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Non-Singular Fractional Derivative Approach for Semi-Analytical Solution of Casson Type Cnts-Based Nanofluid with Heat Transfer

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Abstract: In this manuscript, a mixed free convection Casson nanofluid mixed with carbon nanotubes (CNTs) and water as base fluid flowing on a porous plate is examined with heat transfer. The leading non-dimensional energy and momentum equations are solved by fractional mathematical techniques namely Atangana-Balenau (AB) fractional derivative and Laplace scheme. Different numerical techniques are also utilized for the Laplace inverse. The effects of different constraints with altered values are also analyzed graphically and numerically on governing equations. As a result, we have confabulated that, by enhancing the value of fractional constraint both energy and momentum profiles play decreasing behavior. Furthermore, the increasing rate of nusselt numbers signifies the equivalent boundary conditions.

Keywords: AB derivative, Nanofluids, CNTs, Damped temperature

1 Introduction

Recent trends in thermal engineering promoted the solar energy applications associated with the novel suspension of nanoparticles. The thermal efficiencies are found to be more progressive in nanoparticles and subsequently improve the heating mechanism in the distinct era of industries and engineering. The various major applications like cooling of nuclear reactors, heating devices, fission chemical reactions, nuclear engineering, plasma physics, and extension processes preferred the significances of such nano-materials. With admirable and improved thermo-physical features, nanofluids have become the active and preferable source of energy in many developing countries. Nowadays, the discovery and research on the CNTs-based nanostructure become a fascinating research area due to some of its oblivious properties and characteristics for researchers. (a). Due to good electron field emitters (b). Having highly electrical and thermal conduct ivies (c). Having good tensile strength (d). Due to low thermal expansion coefficient (e). Highly flexible (f). CNTs are about one hundred times stronger than steel (g). Stronger bonding of atoms of CNTs (h). different types of structures with having different values of thickness and elasticity. Firstly, Choi [1] gave the concept of nanofluids in fluid mechanics to enhance the heat transfer and thermal conductivity of the main base fluid. The thermal conductivity of lambda CNTs was investigated by Berber et al. [2] with dependence on the temperature profile. In [3] Hone et al synthesized the CNTs mixed composite material and dig out its thermal conductivity with different fractional derivative techniques. The effects of different constraints on momentum and thermal profile and the behavior of volume fraction of CNTs on the heat transfer were investigated by Hag et al. [4]. They compared their obtained results with different fractional models namely (a). Maxwell model (b). Hamilton and crosser model (c). Xue model of thermal conductivity. Different behavior of shapes and types with characteristics of nanofluids are used by different researchers i.e. Aman et al [5] studied the gold nanoparticles, Ganesh et al [6] investigated Gamma alumina nanofluid, Qasim et al [7] examined methanol-based nanofluid, Khan et al [8] studied the Carreau nanofluid. Single-wall carbon nanotubes (SWCNTs) and multi-wall nanotubes (MWCNTs) have

