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Estimation of Inverse Chen Distribution based on Upper Record Values

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Abstract: According to record values and the associated statistics are interested in many real life applications involving data relating to auction and reverse auction, dynamic purchasing systems and production lines. In this article, we consider the problem of estimating unknown parameters of inverse Chen distribution is obtained based on record values as upper record. We present the maximum likelihood (MLE), Bayes estimators and Lindly approximation method for two parameter of inverse Chen distribution. We have examined Bayes estimates under symmetric loss function based on Monte carlo simulation of record values and numerical Computations. Furthermore the comparisons between the different estimators are given.

Keywords: Point and Bayesian estimation, Inverse Chen distribution, Record values, Monte Carlo Markov Chain, Gibbs-Sampling, Lindly approximation method

1 Introduction

Many authors have been studied record values and associated statistics. Record values and the associated statistics are interest and important in many real life applications involving data relating to weather, economic, Sports data "Olympic records or world records in sport" and life time testing studies. Chandler [1] he formulated the theory of record values as a model for successive extremes in a sequence of independently and identically random variables. Feller [2] gave some examples of record values. Resnick [3] discussed the asymptotic theory of records. Mubarak [4] estimate parameters of frechet distribution based on record. Interested readers may refer to Ahsanullah [5].

For other recent examples, we refer the readers to Sultan [6]. For a review of developments in this characterizing distribution via their two parameters has along history. Some recently published example include: statistical inference about the shape parameter of new two parameter Bathtub-Shaped life time distribution. Bayesian estimation based on progressive Type-II Censoring from two parameter bathtub-shaped life time model: an Markov chain Monte carlo approach Ahmed [7]. Monte Carlo estimation of Bayesian credible and HPD intervals. Estimating the parameters of bathtub-shaped distribution under progressive type-II censoring Manoj [8]. Estimation of two-parameter bathtub-shaped life time distribution with progressive censoring Shuo-Jyo We [9]. Finally, exponentiated Chen distribution: Properties and estimation Sanku Dey [10], Estimation and predication for Chen distribution with bathtub-shape under progressive censoring Tanmay Kayal [11].

Let $(X_{1:n}, X_{2:n}, X_{3:n}, \ldots, X_{n:n})$ be a sequence of independent and identically distributed (iid), random variables with Cumulative distribution function $F(x)$ and probability density $f(x)$. With simplify, we rewrite $(X_1, X_2, \ldots, X_n)$. this sequence if $Y_j < Y_{j-1}$ for $j > 1$. we say Y_j is a upper record value of this sequence if $Y_j > Y_{j-1}$ for $j > 1$. Hence, Y_1 is a upper record value.Let $X_{U_n} = \min\{j | j > U_{n-1}, Y_j > Y_{L_{n-1}}, n \geq 2\}$ with $U_1 = 1$ denote the times of upper record values. For comprehensive accounts of the theory and applications of record values, we refer the readers to Ahsanullah [5] . The sequence $\{X_{U_n}\}_{n=1}^{\infty}$ is known as upper record values and the sequence $\{Un\}_{n=1}^{\infty}$ is known as record times sequence Ahsanullah [12]. In the following, we give some preliminaries. Let X_{U_m} and X_{U_n} for $m < n$ denote the upper record statistics from a given family.The joint probability density function of X_{U_m} and X_{U_n} is given by Arnold et al. [13] . In this article, we will discuss the

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estimation unknown two parameters from Inverse Chen distribution based on three different records. Let $(X_1, X_2, \ldots, X_n)$ be a sequence of (iid) random variables from Inverse Chen distribution with (cdf) and (pdf) are given, respectively, as follows

$$F(x) = 1 - e^{\lambda(1-e^{x^\beta})}, \quad x > 0, \tag{1}$$

and hence the probability distribution function (Pdf) is given by

$$f(x, \lambda, \beta) = \lambda\ \beta\ x^{\beta-1}\ e^{x^\beta + \lambda(1-e^{x^\beta})}, \\ 0 < x < \infty,\ \lambda, \beta > 0. \tag{2}$$

