{-\mu r} dw_k(\mu)$$

and $u^\Lambda(x)$ is an external field which keeps the system in the region Λ ($u^\Lambda(x) = 0$ if $x \in \Lambda$ and $u^\Lambda(x) = +\infty$ if $x \notin \Lambda$). Here $\phi_{i,j}(|x_i - x_j|)$ is symmetric.

3 Solution of the Cauchy Problem for the BBGKY Hierarchy of Quantum Kinetic Equations with generalized Yukawa potential

To obtain the solution of the Cauchy problem defined by (1) and (2) we use a semigroup method [12], [13], [14], [15], [19], [20], [21].

Let $L_2^s(\Lambda)$ be the Hilbert space of functions $\psi_s^\Lambda(x_1, ..., x_s)$, $x_i \in R^3(\Lambda)$, and B_s^Λ be the Banach space of positive-definite, self adjoint nuclear operators $\rho_s^\Lambda(x_1, ..., x_s; x_1', ..., x_s')$ on $L_2^s(\Lambda)$

$$(\rho_s^\Lambda \psi_s^\Lambda)(x_1, ..., x_s) = \int_\Lambda \rho_s^\Lambda(x_1, ..., x_s; x_1', ..., x_s') \times$$

$$\times \psi_s^\Lambda(x_1', ..., x_s') dx_1' ... dx_s',$$

with norm

$$|\rho_s^\Lambda|_1 = sup \sum_{1 \le i \le \infty} |(\rho_s^\Lambda \psi_i^s, \varphi_i^s)|,$$

where the upper bound is taken over all orthonormalied systems of finite, twice differentiable functions with compact support $\{\psi_i^s\}$ and $\{\varphi_i^s\}$ in $L_2^s(\Lambda)$, $s \ge 1$.

We'll suppose that the operators ρ_s^Λ and H_s^Λ act in the space $L_2^s(\Lambda)$ with zero boundary conditions.

Let B^Λ be the Banach space of sequences of nuclear operators

$$\rho^\Lambda = \{\rho_0^\Lambda, \rho_1^\Lambda(x_1; x_1'), ..., \rho_s^\Lambda(x_1, ..., x_s; x_1', ..., x_s'), ...\},$$

where ρ_0^Λ are complex numbers, $\left|\rho_0^\Lambda\right|_1 = \left|\rho_0^\Lambda\right|$ and $\rho_s^\Lambda \subset B_s^\Lambda$,

$$\rho_s^\Lambda(x_1, ..., x_s; x_1', ..., x_s') = 0, \qquad when \qquad s > s_0,$$

where s_0 is finite and the norm is

$$|\rho^\Lambda|_1 = \sum_{s=0}^{\infty} |\rho_s^\Lambda|_1.$$

The Hamiltonian $-\triangle_s + \phi$ with generalized Yukawa potential $\phi(r) = \sum_{k=0}^{N} r^{k-1} \int_{\mu_0}^{\infty} e^{-\mu r} dw_k(\mu)$ is self-adjoint operator on the set $D(-\triangle)$ [7]. Here $\triangle_s$ is $3s$ dimentional Laplacian.

Let $\tilde{B}_s^\Lambda$ be a dense set of "good" elements of B_s^Λ of type $B_s^\Lambda \cap D(H_s^\Lambda) \bigotimes D(H_s^\Lambda)$, where $D(H_s^\Lambda)$ is the domain of the operator H_s^Λ [7] and $\bigotimes$ denote the algebraic tensor product.

We introduce the operators $\omega^{\Lambda}(t)$, $\Omega(\Lambda)$ and $U^{\Lambda}(t)$ on the space B^{Λ} by

$$(\omega^{\Lambda}(t)\rho^{\Lambda})_s(x_1,..,x_s;x'_1,..,x'_s)=$$

$$=(e^{-iH_s^{\Lambda}t}\rho^{\Lambda}e^{iH_s^{\Lambda}t})_s(x_1,..,x_s;x'_1,..,x'_s),$$

$$(\Omega(\Lambda)\rho^{\Lambda})_s(x_1,..,x_s;x'_1,..,x'_s)=$$

$$=\frac{N}{V}(1-\frac{s}{N})\int_{\Lambda}\sum_i\rho^{\Lambda}_{s+1}(x_1,..,x_s,x_{s+1};x'_1,..,x'_s,x_{s+1})\times$$

$$g_i^1(x_{s+1})\tilde{g}_i^1(x_{s+1})dx_{s+1}, \tag{3}$$

$$U^{\Lambda}(t)\rho_s^{\Lambda}(x_1,..,x_s;x'_1,..,x'_s)=$$

$$(e^{\Omega(\Lambda)}e^{-iH^{\Lambda}t}e^{-\Omega(\Lambda)}\rho^{\Lambda}e^{iH^{\Lambda}t})_s(x_1,..,x_s;x'_1,..,x'_s).$$