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been deliberated by Alzahrani et al [9] with accelerating plates. A remarkable focus on non-Newtonian materials is pronounced by researchers due to their novel properties and rheological mechanisms. The non-Newtonian materials are famous owing to the fact of complicated rheology and multidisciplinary behavior. The non-Newtonian fluids have frequent importance in chemical industries, food technology, mechanical phenomenon, petroleum sciences, cosmetics, medicine, drilling processes, wire coating, manufacturing processes etc. The assessed rheological consequences of such liquids are visualized with aim of distinct relations and mathematical models. The famous model amongst non-Newtonian fluids which are known as Casson fluid is famous as it reflects the shear-thinning prospective of non-Newtonian fluids. The yield stress factor and change in the shear stress mechanism are exhibited with the aims of Casson fluid. The behavior of blood and ink is usually classified in the Casson model. Khan et al. [10] addressed the Casson fluid flow with numerical simulations over the accelerated surface. Nandeppanavar et al. [11] inspected the thermal behavior of Casson liquid with nanoparticles over a porous space. The Hall impact along with ion slip features regarding the Casson fluid model was addressed by Bhatti et al. [12]. Prasad et al. [13] claimed the variable properties in inclined channel flow of Casson liquid with slip features. Rehman et al. [14] assumed the Casson fluid in the porous enclosure to analyze the thermal impact with the implementation of a finite element scheme. Leibnitz and L?Hospital [15] developed the idea of the fractional derivative approach. The memory fact is successfully predicted with the fractional approach. The memory function is associated with the kernel of fractional derivatives without imitating any physical progression. The concept of fractional calculus is associated with the differentiation as well as nonlocal integrals [16]. The solution procedure and mathematical modeling of many physical and real-life problems are impacted with fractional calculus. Wang et al. [17] predicted the analytical expressions for a fractional type liquid in terms of velocity and tangential stress. The change in the neighborhood of a point is inspected with classical derivatives while the assessment of complete interval is explained with help of non-integer derivatives. The fractional derivatives reflect the nonlocal nature in contrast to the classical derivatives. These novel features make the fractional derivatives more valuable. This approach can be followed in many problems like vibration problems, polymer, earth quick, fluids flow, etc. The fractional-order derivative parameters are known as rheological parameters which help to control the movement of the fluid. The mathematicians have developed different algorithms and definitions regarding the fractional approach [18, 19]. More new trends about fractional techniques can be seen in [20–27]. The above literature encouraged us to determine the solution of the fractional Casson fluid model. In the 1st section, a brief introduction of nanofluids and fractional derivatives is discussed. In the second section, the formulation of the problem under discussion is constructed mathematically. In the 3rd section, the solution of the transformed problem is calculated by the virtue of the AB-fractional derivative and LT method. In the end, the numerical and graphical results are concluded with physical arguments.

2 Problem formulation

Suppose an unsteady and free convection viscous CNTs-based nanofluid flow on a pours type plate with an isothermal temperature θ_∞.The plate is crammed with MWCNTs and SWCNTs nanofluids in a porous plate filled with half-space. Suppose the flowing CNTs-based nanofluid is conducting electrically, and a magnetic field with strength B_o with the angle of inclination of applied magnetic field γ is applied to the flowing fluid. Initially, the temperature of flowing fluid is θ_∞ and the plate is in a rest state. After the passage of some time, the temperature of the flowing nanofluid escalates from θ_∞ to θ_ω and the erect plate begins to oscillate with $U_oH(t)\cos(\omega t)$. Due to the oscillation of the plate and increase in temperature, the fluid begins to flow on the vibrating plate. The Cauchy stress tensor and rheological stress tensor of the fluid is specified as [28, 30]

$$\underline{T} = \begin{cases} 2\left(\mu_B + \frac{p_r}{\sqrt{2\pi_c}}\right) e_i j; \pi < \pi_c \\ 2\left(\mu_B + \frac{p_r}{\sqrt{2\pi}}\right) e_i j; \pi > \pi_c \end{cases} \tag{1}$$

Where π_c represents the product of μ_b and p_r respectively.
By using equation (1) with the momentum equation, Fourier law of heat conduction [30], Maxwell equation [31], Boussinesqs approximation [28], and Darcy's law [32], the leading equations governed to this problem can be formulated as follows [29]

$$\rho_{nf}\frac{\partial v_{\xi,t}}{\partial t} = \mu_{nf}\left(1+\frac{1}{B_1}\right)\frac{\partial^2 v_{\xi,t}}{\partial \xi^2} - \left(\sigma_{n,f}B_0^2\sin\gamma + \left(1+\frac{1}{B_1}\right)\frac{\mu_{nf\phi}}{\kappa}\right)v_{\xi,t} + g\left(\rho B_T\right)_{nf}\left(\theta_{\xi,t}-\theta_\infty\right) \tag{2}$$

$$\left(\rho C_p\right)_{nf}\frac{\partial \theta}{\partial t} = \kappa_{nf}\frac{\partial^2\theta}{\partial \xi^2} \tag{3}$$

equivalent conditions are specified as

$$v_{(\xi,0)} = 0, \quad \theta_{(\xi,0)} = \theta_\infty; \quad \forall\ \xi \geq 0$$
$$v_{(0,t)} = U_0H(t)\cos(\omega t), \quad \theta_{(0,t)} = \theta_\omega; \quad t > 0$$
$$v_{(xi,t)} \to 0, \quad \theta_{(\xi,t)} \to \theta_\infty; \quad \xi \to \infty, \quad t > 0$$