If a random variables X has a chen distribution function. Then the distribution of $Y = \frac{1}{X}$ may be termed as an inverse Chen distribution (Icd). Its cumulative distribution function (Cdf) is define by

$$F(y) = e^{\lambda(1-e^{y^{-\beta}})}, \quad y > 0, \lambda, \beta > 0. \tag{3}$$

And the probability density function (pdf) of Inverse Chen distribution (ICD) is

$$f(y) = \lambda\beta y^{-1-\beta} e^{y^{-\beta}} e^{\lambda(1-e^{y^{-\beta}})}, \\ y, \lambda, \beta > 0, \tag{4}$$

where λ and β are the shape and scale parameters respectively.

2 Estimation of parameters based on upper Record:

According to the upper record values it will be used in auction for private and general sectors. In this section, we used maximum likelihood estimation and Bayesian estimation with square loss function to estimate the parameters λ and β for Inverse Chen distribution.

2.1 Maximum Likelihood Estimation

Let $\underline{Y} = (y_{u_1}, y_{u_2}, y_{u_3}, \ldots, y_{u_n})$ be the upper record value of size n from the inverse Chen distribution with parameters λ and β. The likelihood function for observed upper records with simplify $\underline{Y} = (y_1, y_2, \ldots, y_n)$ is; is defined according to Arnold et al.[14] as

$$L(\underline{Y}, \lambda, \beta) = f(y_{u_n}) \prod_{i=1}^{n-1} \frac{f(y_{u_i})}{1 - F(y_{u_i})}. \tag{5}$$

The likelihood function with simplify of upper observation records $(Y_1, Y_2, \ldots, Y_n)$ will be written as follows

$$L(\lambda, \beta|\underline{y}) = C \prod_{i=1}^{n} y_i^{-(\beta+1)} e^{\sum_{i=1}^{n} y_i^{-\beta} + \lambda(1-e^{y_i^{-\beta}})} \times \\ \prod_{i=1}^{n-1} \left(1 - e^{\lambda(1-e^{y_i^{-\beta}})}\right)^{-1} \tag{6}$$

where $C = \lambda^n \beta^n$. Hence, the log-likelihood function becomes,

$$L^*(\lambda, \beta|\underline{y}) = n\ln\lambda + n\ln\beta - (\beta+1)\sum_{i=1}^{n} lny_i \\ + \sum_{i=1}^{n} y_i^{-\beta} + \lambda\sum_{i=1}^{n}(1 - e^{y_i^{-\beta}}) \\ - \sum_{i=1}^{n-1} ln\left(1 - e^{\lambda(1-e^{y_i^{-\beta}})}\right). \tag{7}$$

By differentiation with respect to λ and β and equal to zero, to obtain the estimator's λ and β as follows,

$$\frac{n}{\hat{\lambda}} = \sum_{i=1}^{n-1} \frac{e^{\lambda\left(1-e^{y_i^{-\beta}}\right)} e^{y_i^{-\beta}} - e^{\lambda\left(1-e^{y_i^{-\beta}}\right)}}{1 - e^{\lambda\left(1-e^{y_i^{-\beta}}\right)}} \\ - \sum_{i=1}^{n}\left(1 - e^{y_i^{-\beta}}\right), \tag{8}$$

$$\frac{n}{\hat{\beta}} = \frac{\lambda \sum_{i=1}^{n-1} e^{y_i^{-\beta}} y_i^{-\beta} \ln y_i}{\sum_{i=1}^{n-1} ln\left(1 - e^{\lambda(1-e^{y_i^{-\beta}})}\right)} - \sum_{i=1}^{n} \ln y_i \\ + \lambda\sum_{i=1}^{n} e^{y_i^{-\beta}} y_i^{-\beta} \ln y_i - \sum_{i=1}^{n} y_i^{-\beta} \ln y_i \tag{9}$$