In (3) $g_i^1(x_{s+1})$ is a complete orthonormal system of vectors in the one-particle space $L_2(\Lambda)$.

Let

$$(\tilde{\mathscr{H}}^{\Lambda}\rho^{\Lambda})_s(x_1,..,x_s;x'_1,..,x'_s)=[H_s^{\Lambda},\rho_s^{\Lambda}](x_1,..,x_s;x'_1,..,x'_s)+$$

$$\frac{N}{V}(1-\frac{s}{N})Tr_{x_{s+1}}\sum_{1\le i\le s}(\phi_{i,s+1}(|x_i-x_{s+1}|)-$$

$$-\phi_{i,s+1}(|x'_i-x_{s+1}|))\rho^{\Lambda}_{s+1}(x_1,..,x_{s+1};x'_1,..,x_{s+1}).$$

Theorem 1*If potential $\phi(r)=\sum_{k=0}^{N}r^{k-1}\int_{\mu_0}^{\infty}e^{-\mu r}dw_k(\mu)$ is generalized Yukawa potential, the operator $U^{\Lambda}(t)$ generates a strongly continuous semigroup of bounded operators on B^{Λ}, whose generators coincide with the operator $-i\tilde{\mathscr{H}}^{\Lambda}$ on $\tilde{B}^{\Lambda}$ everywhere dense in B^{Λ}.*

*Proof.*According to the general theory of groups of bounded strongly continuous operators, there always exists an infinitesimal generator of the group $U^{\Lambda}(t)$ given by the formula $lim_{t\longrightarrow 0}\frac{U^{\Lambda}(t)\rho^{\Lambda}-\rho^{\Lambda}}{t}$ in the sense of convergence in norm in the space B^{Λ} for ρ^{Λ} that belong to a certain set $D(\tilde{\mathscr{H}}^{\Lambda})$ everywhere dense in B^{Λ} [15]. Therefore, since $U^{\Lambda}(t)$ is a strongly continuous semigroup on B^{Λ} with generator $-i\tilde{\mathscr{H}}^{\Lambda}$ on the right-hand side of the BBGKY hierarchy of quantum kinetic equations on $\tilde{B}_s^{\Lambda}$ which is dense in B_s^{Λ} [11], the abstract Cauchy problem (1)-(2) has the unique solution

$$\rho_s^{\Lambda}(t,x_1,...,x_s;x'_1,...,x'_s)=(U^{\Lambda}(t)\rho^{\Lambda})_s(x_1,...,x_s;x'_1,...,x'_s)$$

$$=(e^{\Omega(\Lambda)}e^{-iH^{\Lambda}t}e^{-\Omega(\Lambda)}\rho^{\Lambda}e^{iH^{\Lambda}t})_s(x_1,...,x_s;x'_1,...,x'_s) \tag{4}$$

for each $\rho_s^{\Lambda}(x_1,...,x_s;x'_1,...,x'_s)\subset\tilde{B}_s^{\Lambda}$. For the initial data ρ_s^{Λ} belonging to a certain subset of B_s^{Λ} (to the domain of definition of $D(-i\tilde{\mathscr{H}}^{\Lambda})$), which is everywhere dense in B_s^{Λ}, (4) is strong solution of Cauchy problem (1)-(2).

This proves the Theorem 1.