Now by setting the non-dimensional variables as

$$v^* = \frac{v}{v_0}, \quad \xi^* = \frac{v_0}{v_f}\xi, \quad t^* = \frac{v_0^2}{v_f}t, \quad T = \frac{\theta-\theta_\infty}{\theta_\omega-\theta_\infty}$$

By presenting these dimensionless variables in the overhead governing equations (2), (3) and neglecting the star symbols, we get

$$a_0 \frac{\partial v_{(\xi,t)}}{\partial \xi^2} = \frac{a_1}{B}\frac{\partial^2 v(\xi,t)}{\partial \xi^2} - \left(a_2 M \sin\gamma + \frac{a_1}{\kappa}\right) v_{(\xi,t)} + a_3 Gr T_{(\xi,t)} \quad (4)$$

$$a_4 P_r \frac{\partial T_{(\xi,t)}}{\partial t} = a_5 \frac{\partial T_{(\xi,t)}}{\partial \xi^2} \quad (5)$$

with transformed conditions

$$v_{(\xi,0)} = 0, \quad T_{(\xi,t)} = 0, \quad \forall \xi \geq 0$$
$$v_{(0,t)} = H(t)\cos(\omega t), \quad T_{(0,t)} = 1, \quad \forall t > 0$$
$$v_{(\xi,t)} \to 0, \quad T_{(\xi,t)} \to 0; \quad \xi \to \infty, t > 0$$

Where

Table 1: Physical characteristics of water and CNTs

Material	Water	SWCNTs	MWCNTs
ρ	997.1	2600	1600
C_p	4179	425	796
σ	0.9	$10^6 - 10^7$	1.9×10^{-4}
κ	0.613	6600	3000
$B_T \times 10^{-5}$	21	27	44

$$M = \frac{v_f \sigma_f B_0^2}{\rho_f U_0^2}, \quad B = \frac{B_1}{1+B_1}, \quad Pr = \left(\frac{\mu C_p}{\kappa}\right)_f,$$
$$\kappa = \frac{\kappa u_0^2}{v_f \phi}, \quad Gr = \frac{g(vB_T)_f(\theta_\omega - \theta_\infty)}{U_0 3},$$
$$a_0 = (1-\phi_{nf}) + \frac{\phi \rho_s}{\rho_f}, \quad a_1 = \frac{1}{1-\phi},$$
$$a_2 = \frac{\sigma hnf}{\sigma_f}, \quad a_3 = (1-\phi) + \frac{\phi(\rho B_T)_s}{(\rho B_T)_f}$$
$$a_4 = (1-\phi) + \frac{\phi(\rho C_p)_s}{(\rho C_p)_f}, \quad a_5 = \frac{\kappa_{nf}}{\kappa_f}$$

are the Magnetic field number, Casson parameter, Prandtl number, Permeability, and Grashof-number respectively. Now the fractional model in AB-fractional sense is given by [33, 34]

$$a_0 {}^{AB}\mathscr{D}_t^\beta v_{(\xi,t)} = \frac{a_1}{B}\frac{\partial^2 v(\xi,t)}{\partial \xi^2} - \left(a_2 M \sin\gamma + \frac{a_1}{\kappa}\right) v_{(\xi,t)} + a_3 Gr T_{(\xi,t)}; \quad 0 < \beta \leq 1 \quad (6)$$

$$a_4 P_r {}^{AB}\mathscr{D}_t^\beta T_{(\xi,t)} = a_5 \frac{\partial^2 T_{(\xi,t)}}{\partial \xi^2}; \quad 0 < \beta \leq 1 \quad (7)$$

Where ${}^{AB}\mathscr{D}_t^\beta$ is AB-time fractional operator defined by [35, 36]

$${}^{AB}\mathscr{D}_t^\beta u_{(\xi,t)} = \frac{1}{1-\beta}\int_0^t E_\beta\left[-\frac{\beta(\tau-t)^\beta}{1-\beta}\right]\frac{\partial u_{(\xi,t)}}{\partial \tau}\,d\tau \quad (8)$$

the LT of equation (8) is acknowledged as [37]

$$\mathscr{L}\left[u_{(\xi,t);s}\right] = \frac{s^\beta \mathscr{L}\left(u_{(\xi,t)}\right) - u(\xi,0)}{(1-\beta)s^\beta + \beta}, \quad 0 < \beta \leq 1 \quad (9)$$