Then the β as the solution of the nonlinear above equation. So, we solve it by simulation of fixed point for function $\psi(\beta) = \beta$. We applying algorithm of fixed point $\psi(\beta_k) = \beta_{k+1}$ and so on which stope when $|\beta_{k+1} - \beta_k| < \varepsilon$ such that $\lim_{n\to\infty} \varepsilon = 0$ (sufficiently small), where, $\psi(\lambda) = \lambda$, and $\psi(\beta) = \beta$, once to obtain $\hat{\lambda}$ and $\hat{\beta}$ with (λ_0, β_0) and repeat the fixed point algorithm T times then,

$$\lambda^* = \frac{1}{T}\sum_{i=1}^{T} \hat{\lambda}_i \quad , \qquad \beta^* = \frac{1}{T}\sum_{i=1}^{T} \hat{\beta}_i.$$

See table (1).

2.2 Bayesian Estimation

In this section, we study Bayes estimators for the unknown parameters λ and β under square error loss function (SELF). Assuming that, the parameters λ and β have the following independent Gamma conjugate prior distribution of the form In this section, we study Bayes estimators for the unknown parameters λ and β under Square error loss function. Assuming that, the parameters λ, β have the following independent Gamma Conjugate prior distribution as the form

$$\Pi_1(\lambda) \propto \lambda^{b-1} e^{-a\lambda}, \quad \lambda > 0, \\ \Pi_2(\beta) \propto \beta^{d-1} e^{-c\beta}, \quad \beta > 0.$$

Then the Joint priors for the parameters λ and β has the following form

$$\Pi(\lambda,\beta) \propto \lambda^{b-1}e^{-a\lambda}\beta^{d-1}e^{-c\beta},\quad \lambda,\beta,\ a,b,p,q>0.$$

The Posterior distribution of λ and β can be written as,

$$\Pi^*(\lambda,\beta|\underline{y}) = \frac{L(\lambda,\beta|\underline{y})\Pi(\lambda,\beta)}{\int_0^\infty\int_0^\infty L(\lambda,\beta|\underline{y})\Pi(\lambda,\beta)\,d\lambda\,d\beta}. \tag{10}$$

$$L\Pi = \lambda^{n+b-1}\beta^{n+d-1}\times \prod_{i=1}^{n} y_i^{-(\beta+1)}e^{-c\beta-a\lambda+\sum_{i=1}^{n}y_i^{-\beta}+\lambda\sum_{i=1}^{n}(1-e^{y_i^{-\beta}})} \times\prod_{i=1}^{n-1}\left(1-e^{\lambda(1-e^{y_i^{-\beta}})}\right)^{-1}, \tag{11}$$

where $L\Pi = L(\lambda,\beta|\underline{y})\Pi(\lambda,\beta)$. Then the Bayes estimate of any function of λ and β say $\eta(\lambda,\beta)$ under the square loss function is given by:

$$\eta_{BS}(\hat{\lambda},\beta) = \int_0^\infty\int_0^\infty \Pi^*(\lambda,\beta|\underline{y})L(\lambda,\beta)d\lambda d\beta$$

Unfortunately, from equation this integration cannot be computed explicitly

$$\Pi_1^*(\beta,\lambda|\underline{y}) \propto \beta^{n+d-1}\prod_{i=1}^{n-1}\left(1-e^{\lambda(1-e^{y_i^{-\beta}})}\right)^{-1}\times y_i^{-(\beta+1)}e^{-c\beta+\sum_{i=1}^{n}y_i^{-\beta}+\lambda\sum_{i=1}^{n}(1-e^{y_i^{-\beta}})}.$$

$$\Pi_2^*(\lambda,\beta|\underline{y}) \propto \lambda^{n+b-1}\prod_{i=1}^{n-1}\left(1-e^{\lambda(1-e^{y_i^{-\beta}})}\right)^{-1}\times e^{-a\lambda+\sum_{i=1}^{n}y_i^{-\beta}+\lambda\sum_{i=1}^{n}(1-e^{y_i^{-\beta}})}.$$

It is clear that, direct generation of a pseudo random number from the posterior density functions of λ and β is not easy, instead, we use the Metropolis Hastings method (MCMC). Therefore, the algorithm of Gibbs sampling is as follows:

1. Take some initial value of λ and β such as λ_0 and β_0.
2. Set $t=1$.
3. Generate λ_t from $\Pi_2^*(\lambda|\beta_{t-1},\underline{X})$ and β_t from $\Pi_1^*(\beta|\lambda_{t-1},\underline{X})$.
4. Set $t=t+1$.
5. Repeat steps 2-4 into T times in order to obtain Bayes estimate of λ and β as

$$\hat{\lambda}_{BS} = \frac{1}{T}\sum_{t=1}^{T}\lambda_t \quad\text{and}\quad \hat{\beta}_{BS} = \frac{1}{T}\sum_{t=1}^{T}\beta_t.$$

But, we can not obtain result for above integration in closed form. So, we use numerical integration technique in maple software in order to solve the above integration.

2.3 Lindly Approximation Method

Lindly proposed an approximation method to compute the ratio of integrals of the form of equation for the specified priors on λ and β under the square error loss function. Consider the ratio of integral $I(y)$ where:

$$I(\underline{y}) = \frac{\int_0^\infty\int_0^\infty u(\lambda,\beta)e^{L^*(\lambda,\beta|y)+\rho(\lambda,\beta)}d\lambda d\beta}{\int_0^\infty\int_0^\infty e^{L(\lambda,\beta|y)+\rho(\lambda,\beta)}d\lambda d\beta}, \tag{12}$$

where;

–$L^*(\lambda,\beta|\underline{y}) \Longrightarrow$ log likelihood function
–$\rho(\lambda,\beta) \Longrightarrow \ln\prod(\lambda,\beta)$.
–$g(\lambda,\beta) \Longrightarrow$ posterior $\prod^*(\lambda,\beta|\underline{y})$.

The ratio of integral $I(y)$ can be written as

$$\begin{aligned}I(\underline{y}) &= \hat{g}(\lambda,\beta)+\frac{1}{2}[(\hat{g}_{\lambda\lambda}+2\hat{g}_\lambda\hat{\rho}_\lambda)\hat{\sigma}_{\lambda\lambda}\\&+(\hat{g}_{\beta\lambda}+2\hat{g}_\beta\hat{\rho}_\lambda)\sigma_{\beta\lambda}+(\hat{g}_{\lambda\beta}+2\hat{g}_\lambda\hat{\rho}_\beta)\hat{\sigma}_{\lambda\beta}\\&+(\hat{g}_{\beta\beta}+2\hat{g}_\beta\hat{\rho}_\beta)\hat{\sigma}_{\beta\beta}]\\&+\frac{1}{2}[(\hat{g}_\lambda\hat{\sigma}_{\lambda\lambda}+\hat{g}_\beta\hat{\sigma}_{\lambda\beta})(\hat{l}_{\lambda\lambda\lambda}\hat{\sigma}_{\lambda\lambda}+\hat{l}_{\lambda\beta\lambda}\hat{\sigma}_{\lambda\beta}\\&+\hat{l}_{\beta\lambda\lambda}\hat{\sigma}_{\beta\lambda}+\hat{l}_{\beta\beta\lambda}\hat{\sigma}_{\beta\beta})\\&+(\hat{g}_\lambda\hat{\sigma}_{\beta\lambda}+\hat{g}_\beta\hat{\sigma}_{\beta\beta})(\hat{l}_{\beta\lambda\lambda}\hat{\sigma}_{\lambda\lambda}+\hat{l}_{\lambda\beta\beta}\hat{\sigma}_{\lambda\beta}\\&+\hat{l}_{\beta\lambda\beta}\hat{\sigma}_{\beta\lambda}+\hat{l}_{\beta\beta\beta}\hat{\sigma}_{\beta\beta})].\end{aligned}$$

$$\sigma(\lambda,\beta) = \frac{L^*(\lambda,\beta|\underline{y})+\rho(\lambda,\beta)}{n}.$$

$$\sigma^*(\lambda,\beta) = \sigma(\lambda,\beta)+\frac{ln\,g(\lambda,\beta)}{n}.$$

$$I_y = \sqrt{\frac{|\Sigma^*|}{|\Sigma|}}\quad e^{n(\sigma^*(\lambda,\beta)-\sigma(\lambda,\beta))}.$$