By a similar argument, one can show that the infinitesimal generator of the group $U(t)$ coincides with the operator that defines the BBGKY chain

$$i\frac{\partial\rho_s(t,x_1,...,x_s;x'_1,...,x'_s)}{\partial t}=[H_s,\rho_s](t,x_1,...,x_s;x'_1,...,x'_s)+$$

$$\frac{1}{v}Tr_{x_{s+1}}\sum_{1\le i\le s}(\phi_{i,s+1}(|x_i-x_{s+1}|)-\phi_{i,s+1}(|x'_i-x_{s+1}|))\times$$

$$\rho_{s+1}(t,x_1,...,x_s,x_{s+1};x'_1,...,x'_s,x_{s+1}),\qquad s\ge 1,\qquad \frac{N}{V}=\frac{1}{v}$$

in the thermodynamic limit ($N\to\infty, V\to\infty$) on an everywhere dense subset of B of finite sequences

$$\rho=\{\rho_0,\rho_1(x_1;x'_1),...,\rho_s(x_1,...,x_s;x'_1,...,x'_s),...\},$$

$$\rho_s=0,\qquad s>s_0,$$

such that $[H_s,\rho_s]$ belongs to B_s together with ρ_s [15].

4 Derivation of Hierarchy of Kinetic Equations for Correlation Matrices with generalized Yukawa Potential and its Solution

Introducing the notation [16]

$$\left(\mathscr{H}^{\Lambda}\rho^{\Lambda}\right)_s(t,x_1,...,x_s;x'_1,...,x'_s)=$$

$$=\left[H_s^{\Lambda},\rho_s^{\Lambda}\right](t,x_1,...,x_s;x'_1,...,x'_s);$$

$$\left(\mathscr{D}^{\Lambda}_{x_{s+1}}\rho^{\Lambda}\right)_s(x_1,\cdots,x_s;x'_1,\cdots,x'_s)=$$

$$=\rho^{\Lambda}_{s+1}\left(x_1,\cdots x_s,x_{s+1};x'_1,\cdots,x'_s,x_{s+1}\right);$$

$$(\mathscr{A}^{\Lambda}_{x_{s+1}}\rho^{\Lambda})_s(t,x_1,...,x_s;x'_1,...,x'_s)=$$

$$=\frac{N}{V}(1-\frac{s}{N})\sum_{1\le i\le s}(\phi_{i,s+1}(|x_i-x_{s+1}|)-$$

$$-\phi_{i,s+1}(|x'_i-x_{s+1}|))\rho_s^{\Lambda}(t,x_1,...,x_s;x'_1,...,x'_s);$$

$$\rho^{\Lambda}(t)=\{\rho_1^{\Lambda}(t,x_1;x'_1),...,\rho_s^{\Lambda}(t,x_1,...,x_s:x'_1,...,x'_s),...\}, \tag{5}$$

where $\rho_s^{\Lambda}=0$, when $s>s_0$, and $s\ge 1$,

we can cast (1) and (2) in the form

$$i\frac{\partial}{\partial t}\rho_s^{\Lambda}(t,x_1,...,x_s;x'_1,...,x'_s)=$$

$$\left(\mathscr{H}^{\Lambda}\rho^{\Lambda}\right)_s(t,x_1,...,x_s;x'_1,...,x'_s)$$

$$+\int_{\Lambda}\left(\mathscr{A}^{\Lambda}_{x_{s+1}}\mathscr{D}^{\Lambda}_{x_{s+1}}\rho^{\Lambda}\right)_s(t,x_1,...,x_s;x'_1,...,x'_s)dx_{s+1},$$

$$\rho_s^{\Lambda}(t,x_1,...,x_s;x'_1,...,x'_s)|_{t=0}=\rho_s^{\Lambda}(0,x_1,...,x_s;x'_1,...,x'_s).$$

For sequences (5) this problem can formulated as

$$i\frac{\partial}{\partial t}\rho^{\Lambda}(t)=\mathscr{H}^{\Lambda}\rho^{\Lambda}(t)+\int_{\Lambda}\mathscr{A}_x^{\Lambda}\mathscr{D}_x^{\Lambda}\rho^{\Lambda}(t)dx, \quad (6)$$

$$\rho^{\Lambda}(t)|_{t=0}=\rho^{\Lambda}(0). \quad (7)$$

Proposition 1*For sequence of correlation matrices*

$$\varphi=\{\varphi_0,\varphi_1(x_1;x_1'),\dots,\varphi_s(x_1,\dots,x_s;x_1',\dots,x_s'),\dots\}, \qquad s\geq 1$$

the hierarchy of kinetic equations has the form:

$$i\frac{\partial}{\partial t}\varphi(t)=\mathscr{H}\varphi(t)+\frac{1}{2}\mathscr{W}(\varphi(t),\varphi(t))+$$