3 Solution of the problem

3.1 Energy Field

The simulation of the thermal field can be analyzed through LT and eq. (9) by exploiting on eq. (7) and using its parallel conditions, we yield that

$$\overline{T}_{(\xi,s)} = \frac{1}{s}e^{-\xi\sqrt{\frac{a_4 P_r}{a_5}\frac{s^\beta}{(1-\beta)s^\beta+\beta}}} \quad (10)$$

above equation (10) can also be written as

$$\overline{T}_{(\xi,s)} = \overline{\chi}_s \overline{\varphi}_{(\xi,s,b_1,b_2)} \quad (11)$$

Where

$$\overline{\chi_s} = \frac{1}{s^{1-\beta}}, \quad b_1 = \frac{a_4 P_r}{a_5(1-\beta)}, \quad b_2 = \frac{\beta}{1-\beta},$$
$$\overline{\varphi}_{(\xi,s,b_1,b_2)} = \frac{1}{s_\beta}e^{-\xi\sqrt{\frac{b_1 s_\beta}{s_\beta + b_2}}}$$

and the convolution theorem,

$$T_{(\xi,t)} = \int_0^t \chi_{(t-\tau)}\varphi_{(\xi,\tau,b_1,b_2)}\,d\tau, \quad 0 < \beta \leq 1 \quad (12)$$

Where

$$\varphi_{(\xi,\tau,b_1,b_2)} = \mathscr{L}^{-1}\{\overline{\varphi}_{(\xi,s,b_1,b_2)}\} = \frac{1}{\pi}\int_0^\infty\int_0^\infty v^\beta \sin(\pi\beta)\, \varphi_{(\xi,t,b_1,b_2)} e^{-r\tau - vr^\beta} e^{\cos(\pi\beta)}\,dr\,dv$$

$$\varphi_{(\xi,t,b_1,b_2)} = \mathscr{L}^{-1}\{\frac{1}{s}e^{-\xi\sqrt{\frac{b_1 s}{s+b_2}}}\} = 1 - 2\frac{b_1}{\pi}\int_0^\infty \frac{\sin(\xi x)}{x(b_1+x^2)}e^{-\frac{b_2 t x^2}{b_1+x^2}}\,dx$$

And

$$\chi_{(t)} = \frac{1}{t^\beta}\Gamma(1-\beta)$$

3.2 Momentum Field

In this sector, applying LT on eq. (6) for the simulation of the momentum field, we obtain

$$\frac{\partial^2 \overline{v}_{(\xi,s)}}{\partial \xi^2} - \left(\frac{b_4 s^\beta + b_5}{s^\beta + b_2}\right) \overline{v}_{(\xi,s)} = -b_3 Gr \overline{T}_{(\xi,s)} \quad (13)$$

Where

$$b_3 = \frac{Ba_3 Gr}{a_1}, \quad b_4 = K_e ff + b_0, \quad K_e ff = \frac{Ba_2}{a_1} M \sin\gamma + \frac{B}{\kappa}$$

$$b_0 = \frac{Ba_0}{a_1(1-\beta)}, \quad b_5 = b_2 K_e ff, \quad b_2 = \frac{\beta}{1-\beta}$$

After utilizing the boundary conditions, we attained the simulation of equation (13) such that

$$\overline{v}_{(\xi,s)} = \frac{s}{s^2+w^2} e^{-\xi\sqrt{\frac{b_4 s^\beta + b_5}{s^\beta + b_2}}} + \frac{b_3}{s}\frac{s^\beta + b_2}{b_6 s^\beta + b_2}\left(e^{-\xi\sqrt{\frac{b_4 s^\beta + b_5}{s^\beta + b_2}}} - e^{-\xi\sqrt{\frac{b_1 s^\beta}{s^\beta + b_2}}}\right) \quad (14)$$

Where

$$b_6 = b_1 - b_4$$

The Laplace inverse of eq. (14) can be analyzed numerically by utilizing Zakian's method [38, 39]

$$v_{(\xi,t)} = \frac{t}{2}\sum_{m=1}^{p} Re\left[\alpha_m \overline{v}\left(\xi, \frac{\beta_m}{t}\right)\right]$$

Where α_m and β_m are the constants and Re(.) is the real part of $\overline{v}_{(\xi,s)}$ function. Another numerical technique i.e. Fourier series method is utilized for comparison of numerical results [40]

$$v_{(\xi,t)} = \frac{e^{bt}}{t}\left[\frac{1}{2}\overline{v}_{(v)_{(\xi,b)}} + Re\sum_{m=2}^{n}\overline{v}\left(b + j\frac{m\pi}{t}\right)(-1)^m\right]$$

Where $j = \sqrt{-1}$ and b, n are constant parameters.