The log-likelihood function

$$\ln L = n\ln\lambda+n\ln\beta-(\beta+1)\sum_{i=1}^{n}\log y_i + \sum_{i=1}^{n}y_i^{-\beta}+\lambda\sum_{i=1}^{n}(1-e^{y_i^{-\beta}}) - \sum_{i=1}^{n-1}\log\left(1-e^{\lambda(1-e^{y_i^{-\beta}})}\right).$$

By using partial derivatives of log likelihood function. Then, we given

$l_\lambda, l_{\lambda\lambda}, l_{\lambda\lambda\lambda}, l_\beta, l_{\beta\beta}, l_{\beta\beta\beta},$
$l_{\lambda\beta}, l_{\lambda\lambda\beta}, l_{\lambda\beta\beta}, l_{\lambda\beta\lambda}, l_{\beta\lambda}, l_{\beta\lambda\lambda}, l_{\beta\lambda\beta}.$

The joint prior function

$$\Pi(\lambda,\beta) \propto \lambda^{b-1}e^{-a\lambda}\beta^{d-1}e^{-c\beta},\quad \lambda,\beta,\ a,b,p,q>0.$$

Log joint prior function

$$\begin{aligned}\rho(\lambda,\beta) &= \log \Pi(\lambda,\beta)\\ &= \ln\left(\lambda^{a-1}e^{-c\lambda}\beta^{b-1}e^{-d\beta}\right),\\ &= (a-1)\ln\lambda + (b-1)\ln\beta\\ &\quad -c\lambda - d\beta.\end{aligned}$$

By using Partial derivatives of Log joint prior function, then we have

$$\hat{\sigma}_{\lambda} = \frac{a-1}{\hat{\lambda}} - c, \quad \hat{\sigma}_{\lambda\lambda} = \frac{-(a-1)}{\hat{\lambda}^2},$$
$$\hat{\sigma}_{\lambda\lambda\lambda} = \frac{-2(a-1)}{\hat{\lambda}^3},$$

and

$$\hat{\sigma}_{\beta} = \frac{b-1}{\hat{\beta}} - d, \quad \hat{\sigma}_{\beta\beta} = \frac{-(b-1)}{\hat{\beta}^2},$$
$$\hat{\sigma}_{\beta\beta\beta} = \frac{-2(b-1)}{\hat{\beta}^3},$$

and

$$\hat{\sigma}_{\lambda\beta} = \hat{\sigma}_{\beta\lambda} = 0$$

We considered the function

$$\delta(\lambda,\beta) = \frac{L^*(\lambda,\beta|\underline{y}) + \rho(\lambda,\beta)}{n}.$$

By using Partial derivatives of $\delta(\lambda,\beta)$ to obtain

$$\delta_{\lambda}, \delta_{\lambda\lambda}, \delta_{\lambda\lambda\lambda}, \delta_{\beta}, \delta_{\beta\beta}, \delta_{\beta\beta\beta},$$
$$\delta_{\lambda\beta\lambda}, \delta_{\lambda\beta\beta}, \delta_{\beta\lambda\lambda}, \delta_{\beta\beta\lambda}, \delta_{\lambda\lambda\beta}.$$

The Bayes estimate of λ under square loss function where $g(\lambda,\beta) = \lambda$ after substituting in $I(y)$ will become

$$\begin{aligned}\hat{\lambda}_{Lndely} &= \hat{\lambda} + \hat{\rho}_{\lambda}\hat{\sigma}_{\lambda\lambda} + \hat{\rho}_{\beta}\hat{\sigma}_{\lambda\beta}\\ &+ \frac{1}{2}\hat{\sigma}_{\lambda\lambda}\left(\hat{l}_{\lambda\lambda\lambda}\sigma_{\lambda\lambda} + 2\hat{l}_{\lambda\beta\lambda}\sigma_{\lambda\beta}\right)\\ &+ \frac{1}{2}\hat{\sigma}_{\beta\lambda}\left(\hat{l}_{\beta\lambda\lambda}\hat{\sigma_{\lambda\lambda}} + \hat{\sigma_{\beta\beta}}\hat{I_{\beta\beta\beta}}\right).\end{aligned}$$