$$+\int_{\Lambda}\mathscr{A}_x\mathscr{D}_x\varphi(t)dx+\int_{\Lambda}(\mathscr{A}_x\varphi\star\mathscr{D}_x\varphi)(t)dx, \quad (8)$$

$$\varphi(t)|_{t=0}=\varphi(0). \quad (9)$$

,

In (8) relation between density matrices and correlation matrices [16],[17], [18] is:

$$\rho(t)=\Gamma\varphi(t)=I+\varphi(t)+\frac{\varphi(t)\star\varphi(t)}{2!}+\cdots\frac{(\star\varphi(t))^s}{s!}+\cdots, \quad (10)$$

where:

$$(\varphi\star\varphi)(X)=\sum_{YCX}\varphi(Y)\varphi(X\setminus Y),$$

$$I\star\varphi=\varphi, \quad (\star\varphi)^s=\underbrace{\varphi\star\varphi\star\cdots\star\varphi}\text{ s times;}$$

$$X=(x_1,\cdots,x_s;x_1',\cdots,x_s'),$$

$$Y=(x_1,\cdots,x_s;x_1',\cdots,x_{s'}'), \quad s'\in s, \quad s=1,2,\cdots;$$

$$\mathscr{W}(\varphi,\varphi)(X)=\sum_{YCX}\mathscr{U}(Y;X\setminus Y)\varphi(Y)\varphi(X\setminus Y),$$

$$(\mathscr{U}\varphi)(X)=\left[\sum_{1\leq i<j\leq s}\phi(x_i-x_j),\varphi\right](X).$$

The prove of the proposition is analogically to [16],[17].

The problem (8), (9) for the system of s particles in the volume V have form:

$$i\frac{\partial}{\partial t}\varphi_s^{\Lambda}(t,x_1,\dots,x_s;x_1',\dots,x_s')=\mathscr{H}^{\Lambda}\varphi_s^{\Lambda}(t,x_1,\dots,x_s;x_1',\dots,x_s')+$$

$$+\frac{1}{2}\mathscr{W}^{\Lambda}\left(\varphi^{\Lambda},\varphi^{\Lambda}\right)_s(t,x_1,\dots,x_s;x_1',\dots,x_s')+$$

$$+\int_{\Lambda}\mathscr{A}_{x_{s+1}}^{\Lambda}\mathscr{D}_{x_{s+1}}^{\Lambda}\varphi_s^{\Lambda}(t,x_1,\dots,x_s;x_1',\dots,x_s')dx_{s+1}+$$

$$+\int_{\Lambda}\left(\mathscr{A}_{x_{s+1}}^{\Lambda}\varphi^{\Lambda}\star\mathscr{D}_{x_{s+1}}^{\Lambda}\varphi^{\Lambda}\right)_s(t,x_1,\dots,x_s;x_1',\dots,x_s')dx_{s+1}, \quad (11)$$

$$\varphi_s^{\Lambda}(t,x_1,\dots,x_s;x_1',\dots,x_s')|_{t=0}=\varphi_s^{\Lambda}(0,x_1,\dots,x_s;x_1',\dots,x_s'). \quad (12)$$

We introduce the quantum operator which is analogy to classical case [13]:

$$U'^{\Lambda}(t)\varphi_s^{\Lambda}(0,x_1,\dots,x_s;x_1',\dots,x_s')=$$

$$=\Gamma exp(\Omega^{\Lambda})\Gamma^{-1}[exp(iH^{\Lambda}t)\Gamma(exp(-\Omega^{\Lambda})\Gamma^{-1}\Gamma\times$$

$$\times\varphi_s(0,x_1,\dots,x_s;x_1',\dots,x_s'))exp(-iH^{\Lambda}t)].$$

Theorem 2*If potential* $\phi(r)=\sum_{k=0}^{N}r^{k-1}\int_{\mu_0}^{\infty}e^{-\mu r}dw_k(\mu)$ *is generalized Yukawa potential, the operator* $U'^{\Lambda}(t)$ *generates a strongly continuous semigroup of bounded operators on* B_+^{Λ}*, whose generators coincide with the operator*

$$-i(\mathscr{H}^{\Lambda}+\frac{1}{2}\mathscr{W}^{\Lambda}+\int_{\Lambda}\mathscr{A}_{x_{s+1}}^{\Lambda}\mathscr{D}_{x_{s+1}}^{\Lambda}dx_{s+1}+$$

$$\int_{\Lambda}\mathscr{A}_{x_{s+1}}^{\Lambda}\star\mathscr{D}_{x_{s+1}}^{\Lambda}dx_{s+1})$$

on $\tilde{B}_+^{\Lambda}$ *everywhere dense in* B_+^{Λ}*.*

Here B_+ is ideal of B [18].