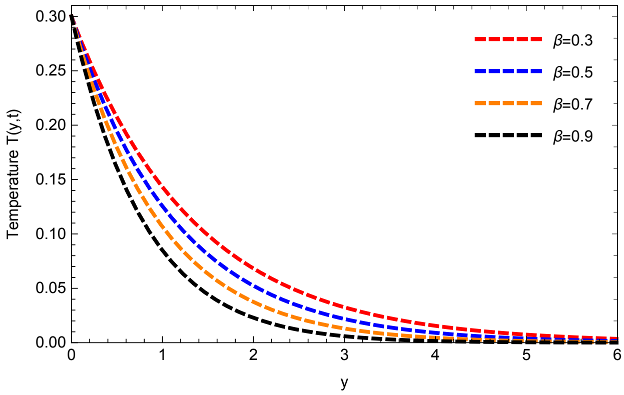

Fig. 1: Temperature field for β with $Pr = 19$

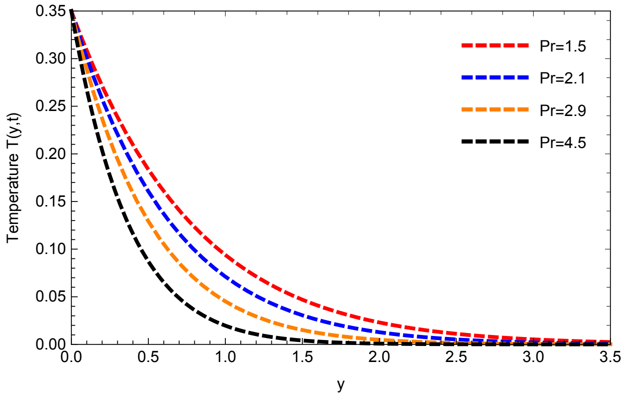

Fig. 2: Temperature field for $\beta = 0.6$ with

Table 2: Numerical analysis of velocity field by different numerical techniques

y	Zakian's Method	Series Method
0.1	0.6589	0.6561
0.2	0.5760	0.5720
0.3	0.5033	0.5001
0.4	0.4395	0.4236
0.5	0.3836	0.3789
0.6	0.3346	0.3239
0.7	0.2916	0.2786
0.8	0.2540	0.2388
0.9	0.2210	0.2025

4 Conclusions

In this paper, a CNTs based Casson nanofluid moving on a porous plate under the influence of an applied magnetic field with velocity and temperature field is discussed. A semi-analytical solution of the momentum field is discovered by fractional mathematical technique namely as AB-fractional derivative and LT scheme. For the inverse of the LT different numerical methods such that Zakian's method and Series method are utilized to investigate the numerical solution of velocity field and temperature distribution. The effective behavior of altered constraints with altered values for velocity and temperature field are plotted represented and compared numerically. The main outcomes from the graphical and numerical representation of governing equations can encapsulate such that

- The solution accomplished by fractional order model for momentum and thermal field is more reliable, saleable for different values of fractional order constraint $0 < \beta \leq 1$.
- The temperature field rose down by raising the rate of fractional parameter β and also decreased by escalating

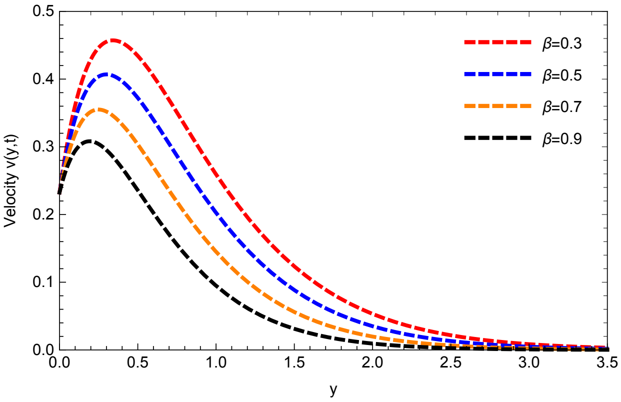

Fig. 3: Velocity field for β with $Pr = 6.3$, $t = 0.5$, $Gr = 3.75$, $M = 1.4$, $\gamma = \frac{\pi}{4}$, $\kappa = 0.9$.