Similarly, The Bayes estimate of λ under square loss function where $g(\lambda,\beta) = \beta$ after substituting in $I(y)$ will become

$$\begin{aligned}\hat{\beta}_{lindly} &= \hat{\beta} + \hat{\rho}_{\lambda}\hat{\sigma}_{\beta\lambda} + \hat{\rho}_{\beta}\hat{\sigma}_{\beta\beta}\\ &+ \frac{1}{2}\hat{\sigma}_{\lambda\beta}\left(\hat{l}_{\lambda\lambda\lambda}\sigma_{\lambda\beta} + 2\hat{l}_{\beta\lambda\lambda}\sigma_{\beta\lambda}\right)\\ &+ \frac{1}{2}\hat{\sigma}_{\beta\beta}\left(\hat{l}_{\beta\lambda\lambda}\hat{\sigma_{\lambda\lambda}} + \hat{\sigma_{\beta\beta}}\hat{I_{\beta\beta\beta}}\right).\end{aligned}$$

See table (1).
The asymptotic variance-covariance of MLE for the two parameters λ and β is the inverse of the fisher information matrix after the ignoring the expectation operators as following

2.4 Approximation Confidence Interval

The exact distribution of MLES cannot be used to obtain explicitly. Therefore, the asymptotic properties [?] under some regularity conditions the MLES can be used to construct the confidence intervals for the parameters.

$$\hat{\theta} = (\hat{\lambda},\hat{\beta}) \rightarrow N_2(0,(I(\theta))^{-1}) \qquad (13)$$

Where $I_{ij}(\theta) \cong \left[-\frac{\partial^2 \ln L}{\partial\theta_i\partial\theta_j}\right]$ is the variance matrix. The asymptotic variance-covariance of MLE for the two parameters λ and β is the inverse of the fisher information matrix after the ignoring the expectation operators as following

$$\begin{aligned}I(\hat{\theta}) &= \begin{bmatrix} var(\hat{\lambda}) & Cov(\hat{\lambda},\hat{\beta}) \\ Cov(\hat{\beta},\hat{\lambda}) & var(\hat{\beta}) \end{bmatrix}\\ &= \begin{bmatrix} -\dfrac{\partial^2 L^*(\lambda,\beta|\underline{y})}{\partial\lambda^2} & -\dfrac{\partial^2 L^*(\lambda,\beta|\underline{y})}{\partial\lambda\partial\beta} \\ -\dfrac{\partial^2 L^*(\lambda,\beta|\underline{y})}{\partial\beta\partial\lambda} & -\dfrac{\partial^2 L^*(\lambda,\beta|\underline{y})}{\partial\beta^2} \end{bmatrix}\end{aligned}$$

The asymptotic normality of the MLE can be used to compute the approximate confidence intervals for parameters λ and β therefore, $(1-\alpha)100\%$ confidence intervals for parameters λ and β, can be written as follows

$$\hat{\lambda} \pm Z_{\frac{\alpha}{2}}\sqrt{Var(\hat{\lambda})},$$

and

$$\hat{\beta} \pm Z_{\frac{\alpha}{2}}\sqrt{Var(\hat{\beta})},$$

where $Z_{\frac{\alpha}{2}}$ is the percentile. See table (1)

conflict of interest

The authors have no conflict of interest regarding this paper

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Table 1: Average estimated values, interval estimates and MSEs of Inverse Chen distribution with $\lambda = 2$ and $\beta = 1.5$ under upper record