*Proof.*Using (10) in (4) and $\Gamma^{-1}\Gamma\varphi(t)=\varphi(t)$ we obtain:

$$\rho_s^{\Lambda}(t,x_1,\dots,x_s;x_1',\dots,x_s')=\Gamma\varphi_s^{\Lambda}(t,x_1,\dots,x_s;x_1',\dots,x_s')=$$

$$=\Gamma exp(\Omega^{\Lambda})\Gamma^{-1}[exp(iH^{\Lambda}t)\Gamma(exp(-\Omega^{\Lambda})\Gamma^{-1}\times$$

$$\times\Gamma\varphi_s^{\Lambda}(0,x_1,\dots,x_s;x_1',\dots,x_s')exp(-iH^{\Lambda}t)]=$$

$$=\Gamma exp(\Omega^{\Lambda})\Gamma^{-1}[exp(iH^{\Lambda}t)\Gamma(exp(-\Omega^{\Lambda})\times$$

$$\times\varphi_s^{\Lambda}(0,x_1,\dots,x_s;x_1',\dots,x_s')exp(-iH^{\Lambda}t)]. \quad (13)$$

Acting to (13) by Γ^{-1} we receive:

$$\varphi_s^{\Lambda}(t,x_1,\dots,x_s;x_1',\dots,x_s')=$$

$$=U'^{\Lambda}(t)\varphi_s^{\Lambda}(0,x_1,\dots,x_s;x_1',\dots,x_s')=$$

$$=exp(\Omega^{\Lambda})\Gamma^{-1}[exp(iH^{\Lambda}t)\Gamma(exp(-\Omega^{\Lambda})\times$$

$$\times\varphi_s(0,x_1,\dots,x_s;x_1',\dots,x_s')exp(-iH^{\Lambda}t)]. \quad (14)$$

The generator of the semigroup $U'^{\Lambda}(t)$ coincides with

$$-i(\mathscr{H}^{\Lambda}+\frac{1}{2}\mathscr{W}^{\Lambda}+\int_{\Lambda}\mathscr{A}_{x_{s+1}}^{\Lambda}\mathscr{D}_{x_{s+1}}^{\Lambda}dx_{s+1}+$$

$$+\int_{\Lambda}\mathscr{A}_{x_{s+1}}^{\Lambda}\star\mathscr{D}_{x_{s+1}}^{\Lambda}dx_{s+1}),$$

on the set $D(H_s^{\Lambda})$.

So, (14) on $D(-\sum_{1\leq i\leq s}\triangle_i)$ is the unique solution of the Cauchy hierarchy of kinetics equations for correlation matrices with generalized Yukawa potential (8),(9).

This proves the Theorem 2.

Acknowledgement

N.(Jr.)Bogoliubov formulated the problem and proposed to apply chain BBGKY to study dynamics of the particle system. He, as well, discussed systematically the obtained results.

M.Rasulova and U.Avazov's research has been done through the grant OT-F2-18 in the Institute of Nuclear Physics of Academy of Sciences of Uzbekistan.