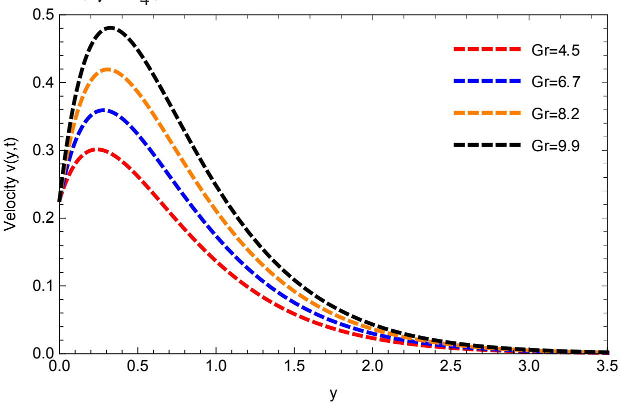

Fig. 4: Velocity field for Gr with $\beta = 0.5$, $Pr = 6.3$, $t = 0.5$, $M = 1.4$, $\gamma = \frac{\pi}{4}$, $\kappa = 0.9$.

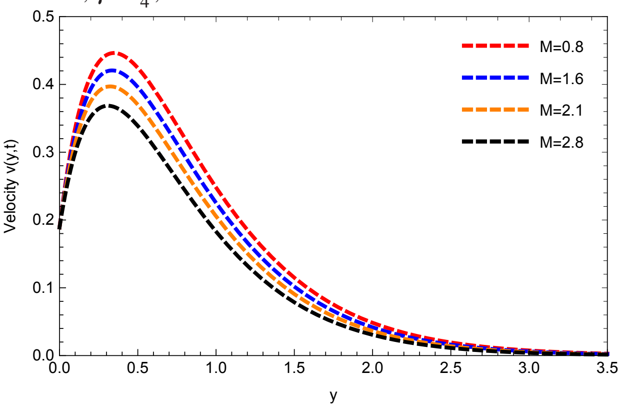

Fig. 5: Velocity field for M with $\beta = 0.5$, $Pr = 6.3$, $t = 0.5$, $Gr = 3.75$, $\gamma = \frac{\pi}{4}$, $\kappa = 0.9$.

the rate of Prandtl number Pr.

• Momentum field increase by enhancing the value of Grashof number Gr, and the slow down with the variation in the angle of applied magnetic field γ.

• All the substantial stream parameters are in admirable alignment with the reported effort patterns and characteristics.

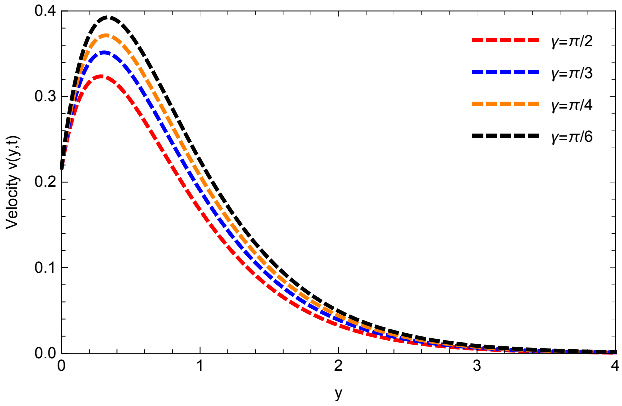

Fig. 6: Velocity field for γ with $\beta = 0.5$, $Pr = 6.3$, $t = 0.5$, $Gr = 3.75$, $M = 1.4$, $\gamma = \frac{\pi}{4}$, $\kappa = 0.9$.

Table 3: Numerical analysis of nusselt number and skin friction

β	Nusselt number	Skin friction
0.1	0.7364	2.2926
0.2	0.7415	2.1327
0.3	0.7500	1.9847
0.4	0.7622	1.8459
0.5	0.7776	1.7146
0.6	0.7959	1.5898
0.7	0.8154	1.4709
0.8	0.8325	1.3576
0.9	0.8395	1.2496

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