		MLE			Bayes (Lindley Approximation)		
(n,k)			MSE	Asy. CI		MSE	Asy. CI
(60,15)	λ	2.8213	(0.7551)	(1.5221,4.1205)	2.2673	(0.7776)	(1.4298 , 4.1290)
	β	1.26025	(0.3055)	(0.5746, 1.9459)	2.378	(1.291)	(1.885, 4.034)
(120,30)	λ	2.44685	(0.3325)	(1.5915, 3.3022)	1.9407	(0.3561)	(1.5347, 3.3090)
	β	1.12555	(0.3891)	(0.7059,1.5452)	2.1249	(0.6147)	(1.8349, 3.2306)
(140,35)	λ	2.07475	(0.1403)	(1.5159, 2.6336)	1.8318	(0.1643)	(1.4461, 2.6374)
	β	1.00025	(0.4918)	(0.6079,1.3926)	2.047	(0.359)	(1.850,2.603)
(200,50)	λ	1.75415	(0.1048)	(1.4069, 2.1014)	1.7607	(0.1364)	(1.3138, 2.0944)
	β	0.76195	(8.990)	(0.5138,1.0101)	2.0241	(0.2858)	(1.8238, 2.1536)
(220,55)	λ	1.76845	(0.1145)	(1.3729, 2.1640)	1.746	(0.150)	(1.271, 2.159)
	β	0.79965	(0.5517)	(0.4941,1.1052)	2.0191	(0.2788)	(1.8806,2.2006)
(280,70)	λ	1.64255	(0.2198)	(1.3583,1.9268)	1.5071	(0.2856)	(1.2550,1.9116)
	β	0.6948	(0.7144)	(0.4340, 0.9556)	1.9440	(0.2061)	(1.8260,2.1319)
(320,80)	λ	1.52	(0.231)	(1.338, 1.702)	1.472	(0.299)	(1.228, 1.668)
	β	0.59875	(0.8044)	(0.4401,0.7574)	1.9563	(0.2139)	(1.8697, 2.1211)
(80, 20)	λ	2.2698	(0.2351)	(1.5321, 3.0075)	2.0039	(0.2577)	(1.4552, 3.0153)
	β	1.27225	(0.2492)	(0.7257,1.8188)	2.143	(0.562)	(1.830, 2.958)
(100,25)	λ	2.01005	(0.06097)	(1.62071, 2.39300)	1.95449	(0.07098)	(1.56707, 2.39410)
	β	0.97365	(0.3535)	(0.6793,1.2680)	2.0943	(0.3767)	(1.9225, 2.3854)
(160,40)	λ	2.0606	(0.1895)	(1.3770, 2.7442)	1.8494	(0.2248)	(1.2685, 2.7490)
	β	0.84465	(0.5038)	(0.4908,1.1985)	2.0834	(0.4161)	(1.8355,2.6940)
(180,45)	λ	1.8382	(0.1729)	(1.3200,2.3564)	1.6965	(0.2242)	(1.1921,2.3567)
	β	0.77755	(0.5693)	(0.4470, 1.1081)	2.0236	(0.2999)	(1.8640,2.3535)
(260,65)	λ	1.7217	(0.1794)	(1.3631, 2.0803)	1.589	(0.232)	(1.259, 2.073)
	β	0.8145	(0.6344)	(0.5302,1.0988)	1.9443	(0.2051)	(1.8573, 2.1356)
(300,75)	λ	1.61315	(0.2034)	(1.3930, 1.8333)	1.5130	(0.2609)	(1.3018, 1.8116)
	β	0.6108	(0.7490)	(0.4389,0.7827)	1.9477	(0.2098)	(1.8037,2.1260)
(340,85)	λ	1.61225	(0.1667)	(1.3838, 1.8407)	1.5789	(0.2148)	(1.2906, 1.8199)
	β	0.6705	(0.6765)	(0.5194,0.8216)	1.9445	(0.1999)	(1.8428, 1.9965)
(240,60)	λ	1.72255	(0.2219)	(1.3203, 2.1248)	1.536	(0.290)	(1.200,2.117)
	β	0.77275	(0.6792)	(0.4768,1.0687)	1.9365	(0.2035)	(1.8318,2.1869)

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