References

[1] F.R.Graziani et al, High Energy Density Phys. **8**, 105-131 (2012).
[2] S.P.Hau-Riege, J.Weisheit, J.I.Castor, R.A.London, H.Scott and D.F.Richards, New Journal of Physics **15**, 1367-2630 (2013).
[3] J. Liouville, Journ. de Math. **3**, 349 (1838).
[4] N.N.Bogolyubov, Problems of a dynamical theory in statistical physics, Gostehizdat, Moscow, 1946.
[5] N.N.Bogolyubov, Lectures on quantum statistics, Radyanska shkola, Kiev, 1949.
[6] N.N.Bogolyubov, N.N.(Jr.) Bogolubov, Introduction to Quantum Statistical Mechanics, Nauka, Moscow, 1984.
[7] M.Reed, B.Saymon, Methods of modern mathematical physics, **3**, Academic Press, New York.-San Francisco-London, 1979.
[8] P.Debye and E. H*ü*ckel, Physikalische Zeitschrift, **24**, 185-206, (1923).
[9] C.Kittel, Introduction to Solid State Physics, John Wiley& Sons, New York 1953.
[10] H. Yukawa, Proc. Phys. Math. Soc. Japan., **17**, 48-56, (1935).
[11] N. N. Bogolyubov (Jr.), M. Yu. Rasulova, U. A. Avazov, Theoret. and Math. Phys., **1**89, 1790-1795 (2016).
[12] D.Ya.Petrina and A.K.Vidybida, Trudi MI AN USSR, **1**36, 370 (1975).
[13] A.K.Vidybida, Theoret. and Math Physics **3**4, 99 (1978).
[14] M.Yu.Rasulova, Preprint **I**TP-44R, Bogolyubov Institute of Theoretical Physics, Kiev, 1976; DAN Uzbek SSR, **2**, 248 (1976).
[15] D.Ya.Petrina, Mathematical Foundation of Quantum Statistical Mechanics, Continuous Systems, Kluwer Acadec Publishers, Dordrecht-Boston-London, 1995.
[16] M.Yu.Rasulova and A.K.Vidybida, Preprint ITP-textbf ITP-27, Bogolyubov Institute of Theoretical Physics, Kiev, 1976.
[17] M.Yu.Rasulova, Theoret. and Math. Physics **4**2, 124 (1980).
[18] D.Ruelle, Statistical Mechanics, Rigorous Results, (Mir, Moscow 1971.
[19] M.Brokate, M.Yu.Rasulova, Physics of Particles and Nuclei, **4**7, 1014 (2010).
[20] I.Gohberg, S.Goldberg and M.A.Kaashoek, Classes of Linear Operators **1**, Birkh*ä*user Verlag, Basel-Boston-Berlin, 1990.
[21] A.Pazy, Semigroups of Linear Operators and Applications to Partial Differential Equations, Springer-Verlag, New York-Berlin, Heidelberg-London-Paris-Tokio-Hong Kong-Barcelona-Budapest, 1983.

Nikolai (Jr.) Bogolubov received the PhD degree in Theoretical Physics from the Moscow State University in 1966 and his D.Sc. in Theoretical Physics from Academy of Sciences of the USSR in 1970. His scientific interests are in general mathematical problems of equilibrium and nonequilibrium statistical mechanics and applications of modern mathematical methods of classical and quantum statistical mechanics to the problems of the polaron theory, superradience theory, and theory of superconductivity. His main works belong to the field of Theoretical and Mathematical Physics, Classical and Quantum Statistical Mechanics, Kinetic theory. Many results have become a part of the modern Mathematical Physics toolbox, namely: the fundamental theorem in the theory of Model Systems of Statistical Mechanics, inequalities for thermodynamical potentials, minimax principle in problem of Statistical Mechanics. He has published more then 150 in the field of Statistical Mechanics, Theoretical and Mathematical Physics.

Mukhayo Rasulova is earned her B.Sc. and M.Sc. in Theoretical Physics from Tashkent State University, Uzbekistan in 1971. She earned her Ph.D. degree from Institute of Theoretical Physics, Ukraine National Academy of Sciences in Kiev, Ukraine 1978 and a doctoral degree of sciences in Mathematics and Physics from Institute of Nuclear Physics, Uzbekistan Academy of Sciences, Tashkent, Uzbekistan, in 1995. Her main research works belong to the field of Theoretical and Mathematical Physics. Her scientific interests are devoted to investigation of kinetic and thermodynamic properties of system of interacting particles and infinite systems of charges and inhomogeneity, the BBGKY's hierarchy of quantum kinetic equations for Bose and Fermi particles with different potentials. Also her current research works are devoted to study statistical and kinetic properties of nonlinear optics and theory of quantum information. She has more then 80 scientific publications in the field of Statistical Physics, Theoretical and Mathematical Physics. She has been an invited speaker in many international conferences.

Umarbek Avazov is earned his B.Sc. and M.Sc. in Mathematical Physics from Tashkent State University, Uzbekistan in 2003. His main research works belong to the field of Theoretical and Mathematical Physics. His scientific interests are devoted to investigation of kinetic properties of system of interacting particles, the BBGKY's hierarchy of quantum kinetic equations.